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Collective phenomena in gauge theories. I. The plasmon effect for Yang-Mills fields*

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This is the first of a series of papers devoted to many-body phenomena in gauge theories. We assume that weak and electromagnetic interactions are described by a spontaneously broken gauge theory and that the strong interactions are described by an asymptotically free non-Abelian gauge theory. The implications of these assumptions for the behavior of matter at finite temperature and/or chemical potential are the subject of this series of papers. Briefly, the three main areas to which our results pertain are (1) renormalization of finite-temperature field theory, (2) the equation of state of strongly interacting matter at high densities in gauge theories, and (3) elementary-particle interactions in the early universe.

I. INTRODUCTION

This is the first of a series of papers on collective phenomena in gauge theories. We assume that weak and electromagnetic interactions are described by a spontaneously broken gauge theory (SBGT)[1] and that the strong interactions are described by an asymptotically free non-Abelian gauge theory.[2] The implications of these assumptions for the behavior of matter at finite temperature and/or chemical potential are the subject of this series of papers.

In the following we present a brief summary of the physical results to be discussed here and in the sequels. The first phenomenon, dealt with in I, is simply the analog in Yang-Mills theories of the well-known plasma effect. That is, a massless gauge boson in a medium at high temperature acquires a mass proportional to temperature. The plasma effect for scalars is, of course, implicit in the works of Ref. 3 where this additional temperature-dependent scalar mass brings about the restoration of the spontaneously broken gauge symmetry. Of equal importance is the recognition that the vectors also have this additional mass contribution. The main consequence of this result is that the gauge bosons in SBGT remain massive at all temperatures. This is in direct contradiction to previous claims in the literature stating that above the critical temperature for restoration of the broken gauge symmetry the gauge quanta are massless and the weak interactions become long range.[3] The implications of the plasma effect are primarily cosmological and are left to the body of this paper.

In paper II (Ref. 4) we examine the renormalizability of finite-temperature and -density field theories (FTD) in perturbation theory. A prescription is given for a mass-, temperature-, and chemical-potential-independent renormalization program. The advantage of this formulation is that the Green's functions in FTD have Callan-Symanzik functions which depend only on the dimensionless coupling constants. This allows a general investigation of the behavior of matter at high FTD via renormalization-group equations.

In paper III (Ref. 5) we examine hadronic matter at high densities assuming the strong interactions are governed by an asymptotically free gauge theory. Renormalization-group equations (RGE) are constructed for pressure and density as a function of chemical potential and temperature. Owing to asymptotic freedom the RGE may be solved to give the correct temperature-dependent equation of state at high densities.

The organization of the remainder of this paper is as follows: In Sec. II some essential kinematics is discussed. The plasmon effect in quantum electrodynamics for a relativistic electron gas[6] is derived in Sec. III. We refer the reader to references[7] for the derivation of the finite-temperature Feynman rules used in the calculations of this paper. We briefly review these. In Sec. IV the plasma effect for the Yang-Mills theory is demonstrated. The plasmon masses calculated in Secs. III and IV are done in lowest-order perturbation theory; however, in Sec. V a self-consistent perturbation calculation of the effective mass is done for ϕ^4. It is shown there that the self-consistent mass agrees with the lowest-order perturbation mass at high temperature. Finally, the last section is devoted to the physical implications of the plasmon effect.

II. KINEMATICS

We consider a gauge vector boson propagating through a medium which is homogeneous and infinite. Let $\Pi_{\mu\nu}$ denote in general a gauge-theory polarization operator (that is, the self-energy tensor) with $\Pi^Q_{\mu\nu}$ and $\Pi^{ab}_{\mu\nu}$ specifying, respectively,

$\Pi_{\mu\nu}$ for quantum electrodynamics and Yang-Mills theory. The four-momentum K^μ of the gauge vector boson is denoted by $(\omega, \vec{K}) = K^\mu$.[8] Because of gauge invariance we have

$$K^\mu \Pi_{\mu\nu}(\vec{K}, \omega) = 0. \tag{1}$$

If the medium is the vacuum then $\Pi_{\mu\nu}$ is a Lorentz tensor and must be of the form

$$\Pi_{\mu\nu}(\vec{K}, \omega) = D(K^2)(K^2 g_{\mu\nu} - K_\mu K_\nu) \tag{2}$$

since this is the only combination which satisfies Eq. (1) and is made up of the available tensors $g_{\mu\nu}, K_\mu K_\nu$. If the infinite-size medium though is a physical system (hereafter we will exclusively consider medium to mean this) then Lorentz covariance is broken. One way to see this is that there is now a preferred Lorentz frame, namely, the medium's center-of-momentum (c.m.) frame. Another way to understand this is to see that, for example,

$$\Pi^Q_{\mu\nu}(x, y) = \Pi^Q_{\mu\nu}(x - y) = i e^2 \langle T(j_\mu(x) j_\nu(y)) \rangle_0 , \tag{3}$$

where now the expectation value $\langle\ \rangle_0$ is averaged over the ensemble of states making up the medium. Each state is weighted by the Boltzmann probability factor so the right-hand side of Eq. (3) has no Lorentz transformation properties; it is defined only in the c.m. frame.

The diagrams defining $\Pi_{\mu\nu}$ in a physical medium shift the poles of the vector-boson propagator. We are interested in obtaining these new masses. In general then $\Pi_{\mu\nu}$ has the form

$$\Pi_{\mu\nu}(\vec{K}, \omega) = \alpha(\vec{K}, \omega) P^T_{\mu\nu} + \beta(\vec{K}, \omega) P^L_{\mu\nu} , \tag{4}$$

where the projection operators are given by

$$P^T_{\mu\nu} = e_\mu e_\nu,\ e_\mu = (K_\lambda K^\lambda)^{-1/2}(K, \omega\hat{K}), \quad K = |\vec{K}|,\quad \hat{K} = \vec{K}/|\vec{K}|, \tag{5}$$

$$P^T_{ij} = \delta_{ij} - \hat{K}_i \hat{K}_j,\quad P^T_{0i} = P^T_{00} = 0, \tag{6}$$

and the functions α, β are to be determined.

The following identity is useful:

$$g_{\mu\nu} = -P^T_{\mu\nu} - P^L_{\mu\nu} + \frac{K_\mu K_\nu}{K^2}. \tag{7}$$

If α and β have negative finite constants then they are respectively the masses of the transverse and longitudinal plasmons. Without deriving the real parts of α and β from Eq. (4) we can show that in a homogeneous medium the longitudinal plasmon mass is equal to the transverse plasmon mass. Let us go to the rest frame of the gauge vector. Then because the medium is isotropic we must have

$$\lim_{|\vec{K}|\to 0} R_{mi} \Pi_{ij} R^{-1}{}_{jl} = \Pi_{ml}, \tag{8}$$

where R is any rotation matrix and Π_{ij} are the space components of $\Pi_{\mu\nu}$. By Schur's lemma[9]

$$\lim_{|\vec{K}|\to 0} \Pi_{ij} = \lambda \delta_{ij}, \tag{9}$$

where λ is a constant. From Eq. (4) then

$$\alpha(\vec{K}, \omega)\Big|_{|\vec{K}|=0} = \beta(\vec{K}, \omega)\Big|_{|\vec{K}|=0}. \tag{10}$$

This expresses the equality of the two masses. In the limit $|\vec{K}| \to 0$ then, by using Eq. (7), we have for Eq. (4)

$$\Pi_{\mu\nu}(\vec{K}, \omega)\Big|_{|\vec{K}|=0} = -\alpha\left(g_{\mu\nu} - \frac{K_\mu K_\nu}{K^2}\right)\Big|_{|\vec{K}|=0}$$

or

$$\tfrac{1}{3}\Pi^\mu_\mu(0, m) = -\alpha(0, m) = m^2, \tag{11}$$

where $\omega = m$ for $|\vec{K}| = 0$. Thus Eq. (11) is an integral equation for m (self-consistently determined) and will be solved to determine the new gauge vector-boson masses. The lowest approximation for m is to neglect the mass dependence on the left-hand side of Eq. (11). We will do this for the vector particles. In Sec. V, however, the scalar theory will be treated self-consistently.

III. QUANTUM-ELECTRODYNAMICAL PLASMON

To order e^2, $\Pi^Q_{\mu\nu}$ is given by Fig. 1 as

$$\Pi^Q_{\mu\nu}(\vec{K}, \omega) = \frac{i e^2}{(2\pi)^4} \int d^4P\, \mathrm{Tr}\, \gamma_\mu S^\beta_F(P+K) \gamma_\nu S^\beta_F(P), \tag{12}$$

where $S^\beta_F(P)$ is the temperature- and chemical-potential-dependent electron propagator. We are interested in the limit of relativistic lepton temperatures where μ_c (chemical potential), m_e (electron mass) can be neglected and

$$N^+(E_P) \approx N^-(E_P) \approx [\exp(\beta P) + 1]^{-1}, \tag{13}$$

where $\beta = 1/T$ and $N^+(E_P)$ $[(N^-(E_P)]$ is the momentum-space distribution for positrons [electrons]. In this limit Eq. (12) becomes

$$\Pi^Q_{\mu\nu} = \frac{i e^2}{(2\pi)^4} \times \int d^4P \frac{4[P_\mu K_\nu + K_\mu P_\nu + 2P_\mu P_\nu - g_{\mu\nu}(P\cdot K + P^2)]}{(P+K)^2 P^2}. \tag{14}$$

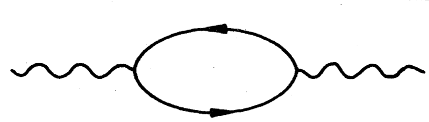

FIG. 1. Plasmon self-energy diagram. The wavy line is the plasmon and solid lines are fermions.

Taking the trace and evaluating it for $|\vec{K}|=0$, $\omega \ll T$ (see Ref. 10) we have

$$\Pi^{Q\mu}_{\mu}\Big|_{\substack{|\vec{K}|=0\\ \omega\ll T}} = \Pi(T)$$

$$= \frac{-ie^2}{(2\pi)^4}\int d^4P\,\frac{8}{P^2}\,. \tag{15}$$

In the finite-temperature formalism[7] we sum over a discrete set of frequencies and integrate over the space components

$$P_0 \to i\omega_N, \quad \frac{1}{P^2-m^2} \to \frac{-1}{\omega_N{}^2+|\vec{P}|^2+m^2},$$

$$\int\frac{d^4P}{(2\pi)^4} \to i\,T\sum_N\int\frac{d^3P}{(2\pi)^3},$$

where $\omega_N = 2\pi N/\beta$ for bosons and Faddeev-Popov ghosts, and $\omega_N=(2N+1)\pi/\beta$ for fermions. Equation (15) thus becomes

$$\Pi(T) = \frac{-8e^2}{(2\pi)^3}\,T\sum_{N=-\infty}^{+\infty}\int\frac{d^3P}{|\vec{P}|^2+(2N+1)^2\Pi^2T^2}\,. \tag{16}$$

Using the identity

$$\sum_{-\infty}^{\infty}[y^2+(2N+1)^2]^{-1} = \frac{\pi}{2y}\tanh\tfrac{1}{2}\pi y$$

Π becomes

$$\Pi(T) = \frac{-8e^2}{(2\pi)^3}\int\frac{d^3P}{2|\vec{P}|}\,\frac{|\vec{P}|}{2T}\,.$$

This has a divergence which is the same one that occurs in the zero-temperature theory and hence $\Pi^Q_{\mu\nu}$ is rendered finite by renormalization of the theory at $T=0$.[11] There remains a temperature-dependent finite part

$$\Pi(T) = \Pi(0) + \frac{e^2T^2}{3}\,.$$

From Eq. (11) we find that the QED plasmon mass in a relativistic fermion gas is

$$M_P{}^2(T) = \frac{e^2T^2}{9}\,. \tag{17}$$

This may be compared with the classical plasma frequency[12] in the low-external-frequency limit (using Heaviside-Lorentz units)

$$\omega_P{}^2 = e^2\int\frac{F(E_P)}{E_P}\left(1-\frac{P^2}{3E_P{}^2}\right)d^3P,$$
$$F(E_P) = \frac{2}{(2\pi)^3}[N^+(E_P)+N^-(E_P)],\quad E_P{}^2 = P^2+m_e{}^2. \tag{18}$$

Taking the high-temperature limit of $\omega_P{}^2$ and using Eq. (13) we see

$$\omega_P{}^2 = \frac{e^2T^2}{9}\,,$$

which agrees with Eq. (17) (as it must).

We have calculated the plasmon mass for a relativistic electron gas. In doing so we neglected terms m_e/T, μ_c/T. Actually the calculation shows more: The plasma effect can occur in a gas with zero chemical potential through the formation of fermion pairs. This gives rise to an interesting effect, a correction $O(e^2)$ to Planck's blackbody radiation law. However, calculation of this correction requires renormalization in finite-temperature field theory which is discussed in Ref. 4. There, the plasmon effect correction to Planck's law is given. Suppose then we consider a black box where, for simplicity, an electron gas in thermal equilibrium is inside. Within the box there are no photons, all having become plasmons. If now a pinhole is stuck into the black box and the emerging photons are observed we would see an energy spectrum which would be Planck's plus $O(e^2)$ corrections. That is, the emerging photons carry information out about the massive plasmon energy distributions inside. Thus a very hot plasma does not radiate as a pure blackbody. For the early universe where T is high the corrections are maximal; again, this effect is fully discussed in Ref. 5.

The corrections to Planck's law are small at normal temperatures. Is there another method of seeing these plasma oscillation quanta experimentally? Because the plasmon mass has chemical-potential dependence as well as temperature, a dense electron gas in a metal may be excited. An electron beam is impinged on a metal surface and the back-scattered electrons are detected. Expectations would be that the energy spectra of the back-scattered electrons would show "Frank-Hertz type dips" at $\hbar\omega_P, 2\hbar\omega_P, \ldots, N\hbar\omega_P$, where energy loss was due to the creation of one, two, ..., N plasmons within the metal. Many of these experiments have been done and have completely confirmed these expectations.[13]

IV. YANG-MILLS PLASMON

We are interested in establishing the plasmon effect for a high-temperature massless gas of self-interacting gauge bosons. Analogous calculations may be done for a gas of scalars and/or fermions. The relevant diagrams are given in Fig. 2. For simplicity we choose a definite gauge group, SU(2). At the lowest approximation we have massless vector bosons in the loops; the more rigorous self-consistent method of having massive vector-boson loops will approach the lowest approximation at high temperatures. We discuss this for ϕ^4 theory below. From Fig. 2 we have for the Yang-Mills polarization operator

$$\Pi^{ab}_{\lambda\tau} = -2ig^2\delta^{ab}\int\frac{d^4P}{(2\pi)^4}\,\frac{5K_\lambda P_\tau + 5P_\lambda K_\tau - 2K_\lambda K_\tau + 10P_\lambda P_\tau + g_{\lambda\tau}(5K^2 + 2P\cdot K + 2P^2)}{P^2(P+K)^2}$$
$$+ 2ig^2\delta^{ab}\int\frac{d^4P}{(2\pi)^4}\,\frac{K_\lambda P_\tau + 2K_\lambda K_\tau + K_\tau P_\lambda}{P^2(P+K)^2} + 12ig^2\delta^{ab}g_{\lambda\tau}\int\frac{d^4P}{P^2}. \tag{19}$$

The three terms in Eq. (19) come respectively from (a), (b), and (c) in Fig. 2.

Taking the trace of Eq. (19) and evaluating for $|\vec{K}|=0$, $\omega \ll T$ (see Ref. 10) we have

$$\Pi^{ab\lambda}_{\lambda}(T) = \Pi^{ab}(T) = 16ig^2\delta^{ab}\int\frac{d^4P}{(2\pi)^4P^2}. \tag{20}$$

We now use the temperature formalism of the last section. Here we are interested in the case $\mu_c \equiv 0$ and very high temperatures. Thus

$$\Pi^{ab}(T) = \frac{16\delta^{ab}g^2}{(2\pi)^3}\int\frac{d^3P}{2|\vec{P}|}\coth\frac{|\vec{P}|}{2T}, \tag{21}$$

where the identity

$$\sum_{N=-\infty}^{+\infty}(\gamma^2+N^2)^{-1} = \frac{\pi}{y}\coth\pi\gamma$$

has been used. As before, this has a divergence at $T=0$ which is absorbed into the renormalization of the charge, leaving a temperature-dependent finite part

$$\Pi^{ab}(T) = \Pi^{ab}(0) + \tfrac{4}{3}g^2T^2\delta^{ab}.$$

From Eq. (11) the Yang-Mills plasmon mass is

$$M_{\rm YM}{}^2 = \tfrac{4}{9}g^2T^2. \tag{22}$$

For SBGT, if temperature is of the order of or higher than $M_0/g \approx T_c$, where M_0 is the $T=0$ spontaneously generated mass, then the total mass of the gauge particles is just the plasma mass.[14] This plasma mass is smaller than T for small coupling constant g so the approximations and calculations done in Sec. IV for a massless gauge boson gas will hold for SBGT when $T \geq T_c$. Thus the mass of the gauge quanta in SBGT at T_c is just the plasma mass $\sim g\,T_c \sim M_0$. In Fig. 3, we plot schematically the mass of the gauge bosons in an SBGT as a function of temperature. This is to be contrasted with Fig. (5) of Weinberg[3], where the plasma effect was not taken into account.

Various cosmological effects centering on Eq. (22) will be discussed in Sec. VI. Here we wish simply to emphasize that this effect rules out a long-range weak force for spontaneously broken gauge theories at high temperature. This follows since every medium, scalar, fermion, or vector boson induces the plasmon effect.

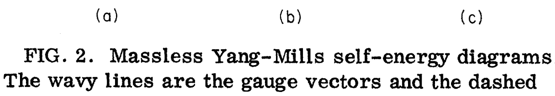

FIG. 2. Massless Yang-Mills self-energy diagrams. The wavy lines are the gauge vectors and the dashed lines are ghosts.

V. ϕ^4 THEORY

The renormalized Lagrangian density is

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \tfrac{1}{2}m_\xi\phi^2 - \frac{\lambda^2}{4!}\phi^4. \tag{23}$$

To lowest order in λ^2 only Fig. 4 contributes to the self-energy. Using the temperature formalism we find that the finite temperature-dependent mass is

$$m_T{}^2 = \frac{\lambda^2}{(2\pi)^2}\int_{m_\xi+m_T}^{\infty}\frac{[E^2-(m_\xi+m_T)^2]^{1/2}dE}{(e^{\beta E}-1)}, \tag{24}$$

where now the mass used in the tadpole is the total mass $m_\xi + m_T$. This is our integral equation to be solved for m_T. Using the variable $\eta = E/T$ Eq. (24) becomes

$$M_T{}^2 = \frac{\lambda^2T^2}{(2\pi)^2}\int_{(m_\xi+m_T)T}^{\infty}\frac{\{\eta^2-[(m_\xi+m_T)/T]^2\}^{1/2}d\eta}{(e^\eta-1)},$$

so

$$M_T{}^2 = \frac{\lambda^2}{(2\pi)^2}T^2f\left(\frac{m_\xi+m_T}{T}\right), \tag{25}$$

where f is the integral function. In the high-temperature limit $m_\xi/T \ll 1$ the solution to Eq. (25) is

$$M_T = CT, \quad C = \text{constant}. \tag{26}$$

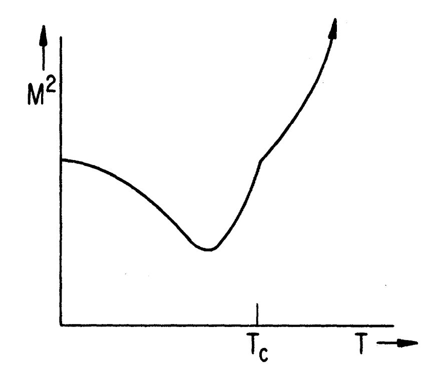

FIG. 3. Schematic representation of the gauge vector masses as a function of temperature in a spontaneously broken gauge theory.

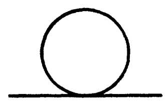

FIG. 4. Lowest-order mass correction for ϕ^4 scalar theory.

By dimensional analysis Eq. (26) is true for the gauge theories and thus Eq. (22) is correct for large temperature.

VI. DISCUSSION

In quantum electrodynamics we evaluate the plasmon mass in a relativistic electron gas. This is just an example of the quasiparticle effect where particle interactions in a medium shift the poles of particle propagators to new masses. In addition to the plasmon there will be quasielectrons whose mass differs from the vacuum value. Another effect is the change in Planck's blackbody radiation law due to pair creations in a photon gas. This result is derived in paper III.[5]

We have demonstrated the plasmon effect for a gas of massless Yang-Mills gauge bosons. The effect may be physically interpreted as explicit scale-invariance breaking by temperature. T acts like an infrared cutoff so there are no infrared divergences at finite temperatures. It is clear that a similar calculation can be done for the Yang-Mills quanta interacting with a gas of fermions and/or scalars. Higher-order effects produce corrections $O(g^4)$ and hence cannot cancel this result. The additional weak boson plasmon mass will, in principle, change the neutrino pair emissivity from stars[6] for SBGT of the weak interaction. In practice though, using $g\sim e$ and $m\sim eT$, temperatures of the order of 10^{14} °K must first be reached. Thus this effect for SGBT is more important for the early universe. As mentioned in the Introduction this effect rules out a long-range weak force for SBGT at high temperatures. In fact, just the opposite occurs where the force becomes shorter due to the heavier gauge quanta. The macroscopic effects of such a long-range weak-interaction force in the early universe have been hypothesized to be the impossibility of isotropy and homogeneity of matter.[3] Additionally such a force was to preclude a closed universe with positive curvature.[3] These important conclusions are now invalid. We wish to emphasize that the plasma effect rules out *any* long-range force in the early universe. This includes the strong-interaction force which is assumed to be mediated by massless colored gluons.[15] This then implies the absence of the color-singlet binding mechanism assumed to arise from the long-range force in pure Yang-Mills theory. Hence the description of "hadron era" in the early universe involves quasiparticles—quarks and Yang-Mills plasmons—rather than the conventional picture[16] of color-singlet hadrons. On the other hand, this does not mean real quarks and gluons exist outside the hot medium. Imagine the early universe as a black box and stick a pinhole in it to watch the spectra of particles emerging from it. Only physical color singlets are observed but the distribution in energy reflects the quasiparticle nature of the quark-gluon "soup" inside. This is in analogy to the electromagnetic black box discussed in Sec. III where the distribution in energy of emerging photons reflects the massive plasmon distribution inside the box. We hope to return to the question of asymptotic freedom, massive quark, and massive gluons, for the early hot universe, in a subsequent investigation

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[8]We use the metric (+1, −1, −1, −1) and units $\hbar=c=k$ (Boltzmann's constant) = 1.

[9]Michael Tinkham, *Group Theory and Quantum Mechanics* (McGraw-Hill, New York, 1974). p. 21.

[10]This corresponds to neglecting any mass dependence on the left-hand side of Eq. (11).

[11]Questions of renormalization are dealt with in paper II of this series.

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Rev. Mod. Phys. 28, 184 (1956); W. Brandt, Phys. Rev. 111, 1042 (1958).

[14]For SBGT the gauge vector mass is $m_V{}^2 = m_1{}^2 + m_2{}^2$, where m_1 is the spontaneously generated mass and m_2 is the effective mass due to polarization of the physical medium. For $T > T_c$, $m_1 = 0$ leaving $m_V = m_2$.

[15]Color is the Yang-Mills gauge group of the strong interactions.

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