Multiparticle ordering as basis for nonperturbative dynamics

P. du T. van der Merwe
Atomic Energy Board, Private Bag x256, Pretoria 0001, Republic of South Africa
(Received 28 October 1975)

The implication of structure analysis is investigated as a starting point for a nonperturbative treatment of the dynamical equations of field theory. The multiparticle ordering entails the arrangement of physical processes according to subclasses of processes embodied by the appropriate correlation functions. Closure among the correlation functions is obtained by exploiting the cluster properties of the correlation functions. The investigation culminates in a method for incorporating self-consistent feedback contributions as nested contributions which result in a method of linearization, in a channel-oriented approach, of the underlying nonlinear equations. The method of approximation exhibits a form-preserving property in that the structure of the equations is maintained in successive steps, thus simulating the underlying nonlinear dynamics by allowing the system to react on itself.

I. INTRODUCTION

In an accompanying paper[1] an analysis of the structural aspects of functional field equations has been given which proposes an ordering of the field equations in terms of its multiparticle attributes. It has been found that the many-particle amplitudes exhibit clustering properties which are the primary mover for generating subclasses of scattering processes. Insofar as the method of structure analysis provides a realization of the mass operator which incorporates for successive actions of the primary interaction all processes of a particular multiparticle nature, the method provides a systematic approach of multiparticle ordering. The purpose of the present paper is to investigate the implications of multiparticle ordering as a basis for nonperturbative dynamics. The method transcends known methods where partial summations of infinite subsets of diagrams of simple structures may be effected. The functional method provides a general method of analysis by recognizing the relationship of functional derivatives of fewer-particle attributes to their multiparticle scattering counterparts.

The general criteria for the relevance of structure analysis as a starting point towards a dynamical approximation method derive from the close connection of the inclusion of subclasses of processes to the systematic inclusion of nearby singularities. Furthermore, the particular ordering by interaction and subclasses of multiparticle scattering, which include all rescattering processes of a particular nature of given particles of a subgroup, would seem to favor a system of particles interacting by short-ranged forces such as the strongly interacting particles found in nature. This conclusion follows in view of the circumstance that the correlations between particles ensure that contributions due to terms of higher order in the interaction which are left out at a particular stage may, in fact, be ineffective as a consequence of the short-ranged forces in question. The foregoing discussion would seem to imply that the method may be less suitable for a system of particles interacting by long-range forces. On the other hand, since the ordering involves multiparticle correlations and interacting strength, and in view of the existence in nature of a reciprocity between force strength, and the range of interaction, it could well be that the weakness of the force syphons off few-particle correlations. The success of the standard perturbation expansion in the strength of interaction in quantum electrodynamics could provide a case in point. Even though the success of the method of multiparticle ordering may thus depend on the particular situation encountered, it may provide a viable method in a large number of physical problems. As a prelude to the investigation of the method in concrete problems, a formulation of the form of the dynamical equations is given in this work.

It is found that the formulation of the exact equations in terms of the multiparticle contributions provides criteria for arriving at successive stages of approximation. The neglect of an appropriate three-particle contribution, in particular, is shown in Sec. II to yield a Bethe-Salpeter equation with Hartree-Fock kernel. In Sec. IIB feedback effects due to the preceding stage are shown to result in approximations analogous to the screened-potential approach,[2] whereas the inclusion of the three-particle contribution incorporates inelastic threshold effects. At this particular stage of development the three-particle contribution is characterized by declustered two-particle contributions acting in tandem. Feynman graphical calculus is employed to illustrate the origin of these features. In order to provide further perspective of the nature of the approximations in the

context of the underlying nonlinear equations, Sec. III provides a brief account of parquet structures, whereupon a nested scattering matrix method is developed. Methods to transcend a channel-oriented approach and aspects of crossing symmetry are discussed in Secs. IV and V.

II. TWO-PARTICLE SCATTERING EQUATIONS

A. Random-phase approximation

In an accompanying paper[1] a functional-derivative technique has been employed to derive exact scattering equations in terms of multiparticle attributes. Recall that the equation for the mass operator,

$$M_{ij}=K(1)_{im,jn}G_{nm}-V_{ip,rq}G_{rk}G_{qn}T_{kn,jm}G_{mp}, \quad (2.1)$$

in conjunction with the scattering equation

$$T_{ib,ja}=K_{ib,ja}-iK_{is,jx}G_{xy}T_{yb,fa}G_{fs} \quad (2.2)$$

and the effective two-particle interaction

$$K_{is,jx}=-i\frac{\partial M_{ij}}{\partial G_{xs}} \quad (2.3)$$

yields

$$T_{ib,ja}=K(1)_{ib,ja}-iK(1)_{is,jx}G_{xy}T_{yb,fa}G_{fs} -iV_{ip,rq}G^{-1}{}_{bw}A_{rqw,jpv}G^{-1}{}_{va}, \quad (2.4)$$

where the three-particle amplitude denotes

$$A_{rqw,jpv}=\frac{\partial}{\partial q_{vw}}(G_{qn}G_{rk}T_{kn,jm}G_{mp}). \quad (2.5)$$

The embedded three-particle contribution and the nonlinear nature of Eq. (2.4) are contributing factors that stand in the way of finding self-consistent solutions to the equation, particularly in view of the fact that the nonlinear structure of the equation itself is a consequence of a self-consistent feedback inherent in the equation.

In order to ease into the discussion of finding self-consistent approximation procedures, consider, first of all, the simplest approximation to the equations in which the effect of scattering on the mass operator is neglected. Retaining the first term of Eq. (2.1) yields the t-channel scattering Bethe-Salpeter equation ("random-phase" approximation)

$$t(1)_{ib,ja}=K(1)_{ib,ja}-iK(1)_{is,jx}G(1)_{xy}t(1)_{yb,fa}G(1)_{fs}. \quad (2.6)$$

In the formulation of the exact equation the Green's function, G, satisfies the equation

$$(D+q+M)_{ik}G_{ks}=\delta_{is}, \quad (2.7)$$

where M denotes the mass operator of Eq. (2.1). The Green's function of Eq. (2.6) denotes the contribution of Eq. (2.7) where the approximation, Eq. (2.6), to the T matrix is invoked. In the discussion in the sequel the symbol for the exact Green's function will be employed throughout, since the particular Green's function implied will be clear from the context in which it enters.

The Bethe-Salpeter equation (2.6) represents an approximation in which repeated binary collisions in the t channel is accommodated as required by unitarity and is known to provide a useful approximation in relativistic problems to describe bound states and resonances and in nonrelativistic problems to describe phenomena such as plasma oscillations and collective excitations at low temperatures like zero sound where detailed correlations are not of importance. The unimportance of feedback effects in the latter case may be traced to the decoupling of the zero-sound mode to its constituents in the event that the Hartree-Fock effective two-particle interaction is itself weak.[3] The breakdown of the Hartree-Fock approximation, on which the derivation of Eq. (2.6) is based, for the mass operator at low temperatures and the accompanying fluctuations in particle number may herald the appearance of phase transition[4] and indicate the importance of feedback effects of the collective excitations on Eq. (2.1) itself. The thermodynamic instability of the Hartree-Fock approximation is reflected as a dynamic instability of the random-phase approximation which implies that the latter is a short-time approximation.

The exact equation (2.4) provides a criterion for the applicability of the approximations of this section in terms of the role of the three-particle amplitude Eq. (2.5). The feedback of the solution of Eq. (2.6), in determining the three-particle amplitude, ushers in self-consistent approximations analyzed in the following sections.

B. Feedback random-phase approximations

An analysis of the structure of the mass operator resulted in a multiparticle ordering of the form[1]

$$M_{ij}=iK(1)_{in,jm}G_{mn}-V_{ib,hy}G_{hk}G_{yc}T(1)_{kc,ja}G_{ab} +iV_{ip,rq}G_{rk}B(1)_{kmq,lnp}V_{ln,jm}+\cdots, \quad (2.8)$$

where

$$T(1)_{kc,ja}=K(1)_{kc,ja}-iK(1)_{ks,jx}G_{xy}T_{yc,fa}G_{fs}. \quad (2.9)$$

In the part of this section which follows the effect of feedback on the integral equation is considered by taking the first two terms of Eq. (2.8) into account in the approximation where Eq. (2.9) is taken as the Bethe-Salpeter equation (2.6). In contrast to the latter t-channel equation, it is appropriate to define the s-channel amplitude suggested by the first two terms of Eq. (2.8),

$$t(1,2)_{ib,ja}=K(1)_{ib,ja}+iV_{ib,hy}G_{hk}G_{yc}t(1)_{kc,ja}, \quad (2.10)$$

so that the first two terms of Eq. (2.8) may be expressed by the equation

$$M(1,2)_{ij}=it(1,2)_{ib,ja}G_{ab}. \quad (2.11)$$

The effective two-particle interaction generated by the last equation reads

$$K(1,2)_{is,jx}=t(1,2)_{is,jx}+U(1,2)_{ibs,jax}G_{ab}, \quad (2.12)$$

where

$$U(1,2)_{ibs,jax}=\frac{\partial}{\partial G_{xs}}t(1,2)_{ib,ja} \quad (2.13)$$

$$=iV_{ib,xy}G_{yc}t(1)_{sc,ja}+iV_{ib,hx}G_{hk}t(1)_{ks,ja}+iV_{ib,hy}G_{hk}G_{yc}U(1)_{kcs,jax}. \quad (2.14)$$

The remaining problem consists in finding the three-particle contribution

$$U(1)_{kcs,jax}=\frac{\partial}{\partial G_{xs}}t(1)_{kc,ja} \quad (2.15)$$

which according to Eq. (2.6) satisfies the integral equation

$$U(1)_{kcs,jax}=-iK(1)_{kd,jx}t(1)_{sc,fa}G_{fd}-iK(1)_{ks,jz}G_{zy}t(1)_{yc,xa}-iK(1)_{kd,jz}G_{zy}G_{fd}U(1)_{ycs,fax}. \quad (2.16)$$

In conjunction with Eq. (2.6), the last equation may be inverted with the result

$$U(1)_{ycs,fax}=-i[t(1)_{yd,fx}t(1)_{sc,za}+t(1)_{ys,fz}t(1)_{dc,xa}]G_{zd}. \quad (2.17)$$

The clustering property of the three-particle amplitude exhibited by the last equation has a simple origin. If we picture the random-phase scattering matrix as consisting of a string of bubbles, the three-particle amplitude, according to Eq. (2.15), results on opening a particular G line, which culminates in the clustered structure of Eq. (2.17)

A graphical representation of the effective two-particle interaction given by Eqs. (2.12), (2.14), and (2.17) is given in Fig. 1. It is seen that the effective two-particle interaction is of the form of the exact one, the exact T matrix being replaced by the "random-phase" scattering matrix of Eq. (2.6). This result is a consequence of the substitution rule[1] which holds for Eq. (2.8) and expresses the form-preserving properties of multichannel ordering. The figure furthermore depicts the nature of the feedback: In addition to the Hartree-Fock contribution, the kernel of the t channel, Eq. (2.2), possesses contributions due to the t-channel Bethe-Salpeter equation (2.6) convoluted in a particular way to render the contribution t-channel two-particle irreducible. The first term itself results in a vertex modification whereas the second term may be characterized as a contribution similar to the screened-potential approximation of Baym and Kakanoff[2] of importance in plasma physics and in describing anomalous features of Fermi liquids due to soft bosons. The following terms describe the contributions due to three-particle processes that exhibit the clustering features of Eq. (2.17). It is expected that they provide an important approximation for incorporating inelastic threshold effects due to feedback of composite particles or collective states generated from the Bethe-Salpeter equation (2.6). The integral equation generated by these terms is analogous to the relativistic equations proposed by the author[5] in a threshold enhancement model for higher excited resonances as corroborated in subsequent work.[6] The main physical features of an equation of this kind derive from the strong forces operative in the proximity of the inelastic threshold. A further important feature of resonances located in the proximity of a threshold[7] is connected with its weak coupling to its constituents (sharp resonances). Considerations based on extended three-particle unitarity provide a complementary way of looking at inelastic threshold effects.[8]

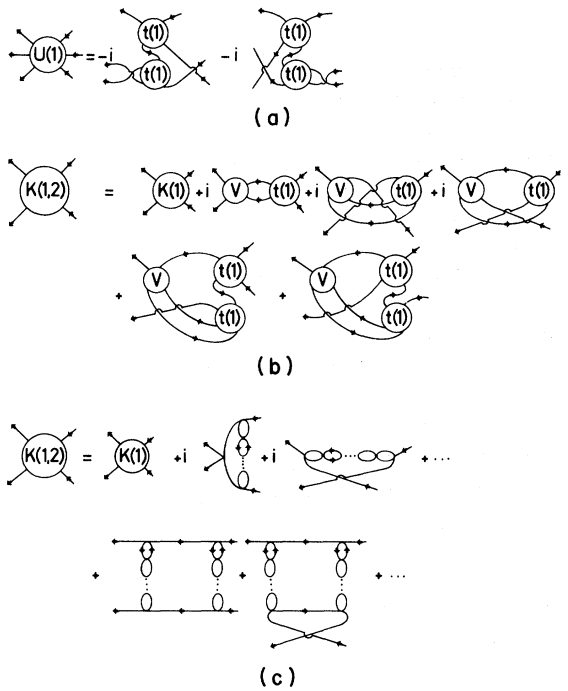

FIG. 1. The effective two-particle interaction in the feedback approximation Eq. (2.12).

III. NESTED SCATTERING-MATRIX APPROXIMATION

It is shown in this section that the effective two-particle interaction derived from the three-particle contribution to the mass operator embodies the effect of the kernel obtained in the feedback random-phase approximation and that the method of multiparticle ordering provides a systematic method of incorporating feedback effects due to fewer-particle dynamics and therefore provides a systematic method of achieving self-consistency.

For the sake of orientation consider, first of all, the contribution to the effective interaction obtained by opening the G line of Eq. (2.5):

$$K'(3)_{is,jx} = V_{ip,xq} B(1)_{smq,lnp} V_{ln,jm}. \tag{3.1}$$

This last equation in conjunction with Eq. (5.5) of the accompanying paper,[1] which incorporates parquet contributions to the three-particle amplitude, yields the following expression for the kernel:

$$\begin{aligned} K'(3)_{is,jx} = &- V_{ip,xq} V_{ln,jm} t(1)_{kb,ca} (G_{mp} G_{qb} G_{an} G_{cl} + G_{mb} G_{qp} G_{an} G_{cl} + G_{mb} G_{an} G_{cp} G_{ql}) \\ &+ i V_{ip,xq} V_{ln,jm} T_{kb,ca} t(1)_{vy,zo} (G_{mv} G_{qy} G_{zb} G_{op} G_{an} G_{cl} + G_{mb} G_{av} G_{qy} G_{zn} G_{op} G_{cl} \\ &+ G_{mb} G_{an} G_{cv} G_{qy} G_{zl} G_{op}). \end{aligned} \tag{3.2}$$

Closer inspection of the last equation reveals that it is of the form of the exact kernel in which the feedback random-phase approximation takes the place of the exact T matrix. It therefore appears that the kernel obtained in successive steps of multichannel ordering systematically incorporates fewer-particle contributions as nested contributions. Equation (3.2), furthermore, provides a general basis for the observation of Roulet, Gavoret, and Nozieres[9] of the construction of parquet diagrams by "bubble" insertions. The foregoing considerations exhibit the parquet approximations as embedded nested contributions.

After these preliminary observations we now proceed to a more general formulation of the nested scattering-matrix method. Equation (2.5) in conjunction with the defining equation for the three-particle amplitude,

$$B(1)_{kmq,lnp} = \frac{\partial}{\partial q_{pq}} [G_{mb} T(1)_{kb,ca} G_{an} G_{cl}], \tag{3.3}$$

yields

$$M_{ij}(3) = - V_{ip,rq} G_{rk} \frac{\partial}{\partial q_{pq}} [G_{mb} T(1)_{kb,ca} G_{an} G_{cl}] V_{ln,jm}. \tag{3.4}$$

Employing the symmetry of the mass operator,

$$\begin{aligned} M(2)_{kj} &= - V_{kb,ca} G_{an} G_{cl} T(1)_{ln,jm} G_{mb} \\ &= - T(1)_{kb,ca} G_{an} G_{cl} V_{ln,jm} G_{mb}, \end{aligned} \tag{3.5}$$

it follows that

$$M(3)_{ij} = - i V_{ip,rq} G_{rk} \left[\frac{\partial}{\partial q_{pq}} M(2)_{kj}\right]. \tag{3.6}$$

The last equation may be cast in the form

$$M(3)_{ij} = V_{ip,rq} G_{rk} K(2)_{kb,ja} F(2)_{aq,bp}, \tag{3.7}$$

where

$$\begin{aligned} F(2)_{aq,bp} &= \frac{\partial G(2)_{ab}}{\partial q_{pq}} \\ &= - G(2)_{ap} G(2)_{qb} \\ &\quad + i G(2)_{ac} G(2)_{qd} t(2)_{cd,fg} G(2)_{fb} G(2)_{gp}. \end{aligned} \tag{3.8}$$

In the last equation the index of the Green's function has been added to indicate that the particular Green's function at this level of approximation derives from the representation of the mass-operator Eq. (3.5). Furthermore, in view of the fact that

$$\begin{aligned} t(2)_{kb,ja} &= K(2)_{kb,ja} \\ &\quad - i K(2)_{ks,jx} G(2)_{xy} t(2)_{yb,fa} G(2)_{fs}, \end{aligned} \tag{3.9}$$

where $K(2)$ denotes the feedback Bethe-Salpeter kernel (with the Hartree-Fock contribution truncated), it follows that

$$M(3)_{ij} = - V_{ip,rq} G_{rk} G_{ap} G_{qb} t(2)_{kb,ja}. \tag{3.10}$$

If we now introduce the amplitude

$$t(2,3)_{ib,ja} = i V_{ib,hy} G_{hk} G_{yc} t(2)_{kc,ja}, \tag{3.11}$$

the procedure of Sec. II B may be followed to yield the result

$$K(2,3)_{is,jx} = t(2,3)_{is,jx} + U(2,3)_{ibs,jax} G_{ab}, \tag{3.12}$$

where

$$U(2,3)_{ibs,jax} = \frac{\partial}{\partial G_{xs}} t(2,3)_{ib,ja} \tag{3.13}$$

$$\begin{aligned} &= i V_{ib,xy} G_{yc} t(2)_{sc,ja} \\ &\quad + i V_{ib,hx} G_{hk} t(2)_{ks,ja} \\ &\quad + i V_{ib,hy} G_{hk} G_{yc} U(2)_{kcs,jax}. \end{aligned} \tag{3.14}$$

The kernel (3.12), derived from the three-particle contribution, exhibits the form-preserving property noted before and demonstrates the nature of the approximation method as successively nested structures if we view (3.12) expressed in terms of the random-phase amplitude $t(1)$ itself. As a consequence of the nested nature of the amplitudes a new feature appears on the three-particle level. This arises in view of the fact that the three-particle amplitude

$$U(2)_{kcs,jax} = \frac{\partial}{\partial G_{xs}} t(2)_{kc,ja} \tag{3.15}$$

$$= \frac{\partial}{\partial G_{xs}} [K(2)_{kc,ja} - iK(2)_{kd,jz} G_{zy} t(2)_{yc,fa} G_{fd}], \tag{3.16}$$

in contrast to the three-particle amplitude $U(1)$ which is characterized by the Hartree-Fock kernel, depends on $t(1)$ as a nested contribution to the effective two-particle interaction $K(2)$. As a consequence, the effective three-particle interaction becomes operative and is defined by

$$L(2)_{kcs,jax} = -\frac{\partial K(2)_{kc,ja}}{\partial G_{xs}}. \tag{3.17}$$

It follows from Eq. (3.16) that the three-particle amplitude satisfies

$$U(2)_{kcs,jax} = H_{kcs,jax} - iK(2)_{kd,jz} G_{zy} G_{fd} U(2)_{ycs,fax}, \tag{3.18}$$

where the inhomogeneous term denotes

$$H_{kcs,jax} = -L(2)_{kcs,jax} + iL(2)_{kds,jzx} G_{zy} t(2)_{yc,fa} G_{fd} - iK(2)_{kd,jx} t(2)_{sc,fa} G_{fd} - iK(2)_{ks,jz} G_{zy} t(2)_{yc,xa}. \tag{3.19}$$

Before deriving the form of the effective three-particle interaction, an inversion of Eq. (3.19) is effected. With this view, it is useful to notice that

$$t(2)_{kd,jz} G_{zy} G_{fd} H_{ycs,fax} = K(2)_{kd,jz} G_{zy} G_{fd} U(2)_{ycs,fax} \tag{3.20}$$

by virtue of Eq. (3.19) and the conjugate equation of Eq. (3.9). The solution may therefore be expressed as

$$U(2)_{kcs,jax} = [\delta_{ok}\delta_{jp} - it(2)_{kn,jm} G_{mo} G_{pn}] H_{ocs,pax}, \tag{3.21}$$

which simplifies to

$$U(2)_{kcs,jax} = -i[t(2)_{kd,jx} t(2)_{sc,za} + t(2)_{ks,jz} t(2)_{dc,xa}] G_{zd} + t(2)_{kn,jm} G_{mo} G_{pn} G_{zy} t(2)_{yc,fa} G_{fd} L(2)_{ods,pzx} - L(2)_{kcs,jax} + it(2)_{kn,jm} G_{mo} G_{pn} L(2)_{ocs,pax} + iG_{zy} t(2)_{yc,fa} G_{fd} L(2)_{kds,jzx}. \tag{3.22}$$

Comparison of the last equation, which is depicted in Fig. 2, to Eq. (2.17) reveals form preservation of the three-particle amplitude in question, apart from the new feature connected with the appearance of the effective three-particle interaction. The latter quantity was introduced[10] in connection with the formulation of relativistic three-particle equations where a perturbative expansion of the three-particle interaction was given. A nonperturbative result is obtained from Eqs. (3.17) and (3.12) by straightforward manipulation:

$$\begin{aligned}
-L(2)_{kcs,jax} = {} & iV_{kc,xy} G_{yf} t(1)_{sf,ja} + iV_{kc,hx} G_{hd} t(1)_{ds,ja} + iV_{kc,hy} G_{hd} G_{yf} U(1)_{dfs,jax} + iV_{kb,ax} t(1)_{cs,jf} G_{fb} \\
& + iV_{ks,ay} G_{yh} t(1)_{ch,jx} + iV_{kb,ay} G_{yh} G_{fb} U(1)_{chs,jfx} + iV_{kb,xa} t(1)_{sc,jg} G_{gb} \\
& + iV_{ks,ha} G_{hf} t(1)_{fc,jx} + iV_{kb,ha} G_{hf} G_{gb} U(1)_{fcs,jgx} \\
& + V_{kb,xy} G_{yf} [t(1)_{sd,ja} G_{zd} t(1)_{cf,zm} + t(1)_{sc,jz} G_{zd} t(1)_{df,am}] G_{mb} \\
& + V_{kb,hx} G_{hg} [t(1)_{gd,ja} G_{zd} t(1)_{cs,zm} + t(1)_{gc,jz} G_{zd} t(1)_{ds,am}] G_{mb} \\
& + V_{kb,hy} G_{hg} G_{yf} [t(1)_{gs,ja} t(1)_{cf,xm} + t(1)_{gc,jx} t(1)_{sf,am}] G_{mb} \\
& + V_{ks,hy} G_{hg} G_{yf} [t(1)_{gd,ja} G_{zd} t(1)_{cf,zx} + t(1)_{gc,jz} G_{zd} t(1)_{df,ax}] \\
& + V_{kb,hy} G_{hg} G_{yf} G_{zd} [U(1)_{dgs,jax} t(1)_{cf,zm} + t(1)_{gd,ja} U(1)_{cfs,zmx} + U(1)_{gcs,jzx} t(1)_{df,am} \\
& \qquad + t(1)_{gc,jz} U(1)_{dfs,amx}] G_{mb},
\end{aligned} \tag{3.23}$$

where $U(1)$ is given by Eq. (2.17).

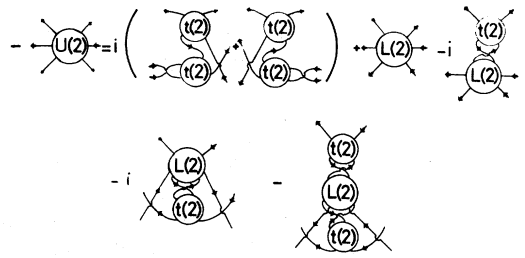

FIG. 2. Schematic presentation of the three-particle contribution defined by Eq. (3.22).

In spite of the complicated appearance of the three-particle interaction its interpretation is transparent. According to Eq. (3.17), it derives from the two-particle interaction by opening G lines and therefore enumerates three-body interactions generated by multiple scattering composed of binary collisions acting in tandem and nested in the appropriate way to render the final contribution irreducible.

The preceding considerations represent a method of self-consistent approximation of the nonlinear integral equations embodied by a nested feedback mechanism. By picturing the action of the system on itself as a step-by-step process in terms of multiparticle ordering, a linearization of the nonlinear dynamics is accomplished; the form-preserving property of the procedure constitutes an essential ingredient in achieving self-consistency. Although the equations appear extremely involved in the general formulation presented, one should keep in mind that in special contexts where dominant features could be exploited considerable simplification can be expected, judging from the situation encountered with regard to parquet structures in the Kondo problem where dominant "bubble" singularities[9] result in nonlinear equations that are tractable.[11]

IV. s-CHANNEL CONSIDERATIONS

In Sec. III an approximation procedure has been developed for incorporating self-consistent corrections as nested contributions. For the sake of clarity, particular emphasis has been placed on t-channel considerations. Even though a channel-oriented approach may be a natural approach from the physical point of view when dominant forces operate in a particular channel, channel orientation entails the breakdown of crossing symmetry. Before taking up these questions in the framework of the exact equations in the next section, s-channel aspects are briefly investigated and lead to results analogous to the "effective potential" method developed in nuclear structure theory.[12]

If the two-particle interaction possesses a hard core, it is necessary to define an "effective potential" by the ladder sum

$$t(s)_{ib,ja} = K(1)_{ib,ja} + iV_{ib,rq}\, G_{rk}\, G_{qn}\, t(s)_{kn,ja}\,. \tag{4.1}$$

If, furthermore, the latter s-channel contribution is singled out in Eq. (2.1), the mass operator becomes

$$M(s)_{ij} = it(s)_{ib,ja}\, G_{ab}\,. \tag{4.2}$$

The effective two-particle interaction therefore reads

$$K(s)_{is,jx} = t(s)_{is,jx} + U(s)_{ibs,jax}\, G_{ab}\,, \tag{4.3}$$

where, according to Eq. (4.1), the three-particle amplitude satisfies

$$\begin{aligned} U(s)_{ibs,jax} = {} & iV_{ib,xq}\, G_{qn}\, t(s)_{sn,jq} \\ & + iV_{ib,rx}\, G_{rk}\, t(s)_{ks,ja} \\ & + iV_{ib,rq}\, G_{rk}\, G_{qn}\, U(s)_{kns,jax}\,. \end{aligned} \tag{4.4}$$

Equations (4.1) and (4.4) suggest that the three-particle amplitude is of the form

$$U(s)_{kns,jax} = it(s)_{kn,ox} X_{os,ja}\,. \tag{4.5}$$

The unknown amplitude of the last equation may be determined by substituting Eq. (4.5) into Eq. (4.4) and by utilizing the symmetry property

$$t(s)_{sp,ja} = -t(s)_{ps,ja} \tag{4.6}$$

to yield the result

$$U(s)_{kns,jax} = it(s)_{kn,ox}\, G_{op}\, t(s)_{ps,ja} \tag{4.7}$$

by virtue of Eq. (4.1).

Equations (4.3) and (4.7) for the kernel, substituted into the integral Eq. (2.2), yield

$$\begin{aligned} t(s,t)_{ib,ja} = {} & t(s)_{ib,ja} + it(s)_{im,oa}\, G_{op}\, t(s)_{pb,jn}\, G_{nm} - it(s)_{is,jx}\, G_{xy}\, t(s,t)_{yb,fa}\, G_{fs} \\ & + t(s)_{im,ox}\, G_{op}\, t(s)_{ps,jn}\, G_{nm} G_{xy}\, t(s,t)_{yb,fa}\, G_{fs}\,. \end{aligned} \tag{4.8}$$

The last equation, which is depicted in Fig. 3, describes the interplay of s- and t-channel effects where the effective potential generated in the s channel serves as driving term for the t-channel integral equation.

The interplay between the channels gives rise to new structures, such as nonparquet contributions, which are absent in the channel-oriented approach. The starting point of this section, based on a screened po-

tential, is known to be of relevance in nuclear structure theory and classical gases. In the next section we return to a study of the coupling between the channels in the framework of the exact equations.

V. CONCLUSIONS

In the previous sections linear approximation methods to the nonlinear equations have been considered by allowing the system to react on itself. Methods that involve channel orientation may be of patent advantage in particular physical situations. In order to gain further understanding of self-consistent crossed-channel fertilization as qualifying aspects of the interplay among the channels and the nature of the nonlinear equations, it is appropriate to view these questions in the framework of the exact equations.

Recall that the neglect of the three-particle amplitude, A, in the exact equation (2.4) results in the random-phase approximation. It is appropriate to express the equation in terms of the amplitude

$$P_{rqw,jmv} = \frac{\partial}{\partial q_{vw}} (G_{qn} G_{rk} T_{kn,jm}) \tag{5.1}$$

which is related to the amplitude, A, by

$$A_{rqw,jpv} = G_{mp} P_{rqw,jnv} - G_{mv} G_{wp} G_{qn} T_{kn,jm} G_{rk} + i G_{qn} G_{rk} T_{kn,jm} G_{mx} G_{wy} T_{xy,zo} G_{zp} G_{ov}, \tag{5.2}$$

in terms of which Eq. (2.4) assumes the form

$$T_{ib,ja} = K(1)_{ib,ja} - iK(1)_{is,jx} G_{xy} T_{yb,fa} G_{fs} + iV_{ib,rq} G_{qn} G_{rk} T_{kn,ja} + V_{is,rq} G_{qn} G_{rk} T_{kn,jx} G_{xy} T_{yb,fa} G_{fs} + P'_{ib,ja}, \tag{5.3}$$

where

$$P'_{ib,ja} = -iV_{ip,rq} G^{-1}{}_{bw} P_{rqw,jmv} G^{-1}{}_{va} G_{mp}. \tag{5.4}$$

Equation (4.3) provides a convenient representation which exhibits the influence of the different channels. It may be expressed in a transparent form in terms of the scattering matrix:

$$T(s)_{ib,ja} = K(1)_{ib,ja} + iV_{ib,rq} G_{rk} G_{qn} T_{kn,ja}. \tag{5.5}$$

The index s of the last equation merely serves as a convenient label for indicating the oriented s-channel sum involved and does not imply an s-variable dependence only, for the scattering matrix of the right-hand side designates the full scattering matrix. Were one to include only the s-channel contributions to the T matrix on the right-hand side, the effective potential method of Sec. IV would obtain, whereas the t-channel contribution on the right-hand side would serve as a starting point for the nested T-matrix method of Sec. II.

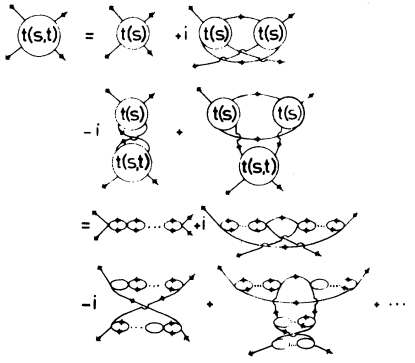

FIG. 3. A simplified version of the two-particle scattering in a linearized approximation to exhibit the interplay among channels.

It is appropriate to point out that Eqs. (2.1) and (4.5), expressed in the form

$$M_{ij} = iT(s)_{ib,ja} G_{ab}, \tag{5.6}$$

provide a convenient starting point for deriving the exact nonlinear scattering equations. According to the last equation, the effective two-particle interaction reads

$$K_{is,jx} = T(s)_{is,jx} + U(s)_{ibs,jax} G_{ab}, \tag{5.7}$$

where

$$U(s)_{ibs,jax} = \frac{\partial}{\partial G_{xs}} T(s)_{ib,ja} \tag{5.8}$$

$$= iV_{ib,xq} G_{qn} T_{sn,ja} + iV_{ib,rx} G_{rk} T_{ks,ja} + iV_{ib,rq} G_{rk} G_{qn} U_{kns,jax}, \tag{5.9}$$

which is the exact form of the effective two-particle interaction.[1] It is worthwhile to emphasize that the latter equations for the kernel are of the same form as encountered in the multiparticle ordering approximations developed in the previous sections. These approximations to the equations entailed a linearization of the equations by allowing the system to react on itself in a channel-oriented fashion. This indicates that nonlinear and crossing aspects of the equations are intricately connected. The coupling among the channels may be made explicit by formulating the equation

that complements Eq. (4.5). Substituting Eq. (4.5) into (4.3), it follows that

$$T_{ib,ja} = T(s)_{ib,ja} - iT(s)_{is,jx} G_{xy} T_{yb,fa} G_{fs} + P'_{ib,ja}. \quad (5.10)$$

Equations (4.5) and (4.10) constitute a set of coupled equations that describe the dynamics of s- and t-channel scattering which serves to gauge the importance of mutual effects of the channels on each other. Unfortunately, short of solving the coupled equations exactly, approximation to the equations is considerably complicated by the presence of the exact Green's functions which in the complete problem must acquire dressing in the different channels simultaneously, the absence of which would induce violation of unitarity. In contrast to the channel-oriented approach where feedback contributions could be systematically elucidated by a nested scattering-matrix method, the crossing aspects locked up in the equations, like captives, only partially emerge.

In conclusion, structure analysis provides a tool for revealing the multiparticle forms that lie hidden in the functional field equations and expresses the qualities as visible forms of subclasses of processes by means of multiparticle ordering. Multiparticle ordering, in turn, makes it possible to assemble the order of multiparticle structures in new arrangements which manifest themselves as nested structures and dictates nonperturbative approximation methods to simulate the underlying nonlinear dynamics in the form of feedback contributions. The progressive steps of the analysis exhibit a measure of form preservation which provides the germ for generating self-consistency. The results found in the initial stages of analysis reproduce familiar approximations and, in effect, provide the natural grain that yield first to analysis in terms of binary scattering approximations, criteria for the validity of which are provided in terms of the importance of multiparticle contributions.

[1]P. du T. van der Merwe, preceding paper, Phys. Rev. D 13, 2383 (1976).
[2]G. Baym and L. P. Kadanoff, Phys. Rev. 124, 287 (1961); G. Baym, *ibid.* 127, 1391 (1962).
[3]P. du T. van der Merwe (unpublished).
[4]N. D. Mermin, Ann. Phys. (N.Y.) 18, 421 (1962); 18, 454 (1962).
[5]P. du T. van der Merwe, Phys. Rev. 145, 1257 (1966).
[6]R. Aaron and R. D. Amado, Phys. Rev. Lett. 27, 1316 (1971); R. Aaron, D. C. Teplitz, R. D. Amado, and J. E. Young, Phys. Rev. 187, 2047 (1969).
[7]P. du T. van der Merwe, Lett. Nuovo Cimento 2, 113 (1969).
[8]L. F. Cook and B. W. Lee, Phys. Rev. 127, 297 (1962); A. M. Gleeson and K. T. Mahanthappa, Nuovo Cimento 48A, 849 (1967); 51A, 40 (1967).
[9]B. Roulet, J. Gavoret, and P. Nozieres, Phys. Rev. 178, 1072 (1969); 178, 1084 (1969).
[10]P. du T. van der Merwe, Phys. Rev. D 5, 353 (1972).
[11]A. A. Abrikosov, Physics (N.Y.) 2, 5 (1965); K. Fukushina, Prog. Theor. Phys. 46, 1307 (1971); R. Sundaram, J. Low Temp. Phys. 12, 25 (1973); 20, 117 (1975).
[12]E.g., A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-particle systems* (McGraw-Hill, New York, 1971). The specialization of the general formulation of this work to the nonrelativistic many-body problem would result in a formalism in which particles and "holes" are treated symmetrically as is the case in the Migdal-Galitskii formulation. In this connection, see also the following: R. F. Bishop, Ann. Phys. (N.Y.) 77, 106 (1973); R. F. Bishop, M. R. Strayer, and J. M. Irvine, Phys. Rev. A 10, 2423 (1974).