Spectrum-generating SU(3) and SU(4) and the leptonic decays of ρ, ω, ϕ, and J (ψ)

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SU(3) and SU(4) are considered as spectrum-generating groups and applied to the decays $\rho \to e\bar{e}$, $\omega \to e\bar{e}$, $\phi \to e\bar{e}$, and $J(\psi) \to e\bar{e}$. The results are compared with the experimental data, which leads to predictions of the form of the electromagnetic transition operator (currents).

I. INTRODUCTION

In a series of recent works[1] that treated the leptonic and semileptonic decays of hadrons in a quantum-mechanical framework, it was suggested to consider the particle classifying SU(3) not as an approximate symmetry group but as a spectrum-generating group—denoted $\mathrm{SU}(3)_E$—whose connection with the Poincaré group is given by the relation (Werle)

$$[\hat{P}_\mu, \mathrm{SU}(3)_E] = 0\,, \quad [L_{\mu\nu}, \mathrm{SU}(3)_E] = 0\,. \tag{1}$$

Here $\hat{P}_\mu = P_\mu M^{-1}$, P_μ are the momentum operators, $M = (P_\mu P^\mu)^{1/2}$ is the mass operator, and $L_{\mu\nu}$ are the generators of the homogeneous Lorentz group. In this approach the mass differences are not only taken into account in the phase space but also the form factors and coupling constants are functions of the masses, and only in the symmetry limit of equal masses does one obtain the usual SU(3) symmetric expressions.

In the present work we shall apply this method to the leptonic decay of vector mesons, $V \to e\bar{e}$. As the dependence upon the mass will be much more pronounced for the new heavy mesons we will extend this spectrum-generating-group approach to the SU(4) and calculate $\Gamma(J(\psi) \to e\bar{e})$ using otherwise the usual SU(4) assumptions.

II. $V \to e\bar{e}$ DECAYS IN THE SPECTRUM-GENERATING-GROUP APPROACH

Following the same procedure that has been described in detail in Sec. III of Ref. 1(a) one obtains for the decay rate

$$\Gamma(V \to e\bar{e}) = 2\pi \int \frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\delta^4(p_V - p_+ - p_-)\frac{1}{2E_V m_V^2} \times \sum_{\mathrm{pol}} |\langle\langle e^+e^- p_+ p_- | T | \hat{p}_V, V\rangle\rangle|^2\,. \tag{2}$$

Here $p_\pm = (E_\pm, \vec{p}_\pm)$ are the momenta of $e^\pm$, $p_V = (E_V, \vec{p}_V)$ is the momentum of the vector meson, m_V its mass, and $\hat{p}_V = p_V/m_V$. The factor $1/m_V^2$ is a consequence of using generalized eigenvectors of $\hat{P}_\mu$ rather than P_μ.

In analogy to (3.15) of Ref. 1(a) and in concord with the usual lowest-order perturbation-theory expression, the transition matrix element $\langle\langle e^+e^- p_+ p_- | T | \hat{p}_V, V\rangle\rangle$ is assumed to be the product of a leptonic part $L^\mu(p_+, p_-)$ and a hadronic part $\langle\langle \sigma | H_\mu | \hat{p}_V, V\rangle\rangle$:

$$\langle\langle e^+e^- p_+ p_- | T | \hat{p}_V, V\rangle\rangle = L^\mu(p_+, p_-)\langle\langle \sigma | H_\mu | \hat{p}_V, V\rangle\rangle\,, \tag{3}$$

σ denotes the vector with the hadron quantum numbers of the vacuum, i.e., the trivial representation of $\mathrm{SU}(3)_E$ and of $\mathcal{P}_{\hat{P}_\mu, L_{\mu\nu}}$.

According to the principles stated in Ref. 1, the transition operator in the hadron space H_μ is connected with the octet of vector and axial-vector operators $V_\mu^\alpha, A_\mu^\alpha$, $\alpha = \pm 1, \pm 2, \pm 3, 0, 8$. For the electromagnetic decays under consideration, it should be given by the electromagnetic component of the vector operator[2]

$$V_\mu^{\mathrm{el}} = V_\mu^{\pi^0} + \frac{1}{\sqrt{3}} V_\mu^\eta\,. \tag{4}$$

Several connections between the transition operator H_μ and the octet operators $V_\mu^\alpha, A_\mu^\alpha$ have been discussed in Ref. 1, which all had the property that in the symmetry limit when the mass operator M is an SU(3) scalar they lead to the usual expressions given by the hadronic currents. We will consider here the ansatz[3]

$$H_\mu = g\{V_\mu^{\mathrm{el}}, M^P\} \tag{5}$$

and will determine P from the fit of the decay rates of ρ, ω, ϕ to the experimental data. The constant g is a coupling constant of $\dim g = \mathrm{mass}^{1-P}$.

A completely equivalent way, which will lead to the same result for $V \to e\bar{e}$ is to assume that

$$H_\mu = g V_\mu^{\mathrm{el}} \text{ and } \{V_\mu^\alpha, M^{-P}\} = \text{octet operator}\,. \tag{5$'$}$$

The leptonic part $L^\mu(p_+, p_-)$—which for the lepton pair $l\bar{\nu}$ was given by[1] $\bar{u}(p_\nu)\gamma^\mu(1-\gamma_5)v(p_e)$—should be obtained from this by replacing $\bar{u}(p_\nu)(1+\gamma_5)$ by the positron spinor $u(p_+)$. Thus, complete analogy to the weak decays would require one to choose for the leptonic part

$$L^{\mu}_{(1)}(p_+,p_-)=\bar{u}(p_+)\gamma_\mu v(p_-)\,. \quad (6)$$

Another possible choice for the leptonic part, in complete analogy to the usual perturbation-theory expression with the one-photon-exchange term, would be

$$L^{\mu}_{(2)}(p_+,p_-)=\bar{u}(p_+)\gamma_\mu v(p_-)\frac{1}{q^2}\,. \quad (6')$$

The difference of (6) and (6′) in the results for $V\to e\bar{e}$ will only be a factor of $1/m_V{}^2$, which can also be obtained by choosing in (5) $(P-2)$ instead of P. Thus, the distinction between (6) and (6′) is irrelevant for the decay rates and can always be expressed by the appropriate choice for the connection between the hadron transition operator H_μ and the octet operators V_μ^α.[4] We will, therefore, fix the definition of the value of P by choosing (6) for the leptonic part.

Inserting (3), with (5) and (6), into (2) gives

$$\Gamma(V\to e\bar{e})=2\pi\int\frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\delta^4(p_V-p_+-p_-)\frac{1}{2E_Vm_V{}^2}\times\sum_{\rm pol}|\bar{u}(p_+)\gamma^\mu v(p_-)gm_V{}^P\times\langle\langle\sigma|V_\mu^{\rm el}|\hat{p}_V,V\rangle\rangle|^2\,. \quad (7)$$

The matrix element of $V_\mu^{\rm el}$ can be written as

$$\langle\langle\sigma|V_\mu^{\rm el}|\hat{p}_V,V\rangle\rangle=C(V,{\rm el},\sigma)\langle\langle\sigma|V_\mu|\hat{p}_V,s,s_3\rangle\rangle\,, \quad (8)$$

where

$$C(V,{\rm el},\sigma)=C(\{8\},\{8\},\{1\};V,\pi^0,\sigma)+\frac{1}{\sqrt{3}}C(\{8\},\{8\},\{1\};V,\eta,\sigma) \quad (9)$$

are SU(3) Clebsch-Gordan coefficients. $\langle\langle\sigma|V_\mu|\hat{p}_V,s,s_3\rangle\rangle$ are SU(3)-invariant reduced matrix elements, as a consequence of (1). As explained in detail in Ref. 1, this is the essential distinction between the usual "broken SU(3) symmetry" and our use of SU(3) as a spectrum-generating group. In the usual approach one has momentum and not velocity eigenvectors in expressions like (8), and $\langle\langle\sigma|V_\mu|p_V,s,s_3\rangle\rangle$ would not be an SU(3)-invariant reduced matrix element but would depend upon the SU(3) quantum numbers through the masses.

The reduced matrix elements can be written

$$\begin{aligned}\langle\langle\sigma|V_\mu|\hat{p}_V,s=1,s_3\rangle\rangle&=\langle\langle\sigma|U(L(\hat{p}_V))V_\mu U(L^{-1}(\hat{p}_V))|\hat{p}_V=0,s=1,s_3\rangle\rangle\\&=L_\mu^{-1\gamma}(\hat{p}_V)\langle\langle\sigma|V_\gamma|\hat{p}_V=0,s=1,s_3\rangle\rangle\,, \quad (10)\end{aligned}$$

$\langle\langle\sigma|V_0|\hat{p}_V=0,s=1,s_3\rangle\rangle=0$ as V_0 is SO(3) scalar, and

$$\langle\langle\sigma|V(r)|\hat{p}_V=0,s=1,s_3\rangle\rangle=C(1,1,0;s_3,r,0)f'\,, \quad (12)$$

where $C(1,1,0;s_3,r,0)$ is an SO(3) Clebsch-Gordan coefficient and f' is the reduced matrix element of the SO(3) vector operator $V(0)=V_3$, $V(\pm1)=\mp(1/\sqrt{2})(V_1\pm iV_2)$. If one defines $e_\mu(\hat{p},s_3)$ by

$$fe_\mu(\hat{p},s_3)=L_\mu^{-1\gamma}(\hat{p})\langle\langle\sigma|V_\gamma|0,s=1,s_3\rangle\rangle\,, \quad (13)$$

where f is a suitable normalization factor related to the reduced matrix element f', one can show—using the properties of the boost $L_\mu^{-1\nu}(p)$ and of the SO(3) Clebsch-Gordan coefficients—that $e_\mu(\hat{p},s_3)$ has the properties of the vector-meson polarization vector, in particular

$$\sum_{s_3}e_\mu(\hat{p},s_3)e_\nu(\hat{p},s_3)=-\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{m_V{}^2}\right). \quad (14)$$

Thus

$$\langle\langle\sigma|V_\mu|\hat{p}_V,s=1,s_3\rangle\rangle=fe_\mu(\hat{p},s_3)\,, \quad (15)$$

where f is a reduced matrix element.

Inserting (15) into (7)

$$\Gamma(V\to e\bar{e})=2\pi\int\frac{d^3p_+}{2E_+}\frac{d^3p_-}{2E_-}\delta^4(p_V-p_+-p_-)\frac{1}{2E_V}\times\sum_{\rm pol}|\bar{u}(p_+)\gamma^\mu v(p_-)e_\mu(p,\lambda)|^2\times\left|\frac{1}{m_V}gm_V{}^PfC(V,{\rm el},\sigma)\right|^2, \quad (16)$$

and doing the usual integration gives

$$\Gamma(V\to e\bar{e})=\tfrac{2}{3}\pi^2|gfC(V,{\rm el},\sigma)|^2m_V{}^{2P-1}\,. \quad (17)$$

The above arguments obviously do not depend upon the choice of SU(3) as the spectrum-generating group, and any other choice of a spectrum-generating group, e.g., SU(4), which fulfills (1) will lead to the same result (17) with $fC(V,{\rm el},\sigma)$ replaced by the appropriate matrix elements of this group.

III. $SU(3)_E$ PREDICTIONS

To determine the value of P and to compare (17) with the experimental data,

$$\begin{aligned}\Gamma(\rho^0\to e\bar{e})&=6.42\pm0.80\ {\rm keV}\ \ \text{(world average)}\\&=6.11\pm0.53\ {\rm keV}\ \ \text{(Orsay)},\\\Gamma(\omega\to e\bar{e})&=0.76\pm0.17\ {\rm keV}\ \ \text{(world average)}\\&=0.76\pm0.08\ {\rm keV}\ \ \text{(Orsay)}, \quad (18)\\\Gamma(\phi\to e\bar{e})&=1.34\pm0.11\ {\rm keV}\ \ \text{(world average)}\\&=1.36\pm0.10\ {\rm keV}\ \ \text{(Orsay)},\end{aligned}$$

TABLE I. Connection between the value of P and the mixing angle of θ.

$2P-1$	2	1	0	−1	−2
θ (deg)	44.4 ± 3.5	40.7 ± 3.3	37.0 ± 3.1	33.4 ± 3.0	30.0 ± 2.9

we have to insert the Clebsch-Gordan coefficients $C(V,\mathrm{el},\sigma)$.

We use

$$|\rho^0\rangle = |\pi^0_V\rangle,$$
$$|\phi\rangle = \cos\theta|\eta_V\rangle + \sin\theta|\sigma_V\rangle, \tag{19}$$
$$|\omega\rangle = -\sin\theta|\eta_V\rangle + \cos\theta|\sigma_V\rangle,$$

and will admit for θ only values which lie in the vicinity of the "ideal" mixing angle $\theta = 35.2°$, the linear-mass-mixing value $\theta = 37.7°$, and the quadratic-mass-mixing value $\theta = 40.2°$. Subsequently, when we have convinced ourselves that these differences are negligible at the present state of experimental accuracy for the decay rates, we will use the ideal mixing angle $\cos\theta = (\frac{2}{3})^{1/2}$, $\sin\theta = (\frac{1}{3})^{1/2}$. The values of the SU(3) Clebsch-Gordan coefficients with V_μ^{el} given by (4) and $|V\rangle$ by (19) are [except for an irrelevant normalization factor, $(-1/\sqrt{8})$, that can be absorbed in f]

$$C(\rho^0,\mathrm{el},\sigma) = 1,$$
$$C(\phi,\mathrm{el},\sigma) = \frac{1}{\sqrt{3}}\cos\theta = \frac{1}{\sqrt{3}}\left(\frac{2}{3}\right)^{1/2}, \tag{20}$$
$$C(\omega,\mathrm{el},\sigma) = \frac{1}{\sqrt{3}}\sin\theta = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}.$$

From the experimental value (world average) and (17),

$$0.567 \pm 0.132 = \frac{\Gamma(\omega)}{\Gamma(\phi)} = \tan^2\theta\left(\frac{m_\omega}{m_\phi}\right)^{2P-1},$$

one obtains the connection between P and θ as given in Table I. Thus the values of $(2P-1)$ which will give mixing angles inside the admitted range are between $2P-1=2$ and $2P-1=-1$, i.e., $P=\frac{3}{2}$ and $P=0$.

None of these values of P is really ruled out by the experimental value of $\Gamma(\rho^0\to e\bar{e})$, though the values for $2P-1=2$ are slightly outside the experimental error bars. This is best demonstrated by calculating

$$\mathrm{const} = \tfrac{2}{3}\pi^2|gf|^2 = \frac{\Gamma(V\to e\bar{e})}{|C(V,\mathrm{el},\sigma)|^2 m_V^{2P-1}} \tag{21}$$

from $\Gamma(\rho\to e\bar{e})$ and $\Gamma(\omega\to e\bar{e})$ for the various admitted values of P:

$2P-1=2$, $\theta=(44.4\pm3.5)°$:

$$\mathrm{const} = 1.08\times10^{-11}\ \mathrm{keV}^{-1}\pm13\%\ \text{ from }\Gamma(\rho\to e\bar{e}),$$
$$\mathrm{const} = 0.76\times10^{-11}\ \mathrm{keV}^{-1}\pm14\% \tag{22}$$
from $\Gamma(\omega\to e\bar{e})$ or $\Gamma(\phi\to e\bar{e})$;

$2P-1$, $\theta=(40.7\pm3.3)°$:

$$\mathrm{const} = 8.34\times10^{-6}\pm13\%\ \text{ from }\Gamma(\rho\to e\bar{e}),$$
$$\mathrm{const} = 6.86\times10^{-6}\pm14\% \tag{23}$$
from $\Gamma(\omega\to e\bar{e})$ or $\Gamma(\phi\to e\bar{e})$;

$2P-1=1$, $\theta=(37.0\pm3.1)°$:

$$\mathrm{const} = 6.42\ \mathrm{keV}\pm13\%\ \text{ from }\Gamma(\rho\to e\bar{e}),$$
$$\mathrm{const} = 6.30\ \mathrm{keV}\pm14\% \tag{24}$$
from $\Gamma(\omega\to e\bar{e})$ or $\Gamma(\phi\to e\bar{e})$;

$2P-1=-1$, $\theta=(33.4\pm3.0)°$:

$$\mathrm{const} = 4.94\times10^{6}\ \mathrm{keV}^{2}\pm13\%\ \text{ from }\Gamma(\rho\to e\bar{e}),$$
$$\mathrm{const} = 5.89\times10^{6}\ \mathrm{keV}^{2}\pm14\% \tag{25}$$
from $\Gamma(\omega\to e\bar{e})$ or $\Gamma(\phi\to e\bar{e})$.

Thus the values $P=1,\frac{1}{2},0$ [restricting oneself only to integer $(2P-1)$] are chosen by the experimental values of the leptonic decays of ρ^0, ϕ, ω. This is remarkable as these are the same values that were also chosen by the leptonic and semileptonic decay data of the pseudoscalar mesons[5] in Ref. 1.

The value $P=0$, i.e., $(2P-1)=-1$, gives in fact the same results as are obtained from the vector-meson pole approximation of the first SU(3) spectral-function sum rule[6] and also from asymptotic SU(3).[7] The value $P=1$, i.e., $2P-1=1$, leads to the sum rule

$$\frac{\Gamma(\omega\to e\bar{e})}{m_\omega} + \frac{\Gamma(\phi\to e\bar{e})}{m_\phi} - \frac{1}{3}\frac{\Gamma(\rho\to e\bar{e})}{m_\rho} = 0, \tag{23'}$$

and the value $P=\frac{1}{2}$, i.e., $2P-1=0$, leads to the sum rule

$$\Gamma(\omega\to e\bar{e})+\Gamma(\phi\to e\bar{e})-\tfrac{1}{3}\Gamma(\rho\to e\bar{e})=0\,. \qquad (24')$$

IV. $SU(4)_E$ PREDICTIONS

Though the assignment of the newly discovered narrow resonance $J(\psi)$ to an SU(4) representation may still be considered too premature, we will apply the results of Sec. II to calculate $\Gamma(J\to e\bar{e})$ making the standard assumptions for the SU(4) properties of J[8,9] and the electromagnetic component of the regular tensor operator.[10,11] As we have seen in Sec. III that the present experimental accuracy does not allow for distinguishing between the various mixing angles and as the narrow width of the J indicates that it is very close to the ideally mixed $|c\bar{c}\rangle$ state,[8] it is sufficient to consider here only the ideal mixing:

$$\begin{aligned}
|\phi\rangle&=\left(\frac{2}{3}\right)^{1/2}|\eta_V\rangle+\frac{1}{2\sqrt{3}}|\eta_{c_V}\rangle-\frac{1}{2}|\sigma_V\rangle,\\
|\omega\rangle&=-\left(\frac{1}{3}\right)^{1/2}|\eta_V\rangle+\left(\frac{1}{6}\right)^{1/2}|\eta_{c_V}\rangle-\frac{1}{\sqrt{2}}|\sigma_V\rangle,\\
|J\rangle&=\frac{\sqrt{3}}{2}|\eta_{c_V}\rangle+\frac{1}{2}|\sigma_V\rangle,\\
|\rho\rangle&=|\pi^0_V\rangle.
\end{aligned} \qquad (26)$$

Here $|\eta_V\rangle$ denotes the SU(3)-octet state with $I_3=Y=\text{charm}=0$, $|\eta_{c_V}\rangle$ denotes the SU(3)-singlet state with $I_3=Y=\text{charm}=0$, and $|\sigma_V\rangle$ denotes the SU(4)-singlet state.

For the electromagnetic component of the SU(4)-covariant transition operator we take the two alternatives[10-12]

$$V_\mu^{\text{el}}=V_\mu^{\pi 0}+\frac{1}{\sqrt{3}}V_\mu^{\eta}-\left(\frac{2}{3}\right)^{1/2}\left(V_\mu^{\eta c}-\frac{1}{\sqrt{3}}V_\mu^{\sigma}\right), \qquad (27a)$$

$$V_\mu^{\text{el}}=V_\mu^{\pi 0}+\frac{1}{\sqrt{3}}V_\mu^{\eta}+2\left(\frac{2}{3}\right)^{1/2}\left(V_\mu^{\eta c}-\frac{1}{\sqrt{3}}V_\mu^{\sigma}\right), \qquad (27b)$$

where $V_\mu^{\pi 0}, V_\mu^{\eta}, V_\mu^{\eta c}$ are the π_0, η, η_c components of the SU(4) regular tensor operator and V_μ^{σ} is an SU(4) scalar operator (the factor $1/\sqrt{3}$ is a convention). Thus, instead of the one reduced matrix element f in (17) we have now two reduced matrix elements f^{15} and f^1. The decay rate is given

$$\Gamma(V\to e\bar{e})=\tfrac{2}{3}\pi^2g^2|\langle\sigma|V^{\text{el}}|V\rangle|^2 m_V^{2P-1}, \qquad (17')$$

where $\langle\sigma|V^{\text{el}}|V\rangle$ are the SU(4) matrix elements, which depend upon the two reduced matrix elements f^{15} and f^1.

If V_μ^{el} is given by (27a), then the SU(4) matrix elements $\langle\sigma|V^{\text{el}}|V\rangle$ are given by

$$\begin{aligned}
\langle\sigma|V^{\text{el}}|\rho^0\rangle&=f^{15},\\
\langle\sigma|V^{\text{el}}|\phi\rangle&=\frac{1}{\sqrt{2}}f^{15}+\frac{1}{\sqrt{3}}f^1,\\
\langle\sigma|V^{\text{el}}|\omega\rangle&=(\tfrac{2}{3})^{1/2}f^1,\\
\langle\sigma|V^{\text{el}}|J\rangle&=-\left(\frac{1}{2}\right)^{1/2}f^{15}+\frac{1}{\sqrt{3}}f^1.
\end{aligned} \qquad (28a)$$

If V_μ^{el} is given by (27b), then the SU(4) matrix elements are given by

$$\begin{aligned}
\langle\sigma|V^{\text{el}}|\rho^0\rangle&=f^{15},\\
\langle\sigma|V^{\text{el}}|\phi\rangle&=-\frac{2}{\sqrt{3}}f^1,\\
\langle\sigma|V^{\text{el}}|\omega\rangle&=f^{15}+(\tfrac{8}{3})^{1/2}f^1,\\
\langle\sigma|V^{\text{el}}|J\rangle&=\sqrt{2}f^{15}-\frac{2}{\sqrt{3}}f^1.
\end{aligned} \qquad (28b)$$

(28a) as well as (28b) will lead to the following relation for the decay rates of ρ, ω, ϕ:

$$\begin{aligned}
&[2m_\phi^{-(2P-1)}\Gamma(\phi\to e\bar{e})]^{1/2}\\
&\quad=|[m_\rho^{-(2P-1)}\Gamma(\rho\to e\bar{e})]^{1/2}-[m_\omega^{-(2P-1)}\Gamma(\omega\to e\bar{e})]^{1/2}|\,.
\end{aligned} \qquad (29)$$

(29) is in excellent agreement with the experimental data for the case $2P-1=0$. For the cases $2P-1=1$ and $2P-1=-1$ the experimental data

TABLE II. Comparison of the right- and left-hand sides of Eq. (29) (w.a. indicates world average).

Case	Left-hand side of (29)	Right-hand side of (29)
$2P-1=-1$ (w.a.)	$(1.45\pm0.15)\times10^3$ keV	$(1.65\pm0.06)\times10^3$ keV
$2P-1=-1$ (Orsay)	$(1.40\pm0.11)\times10^3$ keV	$(1.67\pm0.06)\times10^3$ keV
$2P-1=0$ (w.a.)	1.66 keV$^{1/2}$	1.64 keV$^{1/2}$
$2P-1=+1$ (w.a.)	1.90 ± 0.20	1.62 ± 0.06
$2P-1=+1$ (Orsay)	1.83 ± 0.14	1.63 ± 0.05
$2P-1=2$(w.a.)	$(2.18\pm0.22)\times10^{-6}$ keV$^{-1/2}$	$(1.61\pm0.06)\times10^{-6}$ keV$^{-1/2}$

barely fit (29); in fact, they do not really fit if one takes the Orsay values and not the world averages with their large errors. For all other values of P (29) disagrees with the experimental data.

To make this statement quantitative we list in Table II the right- and left-hand sides of (29) for the various cases.

One can now determine the two reduced matrix elements f^1 and f^{15} from the 3 experimental data $\Gamma(\rho\to e\bar{e})$, $\Gamma(\omega\to e\bar{e})$, $\Gamma(\phi\to e\bar{e})$ for the cases $2P-1=\pm 1$ and $2P-1=0$. With these values for the reduced matrix elements one can calculate the SU(4) matrix element $\langle\sigma|V^{\rm el}|J\rangle$ from (28a) [for the case that $V_\mu^{\rm el}$ is given by (27a)] and from (28b) [for the case that $V_\mu^{\rm el}$ is given by (27b)], and then calculate with (17′) the rate $\Gamma(J\to e\bar{e})$. The results of the calculation are the following:

Case $2P-1=0$. The reduced matrix elements are

$$\left|\frac{\sqrt{3}}{\sqrt{2}\,\pi g}f^1\right| = 1.03\pm 0.04\ \text{keV}^{1/2},$$

$$\left|\frac{\sqrt{3}}{\sqrt{2}\,\pi g}f^{15}\right| = \begin{cases} 2.47\pm 0.11\ \text{keV}^{1/2} & \text{(Orsay)},\\ 2.53\pm 0.32\ \text{keV}^{1/2} & \text{(world average)},\end{cases}$$

$$\operatorname{sign} f^1 = -\operatorname{sign} f^{15}. \tag{30}$$

The predicted decay rate for the case that $V_\mu^{\rm el}$ is given by (27a) is

$$\Gamma(J\to e\bar{e}) = 5.48\pm 0.38\ \text{keV}\quad \text{(using Orsay data)}$$

$$\Gamma(J\to e\bar{e}) = 5.63\pm 1.08\ \text{keV}\quad \text{(using world averages)}. \tag{31a}$$

The predicted decay rate for the case that $V_\mu^{\rm el}$ is given by (27b) is

$$\Gamma(J\to e\bar{e}) = 22.73\pm 4.34\ \text{keV}\quad \text{(using world averages)} \tag{31b}$$

Case $2P-1=-1$. [Here we use only world-average values as (29) is already incompatible with the Orsay data.]

The reduced matrix elements are

$$\left|\frac{\sqrt{3}}{\sqrt{2}\,\pi g}f^1\right| = (0.98\pm 0.10)\times 10^3\ \text{keV},$$

$$\left|\frac{\sqrt{3}}{\sqrt{2}\,\pi g}f^{15}\right| = (2.22\pm 0.14)\times 10^3\ \text{keV}, \tag{32}$$

$$\operatorname{sign} f^1 = -\operatorname{sign} f^{15}.$$

The predicted decay rate for the case that $V_\mu^{\rm el}$ is given by (27a) is

$$\Gamma(J\to e\bar{e}) = 1.47\pm 0.49\ \text{keV} \tag{33a}$$

and the predicted decay rate for $V_\mu^{\rm el}$ given by (27b) is

$$\Gamma(J\to e\bar{e}) = 5.88\pm 1.96\ \text{keV}. \tag{33b}$$

Case $2P-1=+1$. The values for the predicted decay rates are

$$\Gamma(J\to e\bar{e}) = 23.3\ \text{keV}\pm 30\%\quad \text{for } V_\mu^{\rm el} \text{ given by (27a)}, \tag{34a}$$

$$\Gamma(J\to e\bar{e}) = 21.2\ \text{keV}\pm 30\%\quad \text{for } V_\mu^{\rm el} \text{ given by (27b)}. \tag{34b}$$

The latest experimental value[13] for the decay rate of $J(3.1)$ is

$$\Gamma^{\rm exp}(J\to e\bar{e}) = 4.8\pm 0.6\ \text{keV}.$$

(31a), obtained from (27a) (Ref. 10) for $P=\frac{1}{2}$, and (33b), obtained from (27b) (Ref. 11) for $P=0$, are in agreement with this value.

Summarizing the results of this section, we have seen that the SU(4) assumption (27a) with $P=\frac{1}{2}$ gives an excellent fit of the experimental data for the $\rho\to e\bar{e}$, $\omega\to e\bar{e}$, $\phi\to e\bar{e}$, $J\to e\bar{e}$ decays, and the assumption (27b) with $P=0$ cannot be ruled out but gives a comparatively poor fit of the experimental data. That (17′) with $P=\frac{1}{2}$ fits the experimental data well has also been noted by Yennie.[14]

V. SUMMARY AND CONCLUSIONS

SU(3) is not a symmetry group and SU(4) will be much less so if the newly discovered high-mass resonances are classified by it. If these groups are considered as spectrum-generating groups, then the transition operators (currents) are unlikely to be irreducible tensor operators and the question is whether one can find functions of them and the mass and momentum operators that transform irreducibly (according to the regular representation) with respect to the spectrum-generating group. This idea, previously applied to the leptonic and semileptonic decays of pseudoscalar mesons[1] has been applied here to the decays $V\to e\bar{e}$. The ansatz (5) which gave agreement with the semileptonic decay data for the values $P=0,\frac{1}{2},1$ leads to acceptable fits of the $\rho\to e\bar{e}$, $\omega\to e\bar{e}$, $\phi\to e\bar{e}$ data for these three values of P only; the best fit is obtained for $P=\frac{1}{2}$, as was the case for the semileptonic decays.

SU(4), if it should turn out to classify the hadrons, will serve as a much more sensitive test of this kind of symmetry-breaking effects; unfortunately in SU(4) the form of $V_\mu^{\rm el}$ is ambiguous. With the original ansatz[10] for $V_\mu^{\rm el}$, (27a), $P=\frac{1}{2}$ is the only possibility that will fit the experimental data and this fit is excellent. With the ansatz (27b) for $V_\mu^{\rm el}$, $P=0$ is the only possibility that cannot be excluded, though this gives already a rather poor fit of the experimental data for ρ,ω,ϕ.

[1](a) A. Böhm, Phys. Rev. D 13, 2110 (1976).
(b) A. Böhm and J. Werle, Nucl. Phys. (to be published).
(c) A. Böhm, in *Neutrino-75*, proceedings of the Fifth International Conference on Neutrino Science, Balaton, Hungary, 1975, edited by A. Frenkel and G. Marx (Budapest, Hungary, 1976).

[2]We label here the components of the octet operator by the corresponding particle in the pseudoscalar meson octet. The connection with the notation in Ref. 1 is given by $V_\mu^{\pi^0} = V_\mu^0$, $V_\mu^\eta = V_\mu^8$.

[3]This corresponds in the notation of Ref. 1 to the choice $\phi_V(M) = M^P$. The choice $P = \frac{1}{2}$ gave the best fit to the experimental data for the weak semileptonic decays of pseudoscalar mesons [Ref. 1(a)]; $P = 0$ and $P = 1$ gave still reasonable values.

[4]The distinction between (6) and (6′) is of course relevant for the calculation of angular distribution, polarization, etc. However, the $1/q^2$ can always be obtained by a suitable choice of the algebraic relation between H_μ and V_μ^{el}, which will then have to contain not only the mass but also the momentum operator.

[5]The case $P = \frac{1}{2}$ predicts for the suppression factor S_{I_3} (Cabibbo angle) a value of $S_{I_3}^{1/2} = 0.238$ which compares very well with the experimental value of 0.224. $P = \frac{1}{2}$ or any noninteger value of P is, or course, aesthetically very unappealing; however, there is no reason to exclude it. $P = 1$ has the theoretical advantage of requiring a dimensionless coupling constant g.

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