Neutron charge radius and the quark-parton model

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The problem of finiteness and negative sign of the mean square charge radius of the neutron, $\langle r_n^2\rangle$, is studied in the context of three popular quark-parton models.

An analysis of the experimental data on the scattering of thermal neutrons from atomic electrons[1] and the quasielastic e-d scattering[2] reveals that the mean square charge radius of the neutron, $\langle r_n^2\rangle$, could indeed be finite and negative in sign. This implies that the slope of the charge form factor, $(d/dq^2)[G_E^n(q^2)]$, of the neutron is positive.[3] From the available data on the slope, McCarthy[4] has extracted the value of $\langle r_n^2\rangle$ satisfying $-0.16 \leq \langle r_n^2\rangle \leq -0.11$ (fm^2), which is obviously far from small on the scale of the proton charge radius.

This observation has been a subject of great importance from the theoretical point of view. Within the framework of standard quark models, the nucleon charge form factor is proportional to the nucleon charge, which means that the neutron charge form factor and hence also $\langle r_n^2\rangle$ are necessarily zero, in contradiction with the experimental results. Thus a simple version of the quark model is not adequate to account for the negative sign and the nonvanishing magnitude of $\langle r_n^2\rangle$.

Recently, Sehgal has made an attempt[5] to understand the negative sign of $\langle r_n^2\rangle$, on the basis of the quark-parton model and the hypothesis that the transverse-momentum distribution of the partons is nonfactorizable. The result of this investigation is that the negative sign of $\langle r_n^2\rangle$ can be recovered, provided the inequality, $\sigma^{\nu N} - \sigma^{\bar{\nu} N} > 6\int dx(F_2^{ep} - F_2^{en})$, holds. The purpose of the present paper is to calculate $\langle r_n^2\rangle$ explicitly, using Sehgal's formulation with the specific forms for the quark distributions within the framework of (i) modified Kuti-Weisskopf (MKW),[6] (ii) Landshoff-Polkinghorne-Kuti-Weisskopf (LPKW),[7] and (iii) Gunion[8] quark-parton models to see if agreement is obtained with the experimental results. Following Sehgal[5] we can write the mean square charge radius as

$$\langle r^2\rangle = \tfrac{3}{2}\int dx \sum_i e_i \int d^2b\, b^2 h_i(x,b)\,, \tag{1}$$

where $h_i(x,b)$ is a properly normalized spatial distribution function given by the Gaussian form

$$h_i(x,b) = [\pi A(x)]^{-1} f_i(x) \exp[-b^2/A(x)]\,, \tag{2}$$

with $A(x)$ as a decreasing function of x which is chosen to have the linear form $A(x) = 1-\beta x$ ($\beta>0$ and real). The functions $f_i(x)$ represent the mean number density of quarks of type i (and charge e_i) present in the nucleon with a fraction x of the total longitudinal momentum, and b is a transverse coordinate. Using (2) with (1) we have

$$\langle r_n^2\rangle = -\tfrac{3}{2}\beta \int dx\, x \sum_i e_i f_i(x)\,. \tag{3}$$

In the context of the quark-parton models considered here, the functions $f_i(x)$ can be written in terms of quark $[q = u(x), d(x), s(x)]$ and antiquark $[\bar{q} = \bar{u}(x), \bar{d}(x), \bar{s}(x)]$ distributions. Furthermore, these models assume the separability of the valence (v) and the core (c) quark contributions so that $u(x) = u_v(x) + c(x)$, $d(x) = d_v(x) + c(x)$, and $s(x) = c(x) = \bar{u}(x) = \bar{d}(x) = \bar{s}(x)$ (in the symmetric sea limit). With these prescriptions, expression (3) can be recast for the neutron and proton mean square charge radii, respectively, with appropriate $f_i(x)$ as

$$\langle r_n^2\rangle = -\tfrac{1}{2}\beta \int dx\, x[2d_v(x) - u_v(x)]\,, \tag{4}$$

$$\langle r_p^2\rangle = \tfrac{3}{2} - \tfrac{1}{2}\beta \int dx\, x[2u_v(x) - d_v(x)]\,. \tag{5}$$

The explicit structures of the parton distributions in the MKW, LPKW, and Gunion models can be written in the form

$$\text{MKW:}\quad u_v(x) = 1.79 z^3 x^{-1/2}(1+2.3x)\,,\quad d_v(x) = 1.107 z^{3.1} x^{-1/2}\,,\quad c(x) = 0.1\, z^{7/2} x^{-1}\,, \tag{6}$$

$$\text{LPKW:}\quad u_v(x) = 2N(1+a\sqrt{x})\, z^3 x^{-1/2}\left[\frac{\pi}{\Gamma(4)} - \frac{az}{\Gamma(5)}\right],\quad d_v(x) = N(1-a\sqrt{x})\, z^3 x^{-1/2}\left[\frac{\pi}{\Gamma(4)} + \frac{2a\sqrt{z\pi}}{\Gamma(\frac{9}{2})} + \frac{a^2 z}{\Gamma(5)}\right],$$

$$c(x) = \tfrac{1}{6} N z^{7/2} x^{-1}\left[\frac{\pi^{3/2}}{\Gamma(\frac{9}{2})} + \frac{\pi a\sqrt{z}}{\Gamma(5)} - \frac{\sqrt{\pi}\,a^2 z}{\Gamma(\frac{11}{2})} - \frac{a^3 z^{3/2}}{\Gamma(6)}\right],\quad N^{-1} = \frac{\pi^{3/2}}{\Gamma(\frac{9}{2})} + \frac{\pi a}{\Gamma(5)} - \frac{\sqrt{\pi}\,a^2}{\Gamma(\frac{11}{2})} - \frac{a^3}{\Gamma(6)}\,,\quad a = 0.7\,, \tag{7}$$

$$\text{Gunion:}\quad u_v(x)=5z^3+1.89z^7x^{-1/2},\quad d_v(x)=3.5z^3+1.03z^7x^{-1/2},\quad c(x)=0.2z^7x^{-1}, \tag{8}$$

where z stands for $(1-x)$. Using these forms we can now compute expressions (4) and (5) for $\langle r_n^2\rangle$ and $\langle r_p^2\rangle$, respectively. The value of the parameter β, in each case, is found from the experimentally known value of $\langle r_p^2\rangle$ which is taken to be 0.708 fm^2.[9] A fit to the $\langle r_p^2\rangle$ data yields the values 3.3, 3.8, and 3.26 for β for the MKW, LPKW, and Gunion models, respectively. The corresponding values for $\langle r_n^2\rangle$ are obtained to be

$$\begin{aligned}\langle r_n^2\rangle &= 0.13\ \text{fm}^2\ (\text{MKW}),\\ \langle r_n^2\rangle &= 0.15\ \text{fm}^2\ (\text{LPKW}),\\ \langle r_n^2\rangle &= 0.017\ \text{fm}^2\ (\text{Gunion}).\end{aligned} \tag{9}$$

The result obtained in Eq. (9) accounts well for the magnitude of $\langle r_n^2\rangle$ (except perhaps for the Gunion model) but with a wrong sign. So there is an apparent contradiction of our results with the experiments if we take the sign of $\langle r_n^2\rangle$ seriously. Moreover, the calculation of the neutrino cross sections and deep-inelastic structure functions[10] show that Sehgal's inequality is not satisfied in the context of the models considered here. However, it may be noted that the available experimental data[11] seem to satisfy this inequality in a somewhat restricted sense only. For instance, the large errors quoted in the experimental measurements may indeed reverse the sign of the inequality. Therefore, it is difficult to make a categorical statement about the validity of Sehgal's inequality as the relevant experimental information is not completely reliable at present.

In order to reconcile the negative sign of $\langle r_n^2\rangle$ within the framework of the quark-parton models, we suggest a modified-Gaussian-type spatial distribution function having the form

$$h_i(x,b)=[\pi(1+\eta)A(x)]^{-1}[1+\eta b^2/A(x)]\exp[-b^2/A(x)], \tag{10}$$

where η is a parameter to be adjusted from the data. With this modification, the corresponding expression for $\langle r_n^2\rangle$ gets modified by a multiplying factor $(1+2\eta)/(1+\eta)$. The negative sign of $\langle r_n^2\rangle$ can now be restored if the parameter η lies in the range $-1.0<\eta<-0.5$. A suitable choice of η within this range is expected to yield the correct sign and magnitude of $\langle r_n^2\rangle$ in agreement with the experimental evidence.

In conclusion, we have attempted to account for the observed negative sign and the finite magnitude of the neutron mean square charge radius, using three different sets of quark-parton distributions. We find that these models in conjunction with Sehgal's assumption of a Gaussian-type spatial distribution not only give the wrong sign of $\langle r_n^2\rangle$ but also violate Sehgal's inequality. We have shown that the negative sign of $\langle r_n^2\rangle$ can be restored by taking recourse to a modified Gaussian distribution [cf. Eq. (10)]. However, this modification has no effect whatsoever on the validity of the inequality. Since the data on the magnitude of $\langle r_n^2\rangle$ are not conclusive enough at this stage, any theoretical pursuit is bound to incorporate some degree of flexibility. The most fascinating aspect of the quark-parton models lies in the fact that they give excellent agreement with deep-inelastic data. In addition, they are conceptually the easiest to understand.

Note added in proof. After submitting this article for publication, we were informed by Professor S. Pakvasa and Professor S. F. Tuan that the correct normalization entering the expression for $u_v(x)$ in the MKW model distributions [cf. Eq. (6)] is 1.74 as opposed to 1.79. This would not, however, affect the calculations in any significant way. Thus, the conclusions arrived at in this paper remain essentially unaltered.

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$\langle r_n^2 \rangle = 0$ on account of the fact that $u_v(x) = 2d_v(x)$.

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[10]In terms of our notation, the neutrino cross sections and structure functions have the form

$$\sigma^{\nu N} = \int dx\, x[u_v(x) + d_v(x) + \tfrac{8}{3}c(x)],$$

$$\sigma^{\bar{\nu} N} = \tfrac{1}{3}\int dx\, x[u_v(x) + d_v(x) - 8c(x)],$$

$$F_2^{ep} = \tfrac{1}{9}x[4u_v(x) + d_v(x) + 12c(x)],$$

$$F_2^{en} = F_2^{ep}\ (\text{with } u \longleftrightarrow d).$$

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