PHYSICAL REVIEW D VOLUME 13, NUMBER 9 1 MAY 1976

Classification and the leptonic decay widths of new particles in the charm-color scheme

Suk Koo Yun*
Lyman Laboratory, Physics Department, Harvard University, Cambridge, Massachusetts 02138
(Received 9 July 1975; revised manuscript received 20 November 1975)

A classification of new resonances $\psi(3.095)$, $\psi'(3.684)$ and a rather broad $\psi''(4.150)$ is presented under the charm-color $SU(4)\times SU(3)'$ scheme. The masses of the color-excited states are estimated, and the predicted leptonic decay widths and the sum rules are in reasonable agreement with experiment.

I. INTRODUCTION

The discovery of the new narrow resonances $\psi(3.095)$(Ref. 1) and $\psi'(3.684)$(Ref. 2) and a rather broad resonance $\psi''(4.150)$(Ref. 3) prompted many attempts to interpret these new particles. These attempts may be divided into two approaches, namely charm schemes and color schemes. Among the charm schemes, the most interesting is that based on charmonium,[4] the bound states of the charmed quark and the anticharmed quark. In this model, ψ' and ψ'' are regarded as the radial excitations[5] of orthocharmonium ψ.

The attractive feature of the charmed quark in SU(4) symmetry is that the unwanted strangeness-changing neutral hadronic weak current is naturally absent, while the strangeness-conserving neutral current in the gauge theories of weak and electromagnetic interactions is allowed. The colored models are known to predict the correct π^0 lifetime as well as to provide the means for having totally antisymmetric wave functions for fermions. If we accept the fact that both the charm scheme (with charmonium excited states) and the color scheme (with colored excited states) have desirable properties, it is tempting to pursue the combination of the two schemes. There are various color models, such as the Gellmann-Zweig color model (which has only a color singlet) with fractionally charged quarks and the Han-Nambu color model with integrally charged quarks. In this paper we would like to investigate the charm-color scheme (with both charm- and color-excited states) under $SU(4)\times SU(3)'$ symmetry with integrally charged quarks).

This charm-color scheme of $SU(4)\times SU(3)'$ is also useful in incorporating[6] CP violation with the unified renormalizable gauge theory of weak and electromagnetic interactions. There has been some work[7,8] to classify the new particles in the charm and color schemes. We present here a classification based on the charm-color $SU(4)\times SU(3)'$ scheme and its predictions of leptonic decay widths of various vector mesons and their sum rules together with the estimated masses of color excited states.

The theoretical expectation of the asymptotic ratio $R=\sigma(e^+e^-\to \text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)$ is the sum of squares of the charges that participate in the processes. Thus the ratio

$$R=\begin{cases} 2 \text{ for the fractionally charged three-triplet model (Gell-Mann–Zweig color model),}\\ \frac{10}{3} \text{ for the fractionally charged three-quartet model with charmed quarks,}\\ 4 \text{ for the integrally charged three-triplet model (Han-Nambu color model),}\\ 6 \text{ for the integrally charged three-quartet model with charmed quarks [integrally charged}\\ \quad \text{three-quartet } SU(4)\times SU(3)' \text{ model].}\end{cases}$$

The experimental situation[9] of R after subtracting the contributions by resonances is that $R\geq 2$ for the energies above 2.3 GeV. Between 2.3 GeV and 3.8 GeV, R is greater than 2 and less than $\frac{10}{3}$, and it reaches to 4 near 3.9 GeV. Above 3.9 GeV, it surpasses 4 and approaches 6 as $\sqrt{s}$ approaches 8 GeV.

In our scheme, this behavior of R can be explained by the participations of charmed quarks as well as color indices of the integrally charged three quartets of quarks starting around 2.3 GeV. In Sec. II various vector-meson states relevant to the e^+e^- annihilations are given. The leptonic decay widths and the sum rules of these particles are shown in Sec. III. The classification of new particles is carried out in Sec. IV, and discussions are given.

II. THE RELEVANT PARTICLE STATES

We denote the vector-meson particle states in the $SU(4)\times SU(3)'$ group relevant to the e^+e^- annihilation by particle symbols in SU(4) and color SU(3)′ spaces. For example, the particle (ϕ, ω') behaves like ϕ in SU(4) space and also behaves like ω in terms of the color indices ($\iota = 1, 2, 3$) in SU(3)′ space. The singlet and octet states in the SU(3) subspace and the 15-plet state in SU(4) space are assumed to be mixed with the mixing angles θ and ϕ as follows:

$$\begin{aligned} \omega &= -\sin\theta\,\omega_8 + \cos\theta(-\sin\phi\,\omega_{15} + \cos\phi\,\omega_1)\,, \\ \phi &= \cos\theta\,\omega_8 + \sin\theta(-\sin\phi\,\omega_{15} + \cos\phi\,\omega_1)\,, \\ \phi_c &= (\sin\phi\,\omega_1 + \cos\phi\,\omega_{15})\,, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \omega_1 &= \tfrac{1}{2}(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c})\,, \\ \omega_{15} &= \frac{1}{\sqrt{12}}(u\bar{u} + d\bar{d} + s\bar{s} - 3c\bar{c})\,, \\ \omega_8 &= \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - s\bar{s}). \end{aligned} \tag{2}$$

Equation (1) can be expressed in view of Eq. (2) as

$$\begin{aligned} \omega &= a_\omega(u\bar{u} + d\bar{d}) + b_\omega(s\bar{s}) + c_\omega(c\bar{c})\,, \\ \phi &= a_\phi(u\bar{u} + d\bar{d}) + b_\phi(s\bar{s}) + c_\phi(c\bar{c})\,, \\ \phi_c &= a_{\phi_c}(u\bar{u} + d\bar{d}) + b_{\phi_c}(s\bar{s}) + c_{\phi_c}(c\bar{c})\,, \\ \rho &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})\,, \end{aligned} \tag{3}$$

where the coefficients are defined as

$$\begin{aligned} a_\omega &= \frac{1}{\sqrt{6}}(-\sin\theta + \beta\cos\theta)\,, \\ b_\omega &= \frac{1}{\sqrt{6}}(2\sin\theta + \beta\cos\theta)\,, \\ c_\omega &= \frac{3}{\sqrt{6}}\,\alpha\cos\theta\,, \\ a_\phi &= \frac{1}{\sqrt{6}}(\beta\sin\theta + \cos\theta)\,, \\ b_\phi &= \frac{1}{\sqrt{6}}(\beta\sin\theta - 2\cos\theta)\,, \\ c_\phi &= \frac{3}{\sqrt{6}}\,\alpha\sin\theta\,, \\ a_{\phi_c} &= \frac{\alpha}{\sqrt{2}} = b_{\phi_c} \\ c_{\phi_c} &= -\frac{\beta}{\sqrt{2}}\,, \end{aligned} \tag{4}$$

and

$$\begin{aligned} \alpha &\equiv \frac{1}{\sqrt{2}}\left(\sin\phi + \frac{1}{\sqrt{3}}\cos\phi\right), \\ \beta &\equiv \frac{1}{\sqrt{2}}(-\sin\phi + \sqrt{3}\cos\phi). \end{aligned} \tag{5}$$

Similarly, we also assume the mixing of the color singlet and the color octet with a mixing angle θ'.

The relevant vector-meson particle states in our SU(4)×SU(3)′ scheme are for arbitrary θ, ϕ, and θ'

$$\begin{aligned} (\rho, \omega') &= \frac{1}{\sqrt{2}}\left[a_{\omega'}(u_1\bar{u}^1 + u_2\bar{u}^2 - d_1\bar{d}^1 - d_2\bar{d}^2) + b_{\omega'}(u_3\bar{u}^3 - d_3\bar{d}^3)\right], \\ (\rho, \rho') &= \tfrac{1}{2}(u_1\bar{u}^1 - u_2\bar{u}^2 - d_1\bar{d}^1 + d_2\bar{d}^2)\,, \\ (\omega, \omega') &= a_{\omega'}\left[a_\omega(u_1\bar{u}^1 + u_2\bar{u}^2 + d_1\bar{d}^1 + d_2\bar{d}^2) + b_\omega(s_1\bar{s}^1 + s_2\bar{s}^2) + c_\omega(c_1\bar{c}^1 + c_2\bar{c}^2)\right] + b_{\omega'}\left[a_\omega(u_3\bar{u}^3 + d_3\bar{d}^3) + b_\omega(s_3\bar{s}^3) + c_\omega(c_3\bar{c}^3)\right], \\ (\omega, \rho') &= \frac{1}{\sqrt{2}}\left[a_\omega(u_1\bar{u}^1 - u_2\bar{u}^2 + d_1\bar{d}^1 - d_2\bar{d}^2) + b_\omega(s_1\bar{s}^1 - s_2\bar{s}^2) + c_\omega(c_1\bar{c}^1 - c_2\bar{c}^2)\right], \\ (\phi, \omega') &= a_{\omega'}\left[a_\phi(u_1\bar{u}^1 + u_2\bar{u}^2 + d_1\bar{d}^1 + d_2\bar{d}^2) + b_\phi(s_1\bar{s}^1 + s_2\bar{s}^2) + c_\phi(c_1\bar{c}^1 + c_2\bar{c}^2)\right] + b_{\omega'}\left[a_\phi(u_3\bar{u}^3 + d_3\bar{d}^3) + b_\phi(s_3\bar{s}^3) + c_\phi(c_3\bar{c}^3)\right], \\ (\phi, \rho') &= \frac{1}{\sqrt{2}}\left[a_\phi(u_1\bar{u}^1 - u_2\bar{u}^2 + d_1\bar{d}^1 - d_2\bar{d}^2) + b_\phi(s_1\bar{s}^1 - s_2\bar{s}^2) + c_\phi(c_1\bar{c}^1 - c_2\bar{c}^2)\right], \\ (\phi_c, \omega') &= a_{\omega'}\left[a_{\phi_c}(u_1\bar{u}^1 + u_2\bar{u}^2 + d_1\bar{d}^1 + d_2\bar{d}^2) + b_{\phi_c}(s_1\bar{s}^1 + s_2\bar{s}^2) + c_{\phi_c}(c_1\bar{c}^1 + c_2\bar{c}^2)\right] + b_{\omega'}\left[a_{\phi_c}(u_3\bar{u}^3 + d_3\bar{d}^3) + b_{\phi_c}(s_3\bar{s}^3) + c_{\phi_c}(c_3\bar{c}^3)\right], \\ (\phi_c, \rho') &= \frac{1}{\sqrt{2}}\left[a_{\phi_c}(u_1\bar{u}^1 - u_2\bar{u}^2 + d_1\bar{d}^1 - d_2\bar{d}^2) + b_{\phi_c}(s_1\bar{s}^1 - s_2\bar{s}^2) + c_{\phi_c}(c_1\bar{c}^1 - c_2\bar{c}^2)\right], \end{aligned} \tag{6}$$

and similar ones for (ρ, ω, ϕ, or ϕ_c, ϕ') by the replacement $a_{\omega'}$ (or $b_{\omega'}$) $\leftrightarrow a_{\phi'}$ (or $b_{\phi'}$). The primed coefficients are

$$a_{\omega'} = \frac{1}{\sqrt{6}}(-\sin\theta' + \sqrt{2}\cos\theta'),$$
$$b_{\omega'} = \frac{1}{\sqrt{3}}(\sqrt{2}\sin\theta' + \cos\theta'),$$
$$a_{\phi'} = \frac{1}{\sqrt{6}}(\sqrt{2}\sin\theta' + \cos\theta'), \qquad (7)$$
$$b_{\phi'} = \frac{1}{\sqrt{3}}(\sin\theta' - \sqrt{2}\cos\theta').$$

The electric charges of u_ι, d_ι, s_ι, and c_ι quarks are

$$Q = \begin{cases} \begin{array}{cccc} u_\iota & d_\iota & s_\iota & c_\iota \\ 1 & 0 & 0 & 1 \end{array} & \text{for } \iota = 1 \text{ and } 3 \\ \begin{array}{cccc} 0 & -1 & -1 & 0 \end{array} & \text{for } \iota = 2 \end{cases} \qquad (8)$$

III. THE LEPTONIC DECAY WIDTHS

Apart from the over-all constant and neglecting mass differences, the leptonic widths of the vector mesons are for arbitrary θ, ϕ, and θ'

$$\Gamma((\rho, \omega') \to e\bar{e}) \propto \tfrac{1}{2}(2a_{\omega'} + b_{\omega'})^2,$$
$$\Gamma((\rho, \rho') \to e\bar{e}) \propto 0,$$
$$\Gamma((\omega, \omega') \to e\bar{e}) \propto [a_{\omega'}(-b_\omega + c_\omega) + b_{\omega'}(a_\omega + c_\omega)]^2,$$
$$\Gamma((\omega, \rho') \to e\bar{e}) \propto \tfrac{1}{2}(2a_\omega + b_\omega + c_\omega)^2$$
$$\Gamma((\phi, \omega') \to e\bar{e}) \propto [a_{\omega'}(-b_\phi + c_\phi) + b_{\omega'}(a_\phi + c_\phi)]^2, \qquad (9)$$
$$\Gamma((\phi, \rho') \to e\bar{e}) \propto \tfrac{1}{2}(2a_\phi + b_\phi + c_\phi)^2,$$
$$\Gamma((\phi_c, \omega') \to e\bar{e}) \propto [a_{\omega'}(-b_{\phi_c} + c_{\phi_c}) + b_{\omega'}(a_{\phi_c} + b_{\phi_c})]^2,$$
$$\Gamma((\phi_c, \rho') \to e\bar{e}) \propto \tfrac{1}{2}(2a_{\phi_c}, + b_{\phi_c} + c_{\phi_c})^2,$$

and similar ones for (anything, ϕ') states. If we adopt the "ideal" mixing in SU(4) space where ω has no contamination of strange quarks, and ϕ and ϕ_c consist of pure strange quarks and pure charmed quarks respectively, then the mixing angles θ and ϕ are

$$\sin\theta = -\frac{1}{\sqrt{3}}, \quad \cos\theta = (\tfrac{2}{3})^{1/2},$$
$$\sin\phi = -\tfrac{1}{2}, \quad \cos\theta = \frac{\sqrt{3}}{2}. \qquad (10)$$

For this "ideal" mixing in SU(4) space, and to maintain the color mixing angle θ' as a parameter, Eqs. (9) become

$$\Gamma((\rho, \omega') \to e\bar{e}) \propto \tfrac{1}{2}(2a_{\omega'} + b_{\omega'})^2 = \tfrac{3}{2}\cos^2\theta',$$
$$\Gamma((\rho, \rho') \to e\bar{e}) \propto 0,$$
$$\Gamma((\omega, \omega') \to e\bar{e}) \propto \tfrac{1}{2}b_{\omega'}^2,$$
$$\Gamma((\omega, \rho') \to e\bar{e}) \propto 1 \qquad (11)$$
$$\Gamma((\phi, \omega') \to e\bar{e}) \propto a_{\omega'}^2,$$
$$\Gamma((\phi, \rho') \to e\bar{e}) \propto \tfrac{1}{2},$$
$$\Gamma((\phi_c, \omega') \to e\bar{e}) \propto (a_{\omega'} + b_{\omega'})^2,$$
$$\Gamma((\phi_c, \rho') \to e\bar{e}) \propto \tfrac{1}{2},$$

and similar expressions for other states.

The following sum rules of leptonic decay widths are obtained from Eq. (11) for any arbitrary θ':

$$\Gamma((\rho, \omega') \to e\bar{e}) = 2([\Gamma((\phi, \omega') \to e\bar{e})]^{1/2} + \frac{1}{\sqrt{2}}[\Gamma((\omega, \omega') \to e\bar{e})]^{1/2})^2,$$
$$\Gamma((\rho, \omega') \to e\bar{e}) + \Gamma((\omega, \omega') \to e\bar{e}) = \Gamma((\phi, \omega') \to e\bar{e}) + \Gamma((\phi_c, \omega') \to e\bar{e}),$$
$$\Gamma((\rho, \omega') \to e\bar{e}) + \Gamma((\phi_c, \phi') \to e\bar{e}) = 2[2\Gamma((\phi, \omega') \to e\bar{e}) + \Gamma((\omega, \omega') \to e\bar{e})],$$
$$\Gamma((\omega, \omega') \to e\bar{e}) + \Gamma((\phi, \omega') \to e\bar{e}) = \Gamma((\phi_c, \rho') \to e\bar{e}),$$
$$\Gamma((\omega, \rho') \to e\bar{e}) = \Gamma((\phi, \rho') \to e\bar{e}) + \Gamma((\phi_c, \rho') \to e\bar{e}),$$
$$\begin{aligned} \tfrac{1}{3}[\Gamma((\rho, \omega') \to e\bar{e}) + \Gamma((\rho, \phi') \to e\bar{e})] &= \Gamma((\phi, \omega') \to e\bar{e}) + \Gamma((\omega, \omega') \to e\bar{e}) \\ &= \Gamma((\omega, \omega') \to e\bar{e}) + \Gamma((\omega, \phi') \to e\bar{e}) \\ &= \Gamma((\phi, \omega') \to e\bar{e}) + \Gamma((\phi, \phi') \to e\bar{e}) \\ &= \tfrac{1}{3}[\Gamma((\phi_c, \omega') \to e\bar{e})] + \Gamma((\phi_c, \phi') \to e\bar{e}), \end{aligned} \qquad (12)$$
$$\Gamma((\rho, \rho') \to e\bar{e}) = 0,$$

etc.

If we assign the ordinary vector mesons ρ^0 and ω as

$$\begin{aligned}\rho^0 &\equiv(\rho^0, \omega') ,\\ \omega &\equiv(\omega, \omega') ,\end{aligned}\tag{13}$$

and take the leptonic decay widths of ρ and ω as inputs, then the color mixing angle is $\theta'=1°12'$, indicating that there is very little color mixing, if any. The predicted leptonic decay widths of all other particle states are shown in the second column of Table I. The fourth column in Table I represents the ratios of the leptonic decay widths for the case of no color mixing ($\theta'=0$).

IV. CLASSIFICATION OF NEW PARTICLES

If we assume that ψ, ψ', and ψ'' are the hidden charm states in SU(4) space, then the best assignment[10] is

$$\begin{aligned}\psi(3.095) &\equiv(\phi_c, \omega') ,\\ \psi'(3.684) &\equiv(\phi_c, \rho') ,\\ \psi''(4.150) &\equiv(\phi_c, \phi') ,\end{aligned}\tag{14}$$

and

$$\phi(1.019) \equiv(\phi, \omega') ,$$

as shown in the first column of Table I. In this assignment, the sum rules given in Eqs. (12) and the leptonic decay widths are in good agreement with the experiment with the exception of ψ''. The predicted width of ψ'' does not agree with the experimental value. A possible explanation is that the contributions of the radially excited charmonium states are causing the rather higher value of the experimental decay width of ψ''. It is interesting to note that the width ratios are

$$\rho;\omega;\phi;\psi;\psi';\psi''=\begin{cases}9;1;2;8;3;1 \text{ for } \theta'=0\\ 6;0;3;3;3;6 \text{ for } \sin\theta'=-\dfrac{1}{\sqrt{3}}\end{cases}\tag{15}$$

as shown in the last two columns of Table I. Thus the sensitivity of the widths due to the color mixing angle cannot be ignored.

In order to investigate further consequences of our particle assignments, the mass term before the nonet symmetry is broken will be assumed as

$$H=M_0^2(1,1')+\sqrt{3}A(\lambda_8,1')+\sqrt{3}B(1,\lambda_8')+\sqrt{6}C(\lambda_{15},1')+3D(\lambda_8,\lambda_8')+3\sqrt{2}E(\lambda_{15},\lambda_8') ,\tag{16}$$

TABLE I. The leptonic decay widths of various vector-meson states in the SU(4) $\times$SU(3)$'$ charm-color scheme.

Vector-meson particle state	Theoretical width (keV)	Experimental width (keV)	Width ratios for $\theta'=0$	Width ratios for $\sin\theta'=-1/\sqrt{3}$
$\rho^0\equiv(\rho^0,\omega')$	6.45 (input)	6.45 ± 1.18 [a]	9	6
(ρ,ϕ')	0.003	...	0	3
(ρ,ρ')	0	...	0	0
$\omega\equiv(\omega,\omega')$	0.76 (input)	0.76 ± 0.20 [a]	1	0
(ω,ϕ')	1.39	...	2	3
(ω,ρ')	4.30	...	6	6
$\varphi\equiv(\phi,\omega')$	1.39	1.34 ± 0.15 [a]	2	3
(ϕ,ϕ')	0.76	...	1	0
(ϕ,ρ')	2.15	...	3	3
$\psi\equiv(\phi_c,\omega')$	5.82	4.8 ± 0.6 [b]	8	3
$\psi''\equiv(\phi_c,\phi')$	0.63	4.0 ± 1.2 [c]	1	6
$\psi'\equiv(\phi_c,\rho')$	2.15	2.2 ± 0.3 [d]	3	3

[a] Particle Data Group, Phys. Lett. 50B, 1 (1974).
[b] A. Boyarski *et al.*, Phys. Rev. Lett. 34, 1357 (1975).
[c] O. W. Greenberg, invited talk given at Second Orbis Scientiae at the University of Miami, Coral Gables, Florida, 1975 (unpublished).
[d] A. Boyarski *et al.*, paper submitted to the Palermo Conference by SLAC-LBL magnetic detector collaboration, 1975 (unpublished).

which contains six parameters associated with the direct products of matrices, 1, λ_8, and λ_{15} for SU(4), and 1′ and λ_8' for SU(3)′. The expressions of the squared mass for various particle states are given in the second column of Table II for the case of the nonet symmetry. With the further assignment of the ordinary vector meson $K^*(0.892) \equiv (K^*, \omega')$, the masses of all the relevant particles are shown in the last column of Table II. The nonet mass formulas and the SU(4) mass formulas are applied whenever possible by using the mean values of squared masses of (ρ, Y) and (ω, Y) or (z, ρ') and (z, ω'), where Y and Z are anything. Since the knowledge of the specific mass term which further breaks the nonet symmetry is lacking, we assumed the nonet

TABLE II. Classification and masses of various vector mesons in the SU(4) ×SU(3)′ scheme.

Vector-meson particle state	(Mass)^2 for nonet symmetry	Mass (GeV)
$\psi'' \equiv (\phi_c, \phi')$	$M_0^2 - 2B - 3C + 6E$	4.150 [a]
(F^*, ϕ')	$M_0^2 - A - 2B - C + 2D + 2E$	3.599
(D^*, ϕ')	$M_0^2 + \frac{1}{2}A - 2B - C - D + 2E$	3.569
(ϕ, ϕ')	$M_0^2 - 2A - 2B + C + 4D - 2E$	2.947 [b]
(K^*, ϕ')	$M_0^2 - \frac{1}{2}A - 2B + C + D - 2E$	2.909
(ω, ϕ')	$M_0^2 + A - 2B + C - 2D - 2E$	2.873 [b]
(ρ, ϕ')		2.870 [b]
$(\phi_c, K_*^\prime)$	$M_0^2 - \frac{1}{2}B - 3C + \frac{3}{2}E$	3.795
$(F^*, K_*^\prime)$	$M_0^2 - A - \frac{1}{2}B - C + \frac{1}{2}D + \frac{1}{2}E$	3.183
$(D^*, K_*^\prime)$	$M_0^2 + \frac{1}{2}A - \frac{1}{2}B - C - \frac{1}{4}D + \frac{1}{2}E$	3.148
$(\phi, K_*^\prime)$	$M_0^2 - 2A - \frac{1}{2}B + C + D - \frac{1}{2}E$	2.420
$(K^*, K_*^\prime)$	$M_0^2 - \frac{1}{2}A - \frac{1}{2}B + C + \frac{1}{4}D - \frac{1}{2}E$	2.375
$(\omega, K_*^\prime)$	$M_0^2 + A - \frac{1}{2}B + C - \frac{1}{2}D - \frac{1}{2}E$	2.331
$(\rho, K_*^\prime)$		2.327
$\psi' \equiv (\phi_c, \rho')$	$M_0^2 + B - 3C - 3E$	3.684 [a]
$\psi \equiv (\phi_c, \omega')$		3.095 [a]
(F^*, ρ')	$M_0^2 - A + B - C - D - E$	3.050
(F^*, ω')		2.304
(D^*, ρ')	$M_0^2 + \frac{1}{2}A + B - C + \frac{1}{2}D - E$	3.014
(D^*, ω')		2.256
(ϕ, ρ')	$M_0^2 - 2A + B + C - 2D + E$	2.243 [c]
$\phi \equiv (\phi, \omega')$		1.019 [a]
(K^*, ρ')	$M_0^2 - \frac{1}{2}A + B + C - \frac{1}{2}D + E$	2.194
$K^* \equiv (K^*, \omega')$		0.906
(ω, ρ')	$M_0^2 + A + B + C + D + E$	2.146 [c]
$\omega \equiv (\omega, \omega')$		0.783 [a]
(ρ, ρ')		2.141 [c]
$\rho \equiv (\rho, \omega')$		0.770 [a]

[a] Input.
[b] $D=0$ and $C=0$ are assumed, and estimated by $(Z, \phi')^2 - (Y, \phi')^2 = (Z$, mean value of ρ' and $\omega')^2 - (Y$, mean value of ρ' and $\omega')^2$.
[c] Estimated by assuming nonet-symmetry breaking such that $(Z, \rho')^2 - (Z, \omega')^2 = \text{constant}$.

symmetry breaking by equal spacing such that $(z, \rho')^2-(z, \omega')^2=\text{constant}$. The masses of the pseudoscalar counterparts are obtained analogously, and are shown in Table III. Figure 1 shows the mass spectrum of the vector and pseudoscalar mesons in our charm-color scheme.

In this model, the narrowness of ψ is attributed to suppression by Zweig's rule and charm conservation in strong interaction. The decay $\psi' \to \psi + 2\pi$ is a color ($\Delta I'=1$) decay and the color radiative decay $\psi' \to \psi + \gamma$ is forbidden, although the successive electromagnetic decays of $\psi' \to \chi\gamma \to \psi + 2\gamma$ and $\psi'' \to \chi'\gamma \to \psi' + 2\gamma$ are allowed if the color electromagnetic interaction is C-invariant. The decays of ψ', as well as those of ψ and ψ'', to other color-excited states, such as (z, ρ') or (z, ϕ'), are suppressed, and the following decays to pseudoscalar mesons are energetically forbidden, as can be seen from Fig. 1,

$$
\begin{aligned}
&\psi \not\to D\bar{D} \text{ or } F\bar{F},\\
&\psi' \not\to D\bar{D} \text{ or } F\bar{F}\\
&\quad \not\to (D,\pi') + (\bar{D},\pi') \text{ or } (F,\pi') + (\bar{F},\pi'),\\
&\psi'' \not\to (D,\eta'') + (\bar{D},\eta'') \text{ or } (F,\eta'') + (\bar{F},\eta'').
\end{aligned} \tag{17}
$$

However, $\psi'' \to D\bar{D}$ is energetically allowed as well as the decay[11] of ψ'' to ψ. Thus, ψ and ψ' are narrow and ψ'' is rather broad, provided that the pseudoscalar-meson states have substantial mixtures of the color singlet and the color octet which are different from those of the vector-meson states. The states (z, ρ') can decay to either a pair of pseudoscalar mesons or to both pseudoscalar and vector mesons, such as

$$
\begin{aligned}
&(\rho,\rho') \to 2\pi,\\
&(\omega,\rho') \to 3\pi,\\
&(\phi,\rho') \to 3\pi \text{ or } K\bar{K},
\end{aligned} \tag{18}
$$

with some suppression by color mixing. Thus $(\rho, \omega \text{ or } \phi, \rho')$ are expected not to be narrow states. This may explain the fact that these states have not been observed in the e^+e^- annihilation experiment[12]. With similar argument $(\rho, \omega \text{ or } \phi, \phi')$ are also expected to be rather broad, and the experiment has yet to sweep these energy ranges.

In addition to the color-excited states, there are charm-excited states of ψ, ψ', and ψ''. ψ' or ψ'' can decay into the charm-excited $\chi \equiv p$-wave states of $\psi \equiv (\phi_c, \omega')_p$ by ψ' (or ψ'') $\to \chi + \gamma$, consistent with the recent observations of at least two χ states[13] at 3.41 and 3.53 GeV. The observed[14]

TABLE III. Masses of pseudoscalar mesons in the SU(4) × SU(3)′ scheme.

Pseudoscalar-meson particle state	Mass (GeV)
(η_c, η'')	3.800 [a,d]
(F, η'')	3.313
(D, η'')	3.245
(η', η'')	2.742 [b]
(K, η'')	2.659
(η, η'')	2.627 [b]
(π, η'')	2.573 [b]
(η_c, K')	3.499
(F, K')	2.963
(D, K')	2.886
(η', K')	2.307
(K, K')	2.207
(η, K')	2.169
(π, K')	2.103
(η_c, π')	3.500 [a,d]
$\eta_c \equiv (\eta_c, \eta')$	2.800 [a,d]
(F, π')	2.965
$F \equiv (F, \eta')$	2.093
(D, π')	2.888
$D \equiv (D, \eta')$	1.982
(η', π')	2.380 [c]
$\eta' \equiv (\eta', \eta')$	0.958 [a]
(K, π')	2.209
$K \equiv (K, \eta')$	0.684
(η, π')	2.171 [c]
$\eta \equiv (\eta, \eta')$	0.549 [a]
(π, π')	2.104 [c]
$\pi^0 \equiv (\pi, \eta')$	0.135 [a]

[a] Input

[b,c] Analogous with Table II.

[d] We take these values as inputs for an example. As for 2.8 GeV., see B. Wiik, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford,* edited by W. T. Kirk (SLAC, Stanford University, Stanford, California, 1976), p. 69.

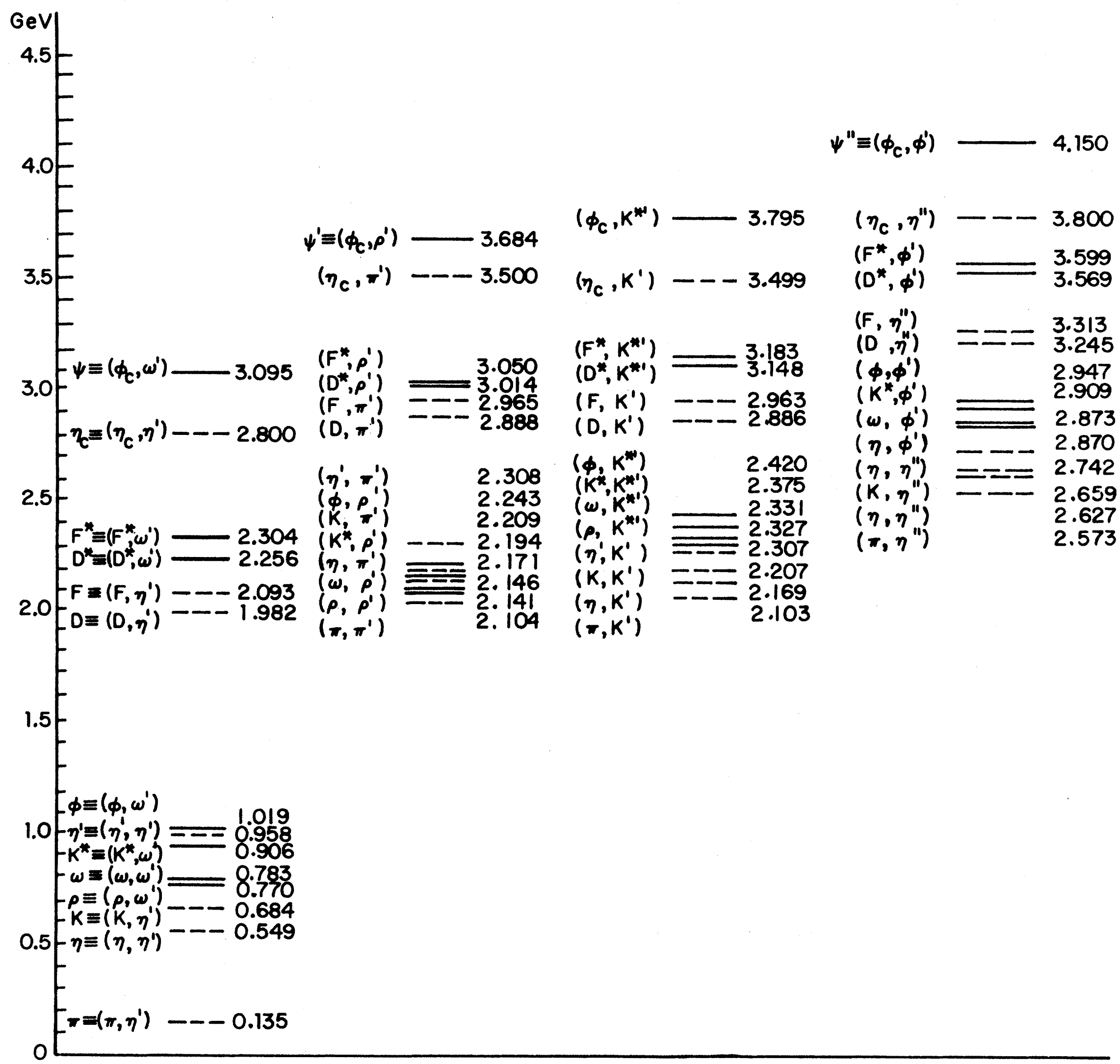

FIG. 1. Mass spectrum of vector (solid lines) and pseudoscalar (dotted lines) mesons in the charm-color scheme.

$\psi'\to\psi+\eta$ decay can be accommodated in this model by further mixing the octet with the singlet in the SU(3) subspace for ϕ_c in Eq. (1). The masses of (Z,ρ') and (Z,ϕ') are not inconsistent with the dependence of the ratio R with respect to energies.

ACKNOWLEDGMENTS

The author would like to acknowledge the useful discussions with R. M. Barnett, Y. Tomozawa, and S. Y. Park. He is also grateful for S. Weinberg's kind arrangement of his visit.

*Permanent address: Department of Physics, Saginaw Valley State College, University Center, Michigan 48710.

[1]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* 33, 1406 (1974); C. Bacci *et al.*, *ibid.* 33, 1408 (1974).

[2]G. S. Abrams *et al.*, Phys. Rev. Lett. 33, 1453 (1974).

[3]J.-E. Augustin *et al.*, Phys. Rev. Lett. 34, 764 (1975).

[4]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); A. De Rújula and S. L. Glashow, *ibid.* 34, 46 (1975).

[5]B. J. Harrington, S. Y. Park, and A. Yildiz, Phys.

Rev. Lett. 34, 168 (1975); 34, 706 (1975); T. Appelquist, A. De Rújula, H. D. Politzer, and S. L. Glashow, *ibid*. 34, 365 (1975).

[6]Y. Tomozawa and S. K. Yun, Phys. Rev. D 11, 3018 (1975).

[7]M. Machacek, Y. Tomozawa, and S. K. Yun, Phys. Rev. D 12, 2925 (1975).

[8]J. C. Pati and A. Salam, University of Maryland Technical Report No. 75-056, 1975 (unpublished); C. A. Nelson, SUNY at Binghamton Report No. SUNY-BING 3/24/75 (unpublished).

[9]J.-E. Augustin *et al.*, Phys. Rev. Lett. 34, 764 (1975). For the behavior of R after subtracting the contributions by resonances, see R. Schwitters, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford*, edited by W. T. Kirk (SLAC, Stanford University, Stanford, California, 1976), p. 5.

[10]These assignments are in contrast with the ones in Ref. 7.

[11]The predicted decay $\psi'' \to \psi + \text{anything}$ with its signature $\psi \to \mu\bar{\mu}$ and $e\bar{e}$ has not been observed.

[12]C. Bemporad, talk given at the conference mentioned in Ref. 9.

[13]G. J. Feldman *et al.*, Phys. Rev. Lett. 35, 821 (1975).

[14]G. J. Feldman, SLAC Report No. SLAC-PUB-1624, 1975 (unpublished).