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Mass of the charmed pseudoscalar meson in a model of broken SU(4) ⊗ SU(4) symmetry

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A mass relation for pseudoscalar mesons is obtained in a broken-chiral-SU(4)-symmetry scheme. An interesting property of the chimeral SU(4) group and the Goldstone nature of some of the pseudoscalar mesons in certain symmetry limits are exploited for the derivation of the relation.

A unified, renormalizable theory of weak and electromagnetic interactions has resulted from the study of spontaneously broken gauge theories[1] and the subsequent detection of neutral currents.[2] However, this theory needs a new degree of freedom,[3] named charm, for hadronic matter. The experimental observation of stable narrow resonances[4] in the 3–4 GeV region and the claim[5] of detection of a charmed baryon lend support to this theory. Inclusion of charm as a quantum number has led to an extensive study[6-8] of the group SU(4)⊗ SU(4) for the classification of hadrons. In the following we exploit some interesting properties of this group to predict the approximate value of the mass of the charmed pseudoscalar meson.

We assume that SU(3) is a good symmetry for normal pseudoscalar mesons[9] ($m_K \approx 500$ MeV, $m_\eta \approx 550$ MeV, and considering π to be an abnormal state). We also assume that these normal pseudoscalar mesons $N(\pi, K, \eta)$ together with the charmed mesons $C(F, D)$ and X^0, η_c belong to $(4,4^*) \oplus (4^*,4)$ representation of the group SU(4)⊗ SU(4).[6-8] Under the above assumption we can take the symmetry-breaking Hamiltonian density, by the simplest generalization of the model of Gell-Mann, Oakes, and Renner (GMOR),[10] as

$$H' = -u_0 - du_{15}\,, \tag{1}$$

where d is a parameter and u_i, v_i $(i = 0,1,2,\ldots,15)$ form a 16-plet of scalar and pseudoscalar densities, respectively, belonging to the representation $(4,4^*) \oplus (4^*,4)$.

The vector and the axial-vector current 15-plets $V_{\mu i}, A_{\mu i}$ have the divergences

$$\partial_\mu V_{\mu i} = d f_{i15k} u_k \quad (i, k = 1,2,\ldots,15)\,, \tag{2}$$

$$\partial_\mu A_{\mu i} = -d_{i0k} v_k - d d_{i15k} v_k \quad (i = 1,2,\ldots,15,\quad k = 0,1,2,\ldots,15)\,, \tag{3}$$

where the symmetry-breaking Hamiltonian density is given by the relation (1). The relation (3) for different values of the index i can be expressed as[7]

$$\partial_\mu A_{\mu i} = -\left(\frac{1}{\sqrt{2}} + \frac{d}{\sqrt{6}}\right) v_i \quad (i = 1,2,\ldots,8)\,, \tag{4a}$$

$$\partial_\mu A_{\mu i} = -\left(\frac{1}{\sqrt{2}} - \frac{d}{\sqrt{6}}\right) v_i \quad (i = 9,10,\ldots,14)\,, \tag{4b}$$

and

$$\partial_\mu A_{\mu 15} = -\left[\frac{1}{\sqrt{2}} - (\tfrac{2}{3})^{1/2} d\right] v_{15} - \frac{d}{\sqrt{2}} v_0 . \tag{4c}$$

We see from relation (4a) that for $d = -\sqrt{3}$

$$\partial_\mu A_{\mu i} = 0 \quad (i = 1,2,\ldots,8). \tag{5}$$

This means that for this particular value of d the normal mesons N are Goldstone bosons satisfying the zero-mass relation

$$\langle N | -u_0 | N\rangle - \sqrt{3}\,\langle N | -u_{15} | N\rangle = 0. \tag{6}$$

Next, at $d = \sqrt{3}$, we have

$$\partial_\mu A_{\mu i} = 0 \quad (i = 9, 10, \ldots, 14) \tag{7}$$

from relation (4b). This means that the charmed pseudoscalar mesons $C(F, D)$ are Goldstone bosons at this point, and we have the relation

$$\langle C | -u_0 | C\rangle + \sqrt{3}\,\langle C | -u_{15} | C\rangle = 0 \tag{8}$$

for the zero-mass condition. Again at $d = \sqrt{3}/2$, the 15th component of the axial-vector current has no v_{15} part. We then expect π_{15} to have vanishing mass, i.e.,

$$\langle \pi_{15} | -u_0 | \pi_{15}\rangle + \frac{\sqrt{3}}{2}\langle \pi_{15} | -u_{15} | \pi_{15}\rangle = 0. \tag{9}$$

When $d = 0$ we have exact SU(4) symmetry, and all the mesons $A(N, C, \pi_{15})$ have equal mass, i.e.,

$$\langle A | -u_0 | A\rangle = m_{15}{}^2. \tag{10}$$

The physical masses for different pseudoscalar mesons (N, C, π_{15}) are given by

$$m_N{}^2 = \langle N | (-u_0 - du_{15}) | N\rangle = \left(1 + \frac{d}{\sqrt{3}}\right) m_{15}{}^2, \tag{11}$$

$$m_C^2 = \langle C|(-u_0 - du_{15})|C\rangle$$
$$= \left(1 - \frac{d}{\sqrt{3}}\right) m_{15}^2, \tag{12}$$

and

$$m_{\pi_{15}}^2 = \langle \pi_{15}|(-u_0 - du_{15})|\pi_{15}\rangle$$
$$= \left(1 - \frac{2d}{\sqrt{3}}\right) m_{15}^2, \tag{13}$$

where we have used the relations (6), (8), and (9). The π_0 meson can have a nonzero mass even in the limit of exact SU(4)⊗SU(4); hence

$$m_{\pi_0}^2 = M^2 + \langle \pi_0|(-u_0 - du_{15})|\pi_0\rangle. \tag{14}$$

There is thus no Goldstone condition that π_0 could be expected to satisfy. However, the combinations

$$\pi_A = \frac{\pi_{15} + \sqrt{3}\pi_0}{\sqrt{4}}, \tag{15}$$

$$\pi_B = \frac{\sqrt{3}\pi_{15} - \pi_0}{\sqrt{4}} \tag{16}$$

belong to $(3,3^*)\oplus(3^*,3)$ and the singlet representation of SU(3)⊗SU(3), respectively. Hence they satisfy

$$\langle \pi_A|(-u_0 + \sqrt{3}u_{15})|\pi_B\rangle = 0. \tag{17}$$

Going back to the point $d = \sqrt{3}$, where the charmed mesons appear as Goldstone bosons, we have 15 conserved currents, which are $V_{\mu i}$ for $i = 1, 2, \ldots, 8$ and 15 and $A_{\mu i}$ for $i = 9, 10, \ldots, 14$. The charges corresponding to these currents now form an SU(4) group which has been called "chimeral" $\overline{\text{SU}}(4)$ group.[6] We have the interesting property of the chimeral SU(4) group that with respect to it the normal pseudoscalar mesons N, the scalar charmed mesons and their antiparticles, and a combination of π_0 and π_{15} given by

$$\overline{\pi}_{15} = \frac{\pi_{15} - \sqrt{3}\pi_0}{\sqrt{4}} \tag{18}$$

form a 15-plet, and the combination

$$\overline{\pi}_0 = \frac{\sqrt{3}\pi_{15} + \pi_0}{\sqrt{4}} \tag{19}$$

forms a singlet. So we get the following relations:

$$\langle \overline{\pi}_{15}|(-u_0 - \sqrt{3}u_{15})|\overline{\pi}_{15}\rangle = \langle N|(-u_0 - \sqrt{3}u_{15})|N\rangle, \tag{20}$$

$$\langle \overline{\pi}_{15}|(-u_0 - \sqrt{3}u_{15})|\overline{\pi}_0\rangle = 0. \tag{21}$$

Combining the relations (17), (20), and (21) we obtain

$$\langle \pi_0|-u_0|\pi_0\rangle = 3m_{15}^2, \tag{22}$$

$$\langle \pi_{15}|-u_{15}|\pi_{15}\rangle = -\frac{2}{\sqrt{3}} m_{15}^2, \tag{23}$$

$$\langle \pi_{15}|-u_{15}|\pi_0\rangle = -m_{15}^2, \tag{24}$$

and

$$\langle \pi_0|-u_{15}|\pi_0\rangle = -\frac{2}{\sqrt{3}} m_{15}^2. \tag{25}$$

From the relations (11) and (12) we obtain

$$m_{15}^2 = \tfrac{1}{2}(m_N^2 + m_C^2) \tag{26}$$

and

$$\beta = \frac{m_C^2 - m_N^2}{m_C^2 + m_N^2}, \tag{27a}$$

where

$$\beta = -d/\sqrt{3}. \tag{27b}$$

Substituting from relations (22)–(26) in relations (14) and (13) we have

$$m_{\pi_0}^2 = M^2 + (3 + 2\beta)m_{15}^2, \tag{28}$$

$$m_{\pi_{15}}^2 = (1 + 2\beta)m_{15}^2, \tag{29}$$

and the transition mass squared

$$m_t^2 = \langle \pi_{15}|(-u_0 - du_{15})|\pi_0\rangle$$
$$= \sqrt{3}\beta m_{15}^2. \tag{30}$$

If we assume that X^0 (~950 MeV) and η_c are orthogonal linear combinations of π_0 and π_{15}, we have to diagonalize the mass matrix

$$\begin{pmatrix} m_{\pi_0}^2 & m_t^2 \\ m_t^2 & m_{\pi_{15}}^2 \end{pmatrix} \tag{31}$$

to obtain the masses of X^0 and η_c. Taking

$$|X^0\rangle = -\sin\theta|\pi_0\rangle + \cos\theta|\pi_{15}\rangle$$

and

$$|\eta_c\rangle = \cos\theta|\pi_0\rangle + \sin\theta|\pi_{15}\rangle, \tag{32}$$

we obtain[11]

$$m_{X^0}^2 = \tfrac{1}{2}(m_{\pi_{15}}^2 + m_{\pi_0}^2)$$
$$-\tfrac{1}{2}[(m_{\pi_{15}}^2 - m_{\pi_0}^2)^2 + 4m_t^4]^{1/2}, \tag{33a}$$

$$m_{\eta_c}^2 = \tfrac{1}{2}(m_{\pi_{15}}^2 + m_{\pi_0}^2)$$
$$+\tfrac{1}{2}[(m_{\pi_{15}}^2 - m_{\pi_0}^2)^2 + 4m_t^4]^{1/2}, \tag{33b}$$

and

$$\tan 2\theta = \frac{2m_t^2}{m_{\pi_0}^2 - m_{\pi_{15}}^2}$$
$$= \frac{2\sqrt{3}\beta m_{15}^2}{M^2 + 2m_{15}^2}. \tag{34}$$

From the relations (26)–(30) and (33a), (33b) we find the mass squared

$$m_C^2=\tfrac{1}{4}\{m_{X^0}^2+m_{\eta_c}^2+2m_N^2+[(m_{X^0}^2+m_{\eta_c}^2+\tfrac{2}{3}m_N^2)^2-\tfrac{16}{3}(m_{X^0}^2m_{\eta_c}^2+\tfrac{1}{3}m_N^4)]^{1/2}\} \quad (35)$$

for the charmed pseudoscalar mesons.

For obtaining an estimate for the mass of charmed pseudoscalar mesons from the relation (35) we take $m_N \approx 0.5$ GeV and $m_{X^0} \approx 0.95$ GeV. Also there is evidence[12] for a state at 2.8 GeV which would be expected to be η_c. We obtain

$$m_C = 1.98 \text{ GeV}, \quad (36)$$

which is consistent with the estimate of Gaillard, Lee, and Rosner.[13] However, the relation (35) gives only a rough estimate for the mass of the charmed pseudoscalar mesons because a more realistic form of the symmetry-breaking Hamiltonian density

$$H' = -u_0 - cu_8 - du_{15} \quad (37)$$

will contain a u_8 term. Unfortunately for the symmetry-breaking Hamiltonian density (37) the relations (17), (20), and (21) are not sufficient to find all unknown matrix elements. However, we feel that the choice of the symmetry-breaking Hamiltonian density (1) gives roughly the charmed-pseudoscalar-meson mass as SU(4) symmetry is expected to be more badly broken[9] than SU(3) symmetry.

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