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Does the $\Delta(3,3)$ resonance factorize?*

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Evidence is presented showing that there are nonresonant coherent backgrounds interfering with $\Delta(3,3)$ resonance production in both πN elastic scattering and photoproduction. The background in each case is predominantly given by the Born term projected into the resonant channel. Interplay between background and direct resonance production results in the observed shifts in the resonant-cross-section peaks.

I. INTRODUCTION

The $\Delta(3,3)$ resonance, first of the elementary particle resonances, was discovered at nearly the same time in elastic πN scattering and in photoproduction.[1] With the advent of precision measurement of the elastic channel this resonance has provided a laboratory for the investigation of different resonance formulas and searches for the complex pole position.[2]

In the usual parametrizations of the (3, 3) phase shift a Breit-Wigner resonance with a momentum-dependent width is assumed. The possible existence of a nonresonant background amplitude is generally ignored because the effect of a small background just shifts the resonance energy and modifies the momentum dependence of the width both of which are already parameterized. There is thus no way to determine if the Breit-Wigner parameters represent the underlying resonance properties or are only "effective" resonance parameters distorted by the presence of a nonresonant background. However, by comparing the resonant photoproduction multipoles with the elastic channel this question can be examined more carefully.

If there are no backgrounds, a multichannel Breit-Wigner resonance amplitude will factorize into formation, propagation, and decay parts. In particular, the resonant peaks in each channel will occur at the same energy neglecting the effects of isospin breaking and differing phase-space factors.[3] In Fig. 1 we compare the total elastic resonant cross section[4]

$$\sigma_{3,3}=\frac{8\pi}{q^2}\sin^2\delta_{33} \tag{1}$$

and the total photoproduction cross section[5,6] in its two resonant multipoles,[7] magnetic dipole

$$\sigma_{M_{1+}}=\frac{8\pi q}{k}\,|M_{1+}|^2 \tag{2}$$

and electric quadrupole

$$\sigma_{E_{1+}}=\frac{24\pi q}{k}\,|E_{1+}|^2\ . \tag{3}$$

From Fig. 1 it is at once apparent that at least one of the resonant amplitudes is accompanied by a large nonresonant background. We will later observe that all three resonant amplitudes contain significant backgrounds and that these backgrounds are basically the Born terms projected into the resonant channel.

In Sec. II we discuss the resonant photoproduction multipoles in detail and in Sec. III our conclusions are presented.

II. PHOTOPRODUCTION

In this section we will first parametrize a resonant photoproduction multipole when there are nonresonant backgrounds in both the multipole and the associated elastic partial wave. Given this parametrization we can draw a number of qualitative conclusions about the existence and size of the backgrounds. Finally, we demonstrate that the signs and magnitudes of the backgrounds can be understood with a simple model.

A. Resonant multipoles with backgrounds

According to Watson's theorem[8] a photoproduction multipole of definite angular momentum and isospin must have the same complex phase as the corresponding elastic partial wave. Thus considered separately, both the resonance and background multipoles must have the same phase as the elastic resonance and background, respectively. In addition the resonance factorizes and the pure resonance multipole is given by

$$qf^r_{12}=Ne^{i\delta_r}\sin\delta_r\ , \tag{4}$$

where δ_r is the elastic resonant phase shift and $N=v_1/v_2$ is the ratio of formation and decay vertices for the specific multipole.

If the elastic background phase shift is δ_e, the multipole background can be parametrized as

$$q\,f^b_{12}=Ne^{i\delta_e}\sin\delta_p, \tag{5}$$

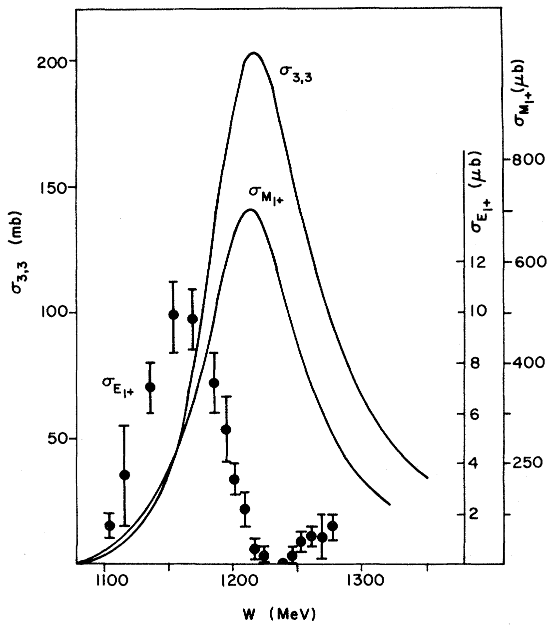

FIG. 1. Total cross sections for resonant elastic scattering in the (3, 3) partial wave, magnetic dipole (M_{1+}) photoproduction and electric quadrupole (E_{1+}) photoproduction. The curves and data points represent data from Refs. 4, 5, and 6. The errors in the (3,3) partial wave and M_{1+} multipole cross sections are comparable to the thickness of the curves.

where $N\sin\delta_p$ is the magnitude of the photoproduction background amplitude. It is not obvious that the background term qf^b_{12} should be bounded by N; however, in practice this always seems to be the case. The observed photoproduction amplitude is then conveniently parametrized as

$$qf_{12} = Ne^{i\delta}\sin\psi, \tag{6}$$

where δ is the observed (3,3) phase shift. We note that $\psi = \delta = \delta_r$ if the background vanishes and $\psi = \delta_p$, $\delta = \delta_e$ in the absence of the resonance.

The sum of the resonance multipole Eq. (4) and the background multipole Eq. (5) will not in general equal the observed multipole Eq. (6). To satisfy the unitarity constraints, we modify the resonant multipole[9] to

$$qf^r_{12} = Ne^{i\phi}\sin\lambda, \tag{7}$$

where ϕ and λ reduce to δ_r if the backgrounds vanish. The resonance-background combination can then be expressed as

$$e^{i\delta}\sin\psi = e^{i\phi}\sin\lambda + e^{i\delta_e}\sin\delta_p.$$

To satisfy the above equation for all values of δ, δ_p, and δ_e we find

$$\phi = \delta + \delta_p,$$
$$\lambda = \delta - \delta_e,$$
$$\psi = \delta + \delta_p - \delta_e,$$

and the total photoproduction multipole can be written as[10]

$$qf_{12} = Ne^{i\delta}\sin(\delta + \delta_p - \delta_e). \tag{8}$$

B. Qualitative conclusions on backgrounds

From Eq. (8) we can observe a number of relationships between the background phases in the resonant processes. Since f_{12} is the multipole amplitude, the total photoproduction cross section corresponding to this multipole is

$$\sigma_{M_{1+}} = \frac{8\pi}{kq} N_M^2 \sin^2(\delta + \delta_p^M - \delta_e),$$
$$\delta_{E_{1+}} = \frac{24\pi}{kq} N_E^2 \sin^2(\delta + \delta_p^E - \delta_e), \tag{9}$$

where we have substituted the parametrization of Eq. (8) in the total cross-section expression of Eqs. (2) and (3).

We first observe from Fig. 1 that the magnetic dipole $\sigma_{M_{1+}}$ peak is roughly similar to the elastic $\sigma_{3,3}$ peak, so by Eq. (9) we expect that

$$\delta_p^M \simeq \delta_e. \tag{10}$$

If on the contrary we assume that there is no elastic background present, i.e., $\delta_e = 0$, then if δ_p^M is nonzero the $\sigma_{M_{1+}}$ peak will be shifted relative to the elastic peak. *Thus by understanding the photoproduction backgrounds it is possible to draw conclusions concerning the nonresonant elastic background.* As we shall see in the next subsection $\delta_p^M \simeq +20°$ near resonance, so by Eq. (9) the $\sigma_{M_{1+}}$ peak would then occur 20° in phase before the elastic peak occurs. Since the elastic peak occurs at $W = 1220$ MeV we would then expect to observe the $\sigma_{M_{1+}}$ peak near 1205 MeV in the absence of an elastic background. From Fig. 1 this is not the case and we infer the existence of elastic nonresonant scattering in the (3, 3) partial wave given the photoproduction background.

The electric quadrupole total cross section $\sigma_{E_{1+}}$ peak is shifted about 60 MeV below the elastic peak as seen in Fig. 1. By Eq. (9) this means that $\delta_p^E - \delta_e$ must be large and positive in the region below the elastic peak. In fact since the elastic phase shift is only about $\delta \simeq 30°$ at the $\sigma_{E_{1+}}$ peak the net background phase $\delta_p^E - \delta_e$ must be about 60°. At $W \simeq 1230$ MeV the $\sigma_{E_{1+}}$ cross section appears to have an approximate double zero. To obtain this double zero Eq. (9) implies that the net background phase must be near 90° as the elastic phase shift resonates. Since the double zero coincides closely with the elastic resonance energy Eq. (8) implies that the real part of the E_{1+} multipole will also have an approximate double zero.

C. A specific model for the backgrounds

To estimate the background amplitudes, we require specific dynamical models for low-energy photoproduction and πN elastic scattering. Fortunately, models are available which can account for all of the nonresonant photoproduction multipoles[11] and elastic low-energy parameters[12] such as subthreshold expansion coefficients, scattering lengths, and low-energy phase shifts. These current-algebra-type models, although employing relatively few parameters, are able to correlate a large amount of physical data.

In the case of photoproduction[11] the model basically consists of the pseudovector Born terms supplemented by (3, 3) resonance coupling to photons, pions, and nucleons. In addition there is also a contribution[13,14] analogous to the σ term in πN elastic scattering. Using the parameter values[11,12]

$$\frac{g_\Delta^2}{4\pi}=0.27m_\pi^{-1},\quad C=0.31m_\pi^{-1},\quad \frac{f^2}{4\pi}=0.079, \tag{11}$$

we show in Fig. 2 the photoproduction background angle δ_p, defined by Eq. (5), for the M_{1+} and E_{1+} multipoles. The solid curve in each case repre-

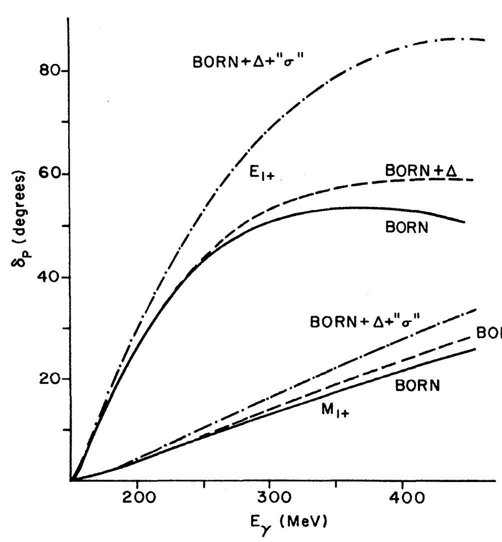

FIG. 2. Photoproduction background angles calculated by the model described in the text.

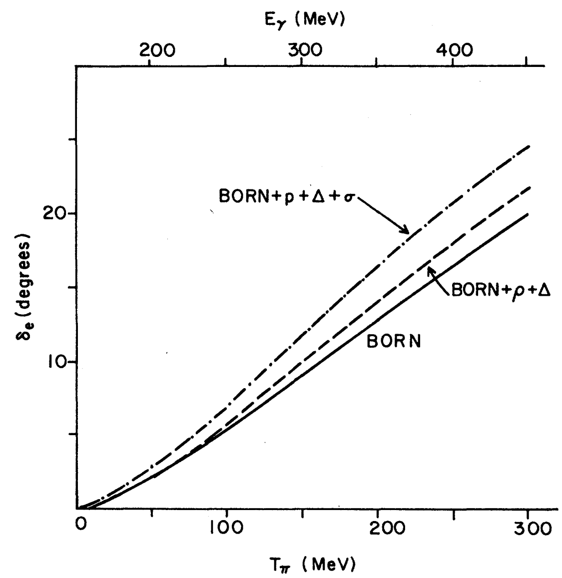

FIG. 3. The elastic background phase calculated using the model described in the text.

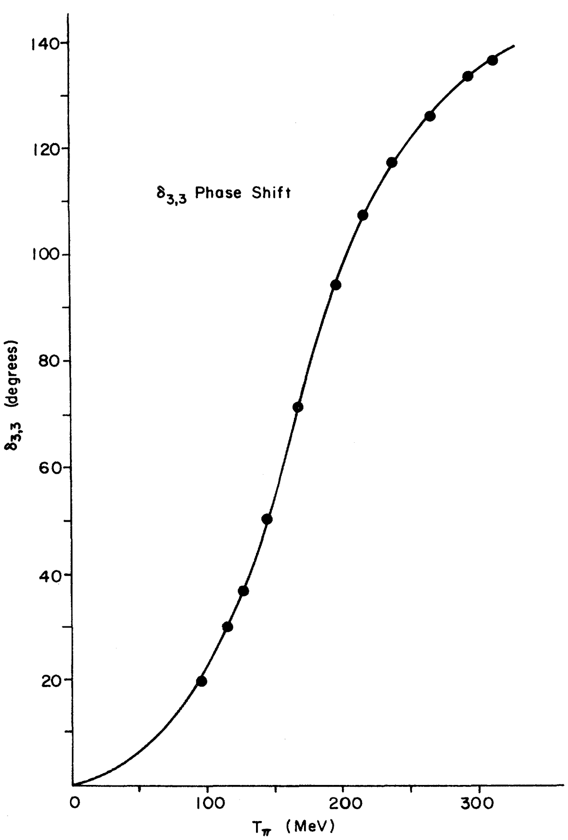

FIG. 4. Experimental (3, 3) phase shift using the data of Ref. 4. The curve is a smooth parametrization of the data including the scattering length.

sents the Born term alone. The dashed curves in Fig. 2 show the effect of adding $\Delta(3,3)$ exchange and a σ term whose magnitude is 20% of the Born term (in analogy to the $\pi N \sigma$ term). The electric quadrupole angle δ_p^E is quite sensitive to small changes in the background since N_E and $q f_{E_{1+}}^{\text{Born}}$ are numerically comparable.

A similar model for low-energy πN elastic has been recently discussed.[12] In this case a Ward identity model is employed with dominant contributions from the equal-time commutator terms, nucleon Born terms, and from $\Delta(3,3)$ exchange. The physical data explained by this model include 28 subthreshold expansion parameters of the invariant amplitudes, the s- and p-wave scattering lengths, and the nonresonant s- and p-wave scattering phase shifts from threshold up through the $\Delta(3,3)$ region. In this model the background amplitude for s-channel $\Delta(3,3)$ resonance production is quite well defined.[15] As seen in Fig. 3 this background is primarily provided by the Born nucleon exchange term. The sigma term gives a smaller contribution and the other exchanges are yet smaller. Comparison of the elastic background angle δ_e in Fig. 3 with the M_{1+} photoproduction background angle δ_p^M in Fig. 2 exhibits the rough required equality discussed in Eq. (10).

With the backgrounds shown in Fig. 2 and Fig. 3 and using the experimentally measured δ_{33} phase shift[4] depicted in Fig. 4 we can use Eq. (8) to predict the resonant multipoles M_{1+} and E_{1+}. Since unitarity[8] constrains the phase of a multipole to be equal to the corresponding elastic phase, we need only consider the real part of each multipole. Several energy-independent multipole analyses are now available; the most recent is that of Berends and Donnachie.[5] The real parts of the M_{1+} and E_{1+} multipoles from the preceding analysis and that of Pfeil and Schwela,[6] extending closer to threshold, are displayed in Fig. 5 and Fig. 6. The curves on these figures correspond to the model expectations. We see that the M_{1+} multipole exhibits good quantitative agreement and the model prediction for the E_{1+} multipole is at least qualitatively correct. In the latter case a somewhat more rapid rise of the background above threshold is indicated. Not only is the model able to account for the backgrounds in a reasonable manner, but the ratio of electric quadrupole to

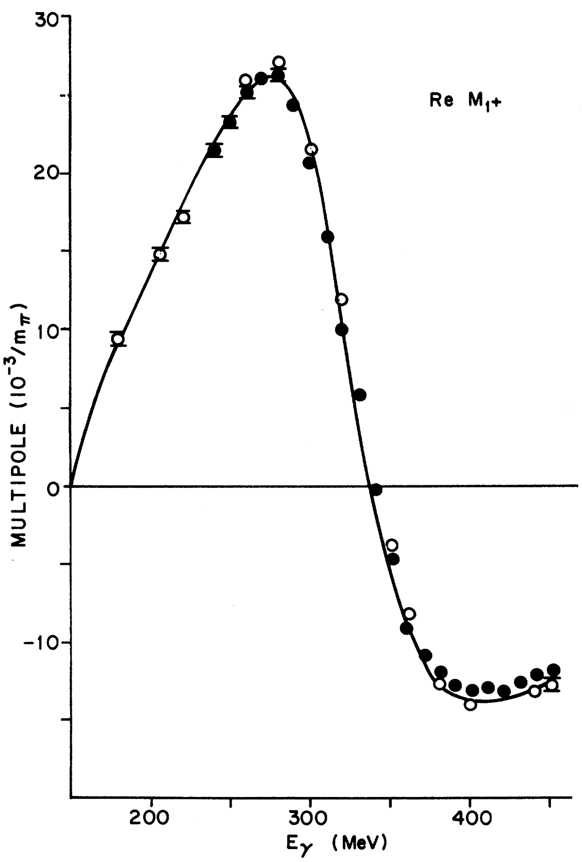

FIG. 5. Real part of the M_{1+} multipole. The open circle data points are from Ref. 6 and the solid data points from Ref. 5. The curve is calculated using the backgrounds of Figs. 2 and 3.

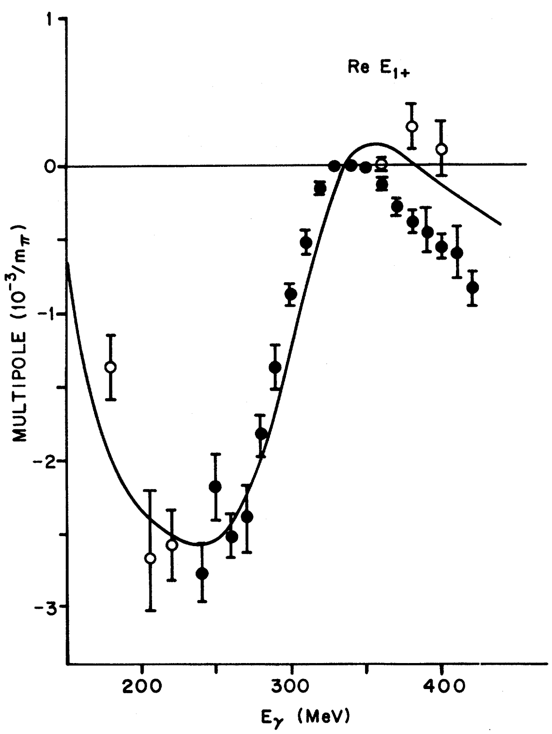

FIG. 6. Real part of the E_{1+} multipole. Data and curve are the same as Fig. 5.

magnetic dipole agrees with the data. Once the coupling constant g_Δ has been fixed in elastic scattering, there is only one new parameter C which enters into resonant photoproduction.

III. CONCLUSIONS

In this paper we stress four main points:

1. Direct observation (i.e., Fig. 1) leads us to believe that the $\Delta(3,3)$ formation amplitudes do not factorize. We infer that nonresonant backgrounds are present.

2. Using the constraints of unitarity, a resonant multipole can be parametrized [Eq. (8)] as

$$q f_{12} = N e^{i\delta} \sin(\delta + \delta_p - \delta_e),$$

where δ is the observed elastic phase shift, δ_p and δ_e represent the photoproduction and elastic backgrounds, and N is the ratio of decay to production vertices of the resonance. This last factor is slowly varying except near photoproduction threshold.

3. The above explicit parametrization of a resonant multipole indicates that there are backgrounds not only in both of the resonant photoproduction multipoles M_{1+} and E_{1+} but also in the elastic channel. Signs of the backgrounds and estimates of their magnitudes can be made without the assumption of a specific model.

4. A current-algebra-type model, which successfully explains the nonresonant low-energy photoproduction and elastic data, can be applied to the resonant multipoles by the use of Eq. (8). The resulting backgrounds and the ratio of resonant multipoles are all fairly consistent with the data.

Finally, the relation between the observed elastic resonant amplitude and the background and underlying resonance amplitudes should be discussed. Up to this point the actual resonance-energy position has never been required. Unfortunately, the question of combining an elastic background with a pure resonance to yield the observed $\delta_{3,3}$ phase shift is model-dependent as unitarity does not give a unique prescription. The combination of the resonant phase shift δ_r with the elastic background phase shift δ_e can be, in general, expressed as

$$\delta = \delta(\delta_r, \delta_e),$$

where the function $\delta(\delta_r, \delta_e)$ obeys a number of more or less obvious requirements. The simplest acceptable function is just the usual phase-shift addition

$$\delta = \delta_r + \delta_e.$$

However, this choice leads to difficulties because a positive background (as in Fig. 3) requires that the underlying resonance energy must lie *above* the observed resonance energy of 1232 MeV. There is considerable evidence on the other hand that the underlying resonance energy must lie near or below 1220 MeV.

One argument supporting a lower-energy resonance is the common conception that the point of maximum speed around the Argand diagram corresponds to the underlying resonance energy. For an elastic resonance the speed is just the rate of change of the phase shift with energy. For the (3, 3) resonance the point of maximum speed is near 1212 MeV. A similar energy is obtained for the real part of the complex resonance pole position.[2] Furthermore, the dispersive analysis of Höhler, Jakob, and Strauss[16] found that the resonance energy should be 1219 MeV and in addition the required $\Delta N\pi$ coupling constant $g_\Delta{}^2/4\pi \simeq 0.264\, m_\pi{}^{-2}$ coincides almost exactly with the value in Eq. (11) obtained from the over-all fit to nonresonant scattering data.[12]

An alternative choice of $\delta(\delta_r, \delta_e)$,

$$\tan\delta(\delta_r, \delta_e) = \frac{\tan\delta_r + \tan\delta_e}{1 + \tan\delta_r \tan\delta_e}, \tag{12}$$

has been previously proposed[10,17] to accommodate a low-energy resonance and fit the observed (3, 3) phase shift. The above function $\delta(\delta_r, \delta_e)$ is valid for small δ_e and for all δ_r. The coupling constant and resonant mass which result from using Eq. (12) are near to the HJS[16] values.

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[7]The photon and pion c.m. momenta are k and q, respectively.

[8]K. M. Watson, Phys. Rev. 95, 228 (1954).

[9]The resonance is modified because this choice yields

a unique result. Any other modification procedure requires a specific decision on how to combine δ_r and δ_e to obtain the elastic phase shift δ.

[10]This parametrization was discussed by M. G. Olsson, Nucl. Phys. B78, 55 (1974), within the context of a specific unitarity model. We see here though that the result is much more general.

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[12]M. G. Olsson and E. T. Osypowski, Nucl. Phys. B101, 136 (1975); M. G. Olsson, Leaf Turner, and E. T. Osypowski, Phys. Rev. D 7, 3444 (1973).

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[14]A comprehensive analysis of low-energy photoproduction with this σ-type term is currently in progress.

[15]The model amplitude although unique is not unitary, hence calculation of the background phase δ_e requires a separate assumption. The reasonable prescriptions $\tan\delta_e = qf_e$ or $\sin\delta_e = qf_e$ yield background phases differing by about one half of one degree at resonance energy.

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