Fermion-pseudoscalar scattering: A source-theory analysis*

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In a pseudoscalar-coupling theory, we calculate the fourth-order fermion-pseudoscalar scattering amplitude using source-theory methods. The space-time extrapolation of one of the double-spectral forms is ambiguous. However, when the ambiguity is shifted to the accompanying single-spectral forms, it is found that their extrapolation is unambiguous, leading to a determinate result for the scattering amplitude.

I. INTRODUCTION

The past few years has seen the development of a new approach to high-energy particle physics, called source theory,[1] which is an alternative to the two now accepted approaches, operator field theory and dispersion relations. Embodying both causal and noncausal techniques, source theory has been successfully applied to a wide range of problems, most notably those of quantum electrodynamics.[2,3] Here we apply the causal methods of source theory to a nonelectromagnetic fourth-order scattering process which has the added feature that the space-time extrapolation of the double-spectral form (DSF) is ambiguous.[4] For definiteness, we consider a theory in which a fermion, "N," with mass m, and a pseudoscalar, "π," with mass μ, interact by means of a local pseudoscalar coupling. We could consider the same process with fermions and pseudoscalars of various masses[5] but all that would be added is algebraic complexity, assuming no anomalous thresholds.[6]

The πN scattering process can be described in terms of two invariant amplitudes. In Sec. II we calculate the DSF weight functions for these amplitudes. We find that one of them implies large contributions to its spectral integral from large values of one of the spectral masses. This spectral mass region actually corresponds to a single-spectral form (SSF) so that the large-mass behavior of the DSF can be removed in favor of an SSF. This can be done in a variety of ways leading to an apparent ambiguity in the space-time extrapolation. We make a definite choice, thereby shifting any possible remaining ambiguity to the accompanying SSF. The amplitudes for causal forward scattering in the NN and πN channels are calculated in Secs. III and IV, respectively. The weight functions for the SSF are then determined by comparison with the DSF evaluated for this kinematic situation. We here find that the space-time extrapolation is unambiguous. Therefore, the combination of double- and single-spectral forms yields a determinate result and only the split up is ambiguous. In Sec. V we complete the calculation of the scattering amplitude by including propagator and vertex correction insertions, while a few concluding remarks are contained in Sec. VI.

II. DOUBLE-SPECTRAL FORM

The interaction of the π (field ϕ) and N (field ψ) is described by the primitive Lagrangian term

$$g\tfrac{1}{2}\psi\gamma^0\gamma_5\psi\phi\,. \tag{1}$$

We start the calculation of the fourth-order πN scattering amplitude by considering four causally localized sources (1 is later and 2 is earlier in time than 3 and 3′) which interact by the exchange of free particles. The causal process is depicted in Fig. 1. The corresponding vacuum amplitude for this process is[7]

$$\langle 0_+|0_-\rangle = g^4\int \frac{(dP_1)}{(2\pi)^4}\frac{(dP_3)}{(2\pi)^4}\frac{(dK_3)}{(2\pi)^4}\frac{(dK_2)}{(2\pi)^4} \times (2\pi)^4\delta(P_1-P_3-K_3-K_2)J \times \phi_3(K_3)\psi_1(-P_1)\gamma^0\langle I\rangle\psi_3(P_3)\phi_2(K_2)\,, \tag{2}$$

where

$$J=\int(dk)\delta((P_1-k)^2+m^2)\,\delta((P_1-K_3-k)^2+m^2) \times\delta((P_3-k)^2+m^2)\,\delta(k^2+\mu^2) \tag{3}$$

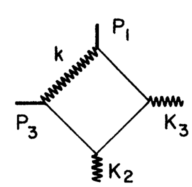

FIG. 1. Causal process leading to the double-spectral form.

and

$$\langle I\rangle=\langle\gamma_5[m-\gamma(P_1-k)]\gamma_5[m-\gamma(P_1-K_3-k)] \times\gamma_5[m+\gamma(k-P_3)]\gamma_5\rangle . \tag{4}$$

Here $\langle\ \rangle$ means an average value and J describes the kinematics of the free internal particles. In terms of the two spectral masses

$$M^2=-(P_3+K_2)^2=m^2(y+1),\quad M'^2=-(K_2+K_3)^2=m^2(x+4), \tag{5}$$

and after mass extrapolation,[8] we find ($\mu^2=m^2z$)

$$J=-\frac{i}{2m^4}\frac{1}{\sqrt{\delta}} \tag{6}$$

with

$$\delta=(x+4)[y^2x-2z(y+2)(x+4) +z^2(x+4y+20)-8z^3]. \tag{7}$$

The dynamics is contained in $\langle I\rangle$. The two invariant amplitudes for the scattering process can be defined as

$$\langle I\rangle=m^3\left[A_1+A_2\frac{1}{m}\gamma(K_2-K_3)\right]. \tag{8}$$

The evaluation of Eq. (4), after mass extrapolation, implies

$$A_1=y\{1-\tfrac{1}{2}d^{-1}[(x+4)(y+2)-z(x+2y+12)+6z^2]\}, \tag{9}$$

$$A_2=1-\tfrac{1}{4}d^{-1}[(x+4)(y+2)^2-z(y+2)(x+12) +2z^2(y+6)-2z^3], \tag{10}$$

where

$$d=(x+4)(y+1)+y^2-2zy-4z+z^2. \tag{11}$$

For large values of y,

$$A_1\sim y, \tag{12}$$

which means that the A_1 amplitude cannot be space-time extrapolated simply, because there are abnormally large contributions from large values of y. As in Ref. 4, the large-y behavior corresponds to an SSF and can be removed in a variety of ways, so that the extrapolation is ambiguous. A convenient rewriting of Eq. (9), in preparation for space-time extrapolation, is

$$A_1=-\frac{1}{m^2}(K^2+m^2)\{1-\tfrac{1}{2}d^{-1}[(x+4)(y+2)-z(x+2y+12)+6z^2]\}\equiv-\frac{1}{m^2}(K^2+m^2)\tilde{A}_1, \tag{13}$$

where $K^2=(P_3+K_2)^2=-M^2$ under the causal conditions. Once this form of A_1 has been chosen, the DSF is unique and the ambiguity has been shifted to the accompanying SSF. The space-time extrapolated DSF is

$$\begin{aligned}\langle 0_+|0_-\rangle=i\frac{g^4}{16\pi^2m}\int d(m^2x)d(m^2y)\frac{1}{\sqrt{\delta}}\,\phi\left(Z+\frac{\xi-\eta}{2}\right)\psi\left(Z+\frac{\xi+\eta}{2}\right)\gamma^0\\ \times\left[-\frac{1}{m^2}(-\partial_\xi{}^2+m^2)\tilde{A}_1+A_2\frac{1}{m}\gamma\frac{1}{i}\overleftrightarrow{\partial}\right]\psi\left(Z+\frac{\eta-\xi}{2}\right)\phi\left(Z-\frac{\eta+\xi}{2}\right)\\ \times\Delta_+[\xi,m^2(y+1)]\Delta_+[\eta,m^2(x+4)](dZ)(d\eta)(d\xi),\end{aligned} \tag{14}$$

where ∂_ξ operates only on $\Delta_+[\xi,m^2(y+1)]$ and $\overleftrightarrow{\partial}=\overrightarrow{\partial}-\overleftarrow{\partial}$ operates only on the pion fields. The region of integration is determined by the nonvanishing of J [Eq. (3)]. If the y integration is performed first, the limits are

$$y>y_+=\frac{1}{x}\{2[zx^2+z(4-3z+z^2)x+z^2(2-z)^2]^{1/2}+zx+2z(2-z)\},\quad x>0, \tag{15}$$

while, if the x integration is performed first, the limits are

$$x+4>\alpha/\beta,\quad y+1>(1+\sqrt{z})^2, \tag{16}$$

where

$$\alpha=4(y^2-z^2y-4z^2+2z^3),\quad \beta=y^2-2zy-4z+z^2. \tag{17}$$

III. *NN* SINGLE-SPECTRAL FORM

A convenient method to calculate the contact terms (that is, the SSF) in the *NN* channel is to first calculate the causal forward scattering amplitude in this channel. The causal process is depicted in Fig. 2. The causal vacuum amplitude for this case is

$$\langle 0_+|0_-\rangle = -\frac{g^4}{4m\pi}\int \frac{(dP_1)}{(2\pi)^4}\frac{(dP_{1'})}{(2\pi)^4}\frac{(dk_2)}{(2\pi)^4}\frac{(dk_{2'})}{(2\pi)^4}(2\pi)^4\delta(P_1+P_{1'}-k_2-k_{2'})\phi_2(k_2)\phi_{2'}(k_{2'})\psi_1(-P_1)\gamma^0 I'\psi_{1'}(-P_{1'})\,, \tag{18}$$

where $(Q=k_2+k_{2'})$

$$I' = 4m\pi\int d\omega_p d\omega_q (2\pi)^4\delta(p+q-Q)\frac{1}{(P_1-p)^2+\mu^2}\left\{\frac{1}{(p-k_2)^2+m^2}\gamma_5(m-\gamma p)\times\gamma_5[m-\gamma(p-k_2)]\gamma_5(m+\gamma q)\gamma_5+(2\to 2')\right\}. \tag{19}$$

For the calculation of I' in the forward direction, defined as the momenta $\vec{k}_2$ and $\vec{P}_1$ being in the same direction, we retain the general vector structure but calculate the coefficients in the forward direction.[9] If we define $[-Q^2=M'^2=m^2(x+4)]$

$$I' = \frac{1}{2}\left(\frac{x}{x+4}\right)^{1/2}\left[B_1+B_2\frac{1}{m}\gamma(k_2-k_{2'})\right], \tag{20}$$

we find for the coefficients

$$B_1 = \frac{2}{x}-\frac{x+4-2z-\gamma}{\rho_-}+2\left\{\frac{\rho_-}{2x}\left(\frac{1}{2}-\frac{2z}{x}\right)+\frac{2z}{\rho_-}[x+(2-z)^2]+\frac{1}{\gamma}[x+(2-z)^2]-\tfrac{1}{2}(x+4-2z-\gamma)\right\}L(-\gamma)+(\gamma\to-\gamma) \tag{21}$$

and

$$B_2 = -\frac{1}{2}\frac{x}{\gamma\rho_-}(x+4-2z-\gamma)+\left[-\frac{z^2}{x+4-4z}+\frac{1}{\rho_-}z^2(x+4-2z+\gamma)+\frac{1}{4}\frac{1}{\gamma}(x^2+4x+8z-4z^2)-\tfrac{1}{4}x-1\right]L(-\gamma) - (\gamma\to-\gamma)\,. \tag{22}$$

Here we have used the notations

$$L(\pm\gamma) = \frac{1}{\rho_\pm}\ln\left(\frac{x+z}{z}\,\frac{x+4-2z\pm\gamma}{x+4-2z\mp\gamma}\right), \tag{23}$$

$$\rho_\pm = x(x+4-2z)\pm(x+2z)\gamma\,. \tag{24}$$

and

$$\gamma^2 = x(x+4-4z)\,. \tag{25}$$

The only possible contact term supplementing the DSF which contributes in this channel is

$$\langle 0_+|0_-\rangle = i\frac{g^4}{32\pi^2 m}\int d(m^2x)\phi\left(Z-\frac{\eta}{2}\right)\psi\left(Z+\frac{\eta}{2}\right)\gamma^0\chi\psi\left(Z+\frac{\eta}{2}\right)\phi\left(Z-\frac{\eta}{2}\right)\Delta_+[\eta,m^2(x+4)](dZ)(d\eta)\,, \tag{26}$$

assuming that the spectral integral exists. Application of the DSF [Eq. (14)] together with the SSF [Eq. (26)] to the causal conditions considered above must reproduce the results presented in Eq. (20). This condition leads to the equations

$$\chi-\int_{y_+}^{\infty}\frac{dy}{\sqrt{\delta}}\tilde{A}_1\left(\frac{C_+}{y+C_+}+\frac{C_-}{y+C_-}\right)=\frac{1}{2}\left(\frac{x}{x+4}\right)^{1/2}B_1\,, \tag{27}$$

$$\int_{y_+}^{\infty}\frac{dy}{\sqrt{\delta}}A_2\left(-\frac{1}{y+C_-}+\frac{1}{y+C_+}\right)=\frac{1}{2}\left(\frac{x}{x+4}\right)^{1/2}B_2\,, \tag{28}$$

where

$$C_\pm = \tfrac{1}{2}(x+4-2z\pm\gamma)\,, \tag{29}$$

and y_+ is given in Eq. (15). Evaluating the integrals, we verify Eq. (28) and calculate χ from Eq. (27):

$$\chi = \left(\frac{x}{x+4}\right)^{1/2}\Bigg\{\frac{1}{x} - \frac{x+4-2z-\gamma}{2\rho_-} + \left[\frac{\rho_-}{2x}\left(\frac{1}{2}-\frac{2z}{x}\right) + \frac{2z}{\rho_-}(x+4-4z+z^2) + z\,\frac{x+4-4z}{2\gamma}\right] L(-\gamma)$$
$$+\left[\frac{x(x+4-3z)+4z(1-z)}{\gamma\rho_-} - 2(1-z)\,\frac{x^2+4x-4xz-4z^2}{\rho_-^{\,2}}\right]\left[-1+\tfrac{1}{2}(x^2+4x+8z-4z^2-x\gamma)L(-\gamma)\right]$$
$$+(\gamma\to-\gamma)\Bigg\}. \tag{30}$$

Since, for large values of x,

$$\chi \sim 1/x, \tag{31}$$

the space-time extrapolation of the SSF contact term [Eq. (26)] is unambiguous and there is no necessity for further local contact terms.

IV. πN SINGLE-SPECTRAL FORM

The four processes that contribute to causal forward scattering in the πN channel are shown in Fig. 3. For this section we will only consider Fig. 3(d) and will return to the remaining processes in Sec. V. The corresponding vacuum amplitude is

$$\langle 0_+|0_-\rangle = -\frac{g^4}{4m\pi}\int \frac{(dP_1)}{(2\pi)^4}\frac{(dk_1)}{(2\pi)^4}\frac{(dP_2)}{(2\pi)^4}\frac{(dk_2)}{(2\pi)^4}(2\pi)^4$$
$$\times\,\delta(P_1+k_1-P_2-k_2)\phi_1(-k_1)\psi_1(-P_1)$$
$$\times\,\gamma^0\tilde{I}\,\psi_2(P_2)\phi_2(k_2), \tag{32}$$

where $(Q=P_2+k_2)$

$$\tilde{I} = 4m\pi\int d\omega_p\, d\omega_k\,(2\pi)^4\delta(Q-p-k)\gamma_5\frac{1}{m+\gamma(p-k_1)}\gamma_5$$
$$\times(m-\gamma p)\gamma_5\frac{1}{m+\gamma(p-k_2)}\gamma_5. \tag{33}$$

Using

$$-Q^2 = M^2 = m^2(y+1), \tag{34}$$

the two invariant amplitudes can be defined as [see Eq. (17) for β]

$$\tilde{I} = \sqrt{\beta}\left[a + b\,\frac{1}{m}\,\gamma(k_1+k_2)\right]. \tag{35}$$

In the forward direction

$$k_1 = k_2, \tag{36}$$

and it is straightforward to find[10]

$$a = \frac{2(y+1)}{l^2-\beta^2}\left(1 + z\,\frac{y+4-3z}{\beta}\right) - \frac{1}{2}\,\frac{y+z}{\beta^2}\ln\frac{l+\beta}{l-\beta}, \tag{37}$$

$$b = \frac{2(y+1)}{l^2-\beta^2}\left[1 + z(4-z)\,\frac{y+2-z}{2\beta}\right]$$
$$-\tfrac{1}{4}y\,\frac{y+2-z}{\beta^2}\ln\frac{l+\beta}{l-\beta}. \tag{38}$$

Here we have defined

$$l = [\beta^2+(y+1)\alpha]^{1/2} = y^2+2(1-z)y-z^2. \tag{39}$$

The SSF contact term in this channel can be defined as

$$\langle 0_+|0_-\rangle = i\,\frac{g^4}{16\pi^2 m}\int d(m^2 y)\,\phi\left(Z+\frac{\xi}{2}\right)\psi\left(Z+\frac{\xi}{2}\right)\gamma^0\left(\bar{\chi}_1+\bar{\chi}_2\,\frac{1}{m}\,\gamma\,\frac{1}{i}\,\bar{\partial}\right)\psi\left(Z-\frac{\xi}{2}\right)\phi\left(Z-\frac{\xi}{2}\right)\Delta_+[\xi, m^2(y+1)](dZ)(d\xi), \tag{40}$$

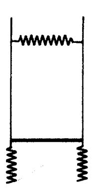

FIG. 2. Causal process for $\pi\pi\to NN$.

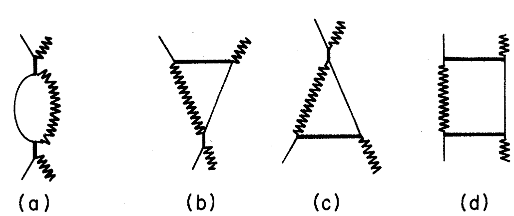

FIG. 3. Contributions to causal πN scattering.

assuming the spectral integral exists. When the DSF, Eq. (14), together with the contact term, Eq. (40), are calculated under the causal conditions considered above, they must reproduce the results of Eq. (35). This implies the equations

$$\bar{\chi}_1 + y\int_{\alpha/\beta-4}^{\infty} \frac{dx}{x+4}\,\frac{1}{\sqrt{\delta}}\,\tilde{A}_1 = \sqrt{\beta}\; a \tag{41}$$

and

$$\bar{\chi}_2 + \int_{\alpha/\beta-4}^{\infty} \frac{dx}{x+4}\,\frac{1}{\sqrt{\delta}}\,A_2 = \sqrt{3}\;\; b\,. \tag{42}$$

Performing the integration on x, we find

$$\bar{\chi}_1 = \bar{\chi}_2 = 0\,, \tag{43}$$

so that there are no contact terms in this channel.

V. INSERTIONS

Besides the processes depicted in Figs. 3(a)–3(c), there remains one more contribution to the scattering amplitude, that of single-particle exchange which is shown in Fig. 4. Here, the πNN form factor, F, is evaluated at the unphysical point of all particles being on their mass shells, with

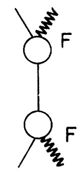

FIG. 4. Single-particle exchange.

$$F(\mu^2, m^2; Q(Q^2=-m^2)) = g(1+\lambda g^2 + \cdots)\,, \tag{44}$$

and we are only interested in the first correction, λ. All of these processes can be viewed as insertions into the lowest-order result,

$$\langle 0_+|0_-\rangle = i g^2 \tfrac{1}{2}\int (dx)(dx')\phi(x)\psi(x)\gamma^0\gamma_5 G_+(x-x')\gamma_5\psi(x)\phi(x)\,. \tag{45}$$

The first of these, Fig. 3(a), is given by the propagator correction. The causal process leading to the correction is shown in Fig. 5. The corresponding vacuum amplitude is

$$\langle 0_+|0_-\rangle = -g^2\int \frac{(dQ)}{(2\pi)^4}\,\psi_1(-Q)\gamma^0\int d\omega_p d\omega_q (2\pi)^4\delta(Q-p-q)\gamma_5(m-\gamma p)\gamma_5\psi_2(Q)\,. \tag{46}$$

When the mass-normalization conditions are imposed, we find that the correction, which is to replace $G_+(x-x')$ in Eq. (45), is

$$G_+(Q) = \frac{1}{m+\gamma Q} \to \bar{G}'(Q) = -\frac{g^2}{16\pi^2 m}\int \frac{dM^2}{M^2+Q^2}\,\frac{\sqrt{\beta}}{y+1}\,\frac{1}{y^2}\left[z + \frac{\gamma Q}{m}\,\frac{y^2 - zy - 2z}{2(y+1)}\right], \tag{47}$$

where we have expressed the result in momentum space and used Eq. (34). We can easily convert this to the two invariant amplitudes if we note that (in a mixed notation)

$$\frac{\gamma Q}{m} = -1 + \frac{1}{2}\frac{1}{m}\gamma\,\frac{1}{i}\,\overleftarrow{\partial}\,. \tag{48}$$

The remaining processes, Figs. 3(b), 3(c), and 4, lead to the introduction of the form factor $F(\mu^2, m^2; Q)$ into Eq. (45). For the calculation of this form factor, we consider the causal process shown in Fig. 6. The vacuum amplitude here is ($Q = P_1 + k_1$)

$$\langle 0_+|0_-\rangle = -g^3\int \frac{(dP_1)}{(2\pi)^4}\,\frac{(dk_1)}{(2\pi)^4}\int d\omega_p d\omega_q (2\pi)^4\delta(Q-p-q)\phi_1(-k_1)\psi_1(-P_1)\gamma^0\gamma_5\,\frac{1}{m+\gamma(p-k_1)}\,\gamma_5(m-\gamma p)\gamma_5\psi_2(Q)\,. \tag{49}$$

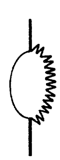

FIG. 5. Fermion propagation function correction.

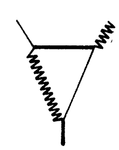

FIG. 6. Form factor for a virtual fermion.

If we define the form factor by means of

$$\langle 0_+|0_-\rangle = ig^2 \int (dx)(dx')\phi_1(x)\psi_1(x)\gamma^0\gamma_5 F(\mu^2, m^2; x-x')\psi_2(x'), \tag{50}$$

then, after imposing the normalization condition, Eq. (44), for $\gamma Q = -m$, we find, in momentum space,

$$F(\mu^2, m^2; Q) = \lambda g^2 - \frac{g^2}{16\pi^2 m}(m+\gamma Q)\int \frac{dM^2}{M^2+Q^2}\frac{\sqrt{\beta}}{y+1}\frac{1}{y}\left(f + h\frac{1}{m}\gamma Q\right), \tag{51}$$

where

$$f = -2y\frac{y+1}{\beta} + \frac{y+1}{\beta}\left[y+2+z\frac{y^2+4(2-z)y+8-6z+z^2}{\beta}\right]\ln\frac{l+\beta}{l-\beta}, \tag{52}$$

$$h = y\frac{y+2-z}{\beta} - \frac{y+1}{\beta}\left[2+z(4-z)\frac{y+2-z}{\beta}\right]\ln\frac{l+\beta}{l-\beta}, \tag{53}$$

and β and l are defined in Eqs. (17) and (39), respectively. Then the replacement in Eq. (45) is

$$G_+(Q) = \frac{1}{m+\gamma Q} \rightarrow V(Q) = \frac{2}{m+\gamma Q}F(\mu^2, m^2; Q). \tag{54}$$

Also, we can use Eq. (48) to write this result in terms of the two invariant amplitudes.

VI. CONCLUSIONS

The total result is given by the double-spectral form, Eq. (14), the single-spectral form, Eq. (26), and the lowest-order result, Eq. (45), with the two replacements, Eqs. (47) and (54). In particular, the πN scattering amplitude is given by

$$\begin{aligned}\langle 1_{p_1\sigma_1}1_{k_1}|1_{p_2\sigma_2}1_{k_2}\rangle &= i(2\pi)^4\delta(p_1+k_1-p_2-k_2)(d\omega_{p_1}d\omega_{p_2}d\omega_{k_1}d\omega_{k_2})^{1/2}\\ &\times u^*_{p_1\sigma_1}\gamma^0\bigg\{2mg^2\gamma_5[G_+(S)+\bar{G}'_+(S)+V(S)+(S\rightarrow U)]\gamma_5 + \frac{g^4}{4\pi^2}\int\frac{dx}{t+x+4}\chi\\ &\quad + \frac{g^4}{4\pi^2}\int dx\,dy\frac{1}{\sqrt{\delta}}\frac{1}{t+x+4}\left[-\tilde{A}_1\left(\frac{s+1}{s+y+1}+\frac{u+1}{u+y+1}\right)+A_2\left(\frac{1}{s+y+1}-\frac{1}{u+y+1}\right)\frac{1}{m}\gamma(k_1+k_2)\right]\bigg\}\\ &\times u_{p_2\sigma_2}, \end{aligned}\tag{55}$$

where

$$\begin{aligned} &S = p_1 + k_1, \quad S^2 = m^2 s,\\ &T = p_1 - p_2, \quad T^2 = m^2 t,\\ &U = p_1 - k_2, \quad U^2 = m^2 u. \end{aligned}\tag{56}$$

From this, the helicity amplitudes and differential cross sections can be easily calculated using the methods of Ref. 1.

The crucial issue under study in the above analysis, as in any causal consideration, is the presence, or absence, of contact terms and their determination. Gauge invariance,[2] charge and mass normalization (see Sec. V), and consistency[11] are important conditions, when applicable, for their calculation. Also, as occurred here and in Ref. 4, the structure of the spectral weight functions can indicate that contact terms are required, that additional information of a more local character is needed. The ambiguity of the DSF implied the necessity of the SSF. However, independent of this ambiguity, it is well known that the DSF need not contain all the nonlocal structure of the scattering amplitude, that SSF are, in general, necessary. So the question reduces to the extrapolation of the SSF which is related to the local character of the interaction. If the SSF is ambiguous, again indicated by abnormal behavior of the weight function for large values of the spectral mass, we learn that a local scattering term must be present in the primitive interaction. On the other hand, if the SSF is unambiguous, as it is here, no further local information is required.

The above calculation can be contrasted with the corresponding dispersive approach.[5] Besides our simpler mass choices which were made for convenience and clarity, the main differences are the following. Our DSF is developed from a causal process which is then mass and space-time extrapolated as opposed to considering a Feynman diagram and applying the Cutkosky rules and unitarity. The accompanying SSF are determined by a comparison with a physical scattering act, that of causal forward scattering, in contrast to a mathematical subtraction at an arbitrary point. The source-theory approach can be characterized as employing physical processes and ideas as compared to mathematical analysis for the dispersive method. As such, source theory is more straightforward and intuitively clearer.

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[7]We use the notations and conventions of Ref. 1.

[8]See Appendix A of Ref. 2.

[9]See Appendix C of Ref. 2.

[10]See Appendix B of Ref. 2.

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