Regge cuts and charge-exchange reactions in the triple-Regge region*

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We estimate the Regge-cut contribution to the inclusive reactions $\pi^-p\to\pi^0X$, $\pi^-p\to\eta X$, and $K^-p\to\bar{K}^0X$ in the triple-Regge region. We find that the magnitude of the cut is less than about one-half that of triple-Regge-pole contribution near $t=-0.5$ (GeV/c)^2. We discuss the implications of this for the interpretation of the t dependence of these reactions.

I. INTRODUCTION

As is well known, the cross sections for some but not all two-body charge-exchange reactions exhibit significant structure as functions of the invariant momentum transfer t. For example, the cross section for $\pi^-p\to\pi^0n$ has dips at $t=0$ and $t=-0.6$ GeV^2. Such structures have been studied extensively in the Regge-pole framework.[1] While it is generally accepted that the dips at $t=0$ are due to the vanishing there of helicity-flip amplitudes, the dips at $t=-0.6$ GeV^2 have been attributed both to nonsense wrong-signature zeros (NWSZ) and to interference between Regge-pole amplitudes without NWSZ and Regge-cut amplitudes.

In this paper we study the possibility of similar dips in charge-exchange inclusive cross sections, specifically those caused by pole-cut interference. We consider $\pi^-p\to\pi^0X$, $\pi^-p\to\eta X$, and $K^-p\to\bar{K}^0X$ in the triple-Regge region with a small momentum transfer t between the two mesons. In this region the graphs shown in Fig. 1 are relevant. We expect the dominant graphs to be $(\alpha_1, \alpha_2, \alpha_3)=(\rho, \rho, P)$ for $\pi^-p\to\pi^0X$, (A_2, A_2, P) for $\pi^-p\to\eta X$, and both (ρ, ρ, P) and (A_2, A_2, P) for $K^-p\to\bar{K}^0X$. Since no helicity flip is involved, none of these graphs should vanish at $t=0$. If the (ρ, ρ, P) coupling has a NWSZ, then the $\pi^-p\to\pi^0X$ cross section will have a dip at $t=-0.6$ GeV^2. Since we want to study the possibility of a pole-cut interference, we will assume here that no such zero is present.

We want to see whether the inclusion of reasonable Regge-cut graphs can produce dips in the cross sections. To calculate such graphs we use the techaniques first introduced by Gribov[2] and later extended to inclusive reactions by several authors.[3,4] We are interested in moderately large values of s, the invariant energy variable, say those available at Fermilab. We can therefore ignore all graphs which decrease like a power of s at fixed $x=1-M^2/s$, where M^2 is the square of the missing mass. Of the cut graphs which give a constant cross section within powers of logs, we believe that the most important are those shown in Fig. 2, where α_2 and α_4 are Pomerons. More complicated graphs involving additional Pomeron interactions are certainly important

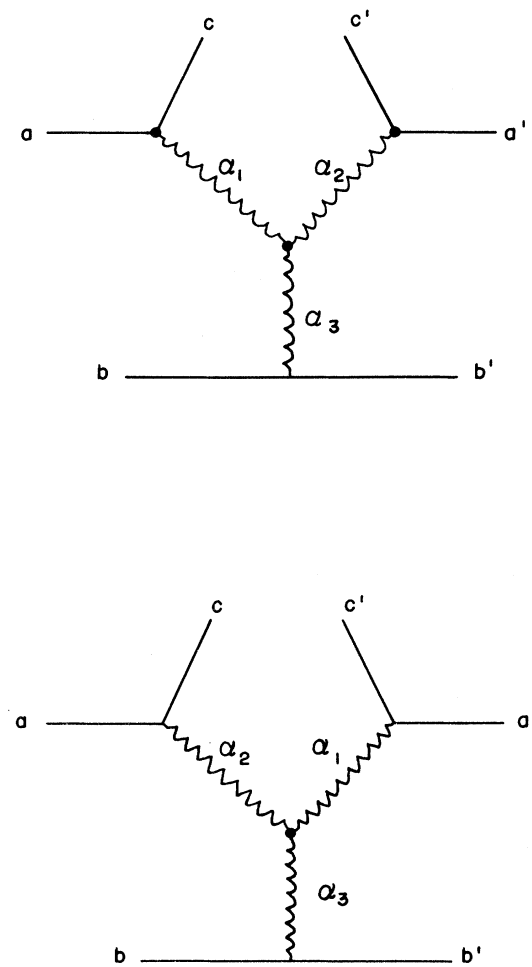
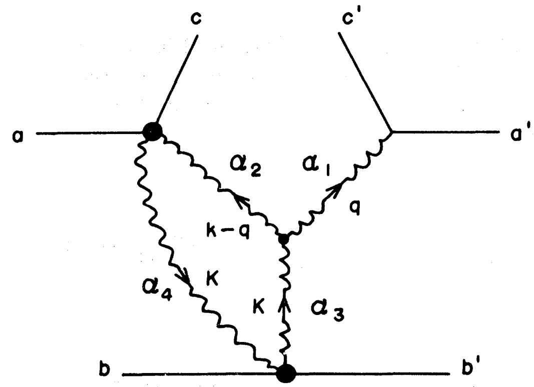


FIG. 1. Triple-Regge-pole graphs.



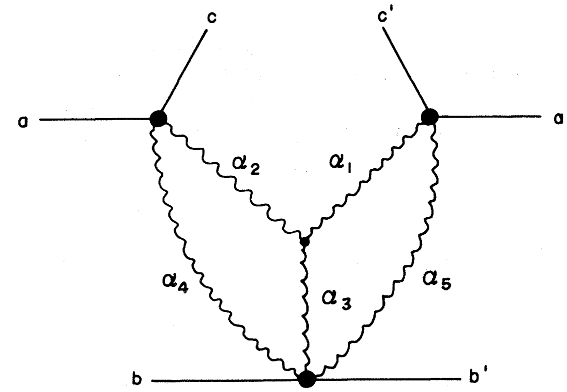


FIG. 3. Higher-order Regge-cut correction to the triple-Regge-pole graph.

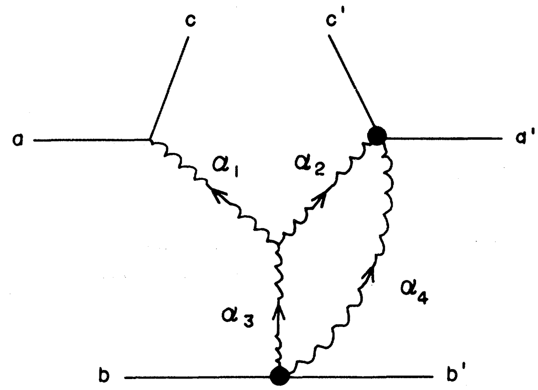


FIG. 2. Leading Regge-cut corrections to the triple-Regge-pole graph.

in the limit $\log s \to \infty$. However, we know that the triple-Pomeron coupling is small in the sense that the unenhanced two-Pomeron-cut graph dominates the enhanced one[5] at moderate values of $\log s$. The meson-Pomeron-meson triple-Regge coupling is not well determined phenomenologically, but there is no evidence that it is large. It thus seems reasonable to drop graphs involving additional Pomeron interactions.

It will be seen below that the graphs shown in Fig. 2 give a negative contribution to the inclusive cross section. Obviously, therefore, if the magnitude of these graphs were to exceed that of the pole graphs, then one would have to include additional graphs, e.g., that shown in Fig. 3. However, we shall see that this does not happen.

In what follows we give a numerical estimate of the cut graphs shown in Fig. 2 for $\pi^- p \to \pi^0 X$, $\pi^- p \to \eta X$, and $K^- p \to \bar{K}^0 X$. We actually calculate the ratio of these graphs to the pole graphs, Fig. 1, so that the over-all normalization is irrelevant. Our main result is that the magnitude of this ratio is fairly small for $|t| \leq 0.5\ (\mathrm{GeV}/c)^2$, reaching about 0.5 at $x = 0.9$. Thus the cut seems too weak to produce a dip in the cross section.

On the basis of our calculation, no dip is expected at $-t \approx 0.6\ (\mathrm{GeV}/c)^2$ in any of the charge-exchange inclusive reactions under consideration. If a dip is found in the inclusive distribution at the above value of momentum transfer, it can only be due to NWSZ in the pole residue and not due to pole-cut interference. This will be checked in an experiment[6] at Fermilab.

II. POLE AND CUT AMPLITUDES

For the reaction $a + b \to c + X$ we take the independent kinematic variables to be

$$s = (p_a + p_b)^2 ,\qquad M^2 = (p_a + p_b - p_c)^2 ,\qquad t = (p_a - p_c)^2 . \tag{2.1}$$

It is also convenient to introduce

$$x = 1 - M^2/s . \tag{2.2}$$

Then the triple-Regge region corresponds to large s and small $(1-x)$ and t. Since we are interested in meson-baryon charge-exchange reactions, we take a and c to have spin zero and b to have spin $s_b = \frac{1}{2}$. Then by Mueller's generalized optical theorem[7] the unpolarized inclusive cross section is

$$\sigma \equiv s\, \frac{d\sigma}{dt\, dM^2} = \frac{1}{2s_b + 1} \sum_{\lambda_b} A_{\lambda_b', \lambda_b} . \tag{2.3}$$

Here $A_{\lambda_b', \lambda_b}$, the Mueller amplitude, is the M^2 discontinuity of the forward $a+b+\bar{c} \to a'+b'+\bar{c}'$ amplitude with s-channel helicities λ_b and λ_b' and with a $(-i\epsilon)$ prescription on the outgoing energy.

The triple-Regge-pole graphs, Fig. 1, have been discussed extensively in the literature.[8] The Mueller amplitude for the sum of the two graphs is

$$A_{\lambda_b' \lambda_b} = \frac{1}{16\pi s} \beta_{a\bar{c}, \alpha_1}(t) \beta_{a\bar{c}, \alpha_2}(t)\, 2\,\mathrm{Re}[\xi_{\alpha_1}(t) \xi^*_{\alpha_2}(t)] g_{\alpha_1, \alpha_2, \alpha_3}(t, t, 0) \beta_{\lambda_b' \lambda_b, \alpha_3}(0) \left(\frac{s}{M^2}\right)^{\alpha_1(t)+\alpha_2(t)} (M^2)^{\alpha_3(0)} , \tag{2.4}$$

where $\alpha_i(t)$ and $\beta_i(t)$ are the usual Regge trajectories and residues, $g_{\alpha_1, \alpha_2, \alpha_3}$ is the triple-Regge coupling, and

$$\xi_i(t) = \frac{\tau + e^{-i\pi\alpha_i(t)}}{-\sin\pi\alpha_i(t)} \tag{2.5}$$

are the signature factors. These have been complex-conjugated for the outgoing Reggeons in accordance with the $(-i\epsilon)$ prescription on the outgoing energy. For $\alpha_1 = \alpha_2 = \alpha$ and $\alpha_3 = \alpha_P$ the corresponding inclusive cross section is

$$\sigma_{\text{pole}} = G_{\alpha\alpha, P}(t, t, 0) \left(\frac{1}{1-x}\right)^{2\alpha(t)-1} , \tag{2.6}$$

where

$$G_{\alpha\alpha, P}(t, t, 0) = \frac{1}{16\pi} \beta_{a\bar{c}, \alpha}{}^2(t) \,|\,\xi_\alpha(t)|^2 g_{\alpha\alpha, P}(t, t, 0)\, \beta_{++, P}(0) \tag{2.7}$$

because $\beta_{++, P} = \beta_{--, P}$. Here we assumed that the intercept of the Pomeron trajectory is at unity.

The Regge-cut graphs shown in Fig. 2 have been calculated in Refs. 3 and 4 using the techniques of the Reggeon calculus. The Mueller amplitude is

$$\begin{aligned} A_{\lambda_b' \lambda_b} = \frac{1}{16\pi s} \int \frac{d^2 k_\perp}{16\pi^2} & \beta_{a\bar{c}, \alpha_1}(t) N_{\alpha_2 c; \alpha_4 a}(t_2, t') \\ & \times [N_{\alpha_3 \lambda_b'; \alpha_4 \lambda_b}(t', t')\, i \xi^*_{\alpha_1}(t) \xi_{\alpha_2}(t_2) \xi_{\alpha_4}(t') + N_{\alpha_4 \lambda_b'; \alpha_3 \lambda_b}(t', t') (-i) \xi_{\alpha_1}(t) \xi^*_{\alpha_2}(t_2) \xi^*_{\alpha_4}(t')] \\ & \times g_{\alpha_1, \alpha_2, \alpha_3}(t, t_2, t') \left(\frac{s}{M^2}\right)^{\alpha_1(t)+\alpha_2(t_2)+\alpha_4(t')-1} (M^2)^{\alpha_3(t')+\alpha_4(t')-1} , \end{aligned} \tag{2.8}$$

where

$$t = -q_\perp{}^2, \quad t' = -k_\perp{}^2, \quad t_2 = -(q_\perp - k_\perp)^2 . \tag{2.9}$$

In this equation N denotes the usual Reggeon-particle vertex and $g_{\alpha_1, \alpha_2, \alpha_3}(t, t_2, t')$ reduces for $t' = 0$ and $t_2 = t$ to the triple-Regge vertex in Eq. (2.4). For $\alpha_1 = \alpha_2 = \alpha$ and $\alpha_3 = \alpha_4 = \alpha_P$ the corresponding inclusive cross section is

$$\begin{aligned} \sigma_{\text{cut}} = \frac{1}{16\pi} \int \frac{d^2 k_\perp}{16\pi^2} & \beta_{a\bar{c}, \alpha}(t) N_{\alpha c; Pa}(t_2, t') N_{P+; P+}(t', t') (-2)\, \mathrm{Im}[\xi^*_\alpha(t) \xi_\alpha(t_2) \xi_P(t')] g_{\alpha\alpha, P}(t, t_2, t') \\ & \times \left(\frac{1}{1-x}\right)^{\alpha(t)+\alpha(t_2)-\alpha_P(t')} (s)^{2\alpha_P(t')-2} . \end{aligned} \tag{2.10}$$

To proceed further it is necessary to introduce a model. For the Reggeon-particle vertices we take

$$N_{\alpha c; Pa}(t_2, t') = \beta_{a\bar{c}, \alpha}(t_2) \beta_{a\bar{a}, P}(t'), \quad N_{P+; P+}(t', t') = \beta_{++, P}{}^2(t') . \tag{2.11}$$

We refer to this as the absorption model, since such a form for N reproduces the usual absorption model for two-body reactions. This model has had considerable success in fitting two-body data. Furthermore, it can be justified at least for the forward Pomeron-Pomeron vertex by relating that vertex to an inclusive cross section. Using the absorption model, we find that Eq. (2.10) becomes

$$\sigma_{\rm cut} = \int \frac{d^2k_\perp}{16\pi^2}\, \beta_{a\bar{a},P}(t')\beta_{++,P}(t')\, \frac{G_{\alpha\alpha,P}(t,t_2,t')}{\mathrm{Re}[\xi_\alpha^*(t)\,\xi_\alpha(t_2)]} \times(-2)\,\mathrm{Im}[\xi_\alpha^*(t)\,\xi_\alpha(t_2)\xi_P(t')]\left(\frac{1}{1-x}\right)^{\alpha(t)+\alpha(t_2)-\alpha_P(t')}(s)^{2\alpha_P(t')-2}\,, \qquad (2.12)$$

where $G_{\alpha\alpha,P}$ is defined in Eq. (2.7).

Now $G_{\alpha\alpha,P}(t_1, t_2, t_3)$ must be symmetric in t_1 and t_2. For $t_3=0$ and $t_1=t_2=t$, it must reduce to $G_{\alpha\alpha,P}(t,t,0)$, which can be studied in inclusive reactions and which behaves roughly like

$$G_{\alpha\alpha,P}(t,t,0)=G_0\,e^{at}\ . \qquad (2.13)$$

(Of course it is the sum of the pole and the cut amplitudes which is related to the cross section, but since that is found to be weak we are justified *a posteriori* in neglecting it. We also assume here that the charge-exchange pole amplitude has a t dependence similar to the RRP amplitude in $pp\to pX$.) The dependence of G on t_3 for fixed t_1 and t_2 cannot be directly observed, but presumably it is like that of a product of two typical Pomeron residues. We are thus led to parameterize G as

$$G(t_1,t_2,t_3)=G_0 e^{a(t_1+t_2)/2+bt_3}\ , \qquad (2.14)$$

with a and b determined as indicated above. Since the integral in Eq. (2.12) is dominated by small $k_\perp{}^2$, the result is not very sensitive to the extrapolation of G to nonzero t_3.

The only remaining quantities in Eq. (2.12) are the Pomeron residues. We parameterize these as

$$\beta_{a\bar{a},P}(t)=\gamma_a\, e^{b_a t}\ ,$$
$$\beta_{++,P}(t)=\gamma_b\, e^{b_b t}\ . \qquad (2.15)$$

Then we can obtain the ratio of the cut to the pole amplitude:

$$\frac{\sigma_{\rm cut}}{\sigma_{\rm pole}} = -\frac{\gamma_a\gamma_b}{16\pi^2}\int_{-\infty}^{0}dt'\int_0^{2\pi}d\varphi\, e^{a(t_2-t)/2}\,e^{(b_a+b_b+b)t'} \times\frac{\mathrm{Im}[\xi_\alpha^*(t)\,\xi_\alpha(t_2)\,\xi_P(t')]}{\mathrm{Re}[\xi_\alpha^*(t)\,\xi_\alpha(t_2)]}\left(\frac{1}{1-x}\right)^{\alpha(t_2)-\alpha(t)-\alpha_P(t')+1}(s)^{2\alpha_P(t')-2}\,, \qquad (2.16)$$

where

$$t_2=t+t'-2(tt')^{1/2}\cos\varphi\ . \qquad (2.17)$$

We shall give numerical results for this ratio in the following section.

III. NUMERICAL RESULTS

For each of the reactions $\pi^-p\to\pi^0X$, $\pi^-p\to\eta X$, and $K^-p\to\bar{K}^0X$ we calculate the ratio $\sigma_{\rm cut}/\sigma_{\rm pole}$ using Eq. (2.16) and evaluating the integrals numerically. We take

$$\alpha_P(t)=1.0+0.28t \qquad (3.1)$$

for the Pomeron trajectory and

$$\gamma_p=10.14,\quad \gamma_\pi=6.18,\quad \gamma_K=4.94 \qquad (3.2)$$

for its residues at $t=0$. These numbers correspond to

$$\sigma_T(pp)=40\text{ mb},\quad \sigma_T(\pi p)=24.4\text{ mb},\quad \sigma_T(Kp)=19.5\text{ mb} \qquad (3.3)$$

for the total cross sections. We assume that the ρ and A_2 trajectories are exchange degenerate, with

$$\alpha(t)=0.5+0.9t\ . \qquad (3.4)$$

We take the parameter a defined in Eq. (2.13) to be the same for the ρ and the A_2 and equal to the logarithmic derivative of the RRP coupling[9] in $pp\to pX$:

$$a=5.88\text{ GeV}^{-2}\ . \qquad (3.5)$$

Finally we note that Eq. (2.16) depends on b_a, b_b, and b only through the combination b_a+b_b+b. We take

$$b_a+b_b+b=6\text{ GeV}^{-2}\ ; \qquad (3.6)$$

varying this parameter within reasonable limits does not change the results substantially.

For $\pi^-p\to\pi^0X$ only ρ exchange contributes. Our numerical results are shown in Fig. 4.

For $\pi^-p\to\eta X$ only A_2 exchange contributes, and the only change from the previous calculation is in the signature factors. Our results are shown in Fig. 5.

For $K^-p\to\bar{K}^0X$ both ρ and A_2 exchange contribute, so both the pole and the cut amplitudes are the sum of two terms. Not knowing the relative

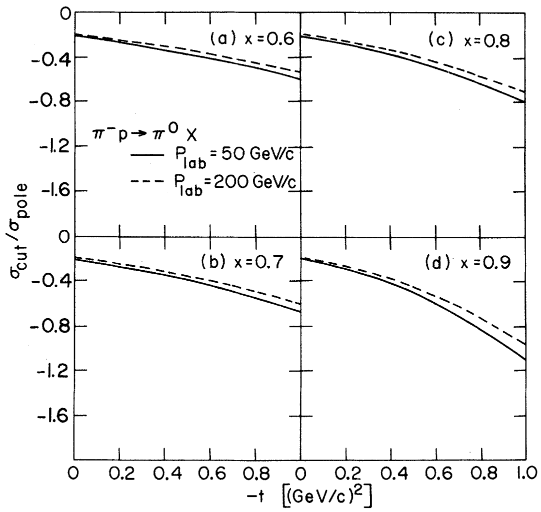


FIG. 4. Relative magnitude of cut and pole graph, $\sigma_{cut}/\sigma_{pole}$, for $\pi^- + p \rightarrow \pi^0 + X$ at $P_{lab} = 50$ and 200 GeV/c, and $x = 0.6$, 0.7, 0.8, and 0.9 as a function of t.

magnitude of the $\rho\rho P$ and A_2A_2P couplings, we take them to be equal. Then we obtain the results shown in Fig. 6.

Several comments are in order:

(i) We have given results for $\sigma_{cut}/\sigma_{pole}$ at values of x as small as 0.6. However, it is clear that the whole triple-Regge formalism is doubtful at such values of x, so that these results are not reliable. They have been included here for completeness.

(ii) The ratio $\sigma_{cut}/\sigma_{pole}$ depends only weakly on

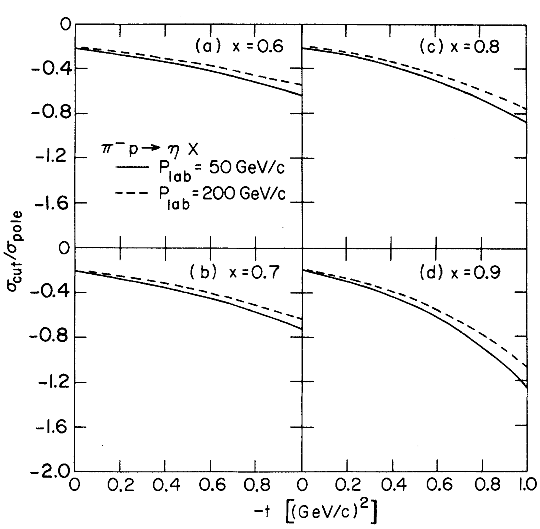


FIG. 5. Same as in Fig. 4 but for $\pi^- + p \rightarrow \eta + X$.

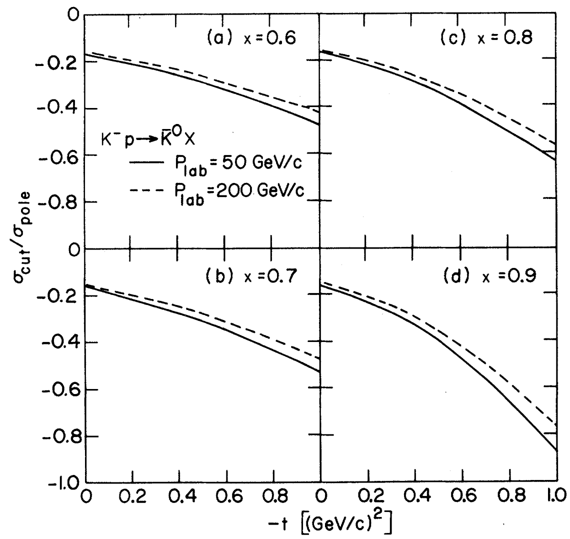


FIG. 6. Same as in Fig. 4 but for $K^- + p \rightarrow \bar{K}^0 + X$.

s and on x except for very small values of $(1 - x)$. As $x \rightarrow 1$, the integral in Eq. (2.16) does become infinite. However, M^2 is small for x near unity except at very large values of s, at which our neglect of Pomeron interactions is not justified.

(iii) Equation (2.16) gives small values for $\sigma_{cut}/\sigma_{pole}$ mainly because the integrand falls off rapidly with $k_\perp^2$, and most of this falloff comes from the t dependence of the various vertex functions. Thus it is important to evaluate the integral numerically rather than calculate just the term of leading order in $\ln s$ and $\ln(1 - x)$.

Using a phenomenological prescription,[10] Pumplin has estimated cut contributions to $\pi^- p \rightarrow \pi^0 X$. His results are similar to ours.

IV. CONCLUSION

We have calculated using what seem to be reasonable assumptions the ratio of the Regge-cut to the Regge-pole amplitude for $\pi^- p \rightarrow \pi^0 X$, $\pi^- p \rightarrow \eta X$, and $K^- p \rightarrow \bar{K}^0 X$. For all three reactions this ratio is less than one, so that the cut cannot cancel the pole and produce a dip in the cross section. Of course, there could be a dip coming from pole-cut interference if our estimate of the cut is too low. In that case the dip should move to smaller $|t|$ as x increases, since $\sigma_{cut}/\sigma_{pole}$ is largest for large x.

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