Importance of the $N\Delta$ inelastic threshold for understanding low-energy NN amplitudes*

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The authors relate three interesting aspects of low-energy NN reactions: (1) The large value of $\Delta\sigma_T = \sigma_T(\uparrow\downarrow) - \sigma_T(\uparrow\uparrow)$ for pp scattering for $P_{\rm LAB}$ below 3 GeV/c; (2) the existence at low energies of large unnatural-parity t-channel amplitudes with nonpion quantum numbers; (3) the anomalous variation with energy of the slope and differential cross section at $t=0$ in $np \to pn$ in the range $P_{\rm LAB} = 0.5$–2 GeV/c. Based on the somewhat crude phase-shift analyses which exist above the single-pion inelastic threshold, it is argued that all three of the above aspects of NN reactions are the reflections of the dominant inelastic channel $pp \to N\Delta$. The well-known possibility that the large amplitudes so generated are related to resonant behavior (in the $^1D_2$ partial wave of $pp \to pp$) is discussed. That some low-energy amplitudes are strongly affected by s-channel threshold effects demonstrates one cannot, in general, equate features of high-energy amplitudes with those of low-energy amplitudes whenever inelastic thresholds are important.

NN scattering below about 3 GeV/c laboratory momentum shows dramatic behavior in several observables. We find the following set of observables particularly interesting: (1) $\phi_1 - \phi_3$, where the s-channel helicity amplitudes $\phi_1 = \langle ++ |\phi| ++\rangle$ and $\phi_3 = \langle +- |\phi| +-\rangle$ can be thought of as observables in the region where phase-shift analyses exist and give the complete S matrix; (2) the difference

$$\Delta\sigma_T = \sigma_T(\uparrow\downarrow) - \sigma_T(\uparrow\uparrow) \tag{1}$$

between the total pp cross section with transversely polarized beam and target spins anti-aligned compared to the cross section with spins aligned, and (3) the forward cross section $(d\sigma/dt)$ and slope for $np \to pn$. We summarize briefly what features of each of these observables we find interesting.

The amplitudes ϕ_1 and ϕ_3 computed from pp phase shifts[1] are surprisingly unequal below 3 GeV/c. This suggests the existence of comparable natural- and unnatural-parity t-channel contributions at these low energies. The unnatural-parity contributions have the quantum numbers of the A_1 state and its even less established partners of opposite G parity and zero isospin. At high energies, there is no evidence for such contributions. An advantage of NN experiments is the possibility of studying amplitudes which cannot be easily isolated in reactions with a less complex spin structure. Here we can compare two nonflip spin amplitudes, whereas in meson-baryon scattering only one such amplitude is available.

The observable $\Delta\sigma_T$ for pp elastic scattering suddenly increases to a value far above that expected from extrapolation of high-energy values as $P_{\rm LAB}$ decreases below 3 GeV/c. This observable has been directly measured at Argonne National Laboratory with a polarized proton beam between 2 and 6 GeV/c.[2] The results are shown in Fig. 1. The difference $\Delta\sigma_T$ is related to the s-

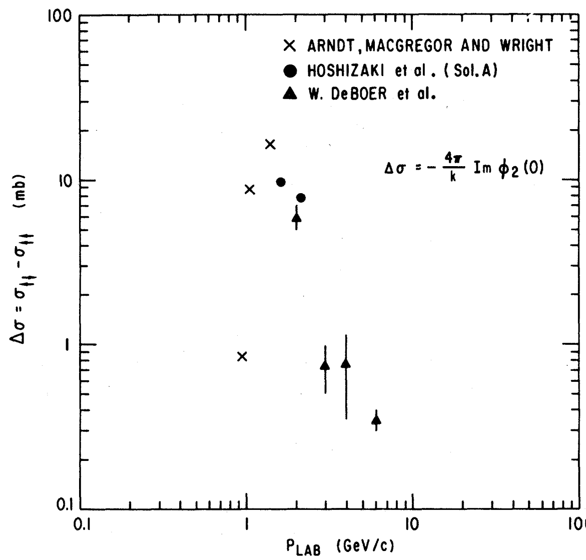

FIG. 1. The quantity $\Delta\sigma_T = \sigma_T(\uparrow\downarrow) - \sigma_T(\uparrow\uparrow) = -(4\pi/k){\rm Im}\phi_2(0)$ is plotted for the pp elastic reaction. The high-energy points (Ref. 2) are measured directly at the ZGS with a polarized beam on a polarized target. The lower-energy points taken from phase-shift results (Refs. 1, 6) use only the central values; errors are not shown. We note that ${\rm Im}\phi_2(0)$ depends upon knowing accurate values for inelastic phases, which at present are poorly determined. For example, a typical error on the (×) points would take the maximum value from 16 mb to 10 mb. Accurate determination of $\Delta\sigma_T$ thus would greatly improve what is known about the phase shifts. In spite of this type of uncertainty, we believe the qualitative trend of the plotted phase shifts is reliable.

channel helicity amplitude $\phi_2 = \langle - - | \phi | + + \rangle$ by

$$\Delta\sigma_T = -\frac{4\pi}{k}\,\mathrm{Im}\phi_2(0)\,. \qquad (2)$$

The behavior of $\Delta\sigma_T$ is surprising for the following reasons.

The usual leading Reggeon contributions all have pole components which vanish in ϕ_2 at $t=0$ since ϕ_2 is an amplitude with helicity flip at each vertex. Therefore, a nonzero $\phi_2(0)$ results only from cuts which could be generated, for example, by absorption. Even then, the π amplitude has a dominantly real pole at small t. The full π amplitude is more real than imaginary, causing $\mathrm{Im}\phi_2(0)$ to be small. Similar arguments hold for $\rho + A_2$ and for $\omega + f$ in this exotic channel. In addition, B exchange must be present by exchange degeneracy with the π. It is mainly imaginary since it has odd signature, and it cancels a significant part of the π contribution. At higher energies (3–6 GeV/c), a model calculation[3] shows that these contributions can account for the measurements, and that their extension to lower energies is determined by their exchange properties. They surely do not account for more than 20% of the observed $\Delta\sigma_T$ at 2 GeV/c. Its origin should be sought in a low-energy effect.

The forward $np \to pn$ charge-exchange cross section shows a sharp peak for $-t < 0.01$ GeV^2 down to very-low energies. This peak is usually associated with π exchange owing to its narrow width. A π-exchange contribution alone must give a zero rather than a peak at $t=0$ since the pseudoscalar pion flips the nucleon helicity. At high energies the peak is present because absorption due to the presence of many inelastic channels modifies the partial waves in the amplitude ϕ_2 in a smooth but differential way, since competition with other channels is most important at small impact parameter. At low energies unitarization of the partial waves produces a similar effect. In either case, the cancellation among partial waves required to give the zero no longer operates and the sharp peak results. As shown in Fig. 2, however, in the region around 1 GeV/c the size and slope of the cross section fluctuate around a smooth energy variation.[4]

To understand what appear to be dramatic low-energy phenomena, first consider $\Delta\sigma_T$. We have taken the low-energy phase shifts[1] and from them computed $\Delta\sigma_T$. As seen in Fig. 1, at 2 GeV/c, where there is an overlap, the agreement is quite reasonable. Moreover, the phase-shift solutions show a quite striking peak at about 1.5 GeV/c lab momentum. The cross section $\Delta\sigma_T$ has a value of about 15 mb, which is comparable to the spin-averaged cross section. We thus have confirmation that the behavior of $\Delta\sigma_T$ is a real effect. We can now easily track down the mechanism which is responsible for large value of $\Delta\sigma_T$.

In the region where pion production begins to be important, the production is known to be dominated by the $N\Delta$ final state.[5] The threshold is at $P_{\mathrm{LAB}} = 1.3$ GeV/c for the central Δ mass, and it is a large effect above about 1.0 GeV/c. The final state will be largely s-wave by the angular momentum barrier, so the orbital angular momentum must be even. The initial state must be antisymmetric, so if it is even in orbital angular momentum it is in an antisymmetric spin state. That is, the production occurs from the singlet spin state. Then $L=J=2$ and the initial state is $^1D_2$.

The singlet scattering contributes equally to $-\phi_1(0)$ and $\phi_2(0)$ an amount[6]

$$5\frac{\eta e^{2i\delta} - 1}{2ik} P_2(0) \equiv {}^1D_2\,. \qquad (3)$$

This is because $\phi_1 = \langle + + | \phi | + + \rangle$ and $\phi_2 = \langle - - | \phi | + + \rangle$ have $J_z = 0$. There is no contribution to $\phi_3 = \langle + - | \phi | + - \rangle$, which has $J_z = 1$. Thus one gets the contribution

$$-\frac{4\pi}{k}\,\mathrm{Im}({}^1D_2) = \frac{10\pi}{k^2}(1 - \eta\cos 2\delta) \qquad (4)$$

to both $\Delta\sigma_T$ and a related total cross section using

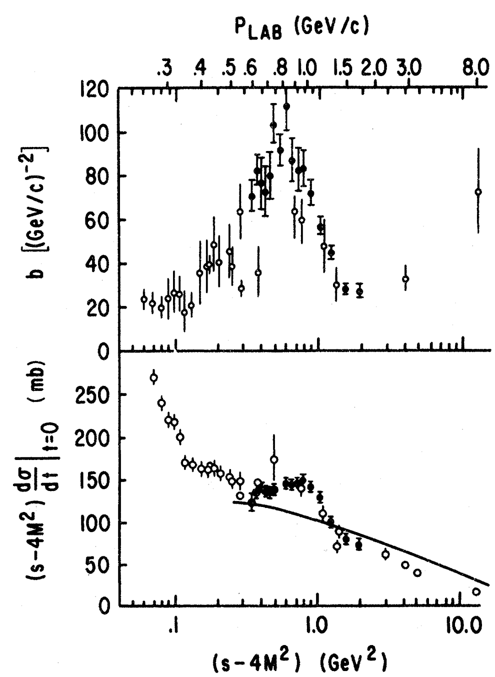

FIG. 2. The slope and $t=0$ normalization of $np \to pn$ are shown plotted vs P_{LAB} (Ref. 4). The curve is $1/s$; its normalization is described in the text.

a longitudinally polarized beam and target:

$$\Delta\sigma_L \equiv \sigma_T(\rightleftarrows) - \sigma_T(\rightrightarrows)$$

$$= \frac{4\pi}{k}\,\mathrm{Im}[\phi_1(0) - \phi_3(0)]\,. \qquad (5)$$

At 1.4 GeV/c contribution (4) is 18 mb, which is large because there is a substantial inelasticity ($\eta \sim \frac{1}{2}$) and the elastic phase is quite small ($\delta \sim 10°$); if η were unity, the contribution would be only about 2 mb. One also sees that the result is quite sensitive to η. When the full partial-wave series is summed, one finds all the remaining contributions to ϕ_2 to be small.

For $\phi_1 - \phi_3$ the situation is more complex; there are other large contributions to ϕ_1 which are nearly cancelled by contributions to ϕ_3. Nevertheless, the end result is qualitatively the same. Above a background, $\Delta\sigma_L$ shows a sharp bump due to the $N\Delta$ threshold. The size of the bump above background is comparable to the effect in $\Delta\sigma_T$.

Thus the large low-energy behavior of $\Delta\sigma_T$ is associated with the opening of the $N\Delta$ inelastic channel and manifests itself in a single partial wave, ${}^1D_2$. Moreover, one necessarily expects a similar behavior in $\Delta\sigma_L$. In terms of amplitudes, $\mathrm{Im}\phi_2$ and $\mathrm{Im}(\phi_1 - \phi_3)$ are receiving large threshold contributions.

Since we see such distinctive behavior in the imaginary parts of forward amplitudes as functions of s, should we perhaps also expect the real parts to have an equally distinctive behavior? We have three sources to learn from: theory, phase shifts, and observables (such as $\Delta\sigma$ or $d\sigma/dt$). In theory, if we wrote a forward dispersion relation for the proper signatured invariant amplitudes, we could extract the real parts. This we cannot do without experimental information about $\bar{p}p \to \bar{p}p$ amplitudes. From phase shifts we can compute the real parts, which we have done. They do show a distinctive behavior, and can be found in Ref. 7. The results depend on very poorly determined $I = 0$ phase shifts. A more direct way is to find an observable which is dominated by, for example, $\mathrm{Re}\phi_2(0)$. One place to look is $np \to pn$ scattering in the forward direction, which is supposed to be dominated by the long-range part of one-pion exchange (i.e. the π cut). This contribution is only to ϕ_2 and is mainly real. The energy dependence of $d\sigma/dt$ indeed generally follows one's expectation for one-pion exchange. Isospin considerations imply that $\mathrm{Im}\phi_2(0)$ in $np \to pn$ is $\frac{1}{2}$ times $\mathrm{Im}({}^1D_2)$ [Eq. (3)]. Thus we might expect some type of anomaly in the energy behavior of $d\sigma/dt$ at $t = 0$.

This is indeed known to be true,[4] as shown in Fig. 2. We have plotted $(s - 4m^2)d\sigma/dt \sim k^2 d\sigma/dt$, so the π exchange will have an energy dependence approximately as $1/s$ for $(s - 4m^2)d\sigma/dt$. The smooth curve is drawn to go as $1/s$, normalized to $d\sigma/dt = 2$ mb/GeV^2 at 8 GeV/c, consistent with the high-energy data. Thus, the smooth curve approximately represents what would happen if the cross section extrapolated smoothly to low energies.

Though there is no direct evidence for a resonant ${}^1D_2$ partial wave (exotic ${}^1D_2$ resonance[8,9]), it is instructive to illustrate how $\mathrm{Re}\phi_2$ and $\mathrm{Im}\phi_2$ would be connected in the event that ϕ_2 were literally a Breit-Wigner resonance. The real part $\mathrm{Re}\phi_2$ will rise ahead of the imaginary part, reaching a maximum half way up where the imaginary part has a maximum derivative, vanishing where the imaginary part has a maximum and zero derivative, and reaching a second maximum of opposite sign where the decreasing imaginary part has a maximum derivative. From Figs. 1 and 2, these points are at 0.95 GeV/c, 1.4 GeV/c, and about 1.8 GeV/c, respectively. At the first point, the extra real part indeed rises maximally above the smooth curve; at the second it coincides with the smooth curve, and the extra real part is consistent with zero; and at the third the interference is indeed of opposite sign. Since the curve for $\mathrm{Im}\phi_2(0)$ rises faster on the low side, the real part should be larger there as observed; the derivative is about three times larger on the low side and the effect is consistent with being three times larger at 0.95 GeV/c than near 1.8 GeV/c. Thus from looking at ϕ_2 alone, one might suspect the existence of a true resonance.

At the least, these arguments suggest that more detailed measurements in the region of 1–2 GeV/c, particularly of $\Delta\sigma_T$ and $\Delta\sigma_L$, may allow a determination of whether the ${}^1D_2$ wave in pp is indeed resonant, with the resonant behavior induced by the strong coupling to an inelastic channel. The qualitative behavior of $\mathrm{Re}\phi_2$ deduced above is, however, a more general consequence of the behavior of $\mathrm{Im}\phi_2$. Even though we cannot write the full definite-signature kinematic-singularity-free amplitude, it is at least qualitatively true that the rapid variation in $\mathrm{Im}\phi_2$ induces a nearby rapid variation in $\mathrm{Re}\phi_2$ just from reasonable analyticity assumptions about hadron scattering amplitudes[10]; the behavior of a Breit-Wigner resonance is just a special case of the general situation.

We would like to end with a few remarks concerning the possible relevance of thresholds to the interpretation of high-energy data and duality.

There is clearly no reason why threshold effects would not be important in other reactions. In pp, we were lucky to find amplitudes which were free from the contamination of other known large contributions. For comparison, note that πN scattering has such a simple spin structure that both resonances and exchanges contribute to all helicity amplitudes. It is well known that there are important thresholds in *KN* scattering where, as in *pp*, nonexotic resonances are excluded.[8] A situation more closely analogous to the *pp* spin complexity but in a nonexotic reaction obtains in pion photoproduction where two single-flip *s*-channel helicity amplitudes are present. These have indeed been found to have experimental energy dependences analogous to ϕ_1 and ϕ_3 in that their difference at high energy is consistent with zero, but is large at low energies, implying a large unnatural-parity exchange (not π-like) contribution,[11] which is exotic or A_1-like.

Thus there appear to be several places where one can look sensitively for anomalous behavior associated with thresholds. It is obvious that when such behavior is present, the energy dependence of an affected amplitude will not show any characteristically high-energy behavior until one is well above the threshold. At energies below 3 GeV, such influence may be very important. By recognizing the existence of such effects, one may alter the interpretation in some cases of finite-energy sum rules. This will surely affect broader issues, e.g. the concepts of duality, and composite structures such as exotic bound states (deuterons) and resonances (D^* etc.). Some specific applications must also be questioned, such as the procedure which has been used of equating (even for imaginary parts) the low-energy behavior in *t* of an amplitude with its high-energy behavior.

NN reactions may be the simplest place to pursue these questions since (1) one has sufficient spin complexity to see consequences associated with threshold phenomena and unitarity effects in amplitudes which would otherwise be expected to be small based on current kinds of models, and (2) resonance and coupled-channel singularities are uncoupled to some extent because *NN* is an exotic channel. It appears to us instructive to systematically study the phenomena associated with threshold effects, and their relevance to duality, Reggeon exchanges, and exotic resonances.

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