Zweig rule and the πN σ term

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Simple applications of the Zweig rule can fix some chiral-symmetry-breaking parameters in the quark-gluon theory of strong interactions. Using a bare-quark-mass ratio of $2m_s/(m_u + m_d) = 25$, we find $\sigma^{\pi\pi}_{NN} = 32 \pm 8$ MeV. This is in disagreement (by a factor of 2) with dispersion-relation calculations based on the new πN phase shifts. Implications of this discrepancy are discussed.

The discoveries of 3.1- and 3.7-GeV narrow resonances in the e^+e^- annihilation and their interpretation as bound states of a heavy quark with its antiquark has dramatically called our attention to the "hairpin" rule of Zweig and others.[1] Here we shall use it to mean the suppression of the matrix elements of any composite operators made up of quark fields which are different from the valence quarks in the states. We remark that such a semiempirical rule should be very useful in the various traditional current-algebra calculations when performed in the context of the quark-gluon theory of strong interactions. In this note we shall address ourselves specifically to the problem of chiral-symmetry-breaking parameters and the πN σ term.

A number of attractive arguments have been advanced that the strong interactions are described by the following Yang-Mills theory:

$$\mathcal{L} = (-\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu} - \bar{q}i\not{D}q) + \bar{q}Mq \equiv (\mathcal{L}_0) + \mathcal{L}'. \qquad (1)$$

$\mathcal{L}$ is exactly invariant under the gauge group $SU(3)_{\text{color}}$ ($G_{\mu\nu}$ being the gluons), and $\mathcal{L}_0$ is also chiral invariant. This (global) chiral symmetry is, however, broken by the (bare) quark mass term:[2]

$$\begin{aligned}\mathcal{L}' &= \bar{q}Mq \\ &= m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + m_c\bar{c}c + \cdots. \end{aligned} \qquad (2)$$

We shall concentrate on the u, d, s terms and also ignore any term proportional to the mass difference of the u and d quarks. Thus we will be working with the usual $(3,\bar{3}) + (\bar{3},3)$ theory of Gell-Mann, Oakes, and Renner,[3] and Glashow and Weinberg[4]:

$$\mathcal{L}' = c_0u_0 + c_8u_8. \qquad (3)$$

In the quark-model language, $c_8 = (1/\sqrt{3})(\hat{m} - m_s)$, $u_8 = (\frac{1}{3})^{1/2}(\bar{u}u + \bar{d}d - 2\bar{s}s)$, etc., where $\hat{m} \equiv \frac{1}{2}(m_u + m_d)$. The pion-nucleon σ term can then be written as

$$\sigma^{\pi\pi}_{NN} = \frac{\hat{m}}{2M_N}\langle N|(\bar{u}u + \bar{d}d)|N\rangle, \qquad (4)$$

where M_N is the nucleon mass.[5]

Using the Zweig rule Eq. (4) may be approximated as[6]

$$\sigma^{\pi\pi}_{NN} \simeq \frac{\hat{m}}{2M_N}\langle N|(\bar{u}u + \bar{d}d - 2\bar{s}s)|N\rangle. \qquad (5)$$

This nucleon matrix element of u_8 can then be related to the baryon mass differences in the SU_3 limit, giving

$$\sigma^{\pi\pi}_{NN} \simeq \frac{\hat{m}}{m_s - \hat{m}}\frac{3}{2M_N}(M_{\Xi}^2 - M_{\Lambda}^2). \qquad (6)$$

With the current-quark mass ratio of $m_s/\hat{m} = 25$, which corresponds to the well-known parameter $c_8/c_0 = -1.25$, from π, K masses,[3] we obtain

$$\sigma^{\pi\pi}_{NN} \simeq 32 \text{ MeV}. \qquad (7)$$

(We also note that a similar calculation yields $\sigma^{KK}_{NN} \simeq 246$ MeV.)

Over four years ago Dashen and the present author first used fixed-t dispersion relations and phase shifts to determine the σ term, which is related to the on-shell (but unphysical) πN amplitude as[7]

$$T(s = M_N^2,\ t = 2\mu_\pi^2) \simeq \sigma^{\pi\pi}_{NN}/f_\pi^2. \qquad (8)$$

(The correction term to this relation is *formally* of the order of μ_π^4. The validity of this estimate will be discussed below.) Subsequently there have been many calculations of the πN amplitude at this unphysical energy-point with widely varying results. (For a review of this subject see Ref. 8.) The main problem, as was realized immediately, is that the then-existing πN phase shifts were not self-consistent; they did not have the correct analyticity properties. In particular, the high-energy and low-energy phase shifts were not compatible.

This situation changed markedly after Carter *et al.*[9] obtained new low-energy πN phase shifts from their extremely accurate pion-proton scattering cross-section measurements[10] in the 90 to 300 MeV/c range of lab kinetic energy [i.e., around the $P_{33}(1232)$ region]. Based upon this set of new phase shifts all the subsequent dispersion calculations[11] of $\sigma_{NN}^{\pi\pi}$ via Eq. (8) basically agree (see Table I):

$$\sigma_{NN}^{\pi\pi}(\text{disp. cal.}) \simeq 63 \text{ MeV}. \tag{9}$$

Thus several developments converge to this interesting situation: It is now widely believed that Eq. (1) has a real chance to be the correct theory of strong interactions and in such a Yang-Mills theory the only manner chiral symmetry can be broken is as in Eq. (2). We have now succeeded in computing the $\pi N\,\sigma$ term in this theory, just when a general consensus has also been reached among dispersion calculations. And there is apparently a factor-of-two discrepancy between Eqs. (7) and (9). In this regard we make the following observations:

(i) The Zweig rule cannot be violated by 100%; in fact, a reasonable expectation will be that Zweig rule and SU_3-symmetric calculation of $\langle N|c_8 u_8|N\rangle$ together can introduce an error of 25% at the most—hence a ($\pm$ 8 MeV) uncertainty in Eq. (7).

(ii) From Table I, we estimate that there is about 15 MeV uncertainty in Eq. (9) at the most.

(iii) The ratio $m_s/\hat{m} = 25$ results from a first order $SU_3 \times SU_3$ chiral perturbation calculation: The axial-vector-current divergence, sandwiched between π, K, and the vacuum state, leads to the relation

$$\frac{m_s}{\hat{m}} = 2\frac{Z_K{}^{-1/2} f_K \mu_K{}^2}{Z_\pi{}^{-1/2} f_\pi \mu_\pi{}^2} - 1 \tag{10}$$

($Z_{K,\pi}$ and $f_{K,\pi}$ being the wave-function renormalization factors and decay constants for K, π, respectively), which to the leading order is

$$\frac{m_s}{\hat{m}} \simeq 2\frac{\mu_K{}^2}{\mu_\pi{}^2} - 1$$

$$= 25. \tag{11}$$

Zee[12] has made the very interesting observation that if we repeat our calculations, Eqs. (4) to (6), for the $\pi\pi\,\sigma$ term and we recall that current algebra itself determines this quantity[13] $\sigma_{\pi\pi}^{\pi\pi} = \frac{1}{2}\mu_\pi$, we recover precisely Eq. (11). This shows that the use of $m_s/\hat{m} = 25$ in our calculation is self-consistent. Our remark (i) would then also lead us to expect a 25% correction to Eq. (11). In this case the correction would most likely revise $m_s/\hat{m}$ upwards [in the unwelcome direction as far as bringing agreement between the results Eqs. (7) and (9) is concerned]. Since $f_K/f_\pi \simeq 1.25$, to have $m_s/\hat{m}$ to be smaller than 25 by any significant amount Eq. (10) would necessarily imply a large departure from unity for Z_K/Z_π. For example, to have $m_s/\hat{m} \simeq 12$ [to bring $\sigma_{NN}^{\pi\pi}$ into agreement with Eq. (9)], one would need $(Z_K/Z_\pi)^{1/2} \simeq 2.4$. A soft-meson calculation will in turn imply $\langle 0|u_8|0\rangle / \langle 0|u_0|0\rangle \approx -0.8$. Such a strong SU_3 symmetry breaking of the vacuum would be contrary to our understanding of how the approximate chiral symmetry is realized in the world.

TABLE I. Results of recent dispersion calculations of $\sigma_{NN}^{\pi\pi}$, via Eq. (8) (Ref. 11).

Authors	$\sigma_{NN}^{\pi\pi}$ (MeV)
Nielsen and Oades (1974)	66 ± 9
Hite and Jacob (1974)	68 ± 12
Langbein (1975)	61 ± 16
Chao, Cutkosky, Kelly, and Alcock (1975)	57 ± 12

(iv) While there is no reliable method of calculating the higher-order corrections, Li and Pagels[14] have raised the general nonanalyticity question in chiral perturbations. They have also advocated that the leading nonanalytic terms, being calculable themselves, should provide us an estimate of these corrections. In this connection it has been shown[15] that the two-soft-pion intermediate state in the t channel can introduce an $O(\mu_\pi{}^3)$ correction term of (+14 MeV) in Eq. (8). A similar type of two-meson calculation for the three-point and two-point functions[16] leads to the conclusion that corrections to $Z_K/Z_\pi \simeq 1$ are negligible and, by way of Eq. (10), $m_s/\hat{m} = 32$. Consequently, if we follow the prescription of Pagels and his co-workers,[17] results in Eqs. (7) and (9) will be modified as

$$\sigma_{NN}^{\pi\pi} \simeq 25 \text{ MeV}, \tag{7'}$$

$$\sigma_{NN}^{\pi\pi}(\text{disp. cal.}) \simeq 49 \text{ MeV}. \tag{9'}$$

Thus the factor-of-two discrepancy persists.[18]

(v) Since it looks unlikely that theoretical corrections can close the gap between Eqs. (7) and (9), we must turn to experiments.[19] While the agreement as displayed in Table I is impressive, different extrapolation procedures based on the new phase shifts yield the same value for the pion-nucleon amplitude at the unphysical energy point of Eq. (9). This certainly demonstrates that the existing phase shifts are reasonably self-consistent and that they satisfy most of the analyticity requirements. Still, this does not prove that they are without bias. In view of the fact such dispersion calculations are most sensitive to the extremely low-energy phase shifts, it is suggested that accurate πN scattering experiments be carried our for lab kinetic energies below 90 MeV. These experiments will clearly be very difficult, but probably not impossible at laboratories such as the Los Alamos Meson Physics Facility.

In summary, we have calculated the $\pi N\,\sigma$ term in the quark-gluon theory and found it to be a factor of two smaller than that obtained from dispersion calculations. With our present understanding of chiral symmetry, it is difficult to see how theoretical corrections can account for this discrepancy. This suggests that a likely resolution will be that further improvement of the low-energy πN data and partial-wave analysis would reveal hidden biases in the existing phase shifts.

Note added in proof. The theoretical conclusion of the paper is too pessimistic. Contrary to our expectations, Zweig rule as applied here may in fact receive a large correction. Preliminary calculations indicate that the correction should be large enough to remove the bulk of the factor-of-2 discrepancy. A key ingredient of these new calculations is that Zweig-violation processes proceed through nonplanar quark graphs.

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[5]Our normalization of states is $\langle p' | p \rangle = (2\pi)^3 2E\delta(\vec{p}\,' - \vec{p})$. In general, $\sigma_{NN}^{\pi\pi}$ is a function of nucleon momentum transfer $t \equiv (p' - p)^2$. In this paper we shall always take $\sigma_{NN}^{\pi\pi}$ to mean $\sigma_{NN}^{\pi\pi}(t=0)$.

[6]The Zweig rule used here only requires $\langle N | (\bar{u}u + \bar{d}d | N\rangle \gg \langle N | \bar{s}s | N\rangle$. It does *not* imply $\hat{m}\langle N | (\bar{u}u + \bar{d}d) | N\rangle \gg m_s \langle N | \bar{s}s | N\rangle$. In fact this last inequality is almost certainly not true since our experience with $\phi \to 3\pi$ (vs $\omega \to 3\pi$) indicates a suppression in the amplitude only at the 10% level. With the large current-quark mass ratios, a good guess will be that *all* the terms (including $m_c \bar{c}c + \cdots$) in Eq. (2) will make comparable contributions when sandwiched between the nucleon states. In other words, we do not assume that the nucleon mass shift $\langle N | \mathcal{H}' | N\rangle$ will vanish in the exact $SU_{(2)} \times SU_{(2)}$ limit. This "strong" version of the Zweig rule was first suggested by M. Gell-Mann, Caltech Report No. CALT-68-244, 1969 (unpublished), in connection with results obtained by J. Kim and F. Von Hippel [Phys. Rev. Lett. 22, 740 (1969)]. Clearly this assumption will yield the same result for the σ term. [See also G. Segrè, Phys. Rev. D 3, 1954 (1971); P. Sinha and V. P. Gautam (unpublished).]

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[18]As a check of these soft-meson corrections, a one-loop calculation in the σ model is being carried out (T. P. Cheng and P. B. James, unpublished).

[19]This is a suggestion by R. F. Dashen (private communication).