Mass spectrum of spin-zero mesons

Michael T. Vaughn
*Physics Department, Northeastern University, Boston, Massachusetts 02115**
and Physics Department, Texas A&M University, College Station, Texas 77843
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The masses of the charmed spin-zero mesons are estimated using a phenomenological Lagrangian in which the fields transform according to the representation $(4,4^*)\oplus(4^*,4)$ of chiral $SU(4)\otimes SU(4)$. One new parameter, which is fixed by the mass of η'', is required to predict the masses of the charmed spin-zero mesons. If $m(\eta'')=2.8$ GeV, then $m(F_P)=2.57$ GeV, $m(D_P)=2.64$ GeV, $m(D_S)=2.99$ GeV, and $m(F_S)=3.1$ GeV are predicted. An analysis of the radiative decays of η'' and $\psi(3.095)$ is given.

I. INTRODUCTION

Various theoretical considerations,[1] and the discovery of the new resonances,[2] have suggested the existence of an SU(4) symmetry group, badly broken in nature, associated with the hadrons. If this suggestion is correct, there must exist hadrons with nonzero values of a new quantum number C ("charm"), which vanishes for the known hadrons. Several authors have attempted to estimate the masses of the charmed hadrons, both from lowest-order SU(4)-symmetry-breaking formulas,[1,3] and from explicit quark models of the hadrons.[1,4]

In the present work I estimate the masses of the charmed spin-zero mesons by assuming that these transform according to the $(4,4^*)\oplus(4^*,4)$ representation of the chiral group $SU(4)\otimes SU(4)$, and that the masses can be obtained from a phenomenological Lagrangian which is both renormalizable and $SU(4)\otimes SU(4)$ invariant, except for terms linear in the scalar fields ("σ terms") which induce the symmetry-breaking. Such a Lagrangian provides a remarkably accurate description of known spin-zero mesons which transform according to the $(3,3^*)\oplus(3^*,3)$ representation of the familiar chiral $SU(3)\otimes SU(3)$ subgroup,[5,6] and it is not unreasonable to expect useful estimates for the charmed mesons as well.

The Lagrangian contains but one parameter not determined by the properties of the noncharmed mesons—a parameter which measures the breaking of SU(4) relative to the breaking of SU(3).

This parameter is constrained by the requirements that (i) the $\psi(3.095)$ be stable against strong decay into charmed meson pairs, and (ii) the new $I=0$, $Y=0$, $C=0$ scalar (σ'') and pseudoscalar (η'') mesons contained in $(4,4^*)\oplus(4^*,4)$ have mass large enough that the expected radiative decay width of $\psi(3.095)$ into $\eta''+\gamma$ (or $\sigma''+\gamma$) not exceed about 10% of the experimental decay width.

The first constraint is enough to guarantee that the lowest-lying charmed spin-zero meson is the pseudoscalar meson F_P (in the terminology of Ref. 1), and the masses satisfy

$$m(F_P)<m(D_P)<m(D_S)<m(F_S)\,.$$

This ordering differs from that predicted by the standard first-order mass formulas[1,3] since the Lagrangian model contains mass terms (at the tree-approximation level) which are quadratic in the SU(4)-breaking parameter, and these are not negligible for the badly broken SU(4) which may describe the hadrons.

If a quark-loop model[7] is used to estimate the decay rate for $\psi\to\eta''+\gamma$, then the second constraint requires

$$m(\eta'')\gtrsim 2.8 \text{ GeV}\,,$$

whence

$$m(F_P)\gtrsim 2.57 \text{ GeV}\,,$$

$$m(D_P)\gtrsim 2.64 \text{ GeV}\,,$$

$$m(D_S)\gtrsim 2.99 \text{ GeV}\,,$$

$$m(F_S)\gtrsim 3.1 \text{ GeV}\,,$$

and it is not surprising that charmed spin-zero mesons have not been seen to date.[8]

The η'' is almost a pure c-$\bar{c}$ state, and, correspondingly, the admixture of c-$\bar{c}$ in the η and η' is small enough that the radiative decays

$$\psi\to\eta(\eta')+\gamma$$

through c-$\bar{c}$ quarks are strongly suppressed.

The σ'' is predicted to have a large mass ($\gtrsim$5.7 GeV), but the admixture of c-$\bar{c}$ in σ and σ' is reasonably large (about 5% in amplitude); the decay modes

$$\psi\to\sigma(\sigma')+\gamma$$

should be significant, perhaps of the order of a few keV partial width.

The $SU(3)\otimes SU(3)$ Lagrangian model is reviewed in Sec. II, and the extension to $SU(4)\otimes SU(4)$ is described in Sec. III. Section IV is devoted to the quark-model analysis of the radiative decays, and some concluding remarks are given in Sec. V.

II. SU(3)⊗SU(3) LAGRANGIAN MODEL

Many authors have discussed phenomenological Lagrangian models for spin-zero mesons which transform according to the $(3,3^*)\oplus(3^*,3)$ representation of the chiral $SU(3)\otimes SU(3)$ group.[5,6,9-14] It is to be hoped that phenomenological Lagrangian can ultimately be derived from a fundamental Lagrangian which includes quarks and other "elementary" constituents of the hadrons, but for the present, I can only assume that the phenomenological Lagrangian has some general properties which may, or may not, be consequences of the fundamental Lagrangian.

In particular, I assume that the phenomenological Lagrangian is (i) renormalizable, and (ii) invariant under chiral $SU(3)\otimes SU(3)$ except for terms linear in the scalar fields ("σ terms") which break the symmetry in such a way that partial current conservation conditions are satisfied. Assumption (i) can be restated as an assumption that the phenomenological Lagrangian contains no terms of canonical dimension greater than 4, so that the breaking of scale invariance is mild.[12]

Then the Lagrangian has the general form

$$\begin{aligned} L = {} & \tfrac{1}{4}\,\mathrm{Tr}[\partial_\mu M^\dagger \partial_\mu M - \mu_0{}^2 M^\dagger M - \lambda(M^\dagger M)^2] \\ & - \tfrac{1}{4}\lambda'[\mathrm{Tr}(M^\dagger M)]^2 - \tfrac{1}{8}\sqrt{3}\,\chi(\det M + \det M^\dagger) \\ & + c_0\sigma^0 + c_8\sigma^8\,, \end{aligned} \tag{1}$$

where

$$M = \sum_A (\sigma^A + i\pi^A)\lambda_A$$

is a 3×3 matrix, which contains the scalar fields σ^A and pseudoscalar fields π^A $(A=0,1,\ldots,8)$, which transforms under the representation $(3,3^*)\oplus(3^*,3)$ of chiral $SU(3)\otimes SU(3)$, and the λ_A are the standard set of Hermitian 3×3 matrices proportional to the generators of SU(3).

The linear terms break the symmetry, and the scalar fields then acquire vacuum expectation values

$$f_A \equiv \langle \mathrm{vac}|\sigma^A|\mathrm{vac}\rangle \quad (A=0,8)\,.$$

In the tree approximation the weak decay constants of π, K, κ are expressed in terms of f_0, f_8 according to

$$F_\pi = (\sqrt{2}\,f_0 + f_8)/\sqrt{3}\,, \tag{2a}$$

$$F_K = (2\sqrt{2}\,f_0 - f_8)/\sqrt{2}3\,, \tag{2b}$$

$$F_\kappa = \sqrt{3}\,f_8/2\,, \tag{2c}$$

and the particle masses are expressed in terms of f_0, f_8 and the coupling constants in the Lagrangian.

The coefficients of the σ terms can also be expressed in terms of f_0, f_8 and the physical particle masses according to

$$\sqrt{2}\,c_0 + c_8 = (\sqrt{2}\,f_0 + f_8)\pi\,, \tag{3a}$$

$$2\sqrt{2}\,c_0 - c_8 = (2\sqrt{2}\,f_0 - f_8)K\,, \tag{3b}$$

$$c_8 = f_8\kappa\,, \tag{3c}$$

where the particle symbol here denotes its mass squared. Elimination of c_0 and c_8 leads to the general mass relation

$$\frac{K-\pi}{\kappa-\pi} = -\frac{3f_8}{2\sqrt{2}\,f_0 - f_8}\,. \tag{4}$$

Thus, if m_π, m_K, and the decay-constant ratio F_K/F_π are known, then f_8/f_0, c_8/c_0, and m_κ^2 are uniquely predicted, and some typical results are shown in Table I.

Note that $c_8/c_0 \approx -\sqrt{2}$ as in the model of Gell-Mann, Oakes, and Renner[15] [the Lagrangian is approximately $SU(2)\otimes SU(2)$ invariant], while $|f_8/f_0|$ is relatively small [the vacuum is approximately SU(3) invariant].

Evidently $F_K/F_\pi \lesssim 1.30$ is required to have the κ mass above 1 GeV, which is reasonable both from the analysis of K-meson decays,[16] and the experimental data on K-π scattering[17] (although the question of the interpretation of tree-approximation masses for very broad resonances is far from settled).

From the general mass formulas

$$K - \pi = \tfrac{3}{2}\chi f_8 - 2\sqrt{2}\,\lambda f_8(f_0 - \sqrt{2}\,f_8)\,, \tag{5a}$$

$$\delta - \pi = \tfrac{4}{3}\lambda(\sqrt{2}\,f_0 + f_8)^2 - \sqrt{2}\,\chi(f_0 - \sqrt{2}\,f_8)\,, \tag{5b}$$

$$\begin{aligned} \eta + \eta' - 2\pi = {} & -2\lambda f_8(2\sqrt{2}\,f_0 - f_8) \\ & - \tfrac{3}{2}\sqrt{2}\,\chi(f_0 - \sqrt{2}\,f_8)\,, \end{aligned} \tag{5c}$$

and

$$\frac{(\eta-\pi)(\eta'-\pi)}{K-\pi} = -\chi(2\sqrt{2}\,f_0 - f_8)\,, \tag{6}$$

TABLE I. Ratio f_8/f_0 of vacuum expectation values, ratio c_8/c_0 of symmetry-breaking parameters, and κ-meson mass m_κ (in GeV) corresponding to various decay-constant ratios F_K/F_π. Input masses are the charge-weighted averages $m_\pi = 0.138\,05$ GeV, $m_K = 0.495\,71$ GeV.

F_K/F_π	f_8/f_0	c_8/c_0	m_κ (GeV)
1.24	−0.1950	−1.286	1.091
1.26	−0.2089	−1.288	1.057
1.28	−0.2225	−1.289	1.027
1.30	−0.2357	−1.291	1.001
1.32	−0.2486	−1.293	0.978

it is clear that in addition to F_K/F_π, one further parameter must be specified in order to determine m_δ, m_η, $m_{\eta'}$, and the pseudoscalar mixing angle.

In Table II are four sets of input parameters and corresponding calculated quantities. The following remarks are in order:

1. Insisting, as in solution C, that $m_\eta = 0.5488$ GeV leads to rather poor results for the calculated masses. On the other hand, allowing the η mass to vary by 1% leads to reasonable predictions for the remaining masses.

2. Solution B shows the effect of using π^0 and K^0 masses as input, rather than charge-weighted averages. The effect is to drive η and η' masses closer to their experimental values, but such a refinement is probably too subtle to include in a model in which electromagnetic interactions are ignored.

3. The pseudoscalar mixing angle is nearly zero, in agreement with analyses of radiative decay modes of pseudoscalar and vector mesons.[7,18] Note that the mass formulas include terms quadratic in the SU(3)-symmetry-breaking parameter f_8. While these terms are small, so is the pseudoscalar mixing of octet and singlet, and I thus prefer the smaller mixing angle deduced here to the value 10° inferred from lowest-order quadratic mass formulas.[17]

4. There is not much to choose between solutions A and D; solution A seems to be a slightly better over-all fit to the masses, and will be used in the subsequent discussion.

To compute the masses of the $I=0$, $Y=0$ scalar mesons σ and σ' requires additional input, since the parameters μ_0^2 and λ' of Lagrangian (1) appear only in the combination

$$\mu_1^2 \equiv \mu_0^2 + 4\lambda'(f_0^2 + f_8^2) \tag{7}$$

outside the σ-σ' mass matrix. Also, as remarked above, the interpretation of tree approximation masses for broad resonances such as σ and σ' is a theoretical question of some importance which has not been definitely answered. Nonetheless, I show in Table III the tree approximation masses for σ and σ', and the scalar mixing angle θ_S, for three values of the parameter μ_0^2, with the remaining parameters taken from solution A of Table II. The masses are not unreasonable.

TABLE II. Solutions to the mass relations for various input parameters (the input parameters are underlined). θ_P is the pseudoscalar mixing angle, λ and χ are coupling constants in the Lagrangian, and f_0 is the vacuum expectation value of the scalar field σ^0. Mass units are GeV.

Solution	A	B	C	D
m_π	$\underline{0.13805}$	$\underline{0.13500}$	$\underline{0.13805}$	$\underline{0.13805}$
m_K	$\underline{0.49571}$	$\underline{0.49770}$	$\underline{0.49571}$	$\underline{0.49571}$
F_K/F_π	$\underline{1.28}$	$\underline{1.28}$	$\underline{1.30}$	$\underline{1.30}$
m_δ	$\underline{0.969}$	$\underline{0.969}$	0.984	0.950
m_η	0.5435	0.5448	$\underline{0.5488}$	0.5438
$m_{\eta'}$	0.967	0.962	1.020	$\underline{0.958}$
θ_P	−2.08°	−2.50°	0.60°	−1.56°
λf_0^2	0.126	0.132	0.090	0.113
χf_0	−0.366	−0.361	−0.415	−0.358

III. SU(4)⊗SU(4) LAGRANGIAN MODEL

The extension of the phenomenological Lagrangian (1) to include a set of spin-zero fields which transform according to the representation $(4,4^*) \oplus (4^*,4)$ of chiral SU(4)⊗SU(4) is straightforward,[19,20] and if the Lagrangian is to be renormalizable and admit partial current conservation, it must have the structure

$$\begin{aligned} L = \tfrac{1}{4}\,\mathrm{Tr}[\partial_\mu M^\dagger \partial_\mu M - \mu_0^2 M^\dagger M - \lambda(M^\dagger M)^2] \\ - \tfrac{1}{4}\lambda'[\mathrm{Tr}(M^\dagger M)]^2 - \tfrac{1}{8}\sqrt{3}\,\chi(\det M + \det M^\dagger) \\ + d_0\sigma^0 + d_8\sigma^8 + d_{15}\sigma^{15}\,, \end{aligned} \tag{8}$$

where now

$$M = \sum_A (\sigma^A + i\pi^A)\lambda_A$$

is a 4×4 matrix, which contains the scalar fields σ^A and pseudoscalar fields π^A $(A=0,1,\ldots,15)$, and the λ_A are the standard set of Hermitian 4×4 matrices[1,19] proportional to the generators of SU(4).

Again the linear terms break the symmetry, and the scalar fields acquire vacuum expectation values

$$h_A \equiv \langle \mathrm{vac}|\sigma^A|\mathrm{vac}\rangle \quad (A=0,8,15)\,.$$

If the scalar fields

$$\sigma_s \equiv \tfrac{1}{2}(\sqrt{3}\,\sigma^0 + \sigma^{15})\,, \tag{9a}$$

$$\sigma_c \equiv \tfrac{1}{2}(\sigma^0 - \sqrt{3}\,\sigma^{15}) \tag{9b}$$

are introduced, then σ_s is equivalent to the scalar field σ^0 of the SU(3)×SU(3) Lagrangian, while σ_c

TABLE III. Tree-approximation masses (in GeV), and scalar mixing angle θ_S, for three values of the parameter μ_0^2 (in GeV^2) in the Lagrangian (1). Other parameters are taken from solution A of Table II.

μ_0^2	0.0	−0.1	−0.2
m_σ	0.685	0.751	0.799
$m_{\sigma'}$	1.175	1.219	1.270
θ_S	30.9°	37.1°	42.9°

corresponds to a pure c-$\bar{c}$ quark-model state. Let

$$h_s \equiv \langle \mathrm{vac} | \sigma_s | \mathrm{vac} \rangle = \tfrac{1}{2}(\sqrt{3}\, h_0 + h_{15}) \,, \tag{10a}$$

$$h_c \equiv \langle \mathrm{vac} | \sigma_c | \mathrm{vac} \rangle = \tfrac{1}{2}(h_0 - \sqrt{3}\, h_{15}) \,. \tag{10b}$$

Then $h_s(=f_0)$ and $h_8(=f_8)$, as well as the coupling constants in the invariant part of the Lagrangian, are determined from the SU(3)$\otimes$SU(3) analysis; the only new parameter introduced is the ratio h_c/h_s, or, equivalently, h_{15}/h_0.

The coefficients of the σ terms are expressed in terms of the h_A and the physical particle masses according to

$$\sqrt{3}\, d_0 + d_{15} + \sqrt{2}\, d_8 = (\sqrt{3}\, h_0 + h_{15} + \sqrt{2}\, h_8)\pi \,, \tag{11a}$$

$$\sqrt{6}\, d_0 + \sqrt{2}\, d_{15} - d_8 = (\sqrt{6}\, h_0 + \sqrt{2}\, h_{15} - h_8)K \,, \tag{11b}$$

$$d_8 = h_8 \kappa \tag{11c}$$

corresponding to (3a)–(3c), and

$$\sqrt{3}\, d_0 - d_{15} - \sqrt{2}\, d_8 = (\sqrt{3}\, h_0 - h_{15} - \sqrt{2}\, h_8)F_P \,, \tag{12a}$$

$$\sqrt{6}\, d_0 - \sqrt{2}\, d_{15} + d_8 = (\sqrt{6}\, h_0 - \sqrt{2}\, h_{15} + h_8)D_P \,, \tag{12b}$$

$$2\sqrt{2}\, d_{15} + d_8 = (2\sqrt{2}\, h_{15} + h_8)D_S \,, \tag{13a}$$

$$\sqrt{2}\, d_{15} - d_8 = (\sqrt{2}\, h_{15} - h_8)F_S \,, \tag{13b}$$

from which can be derived the general mass relations

$$\frac{K-\pi}{\kappa-\pi} = -\frac{3h_8}{2\sqrt{2}\, h_s - h_8} \,, \tag{14}$$

corresponding to (4), and

$$\frac{F_P - D_P}{F_P - \kappa} = \frac{3h_8}{\sqrt{6}\, h_0 - \sqrt{2}\, h_{15} + h_8} \,, \tag{15a}$$

$$\frac{D_P - D_S}{D_P - \pi} = \frac{2(\sqrt{2}\, h_s + h_8)}{2\sqrt{2}\, h_{15} + h_8} \,, \tag{15b}$$

$$\frac{F_S - D_S}{F_S - \kappa} = \frac{3h_8}{2\sqrt{2}\, h_{15} + h_8} \,. \tag{15c}$$

Since $h_8/h_s < 0$ from the SU(3) analysis, and the charmed-meson masses must lie above the κ mass, it is evident that $h_{15}/h_0 < 0$, and the charmed-meson masses are ordered according to

$$F_P < D_P < D_S < F_S$$

as noted in the Introduction. By contrast, the usual first-order mass formulas[1,3] give

$$F_P - D_P = F_S - D_S = K - \pi > 0 \,, \tag{16}$$

but here the terms which are quadratic in h_{15}/h_0 are certainly not negligible.

There is one general linear mass relation between the charmed-meson squared masses,

$$D_S + D_P - F_S - F_P = \pi + \delta - K - \kappa \,, \tag{17}$$

but the remaining masses must be determined from the explicit formulas

$$D_P - \pi = a(h_0 - \sqrt{3}\, h_{15}) + b(\sqrt{3}\, h_0 + h_{15} - 2\sqrt{2}\, h_8) \,, \tag{18a}$$

$$F_P - K = a(h_0 - \sqrt{3}\, h_{15} + \sqrt{6}\, h_8) + b(\sqrt{3}\, h_0 + h_{15} + \sqrt{2}\, h_8) \,, \tag{18b}$$

$$D_S - \delta = a(3\sqrt{3}\, h_0 - h_{15} + 2\sqrt{2}\, h_8)/\sqrt{3} - b(\sqrt{3}\, h_0 + h_{15} - 2\sqrt{2}\, h_8) \,, \tag{18c}$$

$$F_S - \kappa = a(3\sqrt{3}\, h_0 - h_{15} - \sqrt{2}\, h_8)/\sqrt{3} - b(\sqrt{3}\, h_0 + h_{15} + \sqrt{2}\, h_8) \,, \tag{18d}$$

where

$$a = -\sqrt{2}\,\lambda(2\sqrt{2}\, h_{15} + h_8)/\sqrt{3} \,,$$

$$b = \chi(2\sqrt{2}\, h_{15} + h_8)/2\sqrt{6} \,.$$

The charmed-meson decay constants are given in terms of the vacuum expectation values by

$$F(D_P) = (\sqrt{6}\, h_0 - \sqrt{2}\, h_{15} + h_8)/2\sqrt{3} \,, \tag{19a}$$

$$F(F_P) = (\sqrt{3}\, h_0 - h_{15} - \sqrt{2}\, h_8)/\sqrt{6} \,, \tag{19b}$$

$$F(D_S) = (2\sqrt{2}\, h_{15} + h_8)/2\sqrt{3} \,, \tag{19c}$$

$$F(F_S) = (\sqrt{2}\, h_{15} - h_8)/\sqrt{3} \,. \tag{19d}$$

In the absence of observed charmed mesons, the only input to determine the parameter h_{15}/h_0 is the requirement that the new $I=0$, $Y=0$, $C=0$ scalar (σ'') and pseudoscalar (η'') mesons contained in the 4×4 matrix M have mass large enough that $\psi(3.095)$ does not decay into $\eta'' + \gamma$ (or $\sigma'' + \gamma$) with a width greater than about 10–15% of the experimental hadronic decay width of 60 keV. It is also necessary that η'' and σ'' be relatively pure c-$\bar{c}$ states in order that the c-$\bar{c}$ content of η, η', σ, and σ' be small enough to suppress strong decays of $\psi(3.095)$ into these mesons and other hadrons or photons.

It turns out (see Sec. IV below) that these constraints are reasonably well satisfied if $m_{\eta''} \gtrsim 2.8$ GeV, corresponding to

$$h_{15}/h_0 \lesssim -1.0 \,.$$

The actual charmed-meson masses and weak decay constants predicted for some plausible values of the η'' mass are shown in Table IV, along with the amplitudes $\langle \eta_c | \eta \rangle$, $\langle \eta_c | \eta' \rangle$ for the η and η' to be in the c-$\bar{c}$ quark-model state

$$\eta_c \equiv \tfrac{1}{2}(\eta^0 - \sqrt{3}\, \eta^{15}) \,. \tag{20}$$

These amplitudes are small, comparable to the admixture of states other than c-$\bar{c}$ in $\psi(3.095)$, which is about 1% in amplitude. The effect of the mixing on the masses of η and η' is small (less

TABLE IV. Charmed-meson masses (in GeV) and decay-constant ratios predicted for several values of the SU(4)-symmetry-breaking parameter h_{15}/h_0 corresponding to the indicated values of the η'' mass. Also shown are amplitudes $\langle\eta_c|\eta\rangle$ and $\langle\eta_c|\eta'\rangle$ of the c-$\bar{c}$ quark-model state in $\eta(548)$ and $\eta'(958)$. SU(3) parameters are taken from solution A of Table I.

h_{15}/h_0	−1.019	−1.054	−1.062	−1.113
$m(\eta'')$	2.800	3.000	3.050	3.400
$m(F_P)$	2.572	2.769	2.818	3.164
$m(D_P)$	2.639	2.838	2.888	3.237
$m(D_S)$	2.992	3.191	3.241	3.589
$m(F_S)$	3.105	3.303	3.353	3.700
$F(F_P)/F_\pi$	4.765	5.059	5.132	5.644
$F(D_P)/F_\pi$	4.485	4.779	4.852	5.364
$F(D_S)/F_\pi$	3.485	3.779	3.852	4.364
$F(F_S)/F_\pi$	3.205	3.499	3.572	4.084
$\langle\eta_c\|\eta\rangle$	−0.0034	−0.0028	−0.0026	−0.0019
$\langle\eta_c\|\eta'\rangle$	−0.0162	−0.0129	−0.0122	−0.0086

than 1 MeV for η', less than 0.1 MeV for η).

It is pertinent to note that once the η'' mass is constrained, the predictions of the charmed-meson masses do not vary by more than 5–10 MeV as the input parameters to the SU(3) Lagrangian are varied over a reasonable range. Thus the uncertainty in the predicted masses from this source is less than neglected effects due to electromagnetism (or hadronic corrections to the tree approximation), which must already be ~1% in the SU(3) model.

The analysis of the 3×3 mass matrix for the $I=0$, $Y=0$, $C=0$ scalar mesons involves the additional parameter required in the SU(3) sector. For illustration, suppose the solution of Table III corresponding to the parameter μ_0^2 of the SU(3) Lagrangian equal to $-0.1\ \mathrm{GeV}^2$ is used. Then the masses of the scalar mesons and the mixing amplitudes $\langle\sigma_c|\sigma\rangle$ and $\langle\sigma_c|\sigma'\rangle$ of the c-$\bar{c}$ quark-model states in the σ and σ' are shown in Table V (for each of the possible η'' masses of Table IV).

The σ'' mass is quite large (it remains above 5 GeV for any reasonable choice of input), but the masses of σ and σ' are lowered by 40–50 MeV due to mixing with σ_c; the mixing amplitudes are not quite large enough to be excluded by present experimental data. A careful search for the radiative decay modes

$$\psi\to\pi^+\pi^-\gamma \text{ and } \psi\to K\bar{K}\gamma$$

would be revealing.

Note that the parameter μ_0^2 of the SU(4) Lagrangian is not the same as the parameter μ_0^2 of the SU(3) Lagrangian; the two are related by

$$\mu_0^2[\mathrm{SU}(4)]=\mu_0^2[\mathrm{SU}(3)]-4\lambda' h_c^2\,. \tag{21}$$

The sign of $\mu_0^2[\mathrm{SU}(3)]$ is not definitely determined, although it is probably negative; $\mu_0^2[\mathrm{SU}(4)]$ is certainly negative, so that the normal vacuum of the phenomenological SU(4) Lagrangian is unstable, and the $\mathrm{SU}(4)\times\mathrm{SU}(4)$ symmetry would be spontaneously broken even in the absence of the σ terms.

IV. QUARK MODEL AND RADIATIVE DECAY MODES

The fundamental theory from which the phenomenological Lagrangians discussed above may ultimately be derived is not yet known, but it is difficult to imagine such a theory without quarks[21] in one form or another. What is less clear is the role of the low-lying meson states—are they simple composite objects made up of quark and antiquark,[22] or are they fundamental fields[23] which themselves play an essential role in the unified gauge theory of elementary particle interactions which is now under construction?

I have no answer to this question—it may be unanswerable in principle[24]—but I find it reasonable to describe the interaction of the quarks with the

TABLE V. Masses of the $I=0$, $Y=0$, $C=0$ scalar mesons $(\sigma,\sigma',\sigma'')$ in GeV, corresponding to each of the solutions in Table IV, when the parameter μ_0^2 of the SU(3) Lagrangian is fixed at $-0.1\ \mathrm{GeV}^2$. Also shown are the amplitudes $\langle\sigma_c|\sigma\rangle$ and $\langle\sigma_c|\sigma'\rangle$ of the low-mass scalar mesons, and the parameter μ_0^2 (in GeV^2) of the SU(4) Lagrangian.

h_{15}/h_0	−1.019	−1.054	−1.062	−1.113
m_σ	0.692	0.691	0.691	0.689
$m_{\sigma'}$	1.175	1.175	1.175	1.175
$m_{\sigma''}$	5.74	6.16	6.26	6.99
$\langle\sigma_c\|\sigma\rangle$	−0.054	−0.051	−0.050	−0.045
$\langle\sigma_c\|\sigma'\rangle$	−0.055	−0.051	−0.050	−0.045
$\mu_0^2[\mathrm{SU}(4)]$	−4.97	−5.71	−5.91	−7.35

low-mass spin-zero fields by a Lagrangian

$$L_{\text{int}} = G \sum_A \bar{q}(\sigma^A + i\gamma_5 \pi^A)\lambda_A q , \tag{22}$$

which is invariant under the chiral group corresponding to the number of "flavors" of quark ("color" is not relevant to the present discussion).

If the chiral symmetry of the quark Lagrangian is broken only through the σ terms (which is implicit in the point of view of models such as those of Ref. 23), then the masses of the quarks are due entirely to the vacuum expectation values of the scalar fields in L_{int}. The ratio of strange-quark mass to nonstrange-quark mass is then given by

$$\frac{m_s}{m_u} = \frac{\sqrt{2} f_0 - 2f_8}{\sqrt{2} f_0 + f_8} \tag{23}$$

and of charmed-quark mass to nonstrange-quark mass by

$$\frac{m_c}{m_u} = \frac{\sqrt{3} h_0 - 3h_{15}}{\sqrt{3} h_0 + h_{15} + \sqrt{2} h_8} . \tag{24}$$

An alternative interpretation[15,20] is that the σ terms are themselves directly proportional to the quark-mass terms; the quark-mass ratios are then given by Eq. (23) with f_A replaced by c_A, and Eq. (24) with h_A replaced by d_A.

In the SU(3) model Eq. (23) gives

$$m_s/m_u = 1.56 \tag{25}$$

for $F_K/F_\pi = 1.28$, which is in good agreement with the analysis of De Rújula, Georgi, and Glashow,[22] and of Chase and the present author.[25] For the alternative interpretation, $F_K/F_\pi = 1.28$ corresponds to $m_s/m_u \approx 30$.

In the SU(4) model the charmed-quark mass depends on h_{15}/h_0; the mass ratios deduced from Eq. (24) for the solutions considered in Sec. VII are shown in Table VI, along with the predictions for the radiative decay rates discussed below. For alternative interpretation, m_c/m_u ranges from 1150 to 1800.

The decay rate for $\eta'' \to \gamma\gamma$ can be predicted using the quark-loop model of Ref. 7, which gives results equivalent to those of Adler[26] for the decay $\pi^0 \to \gamma\gamma$. In this model the ratio of decay amplitudes is given by

$$\frac{A(\eta'' \to \gamma\gamma)}{A(\pi^0 \to \gamma\gamma)} = \frac{4\sqrt{2}}{3} \frac{m_u}{m_c} \tag{26}$$

for a pure c-$\bar{c}$ η''. The corresponding decay rates for $\eta'' \to \gamma\gamma$, including the small corrections due to the admixture of states other than c-$\bar{c}$ in η'', are shown in Table VI.

If the vector mesons (ρ, ω, ϕ, ψ) interact with the quarks via an SU(4)-invariant minimal coupling, the model of Ref. 7 allows a prediction of the radiative decays of $\psi(3.095)$ into $\eta\gamma$ and $\eta'\gamma$, if it is assumed that ψ is a pure c-$\bar{c}$ state. The amplitude for $\psi_c \to \eta_c\gamma$ is given by

$$\frac{A(\psi_c \to \eta_c\gamma)}{A(\omega \to \pi^0\gamma)} = \frac{4}{3} \frac{m_u}{m_c} \left(\frac{Z_\psi}{Z_\omega}\right)^{1/2} , \tag{27}$$

where Z_ω, Z_ψ are renormalization factors for the coupling of the vector mesons to the appropriate quarks.

There are two reasonable conjectures for the ratio of the renormalization constants at this level of approximation,

$$Z_\psi/Z_\omega = 1 \text{ or } Z_\psi/Z_\omega = m_\psi^2/m_\omega^2 ,$$

corresponding roughly to "mass-mixing" and "current-mixing" models for the vector-meson mixing.[27] The question is somewhat more subtle than implied by this remark, but a complete discussion is beyond the scope of this work.

If the renormalization factor $(Z_\psi/Z_\omega)^{1/2}$ is set equal to unity, then the mass of the η'' is constrained to be $\gtrsim 2.8$ GeV in order that the predicted decay rate $\Gamma(\psi \to \eta''\gamma)$ not be greater than a few percent of the total ψ decay width[28]; for the alternative value m_ψ/m_ω of this factor, the corresponding lower limit on $m_{\eta''}$ is ~3.0 GeV.

The radiative decay rates of ψ into $\eta\gamma$ and $\eta'\gamma$ due to the c-$\bar{c}$ mixing in η and η' are also shown in Table VI (for renormalization factor equal to unity). These rates are sufficiently small that admixtures in the ψ of states other than c-$\bar{c}$ are likely to give significant (perhaps even the dominant) contributions to these decay rates.

V. CONCLUDING REMARKS

If the spectrum of the charmed spin-zero mesons described here is qualitatively correct, it remains

TABLE VI. Quark-mass ratios and radiative decay rates predicted by the analysis of Sec. IV, for the solutions considered in Table IV. The absolute rate for $\eta'' \to \gamma\gamma$ is based on a $\pi^0 \to \gamma\gamma$ rate of 7.40 eV predicted in Ref. 7. The radiative decay rates of ψ(3.095) are based on the assumption that ψ is a pure c-$\bar{c}$ state, and that the renormalization factor discussed in the text is equal to unity.

h_{15}/h_0	−1.019	−1.054	−1.062	−1.113
m_c/m_u	7.97	8.56	8.70	9.73
d_{15}/d_0	−1.708	−1.713	−1.714	−1.719
$\Gamma(\eta'' \to \gamma\gamma)$ (keV)	4.59	4.76	4.80	5.13
$10^5 \frac{\Gamma(\psi \to \eta\gamma)}{\Gamma(\psi \to \pi\gamma)}$	2.02	1.14	1.00	0.41
$10^4 \frac{\Gamma(\psi \to \eta'\gamma)}{\Gamma(\omega \to \pi\gamma)}$	3.68	2.03	1.76	0.70

to explain the broad enhancement[29] in the e^+-e^- cross section between 4.0 and 4.2 GeV, whose width, compared to that of $\psi(3.1)$ and $\psi(3.7)$, strongly suggests a new threshold between 3.7 and 4.0 GeV. Two possible explanations are the following:

1. There is a threshold for the production of charmed particles, which are not spin-zero mesons, but vector mesons (or perhaps even baryons). Note that the mechanism responsible for the enhancement of the nonleptonic decay modes of the K meson need not be operative for vector mesons.[30]

2. The enhancement is related to the μ-e anomaly.[31]

A further test of the model, apart from the obvious particle searches, is the measurement of the branching ratio for the decay modes

$$\psi \to \sigma(\sigma') + \gamma$$

as mentioned above. If the intrinsic matrix element for this decay mode is comparable to that for $\psi \to \eta(\eta') + \gamma$, then the larger c-$\bar{c}$ admixture in $\sigma(\sigma')$ given in Table V leads to an estimated partial width of a few keV for these decay modes.[32]

An outstanding theoretical problem is the derivation of the phenomenological Lagrangians from a fundamental Lagrangian containing elementary constituents such as quarks. I have in this work assumed that the phenomenological Lagrangian is renormalizable and group-invariant apart from the σ terms. While these are plausible assumptions, it would be reassuring to see them derived in some model. It is also relevant to note that the assumptions could not be retained (and be consistent with the experimental mass spectrum) if the chiral group were larger than $SU(4) \otimes SU(4)$, since the term proportional to $\det M + \det M^+$ is not consistent with renormalizability if the meson matrix M is larger than 4×4, but the term is necessary to give the correct η-η' mass matrix [see Eq. (6)].

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*Permanent address.

[1]For a review, see M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. 47, 277 (1975).

[2]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974).

[3]S. Borchardt, V. Mathur, and S. Okubo, Phys. Rev. Lett. 34, 38 (1975); S. Okubo, V. S. Mathur, and S. Borchardt, *ibid.* 34, 236 (1975).

[4]Noteworthy is the "charmonium" model: See T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); A. de Rújula and S. L. Glashow, *ibid.* 34, 46 (1975); T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, *ibid.* 34, 365 (1975); E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, *ibid.* 34, 369 (1975).

[5]S. Gasiorowicz and D. Geffen, Rev. Mod. Phys. 41, 531 (1969), give a review of early phenomenological Lagrangian models.

[6]That the Lagrangian methods described here are so accurate was first demonstrated by R. Olshansky, Phys. Rev. D 4, 2440 (1971); see also J. J. Brehm, *ibid.* 9, 1818 (1974).

[7]N. Chase and M. T. Vaughn, Phys. Lett. (to be published).

[8]For example, the search of A. M. Boyarski *et al.*, Phys. Rev. Lett. 35, 195 (1975), would not have detected charmed mesons of these masses.

[9]S. L. Glashow and S. Weinberg, Phys. Rev. Lett. 20, 224 (1968); W. A. Bardeen and B. W. Lee, Phys. Rev. 177, 2389 (1969).

[10]A. K. Bhargava and T. Dass, Phys. Rev. D 1, 649 (1970); A. K. Bhargava, *ibid.* 6, 1153 (1972); L. M. Brown and H. Munczek, *ibid.* 1, 2595 (1970); C. Cicogna, *ibid.* 1, 1786 (1970); D. W. McKay, W. F. Palmer, and R. F. Sarraya, *ibid.* 8, 2532 (1973).

[11]J. Schecter and Y. Ueda, Phys. Rev. D 3, 168 (1971); 3, 2387 (1971).

[12]P. Carruthers, Phys. Rev. D 2, 2265 (1970); 3, 959 (1971).

[13]P. Carruthers and R. W. Haymaker, Phys. Rev. Lett. 27, 455 (1971); Phys. Rev. D 4, 406 (1971); 4, 1808 (1971); 4, 1815 (1971).

[14]L. H. Chan and R. W. Haymaker, Phys. Rev. D 7, 402 (1973); 7, 415 (1973); 10, 4143 (1974).

[15]M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968).

[16]L.-M. Chounet, J.-M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C, 199 (1972).

[17]Particle Data Group, Phys. Lett. 50B, 1 (1974).

[18]A. Bramòn and M. Greco, Phys. Lett. 48B, 137 (1974).

[19]B. Hu, Phys. Rev. D 9, 1825 (1974).

[20]J. Schechter and M. Singer [Phys. Rev. D 12, 2781 (1975)] have considered a more general Lagrangian which is not renormalizable.

[21]M. Gell-Mann, Phys. Lett. 8, 274 (1964); G. Zweig, in *Symmetries in Elementary Particle Physics*, (Academic, New York, 1965); M. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965).

[22]A recent discussion of this point of view is given by A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975); see also Ref. 4.

[23]There may yet be a role for fundamental spin-zero fields similar to that suggested by I. Bars, M. B. Halpern, and M. Yoshimura, Phys. Rev. Lett. 29, 969 (1972); Phys. Rev. D 7, 1233 (1973); B. de Wit, Nucl. Phys. B51, 237 (1973); B. de Wit, S.-Y. Pi, and J. Smith, Phys. Rev. D 10, 4278 (1974).

[24]Even if quarks are more fundamental than the spin-zero mesons, the phenomenological Lagrangians used here may provide an equivalent description of the observed mesons.

[25]See Ref. 7. The charmed-quark mass inferred in that work from the mass ratio m_ψ / m_ω is smaller than that

inferred here, but that is a reflection of the inadequacy of the one-loop approximation in the presence of large mass ratios.

[26]S. L. Adler, Phys. Rev. 177, 2426 (1969).

[27]The distinction can be traced to the work of S. Coleman and H. J. Schnitzer, Phys. Rev. 134, B863 (1964).

[28]The best experimental limit known to the author at present is due to B. L. Beron *et al.*, SLAC Report No. HEPL-766, 1975 (unpublished).

[29]J.-E. Augustin *et al.*, Phys. Rev. Lett. 34, 764 (1975).

[30]G. Altarelli, N. Cabibbo, and L. Maiani, Phys. Rev. Lett. 35, 635 (1975).

[31]M. Perl, unpublished lectures.

[32]I do not give a more precise estimate because of its dependence on the detailed model of the effective ψ-σ-γ coupling, which is not so unambiguous as the model of the ψ-η-γ coupling.