Structure analysis: A method for analyzing field equations

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A method of analysis for investigating the structural aspects of field equations is developed. An ordering of the field equations in terms of subclasses of multiparticle scattering processes is effected which proposes equivalent realizations of the mass operator. The characterization of the mass operator in terms of subclasses of scattering processes exhibits a form-preserving property which provides a starting point for self-consistent nonperturbative methods as a framework for describing dynamical properties of field equations and many-body problems.

I. INTRODUCTION

The functional-derivative technique constitutes a powerful method for formulating exact field equations.[1] In view of the complexity of the resulting field equations, which for nonlinear quantum systems represent an infinite hierarchy of coupled equations involving correlations among successively larger numbers of particles, the basic problem consists in formulating a closed set of equations that is amenable to solution. The iteration of the exact equations exhibits recurrent topological substructures which in special cases may be resummed into partial subsets described by integral equations. However, the functional-derivative technique itself furnishes the proper way for deriving relationships among the correlation functions and for generating conserving approximations,[2] even in the nonequilibrium situation, by studying the response of the system to an external source.

Rather than undoing the precise functional equations it is complemented in this work by a new method of analysis, referred to as structure analysis, that results in a compact formalism that succeeds in representing the exact field equations, successively, in terms of higher-order correlation functions in a particularly suggestive form. Closure among the infinite sequence of multiparticle amplitudes is achieved by exploiting relationships between the Schwinger mass operator in the presence of external sources and multiparticle correlation functions which are brought to light. The structure that emerges from the analysis is particularly transparent in the case of the mass operator and exhibits the remarkable feature of a self-generating set of equations in which, at each step, a slight mutation occurs which possesses the germ for generating higher-particle attributes. Each successive step of the analysis is terminated by a corresponding multiparticle amplitude. These correlation functions satisfy integral equations in which the kernel and inhomogeneous terms consist of amplitudes of lower-particle attributes which enter in a de-clustered form. A brief account of the method with reference to three-particle equations[3] recently appeared in print.[4]

In the next section, a generating functional is defined and its application to the derivation of single- and two-particle equations considered, particularly with a view of obtaining appropriate equations for the mass operator which is the central object of interest in the analysis. The latter quantity, in turn, defines the effective two-particle interaction which provides the starting point for arriving at two-particle scattering processes. Elimination of the transient aspects related to the presence of an external source results in a hierarchic order of multiparticle amplitudes which shed light on the crossing and nonlinear aspects of the equations. In Sec. III, three-particle equations are investigated and the analysis of multichannel contributions to the mass operator is taken up in Sec. IV. This results in a multichannel ordering, the structural aspects of which are exhibited in the last section.

II. GENERATING FUNCTIONAL AND TWO-PARTICLE EQUATIONS

The description of relativistic microscopic particles resulted in promoting the classical fields to operator-field variables with specified commutators. The differential equations for the corresponding fields supply a system of equations for the particle correlation functions that contain the dynamical properties of the particles. Functional methods provide a convenient method for deriving the quantum-field equations in question by considering the response of the system to classical external sources.

Consider a system of spin-$\frac{1}{2}$ particles in interaction, described by the equation of motion

$$D_{ik}\psi_k + V_{im,ln}\overline{\psi}_m\psi_n\psi_l = 0\,,$$
$$V_{im,ln} = \tfrac{1}{2}(\overline{V}_{im,ln} - \overline{V}_{mi,ln})\,, \tag{2.1}$$

where D_{ik} may represent a Schrödinger operator or Dirac operator of the form

$$D_{ik}\psi_k = -i(\gamma^\mu)_{ik}\delta(x_i - x_k)\frac{\partial}{\partial x_k^\mu}\psi(x_k)\,. \tag{2.2}$$

A compact notation is employed, repeated indices are summed, and the Latin indices denote the space-time and spin variables. The equation of motion, which may serve as a starting point[5,6] in many-body theory or in a nonlinear theory of self-interacting particles, is to be understood as a symbolic equation for arriving at appropriate equations for the multiparticle Green's function that defines the dynamical properties of the system.

Consider the generating functional

$$U = \exp(iq_{lk}\psi_k\overline{\psi}_l)\,,$$
$$W = \langle 0|TU|0\rangle\,, \tag{2.3}$$

where q_{lk} denotes a classical nonlocal source which removes a particle from the system at one point and restores it at another. In particular, the linear response of the single-particle Green's function

$$G_{kl} = iW^{-1}\langle 0|T\psi_k\overline{\psi}_l U|0\rangle = \frac{\partial}{\partial q_{lk}}\ln W \tag{2.4}$$

defines the four-point function or two-particle correlation function

$$F_{km,ln} = \frac{\partial G_{kl}}{\partial q_{nm}} = G_{km,ln} - G_{kl}G_{mn}\,, \tag{2.5}$$

where the two-particle Green's function is given by

$$G_{km,ln} = W^{-1}\langle 0|T\psi_k\psi_m\overline{\psi}_l\overline{\psi}_n U|0\rangle\,. \tag{2.6}$$

The inclusion of effects of the external source by generating the correlation functions as functional derivatives guarantees the maintenance of conservation laws.[2]

The set of differential equations for the Green's functions is generated from the equation of motion and the commutation relations of the field operators. Thus, the equation of motion implies that

$$(D+q)_{ik}G_{ks} + iV_{im,kn}G_{kn,sm} = \delta_{is} \tag{2.7}$$

provided that the following equal-time commutation relation is satisfied[6]:

$$\delta(t_k - t_s)(\psi_k\overline{\psi}_s + \overline{\psi}_s\psi_k) = \delta_{ks}\,. \tag{2.8}$$

Continuation of this procedure would yield an infinite set of equations involving successively higher-order correlation functions. On taking the relationships between the higher-point functions into account and truncating the set of equations by neglecting correlation functions of more than, say, n variables, a Tamm-Dancoff-type approximation would result. The disadvantage of this method is that self-energy effects are not treated self-consistently nor are the clustering properties of multichannel correlation functions taken into account.

This can be effected by adopting the mass-operator formalism. Accordingly, define the mass operator by

$$M_{ik}G_{ks} = iV_{im,kn}G_{kn,sm}\,, \tag{2.9}$$

so that

$$(D+q+M)_{ik}G_{ks} = \delta_{is} = G^{-1}{}_{ik}G_{ks}\,. \tag{2.10}$$

Forming the functional derivative of the last equation with respect to the external source q, and employing the fact that the dependence of the mass operator on the source is only via the Green's function, it follows that

$$F_{iq,mp} = -G_{ip}G_{qm} - iG_{ik}K_{ks,lr}F_{rq,sp}G_{lm} \tag{2.11}$$

in view of the definition of the effective two-particle interaction by

$$K_{ks,lr} = -i\frac{\partial M_{kl}}{\partial G_{rs}}\,. \tag{2.12}$$

It is convenient to transform from the Bethe-Salpeter equation for the correlation function to the two-particle equation for the scattering matrix defined by

$$F_{iq,mp} = -G_{ip}G_{qm} + iG_{ij}G_{qn}T_{jn,fg}G_{fm}G_{gp}\,, \tag{2.13}$$

which satisfies

$$T_{ib,ja} = K_{ib,ja} - iK_{is,jx}G_{xd}T_{db,ca}G_{cs}\,. \tag{2.14}$$

Utilizing Eqs. (2.5), (2.9), and (2.13) yields the following exact equation for the mass operator[7]:

$$M_{ij} = i(V_{im,jn} - V_{im,nj})G_{nm} - V_{ip,rq}G_{rk}G_{qn}T_{kn,jm}G_{mp}\,. \tag{2.15}$$

The relationship between the two-particle attributes may be drawn closer by noticing that the T matrix may be regarded as the linear response of the mass operator. This can be shown as follows. Consider

$$\frac{\partial M_{kl}}{\partial q_{pq}} = iK_{ks,lr}F_{rq,sp}\,, \tag{2.16}$$

which in virtue of Eq. (2.13) and (2.14) yields

$$\frac{\partial M_{kl}}{\partial q_{pq}} = -iG_{qn}T_{kn,lg}G_{gp}\,. \tag{2.17}$$

Substituting this result into Eq. (2.15) results in an integro-differential equation for the mass operator

$$M_{ij} = i(V_{im,jn} - V_{im,nj})G_{nm} - iV_{im,kn}G_{ky}\left(\frac{\partial M_{yj}}{\partial q_{mn}}\right). \tag{2.18}$$

The above equations constitute a complete set of equations for determining the single- and two-particle attributes of the system in the presence of an external source. Equation (2.18) in conjunction with the Dyson equation, derived from Eq. (2.10), provides coupled equations for the mass operator and the single-particle Green's function. The solution of these equations (with appropriate boundary conditions) in the presence of the external source would provide a starting point for arriving at two-particle attributes by functional variation with respect to the external source.

One may also look upon the external-source technique as a device for arriving at the multichannel features of the theory. As soon as the dependence on the source is not explicit, the multichannel aspects of the scattering theory become manifest. The connection between Eqs. (2.15) and (2.18) may be looked upon as the simplest example of this feature of the hierarchy of equations which will be encountered in the development in the following sections.

We ease into the discussions by deriving the exact equation for the two-particle T matrix. This is accomplished by finding the effective two-particle interaction, which according to Eqs. (2.12) and (2.15) assumes the form

$$\begin{aligned} K_{is,jx} = K(1)_{is,jx} &+ iV_{ip,xq}G_{qn}T_{sn,jm}G_{mp} \\ &+ iV_{ip,rx}G_{rk}T_{ks,jm}G_{mp} + iV_{is,rq}G_{rk}G_{qn}T_{kn,jx} \\ &+ iV_{ip,rq}G_{rk}G_{qn}G_{mp}U_{kns,jmx}, \end{aligned} \tag{2.19}$$

where

$$U_{kns,jmx} = \frac{\partial T_{kn,jm}}{\partial G_{xs}}. \tag{2.20}$$

Furthermore,

$$K(1)_{is,jx} = V_{is,jx} - V_{is,xj} \tag{2.21}$$

represents the contribution to the effective interaction in a Hartree-Fock approximation to the mass operator Eq. (3.3) by equating $T=0$. This assertion assumes that rescattering is not of consequence. In the exact analysis, the remaining terms of Eq. (2.19) express the effect of scattering on the mass operator. Since differentiation with respect to a G line corresponds to the removal of this line from the corresponding diagram of the amplitude, Eq. (2.20) represents the effect of three-particle processes.

In conjunction with Eq. (2.19), the two-particle T matrix, Eq. (2.14), satisfies

$$T_{ib,ja} = K(1)_{ib,ja} - iK(1)_{is,jx}G_{xy}T_{yb,fa}G_{fs} + I_{ib,ja} + J_{ib,ja}, \tag{2.22}$$

where

$$\begin{aligned} I_{ib,ja} = {} & iV_{ip,aq}G_{qn}T_{bn,jm}G_{mp} + iV_{ip,ra}G_{rk}T_{kb,jm}G_{mp} + iV_{ib,rq}G_{rk}G_{qn}T_{kn,ja} \\ & + (V_{ip,xq}G_{qn}T_{sn,jm}G_{mp} + V_{ip,rx}G_{rk}T_{ks,jm}G_{mp} + V_{is,rq}G_{rk}G_{qn}T_{kn,jx})G_{xy}T_{yb,fa}G_{fs} \end{aligned} \tag{2.23}$$

and

$$J_{ib,ja} = iV_{ip,rq}G_{rk}G_{qn}G_{mp}(U_{knb,jma} - iU_{kns,jmx}G_{xy}T_{yb,fa}G_{fs}). \tag{2.24}$$

Before interpreting the equations, it is appropriate to simplify Eq. (2.22). With this view, consider the quantity defined by the equation

$$\Lambda_{knb,jma} = -G^{-1}{}_{bi}\frac{\partial T_{kn,jm}}{\partial q_{ei}}G^{-1}{}_{ea} \tag{2.25}$$

$$= -G^{-1}{}_{bi}U_{knd,jmc}F_{ci,de}G^{-1}{}_{ea} \tag{2.26}$$

$$= U_{knb,jma} - iU_{kns,jmx}G_{xy}T_{yb,fa}G_{fs}, \tag{2.27}$$

so that Eq. (2.24) reads

$$J_{ib,ja} = iV_{ip,rq}G_{rk}G_{qn}\Lambda_{knb,jma}G_{mp}. \tag{2.28}$$

It therefore follows that whereas the three-particle attribute that enters into the equation for the effective two-particle interaction derives from variation with respect to the exact single-particle Green's function, the corresponding quantity in the scattering matrix results from variation due to the external source.

In concluding the analysis of the two-particle equations, it is appropriate to find the relationship of the vertex function to known quantities. The vertex function is defined by

$$\Gamma_{bc,ad} = \frac{\partial G^{-1}{}_{ba}}{\partial q_{dc}} = -G^{-1}{}_{bi}F_{ic,ed}G^{-1}{}_{ea}. \tag{2.29}$$

The meaning of the system of two-particle equations is now discussed using Fig. 1 as a guide. Equation (2.14) is an exact equation for the two-particle T matrix, in the t channel, in terms of the effective two-particle interaction given by Eq. (2.19). Equations (2.22) to (2.28) express the T matrix in terms of the initial interaction, the first terms sum the bubbles and ladders whereas the nonlinearity of the equation which results in combinations of ladders and bubbles (vertex corrections) is a consequence of a feedback mechanism as pointed out below. Whereas the T matrix generates both reducible and irreducible contributions, the t-channel kernel itself only contains contributions which are two-particle irreducible in the t channel. These contributions consist of fully irreducible ones such as the Hartree-Fock term and the three-particle contribution, plus further contributions which may be two-particle reducible in the crossed channel (s or u channels). The latter contributions have a particularly suggestive form and appear as two-particle crossed-channel convolutions of the starting interaction with the two-particle T matrix. This implies that these contributions arise because of a feedback phenomenon: In spite of the fact that the full T matrix contains two-particle reducible contributions in the channel under consideration, the feedback nevertheless results in contributions to the two-particle kernel, because the presence of the convolution in the crossed channel renders the total contribution two-particle irreducible in the desired channel. The nonlinearity of the two-particle equation is an immediate consequence of the feedback phenomenon and is an important agent in achieving crossing symmetry and self-consistency. Although a detailed consideration of the numerous symmetries inherent in the equations is beyond the scope of this investigation, it nevertheless provides new insight into questions of crossing-symmetric two-particle equations, particularly in view of the fact that in the formulation of crossing-symmetric equations[8] to date, the kernels of the equations are only implicitly known.

The analysis leaves the multichannel contributions undetermined, the determination of which is the task of structure analysis developed in the following sections.

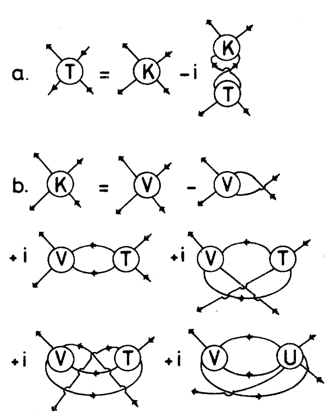

FIG. 1. (a) Bethe-Salpeter equation for the t channel. (b) Kernel of Bethe-Salpeter equation which depicts embedded s-channel contributions.

III. THREE-PARTICLE ASPECTS

In Sec. II a formulation of the two-particle equations was developed which indicated a hierarchic order of multichannel aspects. In this section the analysis is subordinated to an investigation of some of the three-particle aspects.

It was found that the two-body T matrix satisfies the equation

$$T_{ib,ja} = K(1)_{ib,ja} - iK(1)_{is,jx} G_{xy} T_{yb,fa} G_{fs} + (I+J)_{ib,ja}\,, \tag{3.1}$$

where

$$J_{ib,ja} = -iV_{ip,rq} G_{rk} G_{qn} G_{mp} G^{-1}{}_{bw} \left(\frac{\partial T_{kn,jm}}{\partial q_{vw}}\right) G^{-1}{}_{va}\,. \tag{3.2}$$

In order to indicate the role of the multiple parts of three-particle processes, consider first of all the three-particle amplitude defined by

$$A_{rqw,jpv} = \frac{\partial}{\partial q_{vw}} (G_{qn} G_{rk} T_{kn,jm} G_{mp}) \tag{3.3}$$

which, in conjunction with the equations of the previous section, may be expressed as

$$J_{ib,ja} = -I_{ib,ja} - iV_{ip,rq} G^{-1}{}_{bw} A_{rqw,jpv} G^{-1}{}_{va}\,, \tag{3.4}$$

so that

$$T_{ib,ja} = K(1)_{ib,ja} - iK(1)_{is,jx} G_{xy} T_{yb,fa} G_{fs} - iV_{ip,rq} G^{-1}{}_{bw} A_{rqw,jpv} G^{-1}{}_{va}\,. \tag{3.5}$$

The advantage of the latter representation is connected to the fact that it may serve as a starting point for initiating a nonperturbative approximation procedure. The assumption of a vanishing three-particle amplitude Eq. (3.3) results in a "random-phase approximation" for the two-particle T matrix. An approximation of this kind which singles out a particular channel above others naturally results in the breakdown of crossing symmetry. A more natural starting point towards crossing symmetry would correspond to a vanishing three-particle amplitude Eq. (3.2) according to the discussion of the previous section. These remarks serve to illustrate the role that knowledge of the nature of multichannel processes could provide in deriving criteria for effecting nonperturbative approximations.

As a preparation for the analysis of the mass operator, which accentuates the process of multiparticle ordering, a number of three-particle amplitudes are introduced which arise in this connection:

$$B_{kmq,lnp} = \frac{\partial}{\partial q_{pq}} (G_{mb} T_{kb,ca} G_{an} G_{cl})\,, \tag{3.6}$$

$$E_{kmq,cnp}=\frac{\partial}{\partial q_{pq}}(G_{mb}T_{kb,ca}G_{an}). \quad (3.7)$$

The connection between the amplitudes is expressed by

$$B_{kmq,lnp}=b_{kmq,lnp}+E_{kmq,cnp}G_{cl}, \quad (3.8)$$

where

$$b_{kmq,lnp}=-G_{mb}T_{kb,ca}G_{an}G_{cp}G_{ql}+iG_{mb}T_{kb,ca}G_{an}G_{cx}G_{qy}T_{xy,zo}G_{zl}G_{op} \quad (3.9)$$

and

$$A_{rqw,jpv}=a_{rqw,jpv}+G_{rk}E_{kqw,jpv}, \quad (3.10)$$

where

$$a_{rqw,jpv}=-G_{rv}G_{wk}G_{qn}T_{kn,jm}G_{mp}+iG_{ry}G_{wc}T_{yc,fd}G_{fk}G_{dv}G_{qn}T_{kn,jm}G_{mp}. \quad (3.11)$$

The quantity a corresponds to the amplitude b when the indices are read in the reverse order, so that the three-particle amplitudes defined by Eqs. (3.3) and (3.6) are conjugate to each other.

In order to gain further knowledge of the nature of three-particle processes a derivation of the three-particle equation satisfied by the amplitude Eq. (3.6) is considered in the concluding part of the section. It follows from Eq. (2.17) that Eq. (3.6) may be cast in the form

$$B_{kmq,lnp}=i\frac{\partial}{\partial q_{pq}}\left[\left(\frac{\partial M_{kc}}{\partial q_{nm}}\right)G_{cl}\right], \quad (3.12)$$

so that

$$B_{kmq,lnp}=G_{mb}T_{kb,ca}G_{an}F_{cq,lp}+R_{kmq,lnp}, \quad (3.13)$$

where

$$R_{kmq,lnp}=i\left(\frac{\partial^2 M_{kc}}{\partial q_{pq}\partial q_{nm}}\right)G_{cl} \quad (3.14)$$

$$=i\left(\frac{\partial}{\partial q_{pq}}iK_{ky,cx}F_{xm,yn}\right)G_{cl} \quad (3.15)$$

$$=L_{kyh,cxg}F_{gq,hp}F_{xm,yn}G_{cl}-K_{ky,cx}Q_{xmq,ynp}G_{cl}. \quad (3.16)$$

In the above derivation free use has been made of the chain rule and the fact that the source dependence enters via the Green's function. Furthermore, an effective three-particle interaction is defined by

$$L_{kyh,cxg}=-\partial K_{ky,cx}/\partial G_{gh}, \quad (3.17)$$

and Q denotes

$$Q_{xmq,ynp}=\frac{\partial}{\partial q_{pq}}F_{xm,yn}, \quad (3.18)$$

which equals

$$Q_{xmq,ynp}=\frac{\partial}{\partial q_{pq}}(-G_{xn}G_{my}+iG_{xf}G_{md}T_{fd,gh}G_{gy}G_{hn}) \quad (3.19)$$

$$=-F_{xq,np}G_{my}-G_{xn}F_{mq,yp}+iF_{xq,fp}G_{md}T_{fd,gh}G_{gy}G_{hn}-G_{xf}B_{fmq,ynp}. \quad (3.20)$$

The above equations yield the three-particle equation

$$B_{kmq,lnp}=I_{kmq,lnp}-iK_{ky,cx}G_{cl}G_{xf}B_{fmq,ynp}, \quad (3.21)$$

where the inhomogeneous term is given by

$$\begin{aligned}I_{kmq,lnp}=&-G_{ma}T_{ka,cb}G_{bn}G_{cp}G_{ql}+iG_{ma}T_{ka,cb}G_{bn}G_{ci}G_{qj}T_{ij,rs}G_{rl}G_{sp}+L_{kyh,cxg}G_{xn}G_{my}G_{cl}G_{gp}G_{qh}\\&-iL_{kyh,cxg}G_{gp}G_{qh}G_{xa}G_{mb}T_{ab,de}G_{dy}G_{en}G_{cl}-iL_{kyh,cxg}G_{xn}G_{my}G_{gf}G_{qe}T_{fe,st}G_{sh}G_{tp}G_{cl}\\&-L_{kyh,cxg}G_{xa}G_{mb}T_{ab,de}G_{dy}G_{en}G_{gf}G_{qo}T_{fo,st}G_{sh}G_{tp}G_{cl}-K_{ky,cx}G_{cl}(G_{xp}G_{qn}G_{my}+G_{xn}G_{mp}G_{qy})\\&+iK_{ky,cx}G_{cl}(G_{my}G_{xi}G_{rn}+G_{xn}G_{mi}G_{ry})G_{qj}T_{ij,rs}G_{sp}+iK_{ky,cx}G_{cl}G_{xp}G_{qf}G_{md}T_{fd,gh}G_{gy}G_{hn}\\&+K_{ky,cx}G_{cl}G_{xi}G_{qj}T_{ij,rs}G_{rf}G_{sp}G_{md}T_{fd,gh}G_{gy}G_{hn}.\end{aligned} \quad (3.22)$$

The last equation, which is depicted in Fig. 2, provides an integral equation for the three-particle amplitude expressed in terms of two-particle attributes and an effective three-particle interaction denoted by L. A most important feature of the equation as far as its structural aspect is concerned, is the clustering property displayed by the inhomogeneous term which exhibits the three-particle contribution as a two-particle T matrix accompanied by a spectator particle or an rescattering of particles generated by the two-particle T matrices acting in succession. This feature, which reappears again on a higher-particle level, culminates in a kind of multiparticle ordering.

IV. MASS-OPERATOR FORMALISM AND STRUCTURE ANALYSIS

In this section it is shown that the mass operator plays a central role in providing closure among the multiparticle correlation functions.

In the previous sections an integro-differential equation for the single-particle Green's function has been formulated which defined the mass operator

$$M_{ik}G_{ks} = iV_{im,kn}G_{nk,sm}\,. \tag{4.1}$$

With the aid of the conjugate equation of motion an alternative equation for the mass operator may be formulated which expresses a simple symmetry constraint

$$G_{ik}M_{ks} = iG_{im,kn}V_{kn,sm}\,. \tag{4.2}$$

Employing the result

$$G_{im,kn} = G_{ik}G_{mn} - G_{in}G_{mk} + iG_{ia}G_{mb}T_{ab,cd}G_{ck}G_{dn}\,, \tag{4.3}$$

which is a consequence of Eqs. (2.5) and (2.13), it follows from Eq. (4.2) that

$$M_{rs} = i(V_{rn,sm} - V_{nr,sm})G_{mn} - G_{mb}T_{rb,cd}G_{ck}G_{dn}V_{kn,sm}\,. \tag{4.4}$$

It follows from Eqs. (2.17), (2.18), and (4.3) that

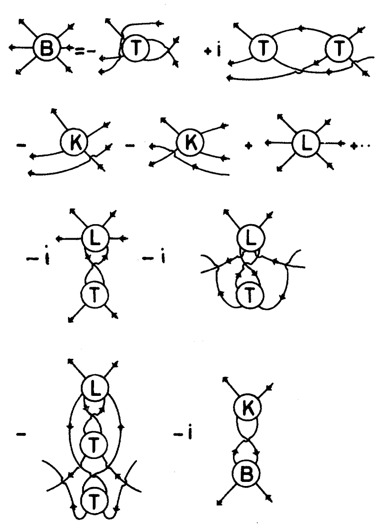

FIG. 2. Graphical representation of the three-particle equation (3.21).

$$M_{ij} = iO_{in,jm}G_{mn} - iV_{ip,rq}G_{rk}\left(\frac{\partial M_{kj}}{\partial q_{pq}}\right) \tag{4.5}$$

and

$$M_{kj} = iN_{kn,jm}G_{mn} - i\left(\frac{\partial M_{kc}}{\partial q_{nm}}\right)G_{cl}V_{ln,jm}\,, \tag{4.6}$$

where

$$O_{in,jm} = V_{in,jm} - V_{in,mj} \tag{4.7}$$

and

$$N_{kn,jm} = V_{kn,jm} - V_{nk,jm} = O_{kn,jm}\,. \tag{4.8}$$

Although there are further symmetries inherent in the theory as a result of the symmetries of the Green's functions, we shall not enter into symmetry considerations in detail, for this could well mask the simplicity of the method of structure analysis.

Forming the functional derivative of Eq. (4.6), yields

$$\frac{\partial M_{kj}}{\partial q_{pq}} = iO_{kn,jm}F_{mq,np} - V_{ln,jm}B_{kmq,lnp}\,, \tag{4.9}$$

where B denotes a three-particle scattering amplitude given by

$$B_{kmq,lnp} = i\frac{\partial}{\partial q_{pq}}\left[\left(\frac{\partial M_{kc}}{\partial q_{nm}}\right)G_{cl}\right] \tag{4.10}$$

which is closely connected to the three-particle amplitude, Eq. (3.6), encountered in formulating exact two-particle equations. This can be shown by utilizing Eq. (2.17), with the result

$$B_{kmq,lnp} = \frac{\partial}{\partial q_{pq}}(G_{ma}T_{ka,cb}G_{bn}G_{cl})\,. \tag{4.11}$$

Information about the structure of the mass operator is obtained by substituting Eq. (4.9) into Eq. (4.5)

$$M_{ij} = M_{ij}(1) + iV_{ip,rq}G_{rk}B_{kmq,lnp}V_{ln,jm}\,, \tag{4.12}$$

where

$$M_{ij}(1) = iO_{in,jm}G_{mn} + V_{ip,rq}G_{rk}O_{kn,jm}F_{mq,np}\,. \tag{4.13}$$

The last equation is the expression of an important connection in structure analysis which may be expressed as follows. It is, first of all, to be noticed that

$$iK(1)_{ib,ja} = \frac{\partial}{\partial G_{ab}}M(0)_{ij} = \frac{\partial}{\partial G_{ab}}(iO_{in,jm}G_{mn}) \tag{4.14}$$

yields

$$K(1)_{ib,ja} = O_{ib,ja} = V_{ib,ja} - V_{ib,aj}\,. \tag{4.15}$$

The effective two-particle interaction defined by

Eq. (4.15) corresponds to a Hartree-Fock approximation which results from Eq. (4.3) by equating $T=0$, which assumes that rescattering is not of consequence for the mass operator. In an exact analysis, the effect of two-particle scattering on the mass operator is expressed by Eq. (4.13). In order to demonstrate this fact, consider

$$K(1)_{ks,jx}F_{xq,sp}, \tag{4.16}$$

which according to Eq. (2.11) equals

$$-G_{qb}G_{ap}[K(1)_{kb,ja}-iK(1)_{ks,jx}G_{xy}T_{yb,fa}G_{fs}], \tag{4.17}$$

so that

$$K(1)_{ks,jx}F_{xq,sp}=-G_{qb}G_{ap}T(1)_{kb,ja}, \tag{4.18}$$

where, according to Eq. (3.1), we have that

$$\begin{aligned} T(1)_{ib,ja} &= T_{ib,ja}-(I+J)_{ib,ja} \\ &= T_{ib,ja}+iV_{ip,rq}G^{-1}{}_{bw}A_{rqw,jpv}G^{-1}{}_{va} \\ &= K(1)_{ib,ja}-iK(1)_{is,jx}G_{xy}T_{yb,fa}G_{fs} \end{aligned} \tag{4.19}$$

with the result that

$$M_{ij}(1)=iK(1)_{in,jm}G_{mn}-V_{ip,rq}G_{rk}T(1)_{kb,ja}G_{qb}G_{ap}. \tag{4.20}$$

It therefore follows that the two-particle T matrix which enters in the successive steps of structure analysis is not the nonlinear object with embedded three-particle structure of Eqs. (2.21), (2.22), and (2.26), but rather as shown by Eq. (4.19), the effective kernel being determined by the Hartree-Fock kernel (4.15). In particular, if three-particle processes characterized by the amplitude, A of Eq. (3.3), in a given physical situation is not of importance, $T(1)$ would correspond to a "random-phase" generated scattering matrix. This represents a useful approximation in the description of bound states or collective excitations in which correlations due to rescattering are not of importance as in the case of zero sound or plasmons. The contribution of more detailed correlations is incorporated by the last term of the exact equation for the mass operator which, according to Eqs. (4.11), (4.12), and (4.20), reads

$$\begin{aligned} M_{ij} &= iK(1)_{in,jm}G_{mn}-V_{ip,rq}G_{rx}G_{qy}T(1)_{xy,jw}G_{wp} \\ &\quad +iV_{ip,rq}G_{rk}B_{kmq,lnp}V_{ln,jm}. \end{aligned} \tag{4.21}$$

The form of the last equation demonstrates the emergence of a substitution rule: The exact Eq. (4.21) results from the exact Eq. (2.15) by replacement of the exact two-particle T matrix by the matrix $T(1)$ which satisfies Eq. (4.19); the residual term of Eq. (4.21) thus incorporates rescattering and many-channel contributions that are left out of consideration when two-particle scattering is described by $T(1)$, which one may wish to consider for practical reasons, instead of the full T matrix. We shall find on further analysis that the substitution rule persists in the higher-particle sector and that the investigation of the multiparticle structure of the mass operator may be pictured as a process of multiparticle ordering.

The preceding formulation of this section completes the first phase of structure analysis which provides two equivalent realizations of the mass operator. The second representation, Eq. (4.21), is expressed in terms of a two-particle T matrix which satisfies a simplified Bethe-Salpeter equation in which dressing of the effective two-particle interaction is not of consequence and in which possible three-particle contributions can be separated in a concise manner. The next phase of the analysis demonstrates that a corresponding simplification of the three-particle structures may be effected by making explicit the nature of four-channel processes.

Forming the functional derivative of Eq. (4.21) with respect to the external source, yields the result

$$\begin{aligned} \frac{\partial M_{ic}}{\partial q_{vw}} &= iK(1)_{in,cm}F_{mw,nv}-V_{ip,rq}A(1)_{rqw,cpv} \\ &\quad +iV_{ip,rq}C_{rmqw,lnpv}V_{ln,cm}, \end{aligned} \tag{4.22}$$

where the four-particle amplitude is defined by

$$C_{rmqw,lnpv}=\frac{\partial}{\partial q_{vw}}(G_{rk}B_{kmq,lnp}) \tag{4.23}$$

and

$$A(1)_{rqw,cpv}=\frac{\partial}{\partial q_{vw}}[G_{qy}G_{rx}T(1)_{xy,co}G_{op}]. \tag{4.24}$$

The three-particle amplitude defined by the last equation differs from the exact three-particle amplitude Eq. (4.11) in two respects: The most important difference which is responsible for the separation of purely three-channel effects is the occurrence of the matrix $T(1)$ of Eq. (4.19) instead of the exact T matrix; the second difference pertains to the particular dependence on the single-particle Green's function which results in a conjugate three-particle amplitude.

Substituting Eq. (4.22) into Eq. (4.6) yields

$$\begin{aligned} M_{zi} &= iK(1)_{zn,im}G_{mn}-G_{wy}T(1)_{zy,jg}G_{gv}G_{jc}V_{cv,iw} \\ &\quad +iV_{zp,rq}A(1)_{rqw,jpv}G_{jc}V_{cv,iw} \\ &\quad +V_{zp,rq}V_{ln,jm}C_{rmqw,lnpv}G_{jc}V_{cv,iw}. \end{aligned} \tag{4.25}$$

The second phase of the analysis, therefore, leaves the first phase intact and verifies the substitution law whereby the exact three-particle amplitude is replaced by Eq. (4.24), providing at the same time a statement of the four-particle aspects involved.

By recalling the defining relationships Eqs. (4.24) and (4.19) it follows that

$$A(1)_{rqw,jpv} = F_{rw,xv}\, G_{qy}\, T(1)_{xy,jo}\, G_{op} + G_{rx}\frac{\partial}{\partial q_{vw}}\{G_{qy}[K(1)_{xy,jo} - iK(1)_{xs,jd}\, G_{dc}\, T_{cy,fo}\, G_{fs}]\, G_{op}\}, \tag{4.26}$$

which after some manipulation provides the three-particle equation required:

$$A(1)_{rqw,jpv} = I(1)_{rqw,jpv} - iG_{rx}K(1)_{xs,jd}\, G_{fs}A(1)_{dqw,fpv}, \tag{4.27}$$

where

$$\begin{aligned} I(1)_{rqw,jpv} = &-G_{rv}\, G_{wx}\, G_{qy}\, T(1)_{xy,jo}\, G_{op} + iG_{ra}\, G_{wb}\, T_{ab,cd}\, G_{cx}\, G_{dv}\, G_{qy}\, T(1)_{xy,jo}\, G_{op} \\ &- G_{rx}K(1)_{xy,jo}(G_{qv}\, G_{wy}\, G_{op} + G_{qy}\, G_{ov}\, G_{wp}) + iG_{rx}\, G_{qa}\, G_{wb}\, T_{ab,cd}\, G_{cy}\, G_{dv}\, G_{op}K(1)_{xy,jo} \\ &+ iG_{rx}\, G_{qy}K(1)_{xy,jo}\, G_{oa}\, G_{wb}\, T_{ab,cd}\, G_{cp}\, G_{dv} + iK(1)_{xs,jd}\, G_{rx}\, G_{fv}\, G_{ws}\, G_{dc}\, T(1)_{cy,fo}\, G_{qy}\, G_{op} \\ &+K(1)_{xs,jd}\, G_{rx}\, G_{fa}\, G_{wb}\, T_{ab,he}\, G_{hs}\, G_{ev}\, G_{dc}\, T(1)_{cy,fo}\, G_{qy}\, G_{op}. \end{aligned} \tag{4.28}$$

By comparing Eqs. (3.21) and (4.28), it follows that the major simplifying feature of the last equation is the absence of the effective three-particle interaction so that the last equation is a linear realization of the three-particle dynamics in terms of two-particle scattering. Before deriving the equation satisfied by the four-particle amplitude, the next phase of the analysis is considered.

Accordingly, forming the functional derivative of Eq. (4.25) yields

$$\frac{\partial M_{zj}}{\partial q_{vw}} = -iG_{wy}\, T(1)_{zy,jg}\, G_{gv} - B(1)_{zhw,clv}\, V_{cl,jh} + iV_{zp,rq}\, D(1)_{rqhw,cplv}\, V_{cl,jh} + R_{zw,jv}, \tag{4.29}$$

where

$$D(1)_{rqhw,cplv} = \frac{\partial}{\partial q_{vw}}[A(1)_{rqh,ipl}\, G_{ic}] \tag{4.30}$$

and

$$R_{zw,jv} = V_{zp,rq}\, V_{fn,om}\frac{\partial}{\partial q_{vw}}(C_{rmqh,fnpl}\, G_{oc})V_{cl,jh}. \tag{4.31}$$

The last equations in conjunction with Eq. (4.5), in turn, furnish

$$\begin{aligned} M_{ij} = &\, iK(1)_{in,jm}G_{mn} - V_{iv,rw}G_{rz}\, G_{wy}\, T(1)_{zy,jg}\, G_{gv} + iV_{iv,rw}G_{rz}\, B(1)_{zhw,clv}\, V_{cl,jh} \\ &+ V_{iv,rw}G_{rz}\, V_{zp,oq}D(1)_{oqhw,cplv}\, V_{cl,jh} - iV_{iv,gw}G_{gz}\, R_{zw,jv}. \end{aligned} \tag{4.32}$$

The relationship between the three-particle amplitude of Eqs. (4.11) and (4.27) and the approximate three-particle amplitude of the realization (4.32) may be established as follows:

$$B(1)_{kmq,lnp} = \frac{\partial}{\partial q_{pq}}[G_{mb}\, T(1)_{kb,ca}\, G_{an}\, G_{cl}], \tag{4.33}$$

which according to Eq. (4.19) equals

$$\begin{aligned} B(1)_{kmq,lnp} &= F_{cq,lp}\, G_{mb}\, T(1)_{kb,ca}\, G_{an} + G_{cl}\frac{\partial}{\partial q_{pq}}\{G_{mb}[K(1)_{kb,ca} - iK(1)_{ks,cr}\, G_{rh}\, T_{hb,fa}\, G_{fs}]\, G_{an}\} \\ &= \mathcal{G}(1)_{kmq,lnp} - iK(1)_{ks,cr}\, G_{rh}\, B_{hmq,snp}\, G_{cl}, \end{aligned} \tag{4.34}$$

where

$$\begin{aligned} \mathcal{G}(1)_{kmq,lnp} = &-T(1)_{kb,ca}\, G_{an}\, G_{cp}\, G_{ql}\, G_{mb} - K(1)_{kb,ca}(G_{mp}\, G_{qb}\, G_{an} + G_{mb}\, G_{ap}\, G_{qn})G_{cl} \\ &+ iT(1)_{kb,ca}\, G_{mb}\, G_{an}\, G_{cx}\, G_{qy}\, T_{xy,fo}\, G_{fl}\, G_{op} + iK(1)_{kb,ca}(G_{mx}\, G_{qy}\, G_{fb}\, G_{op}\, G_{an} + G_{mb}\, G_{ax}\, G_{qy}\, G_{fn}\, G_{op})T_{xy,fo}\, G_{cl} \\ &+K(1)_{ks,cr}\, G_{cl}\, G_{rx}\, G_{qy}\, T_{xy,zo}\, G_{zh}\, G_{op}\, G_{mb}\, T_{hb,fa}\, G_{fs}\, G_{an} + iK(1)_{ks,cr}\, G_{cl}\, G_{rp}\, G_{qh}\, G_{mb}\, T_{hb,fa}\, G_{fs}\, G_{an}. \end{aligned} \tag{4.35}$$

Finally, the equation satisfied by the four-particle amplitude is derived. It follows from the defining relationship Eq. (4.30) and Eq. (4.27) that

$$D(1)_{rghw,cplv} = A(1)_{rgh,ipl} F_{iw,cv} + G_{ic} \frac{\partial}{\partial q_{vw}} [I(1)_{rgh,ipl} - i G_{rx} K(1)_{xs,id} A(1)_{dgh,fpl} G_{fs}], \tag{4.36}$$

which yields

$$\begin{aligned} D(1)_{rqhw,cplv} = {} & G_{ic} I(1)_{rqhw,iplv} - A(1)_{rqh,ipl} G_{iv} G_{wc} + iA(1)_{rqh,ipl} G_{ia} G_{wb} T_{ab,de} G_{dc} G_{ev} \\ & + i G_{ic} G_{rv} G_{wx} K(1)_{xs,id} A(1)_{dqh,fpl} G_{fs} + G_{ic} G_{ra} G_{wb} T_{ab,ek} G_{ex} G_{kv} K(1)_{xs,id} A(1)_{dgh,fpl} G_{fs} \\ & - i G_{ic} G_{rx} K(1)_{xs,id} D(1)_{dqhw,splv}, \end{aligned} \tag{4.37}$$

where

$$I(1)_{rqhw,iplv} = \frac{\partial}{\partial q_{vw}} I(1)_{rqh,ipl}. \tag{4.38}$$

The exact four-particle amplitude satisfies an equation quite similar to the preceding equation, the only difference being the appearance of an effective four-particle interaction which is defined in analogy to Eq. (3.17). Furthermore, the clustering property of the four-particle amplitude is exhibited, which, as is shown in the following section, is the basis for the grouping into compact structures and of a multiparticle ordering.

V. MULTIPARTICLE ORDERING

In the previous section an analysis of some of the multichannel aspects of the mass operator has been given which results in a multiparticle ordering of the form:

$$\begin{aligned} M_{ij} = {} & iK(1)_{in,jm} G_{mn} - V_{ix,hy} G_{hk} G_{yb} T(1)_{kb,ja} G_{ax} \\ & + i V_{ip,rq} G_{rk} B(1)_{kmq,lnp} V_{ln,jm} \\ & + V_{iv,hw} G_{hd} V_{dn,rm} D(1)_{rmqw,znpv} V_{zp,jq} \\ & + \cdots. \end{aligned} \tag{5.1}$$

The analysis proposed equivalent realizations of the exact mass operator expressed successively in terms of higher-particle amplitudes. The different realizations are connected by the existence of a substitution rule which is an expression of the fact that the hierarchic cycle of equations possesses a form-preserving property. The substitution law calls for the replacement of a given multichannel amplitude by a simplified one in which the multichannel contribution of the next order of complexity has been truncated and the latter part in turn serves as the generator in the following step of the analysis. This process is illustrated in Fig. 3 and can be seen to represent a nonperturbative ordering in terms of the primary interaction and multiparticle-scattering amplitudes. The particular choice of the scattering amplitude to initiate this process is not predetermined and the ordering scheme therefore presents a viable method which may be adapted to accommodate particular physical situations that may be of relevance in different circumstances.

A particularly important aspect of the structure of multiparticle equation, which has been pointed out in the previous sections, relates to the cluster properties of the amplitudes that imply the presence of substructures generated by rescattering of a particular nature. This culminates in a systematic method of incorporating subclasses of scattering contributions. In the previous section the formulation was given in terms of a family of multiparticle amplitudes in terms of which the substitution rule assumes a particularly transparent form. In order to exhibit the substructures accompanying the ordering process it is appropriate to express the multiparticle amplitudes of Eq. (5.1) in terms of the three-particle correlation

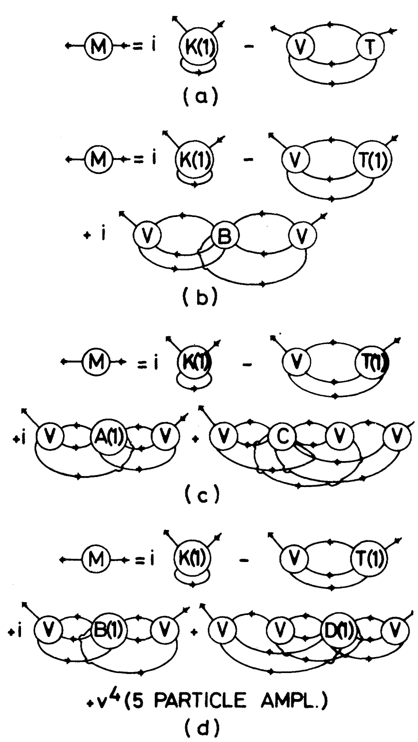

FIG. 3. Graphical representation of the realizations of the mass operator that exhibits multiparticle ordering and the form-preserving property.

function

$$Z_{lno,jme} = \frac{\partial}{\partial q_{eo}} T_{ln,jm} . \tag{5.2}$$

In particular, we have that

$$T(1)_{kb,ja} = T_{kb,ja} - I_{kb,ja} + iV_{kp,rq} G_{rl} G_{qn} G_{mp} G^{-1}{}_{bo} Z_{lno,jme} G^{-1}{}_{ea} , \tag{5.3}$$

where I is defined by Eq. (2.23)

Furthermore, according to Eqs. (3.6) and (5.2) it follows that

$$\begin{aligned} B(1)_{kmq,lnp} &= \frac{\partial}{\partial q_{pq}} [G_{mb} T(1)_{kb,ca} G_{an} G_{cl}] \\ &= F_{mq,bp} T(1)_{kb,ca} G_{an} G_{cl} + G_{mb} T(1)_{kb,ca} F_{aq,np} G_{cl} + G_{mb} T(1)_{kb,ca} G_{an} F_{cq,lp} + G_{mb} Z(1)_{kbq,cap} G_{an} G_{cl}, \end{aligned} \tag{5.4}$$

which by virtue of Eq. (2.13) may be expressed in the following form:

$$\begin{aligned} B(1)_{kmq,lnp} = &- T(1)_{kb,ca}(G_{mp} G_{qb} G_{an} G_{cl} + G_{mb} G_{ap} G_{qn} G_{cl} + G_{mb} G_{an} G_{cp} G_{ql}) \\ &+ iT(1)_{kb,ca}(G_{mx} G_{qy} G_{zb} G_{op} G_{an} G_{cl} + G_{mb} G_{ax} G_{qy} G_{zn} G_{op} G_{cl} + G_{mb} G_{an} G_{cx} G_{qy} G_{zl} G_{op}) T_{xy,zo} \\ &+ G_{mb} Z(1)_{kbq,cap} G_{an} G_{cl} . \end{aligned} \tag{5.5}$$

The importance of the latter representation of the three-particle amplitude derives from the separation of terms which incorporate the cluster property expressed in terms of the two-particle T matrix as depicted in Fig. 4. Before continuing the analogous discussion for the higher particle amplitudes, it is appropriate to consider the application of the results to the mass operator.

The first term of Eq. (5.1) expresses the familiar Hartree-Fock contribution in terms of the exact single-particle Green's function.

Consider the second term of Eq. (5.1), denoted by $M(2)$, which according to Eq. (4.19) may be expressed as

$$M(2)_{ij} = - V_{ix,hy} G_{hd} G_{yb} [K(1)_{db,ja} - iK(1)_{ds,jr} G_{rg} T_{gb,fa} G_{fs}] G_{ax} . \tag{5.6}$$

This result shows that the latter contribution is of the form of the exact result, Eq. (2.15), with the effective two-particle interaction replaced by the Hartree-Fock effective two-particle interaction. The last equation represents contributions to the mass operator generated by a random-phase two-particle scattering matrix with embedded "vertex" modifications.

The three-body generated contribution to the mass operator, according to Eq. (5.4), assumes the form

$$\begin{aligned} M(3)_{ij} = &- iV_{ip,rq} G_{rk} T(1)_{kb,ca}(G_{mp} G_{qb} G_{an} G_{cl} + G_{mb} G_{ap} G_{qn} G_{cl} + G_{mb} G_{an} G_{cp} G_{ql}) V_{ln,jm} \\ &- V_{ip,rq} G_{rk} T(1)_{kb,ca}(G_{mx} G_{qy} G_{zb} G_{op} G_{an} G_{cl} + G_{mb} G_{ax} G_{qy} G_{zn} G_{op} G_{cl} + G_{mb} G_{an} G_{cx} G_{qy} G_{zl} G_{op}) T_{xy,zo} V_{ln,jm} \\ &+ iV_{ip,rq} G_{rk} G_{mb} Z(1)_{kbq,cap} G_{an} G_{cl} V_{ln,jm} . \end{aligned} \tag{5.7}$$

The last equation provides a compact representation of three-body contributions to the mass operator in which the cluster property is manifest and in general characterizes numerous parquet-type contributions to the mass operator. According to Figs. 3 and 4, three-particle contributions are included which consist of the two-body T matrix accompanied by a spectator particle or multiple-scattering contributions as described by the two-particle scattering matrix acting in succession. In order to make more explicit the nature of the processes incorporated, some representative contributions are depicted

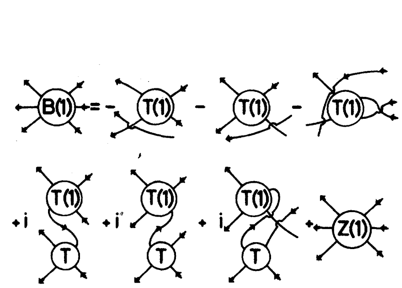

FIG. 4. Graphical representation of Eq. (5.5) which exhibits clustering aspects of the three-particle amplitude.

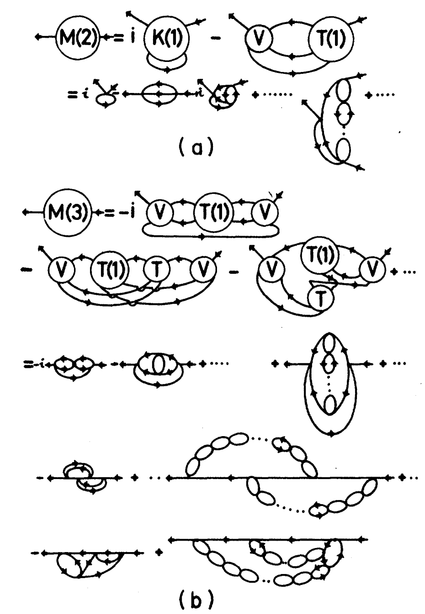

FIG. 5. Three-particle contribution to the self-energy in the "parquet" approximation.

in Fig. 5, where for the sake of economy in presentation the two-particle scattering is taken as the random-phase-generated contribution, thus neglecting crossing and vertex modifications. In general, the parquet contribution may be pictured as particular three-particle correlations in which independent interaction is favored as determined by the underlying two-particle dynamics. Similar views on parquet structures have also been given particularly by Nozieres and co-workers.[9] The importance of the summation of "parquet contributions" has been recognized in a number of problems[10] such as the extensive studies in connection with the Kondo effect[11] and has led to particular infrared features with its accompanying low-temperature aspects[12]; in connection with the role of soft bosons in the anomalous low-temperature features of Fermi liquids[13] and in connection with an infrared driven electromagnetic bootstrap where "parquet" contributions act as a feedback mechanism.[14]

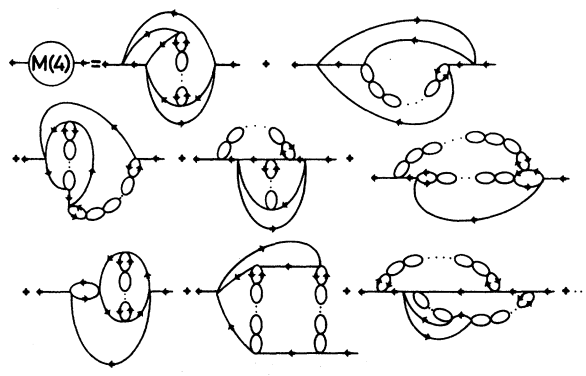

FIG. 6. Four-particle contributions to the self-energy.

The parquet contributions of Fig. 5 may be divided into two classes: class I where two particles interact repeatedly but independent of a third, or in class II where families of paired particles interact any number of times but independently and link up with a third.

In concluding the analysis, the four-particle contribution to the mass operator is given:

$$M(4)_{ij}=V_{iv,hw}G_{hd}\,V_{dn,rm}D(1)_{rmqw,znpv}\,V_{zp,jq}\,. \quad (5.8)$$

In order to exhibit the clustering property of the four-particle scattering it is appropriate to express the four-particle amplitude in the following form:

$$D(1)_{rmqw,znpv}=\frac{\partial}{\partial q_{vw}}[A(1)_{rmq,cnp}\,G_{cz}]$$
$$=[D'(1)+D''(1)]_{rmqw,znpv}, \quad (5.9)$$

where

$$D'(1)_{rmqw,znpv}=A(1)_{rmq,cnp}\,F_{cw,zv} \quad (5.10)$$

and

$$D''(1)_{rmqw,znpv}=G_{cz}\,\frac{\partial}{\partial q_{vw}}[A(1)_{rmq,cnp}]\,. \quad (5.11)$$

By carrying out the relevant functional derivatives, the three-particle amplitude in question may be expressed in the form

$$A(1)_{rmq,cnp}=G_{rk}\,T(1)_{kb,ca}\,G_{an}(-G_{mb}G_{qb}+iG_{mx}G_{qy}T_{xy,fo}G_{fb}G_{op})$$
$$+G_{mb}\,T(1)_{kb,ca}\,G_{an}(-G_{rp}G_{qk}+iG_{rx}G_{qy}T_{xy,fo}G_{fk}G_{op})$$
$$+G_{mb}G_{rk}\,T(1)_{kb,ca}(-G_{ap}G_{qn}+iG_{ax}G_{qy}T_{xy,fo}G_{fn}G_{op})+G_{mb}G_{rk}Z(1)_{kbq,cap}\,G_{an}\,. \quad (5.12)$$

The contribution of the four-particle amplitude (5.10) to the mass operator therefore reads

$$M'(4)_{ij}=[V_{iv,hw}G_{hd}\,V_{dn,rm}V_{zp,jq}G_{cv}C_{wz}\,T(1)_{kb,ca}+iV_{iv,hw}G_{hd}\,V_{dn,rm}V_{zp,jq}\,T(1)_{kb,ca}G_{cs}G_{wt}\,T_{st,eu}G_{ez}G_{uv}]$$
$$\times[G_{rk}G_{an}G_{mp}G_{qb}+G_{mb}G_{an}G_{rp}G_{qk}+G_{mb}G_{rk}G_{ap}G_{qn}$$
$$-iT_{xy,fo}(G_{rk}G_{an}G_{mx}G_{qy}G_{fb}G_{op}+G_{mb}G_{an}G_{rx}G_{qy}G_{fk}G_{op}+G_{mb}G_{rk}G_{ax}G_{qy}G_{fn}G_{op})]\,. \quad (5.13)$$

Further contributions due to clusters arise from Eqs. (5.8) and (5.11), with the effect

$$M''(4)_{ij} = V_{iv,hw} G_{hd} V_{dn,rm} V_{zp,jq} [G_{cz} G_{rk} T(1)_{kb,ca} G_{an} G_{qb} (G_{mv} G_{wp} - iG_{ms} G_{wt} T_{st,eu} G_{ep} G_{uv}) + iG_{cz} G_{rk} T(1)_{kb,ca} G_{an} G_{mx} G_{qy} T_{xy,fo} G_{op} (-G_{fv} G_{wb} + iG_{fs} G_{wt} T_{st,eu} G_{eb} G_{uv}) + G_{cz} G_{mb} T(1)_{kb,ca} G_{an} G_{qk} (G_{rv} G_{wp} - iG_{rs} G_{wt} T_{st,eu} G_{ep} G_{uv}) + iG_{cz} G_{mb} T(1)_{kb,ca} G_{an} G_{rx} G_{qy} T_{xy,fo} G_{op} (-G_{fv} G_{wk} + iG_{fs} G_{wt} T_{st,eu} G_{ek} G_{uv})] + \cdots . \tag{5.14}$$

Representative terms of the last two equations are given in Fig. 6. In view of the clustering property of the four-particle amplitude, which has been utilized, it follows that the contribution may be characterized by nested subclasses of binary scattering events acting in tandem.

In this work a method of analysis of the multichannel aspects of field equations has been developed which provides a systematic method for including subclasses of physical processes. In an accompanying paper[15] the implications of multiparticle ordering in arriving at nonperturbative dynamical approximations are investigated.

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