μ^- capture on C^{12} and the tensor form factor*

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Recent experiments in nuclear beta decay have indicated a large tensor form factor in the decay $B^{12} \rightarrow C^{12} + e^- + \bar{\nu}_e$. Using elementary-particle methods the implications of this result are studied for the μ-capture reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu_\mu$. Agreement is hard to obtain unless the induced pseudoscalar is somewhat larger than its simple PCAC (partial conservation of axial-vector current) value.

Recent measurements of the rate and recoil polarization for the μ-capture reaction[1,2]

$$\mu^- + C^{12} \rightarrow B^{12} + \nu_\mu$$

provide an interesting laboratory for the study of the semileptonic weak interaction. Such a transition between states of spin 0 and spin 1 involves four independent form factors—three axial and one polar. We define

$$\begin{aligned}\langle B^{12}_{p_2}| A_\lambda | C^{12}_{p_1}\rangle = & -g_A(q^2)\frac{1}{4M^2}(\xi^*_\lambda P^2 - P_\lambda \xi^*\cdot P)\\ & -g_T(q^2)\frac{1}{4M^2}(\xi^*_\lambda P\cdot q - P_\lambda \xi^*\cdot q)\\ & -g_P(q^2)\frac{1}{(4M^2)^2} q_\lambda \xi^*\cdot q\, P\cdot(P-q),\end{aligned} \qquad (1)$$

$$\langle B^{12}_{p_2}| V_\lambda | C^{12}_{p_1}\rangle = g_M(q^2)\frac{i}{4M^2}\epsilon_{\alpha\beta\lambda\delta}\xi^{*\alpha}q^\beta p^\delta,$$

where $P = p_1 + p_2$, $q = p_1 - p_2$, and $M = \frac{1}{2}(M_1 + M_2)$. Here g_A is the usual Gamow-Teller form factor, g_P is the induced pseudoscalar, while g_M is the weak-magnetism term. The additional structure function g_T is the induced tensor and is generally neglected in discussions of muon capture.[3]

We may extract $g_A(0)$ from the ft value for B^{12} decay:

$$\begin{aligned}|g_A(0)| &= \left(\frac{2\pi^3 \ln 2}{G^2\cos^2\theta_C\, m_e^5 ft}\right)^{1/2}\\ &= (0.721 \pm 0.002) \text{ for } ft = 11\,890 \pm 60 \text{ sec}\end{aligned} \qquad (2)$$

(see Ref. 4). The value of the weak-magnetism form factor can be found from CVC,[5] which predicts

$$\begin{aligned}|g_M(0)| &= \left[\frac{6M^2\Gamma(C^{12*}(15.11 \text{ MeV}) \rightarrow C^{12} + \gamma)}{\alpha E_\gamma^3}\right]^{1/2}\\ &= (3.84 \pm 0.05)|g_A(0)|A \text{ for } \Gamma = (37.0 \pm 1.1) \text{ eV}\end{aligned} \qquad (3)$$

(see Ref. 6) or from the measured slope of the $B^{12} \rightarrow C^{12} + e^- + \bar{\nu}_e$ shape factor[7]

$$\begin{aligned}\alpha &= \frac{4}{3M}\frac{g_M(0)}{g_A(0)}\\ &= (5.5 \pm 1.0)\times 10^{-3}/\text{MeV} - \frac{10}{3M},\end{aligned} \qquad (4)$$

(see Ref. 8), which yields $g_M(0)/g_A(0) = (3.63 \pm 0.70)A$, in extremely good agreement with the CVC (conserved vector current) value. Finally, the experiment of Sugimoto, Tanihata, and Goring provides information about $g_T(0)$

$$\begin{aligned}\beta &= \frac{g_M(0) - g_T(0)}{3Mg_A(0)}\\ &= (3.1 \pm 0.6)\times 10^{-3}/\text{MeV}\end{aligned} \qquad (5)$$

(see Ref. 9), which gives $g_T(0)/g_A(0) = (-4.86 \pm 1.68)A$ using CVC.

The value of the induced pseudoscalar is *a priori* unknown. However, PCAC (partial conservation of axial-vector current) predicts[10]

$$g_P(q^2) = \frac{4M^2 g_A(q^2)}{m_\pi^2 - q^2}, \qquad (6)$$

although recent work on nuclear effects suggests a value somewhat smaller.[11] Equation (6) yields the canonical Goldberger-Treiman value[12]

$$f_P = \frac{m_\mu}{2MA}\frac{g_P(q^2 = -0.74\, m_\mu^2)}{g_A(q^2 = -0.74\, m_\mu^2)} = 7.1. \qquad (7)$$

For muon-capture work we need the values of all form factors at $q^2 = -0.74\, m_\mu^2$. However, experimental evidence is available only for $g_M(q^2)$ in the form of inelastic electron scattering data (using CVC)

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{4M^2}|g_M(q^2)|^2\\ &\times\left[\frac{(E'+E)^2 - 2EE'(1+\cos\theta) + (E - E'\cos\theta)^2}{4E^2(1-\cos\theta)^2}\right],\end{aligned} \qquad (8)$$

where E, E' are the lab energies of the electron before and after scattering and θ is the laboratory scattering angle. This yields[13]

$$\frac{g_M(q^2=-0.74\,m_\mu^2)}{g_M(q^2=0)}=0.745\pm 0.015. \tag{9}$$

Now in impulse approximation we have

$$\frac{g_A(q^2)}{g_A(0)}\cong\frac{\langle C^{12}\|\sum_i \tau_i^+\sigma_i j_0(qr)\|B^{12}\rangle}{\langle C^{12}\|\sum_i\tau_i^+\sigma_i\|B^{12}\rangle}\approx\frac{g_M(q^2)}{g_M(0)}, \tag{10}$$

which we shall take to be valid in the following.[3] However, note that the predicted capture rate is sensitively dependent on this assumption.

The μ^- capture rate is calculated to be

$$\Gamma=\frac{G^2\cos^2\theta_C}{2\pi^2}\left(\frac{Z\alpha m_\mu M_1}{M_1+m_\mu}\right)^3\frac{k_0^2}{1+k_0/E_2}\frac{E_2+M_2}{2E_2}C \times(3G_A^2+2G_AG_P+G_P^2), \tag{11}$$

where k_0, E_2 are the neutrino and final hadron energies, $\Delta=M_1-M_2$ is the mass difference, and

$$G_A=g_A\left(1+\frac{\Delta^2-q^2}{4M^2}\right)+g_T\frac{\Delta}{2M}+g_M\frac{k_0}{2M}\left(1+\frac{\Delta}{2M}\right),$$

$$G_P=\frac{k_0}{2E_2}\left[g_A\left(1-\frac{m_\mu+2\Delta}{2M}+\frac{2\Delta^2-m_\mu\Delta-2q^2}{4M^2}\right)-g_M\frac{E_2}{M}\left(1+\frac{\Delta}{2M}\right)\right.$$
$$\left.-g_T\left(1+\frac{m_\mu}{2M}+\frac{\Delta(m_\mu+\Delta)}{4M^2}\right)-g_P\frac{m_\mu}{2M}\left(1+\frac{\Delta}{2M}\right)\left(1-\frac{\Delta}{2M}+\frac{\Delta^2-q^2}{4M^2}\right)\right], \tag{12}$$

where $C=0.885$ is a correction factor due to the finite nuclear size.[3]

The predicted capture rate is shown in Fig. 1 as a function of f_P to be compared with the experimental value[1]

$$\Gamma=(6.2\pm 0.3)\times 10^3\ \text{sec}^{-1}.$$

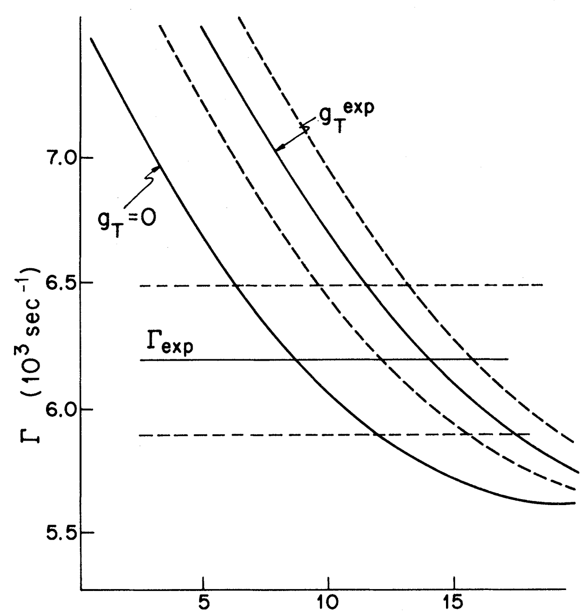

FIG. 1. The muon-capture rate as a function of f_P. Note that $g_A(q^2)$ has been corrected for nonzero values of $g_T(q^2)$ for the effect of the induced tensor on the ft value. The dashed lines represent the one-standard-deviation values.

We note that with the Sugimoto *et al.* value of g_T agreement is obtained only if

$$10<f_P<19\ ,$$

while if $g_T=0$ the one-standard-deviation values are

$$6<f_P<11,$$

which includes the Goldberger-Treiman prediction.

Now consider the recoil B^{12} polarization for which the Louvain group finds[2]

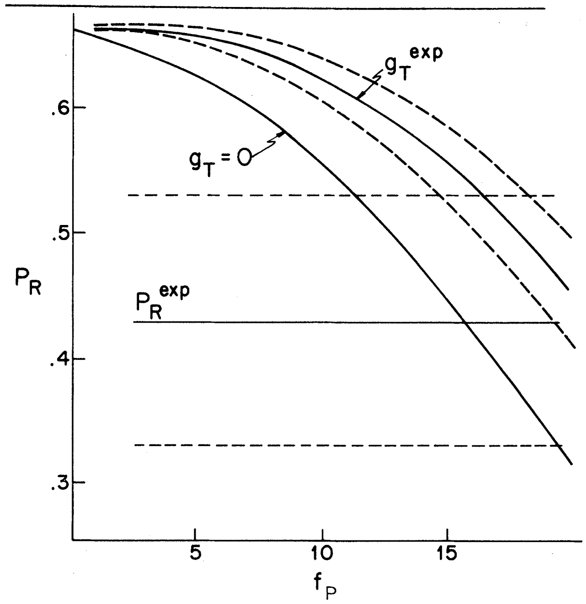

FIG. 2. The recoil polarization as a function of f_P. The dashed lines represent the one-standard-deviation values.

$P_R = 0.43 \pm 0.10.$

The calculated polarization is

$$P_R = \frac{2}{3}\,\frac{3G_A^2 + 2G_AG_P}{3G_A^2 + 2G_AG_P + G_P^2}\,, \qquad (13)$$

for which numerical results are shown in Fig. 2. With $g_T(q^2)/g_A(q^2) = -4.86A$ agreement is obtained for

$16 < f_P < 25,$

while if $g_T = 0$ we find

$11 < f_P < 19.$

Clearly the situation is unsatisfactory and further work is needed. In the case of the capture rate we have not assigned a theoretical uncertainty to our predictions. However, the major source of error [given the correctness of Eq. (10)] is the 2% uncertainty in $g_M(q^2)/g_M(0)$, which becomes a 4% uncertainty for the capture rate. Then with the tensor form factor required to understand the Japanese experiment there exists a possible disagreement with the rate predicted with the PCAC value for g_P, especially if the nuclear effects are taken into account. On the other hand, if we neglect the tensor, agreement can be obtained. Independent of the implications for second-class currents then it is important to verify the tensor term given by Sugimoto *et al.*

The recoil polarization is not subject to uncertainties of the q^2 dependence. Here the results appear anomalous in that for neither value of g_T can the experimental results be accounted for by the expected pseudoscalar. It is important to confirm the polarization measurements, as this correlation is particularly sensitive to f_P provided the induced tensor is known.

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[1]G. H. Miller, M. Eckhause, F. R. Kane, P. Martin, and R. E. Welsh, Phys. Lett. 41B, 50 (1972).

[2]A. Possoz, D. Favart, L. Grenacs, J. Lehmann, P. Macq, D. Meda, L. Pallfy, J. Julien, and C. Samour, Phys. Lett. 50B, 438 (1974).

[3]C. W. Kim and H. Primakoff, Phys. Rev. 140, B566 (1965); L. Foldy and J. D. Walecka, *ibid.* 140, B1339 (1965); K. Kubodera and C. W. Kim, Phys. Lett. 43B, 275 (1973).

[4]F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. A114, 1 (1968).

[5]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[6]B. T. Chertok, C. Sheffield, J. Lightbody, S. Penner, and D. Blum, Phys. Rev. C 8, 23 (1973).

[7]Note that although some authors apply a finite-nuclear-size correction to these data in extracting the weak-magnetism contribution, this is incorrect since the Fermi function used already includes such effects. We thank Professor F. P. Calaprice for a discussion of this point.

[8]Y. K. Lee, L. W. Mo, and C. S. Wu, Phys. Rev. Lett. 10, 253 (1963); C. S. Wu, Rev. Mod. Phys. 36, 618 (1964).

[9]K. Sugimoto, I. Tanihata, and J. Goring, Phys. Rev. Lett. 34, 1533 (1975).

[10]See, e.g., B. Holstein, Rev. Mod. Phys. 46, 789 (1974).

[11]C. W. Kim and J. S. Townsend, Phys. Rev. D 11, 656 (1975); K. Ohta and M. Wakamatsu, Phys. Lett. 51B, 325 (1974); 51B, 337 (1974).

[12]M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).

[13]K. Kubodera and C. W. Kim, Ref. 3.