Double-beta decay and a massive Majorana neutrino*

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We consider the possibility that nuclear no-neutrino double-beta decay is mediated by a universally coupled heavy Majorana electron neutrino. From the known rates of nuclear double-beta decays we conclude that the mass of this neutrino must be at least 10^4 GeV.

It is well known that if the electron-neutrino emitted in nuclear single-beta decay is a γ_5-non-invariant Majorana particle, then nuclear no-neutrino double-beta decay

$$(A, Z) \to (A, Z+2) + 2e^- \qquad (1)$$

will occur as a second-order weak effect. The Majorana property makes possible the exchange of a virtual neutrino between two neutrons in the nucleus, and the lack of over-all γ_5 invariance ensures that the decay amplitude will not vanish merely for reasons of helicity. Invariance of the Hamiltonian under γ_5 transformations of the neutrino field breaks down when the leptonic weak current contains a small admixture of $(1-\gamma_5)$ coupling

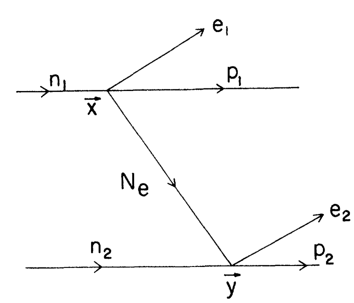
$$L_\lambda' = [\bar{e}\gamma_\lambda((1+\gamma_5)+\eta(1-\gamma_5))\nu_e], \qquad (2)$$

or when the neutrino rest mass does not vanish, or when both conditions are met.

Theoretical estimates of the nuclear no-neutrino and two-neutrino double-beta decay rates[1] have shown that an upper limit on η in the range 10^{-3} to 10^{-4} is consistent with the available experimental values of the double-beta decay half-lives of $Te^{130} \to Xe^{130}$, $Te^{128} \to Xe^{128}$, and $Se^{82} \to Kr^{82}$ and the available experimental limits on the no-neutrino double-beta decay rates of $Ca^{48} \to Ti^{48}$, $Ge^{76} \to Se^{76}$, and $Se^{82} \to Kr^{82}$.[2] On the other hand, taking the electron-neutrino mass to be 60 eV, its experimental upper limit, and setting $\eta = 0$ yields nuclear no-neutrino double-beta decay rates some 300 times smaller than those which correspond to $\eta = 5\times 10^{-4}$. This estimate would change considerably if either the electron neutrino ν_e were much heavier than 60 eV (which, of course, is not possible), or if there were another, heavy Majorana neutrino N_e coupled to the electron with the same strength as ν_e and with a definite helicity,

$$L_\lambda'' = [\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e] + [\bar{e}\gamma_\lambda(1\pm\gamma_5)N_e]. \qquad (3)$$

The existence of neutrinos such as N_e has been discussed in the context of vectorlike gauge theories of elementary particle interactions[3]; they are expected to be too heavy $[M(N_e) \approx$ a few GeV$]$ to be produced in low-energy processes, but they can serve as intermediaries for nuclear no-neutrino



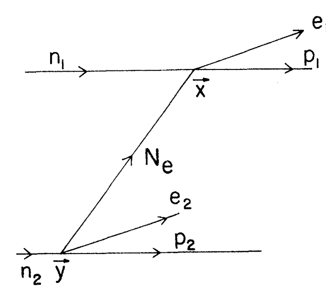


FIG. 1. Diagrams for nuclear no-neutrino double-beta decay involving the exchange of a neutrino between two hadrons within the nucleus.

double-beta decay. Here we propose to examine their role in this phenomenon, and to show that the available experimental limits on the nuclear no-neutrino double-beta decay rates can be used to set a lower limit on the mass of N_e.

The basic mechanism of nuclear no-neutrino double-beta decay is generally thought to involve the exchange of a neutrino between two hadrons within the nucleus, in the first instance between two neutrons: $n_1+n_2 \to p_1+p_2+2e^-$. Two neutrons are necessary because the isospin of the nucleon system is $\frac{1}{2}$, and so, no matter what the internal structure of the nucleon may be, $\Delta Q = 2$ transitions *within* one nucleon are impossible. However, should the nucleus contain an admixture of $N^*(3,3)$ (see Ref. 4) then $\Delta Q = 2$ transitions from nucleon to N^* would be possible; e.g.,

$$n \to N^{*++} + 2e^-. \tag{4}$$

In the quark model for the n and N^{*++}, this process must involve the exchange of a neutrino between two quarks because the constituent quarks of n and N^{*++} are again objects with isospin $\frac{1}{2}$. Thus, as long as there is no fundamental $\Delta Q = 2$ interaction, we must calculate the amplitude for nuclear no-neutrino double-beta decay from diagrams such as those in Fig. 1 in which the hadronic participants may be either nucleons or quarks.[5]

Schematically, the amplitude calculated from the diagrams of Fig. 1 can be written in noncovariant perturbation theory as[4]

$$\mathfrak{M}_{\beta\beta} = \int d^3x\, d^3y \Bigg[\sum_{\vec{p}(N_e),k} \frac{\langle \Psi_f e_2 e_1 | H_w(\vec{y}) | \Psi_k e_1 N_e\rangle \langle \Psi_k e_1 N_e | H_w(\vec{x}) | \Psi_i\rangle}{E(N_e)+E_1+E_k-E_i} + \sum_{\vec{p}(N_e),l} \frac{\langle \Psi_f e_1 e_2 | H_w(\vec{x}) | \Psi_l e_2 N_e\rangle \langle \Psi_l e_2 N_e | H_w(\vec{y}) | \Psi_i\rangle}{E(N_e)+E_2+E_l-E_i} - (e_1 \leftrightarrow e_2)\Bigg], \tag{5}$$

where $H_w(\vec{z})$ is the weak-interaction Hamiltonian density, Ψ_i and Ψ_f are the initial and final hadronic states, and Ψ_k and Ψ_l are the intermediate states. The energies of the intermediate neutrino and the electrons are denoted by $E(N_e)$ and E_1, E_2, and the energies of the hadronic states are represented by E_t $(t \equiv k, l, i, f)$. In general, the hadronic energy differences $(E_k - E_i)$, $(E_l - E_i)$ and the electron energies $E_{1,2}$ are of the order of a few MeV, while the energy of the intermediate neutrino is much larger. It is therefore reasonable to approximate the energy denominators in Eq. (5) by $E(N_e)$, and to carry out the sum over intermediate hadronic states by closure; this approximation becomes more accurate as the mass of N_e increases. With the usual assumption of proportionality of $H_w(\vec{z})$ to the product of a hadronic weak current $J_\lambda(\vec{z})$ times a leptonic weak current $L_\lambda(\vec{z})$, the amplitude becomes

$$\mathfrak{M}_{\beta\beta} = \left(\frac{G}{\sqrt{2}}\right)^2 \int d^3x\, d^3y \sum_{\vec{p}(N_e)} \frac{1}{E(N_e)} \Big[\langle \Psi_f | J_\lambda(\vec{y}) J_\mu(\vec{x}) | \Psi_i\rangle \langle e_2 | L_\lambda(\vec{y}) | N_e\rangle \langle e_1 N_e | L_\mu(\vec{x}) | 0\rangle + \langle \Psi_f | J_\mu(\vec{x}) J_\lambda(\vec{y}) | \Psi_i\rangle \langle e_1 | L_\mu(\vec{x}) | N_e\rangle \langle e_2 N_e | L_\lambda(\vec{y}) | 0\rangle - (e_1 \leftrightarrow e_2)\Big], \tag{6}$$

where G is the universal weak-interaction coupling constant.

To compare the case in which γ_5 invariance is broken directly in the current with the case in which the breaking is due to the neutrino mass, we evaluate the lepton factors in Eq. (6) first for the current L'_λ of Eq. (2), and then for the current L''_λ of Eq. (3). For a zero-mass neutrino and the current L'_λ of Eq. (2) we have[4]

$$\sum_{\vec{p}(\nu_e)} \frac{1}{|\vec{p}(\nu_e)|} \langle e_2 | L'_\lambda(\vec{y}) | \nu_e\rangle \langle e_1 \nu_e | L'_\mu(\vec{x}) | 0\rangle = \int \frac{d^3q}{(2\pi)^3}\, e^{-i(\vec{p}_2\cdot\vec{y}+\vec{p}_1\cdot\vec{x})} e^{i\vec{q}\cdot(\vec{y}-\vec{x})} (4\eta) \left(\bar{u}_2 \gamma_\lambda \frac{i\gamma_\rho q_\rho}{|\vec{q}|^2} \gamma_\mu \mathfrak{C} \tilde{u}_1 \right)$$
$$= \frac{i\eta}{\pi_2} e^{-i(\vec{p}_2\cdot\vec{y}+\vec{p}_1\cdot\vec{x})} \left\{ \bar{u}_2 \gamma_\lambda \left[\frac{i\pi}{2|\vec{y}-\vec{x}|^3} \vec{\gamma}\cdot(\vec{y}-\vec{x}) + \frac{i}{|\vec{y}-\vec{x}|^2} \gamma_4 \right] \gamma_\mu \mathfrak{C} \tilde{u}_1 \right\}. \tag{7}$$

Equations (7) and (6) and the standard nucleons-only impulse approximation for the hadronic weak current yield $(\sum'_{n,m} \cdots \equiv \sum_{n,m;\, n\neq m} \cdots)$

$$\mathfrak{M}_{\beta\beta}(\eta) = \left(\frac{-G^2\eta}{2\pi}\right) \left\langle \Psi_f \right| \sum_{n,m}{}' \tau_m^{(+)} \tau_n^{(+)} (\Gamma_\lambda)_m (\Gamma_\mu)_n e^{-i(\vec{p}_2\cdot\vec{r}_m+\vec{p}_1\cdot\vec{r}_n)} \left[\bar{u}_2 \gamma_\lambda \frac{\vec{\gamma}\cdot(\vec{r}_m-\vec{r}_n)}{|\vec{r}_m-\vec{r}_n|^3} \gamma_\mu \mathfrak{C} \tilde{u}_1 \right] \left| \Psi_i \right\rangle, \tag{8}$$

where n and m denote distinct neutrons in the nucleus, $\tau^{(+)}$ is the isospin-raising operator, and Γ_λ is the $V-A$ operator for the nucleon. All known examples of nuclear double-beta decay occur in even-even nuclei and are $0^+ \to 0^+$ with respect to spin and parity. Consequently, Eq. (8) becomes

$$\mathfrak{M}_{\beta\beta}(\eta)=\left(\frac{-G^2\eta}{6\pi}\right)\left\langle\Psi_f\right|\sum_{n,m}{}'\tau_m^{(+)}\tau_n^{(+)}(\Gamma_\lambda)_m(\Gamma_\mu)_n\frac{1}{|\vec{r}_m-\vec{r}_n|}\left|\Psi_i\right\rangle[\bar{u}_2\gamma_\lambda\vec{\gamma}\cdot(\vec{p}_2-\vec{p}_1)\gamma_\mu\mathfrak{C}\tilde{u}_1], \tag{9}$$

and we see that the exchange of a zero-mass neutrino gives rise to an effective Coulomb potential.

For a heavy Majorana neutrino of mass $M(N_e)\equiv M$ and the current L_λ'' of Eq. (3), the lepton factor in $\mathfrak{M}_{\beta\beta}$ [Eq. (6)] is given by

$$\sum_{\vec{p}(N_e)}\frac{1}{\{[\vec{p}(N_e)]^2+M^2\}^{1/2}}\langle e_2|L_\lambda''(\vec{y})|N_e\rangle\langle e_1N_e|L_\mu''(\vec{x})|0\rangle$$

$$=\int\frac{d^3q}{(2\pi)^3}e^{-i(\vec{p}_2\cdot\vec{y}+\vec{p}_1\cdot\vec{x})}e^{i\vec{q}\cdot(\vec{y}-\vec{x})}\left\{\bar{u}_2\gamma_\lambda\frac{M}{([\vec{p}(N_e)]^2+M^2)}\gamma_\mu(1\pm\gamma_5)\mathfrak{C}\tilde{u}_1\right\}$$

$$=\frac{M}{4\pi}e^{-i(\vec{p}_2\cdot\vec{y}+\vec{p}_1\cdot\vec{x})}\frac{e^{-M|\vec{y}-\vec{x}|}}{|\vec{y}-\vec{x}|}[\bar{u}_2\gamma_\lambda\gamma_\mu(1\pm\gamma_5)\mathfrak{C}\tilde{u}_1]. \tag{10}$$

As expected, the amplitude is now proportional to M instead of η; in addition, the exchange of the heavy neutrino gives rise to an effective Yukawa potential instead of a Coulomb potential. The amplitude for $0^+\to0^+$ nuclear decays via the two-neutron mechanism is given by

$$\mathfrak{M}_{\beta\beta}(M)=\left(\frac{G^2M}{4\pi}\right)\left\langle\Psi_f\right|\sum_{n,m}{}'\tau_m^{(+)}\tau_n^{(+)}(\Gamma_\lambda)_m(\Gamma_\mu)_n\frac{e^{-M|\vec{r}_m-\vec{r}_n|}}{|\vec{r}_m-\vec{r}_n|}\left|\Psi_i\right\rangle[\bar{u}_2\gamma_\lambda\gamma_\mu(1\pm\gamma_5)\mathfrak{C}\tilde{u}_1]. \tag{11}$$

Comparing Eq. (9) and (11), we see that the $\mathfrak{M}_{\beta\beta}(\eta)$ amplitude is of order

$$\eta\{|\vec{p}_2-\vec{p}_1|\}_{\rm Av}\left\langle\frac{1}{|\vec{r}_m-\vec{r}_n|}\right\rangle,$$

while the $\mathfrak{M}_{\beta\beta}(M)$ amplitude is of order

$$M\left\langle\frac{e^{-M|\vec{r}_m-\vec{r}_n|}}{|\vec{r}_m-\vec{r}_n|}\right\rangle.$$

Since both amplitudes must be fitted to the same experimental limit, the mass M must be such that

$$M\left\langle\frac{e^{-M|\vec{r}_m-\vec{r}_n|}}{|\vec{r}_m-\vec{r}_n|}\right\rangle\approx\eta\{|\vec{p}_2-\vec{p}_1|\}_{\rm Av}\left\langle\frac{1}{|\vec{r}_m-\vec{r}_n|}\right\rangle,$$
$$\langle f(|\vec{r}_m-\vec{r}_n|)\rangle\equiv\int f(|\vec{r}|)P(|\vec{r}|)d\vec{r}, \tag{12}$$

where $P(|\vec{r}|)$ is the nucleon-nucleon correlation function. Then, if the nucleon-nucleon potential has a hard core of radius $r_c\cong0.5$ F $\cong(3m_\pi)^{-1}$, and if the nuclear radius $R\cong(1.2\text{ F})A^{1/3}\cong(1.2m_\pi)^{-1}A^{1/3}$, we can approximate $P(|\vec{r}|)$ by $\{\frac{4}{3}\pi[(2R)^3-r_c^{\,3}]\}^{-1}$ for $r_c\leqslant|\vec{r}|\leqslant2R$ and by 0 for $|\vec{r}|<r_c$ and $|\vec{r}|>2R$ and calculate $\langle f(|\vec{r}_n-\vec{r}_m|)\rangle$ immediately. Also, the average difference between the electron momenta is ≈2 MeV $\cong m_\pi/70$. The condition on M can then be written as

$$M\left(\frac{M^{-2}[e^{-Mr_c}(1+Mr_c)-e^{-M2R}(1+M2R)]}{\frac{1}{3}[(2R)^3-r_c^{\,3}]}\right)\approx\eta\left(\frac{m_\pi}{70}\right)\frac{\frac{1}{2}[(2R)^2-r_c^{\,2}]}{\frac{1}{3}[(2R)^3-r_c^{\,3}]} \tag{13}$$

or

$$M\left\{M^{-2}\left[e^{-M/3m_\pi}\left(1+\frac{M}{3m_\pi}\right)-e^{-MA^{1/3}/0.6m_\pi}\left(1+\frac{MA^{1/3}}{0.6m_\pi}\right)\right]\right\}\approx\frac{\eta A^{2/3}}{50m_\pi}, \tag{14}$$

which, with $\eta\lessapprox5\times10^{-4}$ and $A=100$, yields

$$M\lessapprox1\text{ keV},\qquad M\gtrapprox3\text{ GeV}, \tag{15}$$

where the larger of the two M values is appropriate to the N_e of Ref. 3. Our assumption of a hard core between nucleons provides us with a minimum estimate for the mass of such an N_e. Thus, as the core softens, the value of the mass estimate increases Eq. (14), for example, yielding $M(N_e)\gtrapprox700$ GeV for $r_c=0$; alternatively, and again using Eq. (14) with $r_c=0$, $M(N_e)=2$ GeV, corresponds to $\eta\approx350\times(5\times10^{-4})$ and so to a nuclear no-neutrino double-beta decay rate some 10^5 times larger than the experimental limit. In addition, η is treated here as a purely phenomenological parameter chosen to fit the experimental limit on the no-neutrino double-beta decay rate via Eq. (9); thus our estimate of M [in Eq. (15)] is essentially independent of the exact structure of the nuclear matrix element for double-beta decay.

Suppose now that instead of the two-neutron process inside the nucleus, we consider the $n\to N^{*++}$ transition of Eq. (4) which we assume to take place between two quarks inside the nucleon-N^* system. Then, instead of the amplitudes of Eqs. (11) and (9), we shall have amplitudes of the form[4]

$$\mathfrak{M}'_{\beta\beta}(M)=\left(\frac{G^2M}{4\pi}\right)P_{N^*}{}^{1/2}\left\langle N^{*++}\right|\sum_{l,k}{}'\tau_k^{(+)}\tau_l^{(+)}(\Gamma_\lambda)_k(\Gamma_\mu)_l\frac{e^{-M|\vec{R}_k-\vec{R}_l|}}{|\vec{R}_k-\vec{R}_l|}\left|n\right\rangle\langle\Phi_f|\Phi_i\rangle[\bar{u}_2\gamma_\lambda\gamma_\mu(1\pm\gamma_5)\mathfrak{C}\tilde{u}_1],$$

$$\mathfrak{M}'_{\beta\beta}(\eta)=\left(-\frac{G^2\eta}{6\pi}\right)P_{N^*}{}^{1/2}\left\langle N^{*++}\right|\sum_{l,k}{}'\tau_k^{(+)}\tau_l^{(+)}(\Gamma_\lambda)_k(\Gamma_\mu)_l\frac{1}{|\vec{R}_k-\vec{R}_l|}\left|n\right\rangle\langle\Phi_f|\Phi_i\rangle[\bar{u}_2\gamma_\lambda\vec{\gamma}\cdot(\vec{p}_2-\vec{p}_1)\gamma_\mu\mathfrak{C}\tilde{u}_1]. \qquad (16)$$

Here $\langle\Phi_f|\Phi_i\rangle$ is the overlap factor between the initial and final nuclear wave functions, $P_{N^*}\approx10^{-2}$ represents the probability of finding an N^* inside the nucleus, k, l denote the quarks inside the nucleon or N^*, and $\vec{R}_k$, $\vec{R}_l$ are their position vectors. We note that for $P_{N^*}\approx10^{-2}$ we have $\mathfrak{M}'_{\beta\beta}(\eta)\approx\mathfrak{M}_{\beta\beta}(\eta)$; this follows from the fact that

$$\left\langle N^{*++}\right|\sum_{l,k}{}'\tau_k^{(+)}\tau_l^{(+)}(\Gamma_\lambda)_k(\Gamma_\mu)_l\left|n\right\rangle\langle\Phi_f|\Phi_i\rangle \approx\left\langle\Psi_f\right|\sum_{n,m}{}'\tau_m^{(+)}\tau_n^{(+)}(\Gamma_\lambda)_m(\Gamma_\mu)_n\left|\Psi_i\right\rangle$$

and

$$\left\langle\frac{1}{|\vec{r}_m-\vec{r}_n|}\right\rangle\approx R^{-1},\quad\left\langle\frac{1}{|\vec{R}_k-\vec{R}_l|}\right\rangle\approx\left(\frac{R}{2A^{1/3}}\right)^{-1}\cong10R^{-1}.$$

We proceed to consider the fit of $\mathfrak{M}'_{\beta\beta}(M)$ and $\mathfrak{M}_{\beta\beta}(\eta)$ to the same experimental limit. This gives as the condition on M the analog of Eq. (12)

$$P_{N^*}^{1/2}M\left\langle\frac{e^{-M|\vec{R}_k-\vec{R}_l|}}{|\vec{R}_k-\vec{R}_l|}\right\rangle\approx\eta\{|\vec{p}_2-\vec{p}_1|\}_{\rm Av}\left\langle\frac{1}{|\vec{r}_m-\vec{r}_n|}\right\rangle \approx\eta\left(\frac{m_\pi}{70}\right)\left(\frac{m_\pi}{A^{1/3}}\right). \qquad (17)$$

To calculate the left-hand side of Eq. (17) we assume that an effective quark-quark potential deduced on the basis of quantum chromodynamics[6] exists inside the nucleon and the N^*; such a potential confines the quarks to distances $\lesssim(2m_\pi)^{-1}$ and does not provide any hard-core repulsion. As a result, the wave function of the quarks does not vanish as $|\vec{R}_k-\vec{R}_l|\to0$ and

$$\left\langle\frac{e^{-M|\vec{R}_k-\vec{R}_l|}}{|\vec{R}_k-\vec{R}_l|}\right\rangle$$

is not exponentially damped. Indeed, the quark-quark correlation function $P'(|\vec{R}_k-\vec{R}_l|)$ can in this case be approximated by

$$P'(|\vec{R}_k-\vec{R}_l|)=(\pi a^3)^{-1}\exp(-2|\vec{R}_k-\vec{R}_l|/a)$$

so that Eq. (17) becomes

$$P_{N^*}^{1/2}M\left[\frac{1}{a(Ma/2+1)^2}\right]\approx\eta\left(\frac{m_\pi}{70}\right)\left(\frac{m_\pi}{A^{1/3}}\right), \qquad (18)$$

which, with $\eta\leq5\times10^{-4}$, $A=100$, $a=0.7$ F $\cong(2m_\pi)^{-1}$, and $P_{N^*}=10^{-2}$ yields

$$M\lesssim1\text{ keV},\qquad M\gtrsim3\times10^5\text{ GeV}. \qquad (19)$$

Equation (19) shows that $M(N_e)\gtrsim3\times10^5$ GeV; this is, of course, much too large to be considered a reasonable physical possibility. Alternatively, Eq. (18) with $M(N_e)=2$ GeV corresponds to $\eta\approx(8\times10^4)\times(5\times10^{-4})$ and so to a nuclear no-neutrino double-beta decay rate some 10^{10} times larger than the experimental limit. We may also remark that in a theory where the heavy Majorana neutrino is coupled to the muon rather than to the electron $[(\bar{e}\gamma_\lambda(1\pm\gamma_5)N_e)\to(\mu\gamma_\lambda(1\pm\gamma_5)N_\mu)]$ the only energetically allowed analogs of nuclear no-neutrino double-beta decay, namely $K^\mp\to\pi^\pm+\mu^\mp+\mu^\mp$ and $\Sigma^-\to p+\mu^-+\mu^-$, have completely negligible branching ratios relative to $K^\mp\to\pi^0+\mu^\mp+\bar{\nu}_\mu(\nu_\mu)$ and $\Sigma^-\to n+\mu^-+\bar{\nu}_\mu$ for all values of $M(N_\mu)$ [from Eq. (16) with $G=10^{-5}/m_p{}^2\cong\frac{1}{3}10^{-5}/(4m_\pi)^2$ and Eq. (18) with $a\cong(2m_\pi)^{-1}$ we have

$$\frac{\Gamma(\Sigma^-\to p+\mu^-+\mu^-)}{\Gamma(\Sigma^-\to n+\mu^-+\bar{\nu}_\mu)}\approx\left\{\frac{1}{4\pi}\left[\frac{1}{3}\frac{10^{-5}}{(4m_\pi)^2}\right]M(N_\mu)\left[\frac{\frac{1}{2}(4m_\pi)}{(M(N_\mu)/4m_\pi+1)^2}\right]\right\}^2\left[\frac{\frac{1}{4}\pi m_\mu{}^3(m_\Sigma-m_p-2m_\mu)^2}{\frac{1}{30}(m_\Sigma-m_n)^5}\right]$$
$$\approx10^{-15}\left[\frac{M(N_\mu)}{4m_\pi}\right]^2\frac{1}{[M(N_\mu)/4m_\pi+1]^4}\leq10^{-15},$$

and similarly for

$$\frac{\Gamma(K^\mp\to\pi^\pm+\mu^\mp+\mu^\mp)}{\Gamma(K^\mp\to\pi^0+\mu^\mp+\bar{\nu}_\mu(\nu_\mu))}. \qquad (20)$$

In conclusion we see that if there is a 1% probability of finding an $N^*(3,3)$ in the nucleus then any Majorana neutrino coupled universally to the electron must be unreasonably heavy (neutrino mass $\gtrsim10^5$ GeV) in order to fit the present experimental limits on nuclear no-neutrino double-beta decay. Even if this 1% probability of finding the N^* were too large by a factor of 100, the neutrino mass would still be $\gtrsim10^4$ GeV ($M(N_e)$ is very closely proportional to $P_{N^*}{}^{1/2}$ [Eq. (18)]).

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