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Spin-unitary-spin splitting of SU(8) supermultiplets*

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The surprising narrowness of the J or $\psi(3.1)$ is interpreted as indication of a pure $c\bar{c}$ state, and hence as evidence for the $\mathrm{SU}(8)\to\mathrm{SU}(6)\times\mathrm{SU}(2)_{S_c}\times\mathrm{U}(1)_{Y_c}$ symmetry-breaking chain ($\vec{S}_c$ = charmed-quark-spin generators, Y_c = hypercharm generator) instead of an approximate $\mathrm{SU}(8)\to\mathrm{SU}(4)\times\mathrm{SU}(2)_S$ chain ($\vec{S}$ = quark spin generators) which would imply strong mixings. Decompositions under both chains of the s-wave $q\bar{q}$ meson states of the $\underline{64}=\underline{1}+\underline{63}$ of SU(8) and of the $3q$ baryon states of the three-particle symmetric $\underline{120}$ representation are given. The most general mass-splitting operators with breaking in the Y and Y_c directions for these two multiplets are derived, which commute with the Casimir operators of the $\mathrm{SU}(6)\times\mathrm{SU}(2)_{S_c}\times\mathrm{U}(1)_{Y_c}$ chain, which contain only one- and two-body operators, and which are invariant under rotations. Two independent mass relations follow for mesons containing charmed quarks; six, for baryons containing charmed quarks. In an appendix, for reference relative to previous SU(6)-symmetric quark-model mass analyses, the reduced numerical coefficients as determined by the meson $\underline{36}$ of SU(6) are listed.

I. INTRODUCTION

Our purpose is to discuss here two topics from the standpoint of the charmed symmetric quark model:

(i) The ground state $\underline{64}=\underline{1}+\underline{63}$ (meson) and $\underline{120}$ (baryon) representations of SU(8) together with their decompositions under the subgroups SU(6) $\times\mathrm{SU}(2)_{S_c}\times\mathrm{U}(1)_{Y_c}$ and $\mathrm{SU}(4)\times\mathrm{SU}(2)_S$ where S stands for spin. The $\mathrm{SU}(2)_{S_c}$ subgroup acts on the c quark's spin, and Y_c is the hypercharm operator (see Sec. III) with eigenvalues $-\frac{1}{4}$, $-\frac{1}{4}$, $-\frac{1}{4}$, and $\frac{3}{4}$ for, respectively, the $\mathcal{P}$, $\mathcal{N}$, λ, and c type quarks.

(ii) The mass operator for these states in the $\mathrm{SU}(6)\times\mathrm{SU}(2)_{S_c}\times\mathrm{U}(1)_{Y_c}$ chain which is derived by extending the one- and two-body force analysis of the SU(6)-symmetric quark model[1] which previously gave the successful mass formulas[1-3] for the baryons, e.g., the Gürsey-Radicati formula[4] for the $\underline{56}$ of SU(6) theory.[4,5] Electromagnetic effects will be ignored.[6]

The motivation, of course, is the recent discovery[7] of narrow resonances J or $\psi(3.1)$, and $\psi'(3.7)$, which can be interpreted as charmed[8] quark-antiquark objects, $J^{PC}=1^{--}$, $I^G=0^-$, with $N=0$ and 2 harmonic-oscillator quanta excited, respectively. The $N=2$ state is either a radial or an orbital excitation. It is important to recognize that present difficulties with the charm interpretation (the rise of R to 5.3 ± 0.6 at 7.8 GeV, the absence of narrow peaks in missing-mass plots, the absence of increased kaon to pion production ratio, etc.), principally involve phenomena in $e\bar{e}\to$ hadrons above the transition region at about 3.6–4.1 GeV. Hence, these difficulties may, in fact, not exist if the transition region is due to excitation of first the charm and then the color degrees of freedom at about 3.9 GeV, which would be a natural occurrence[9-11] in the Han-Nambu version of the three-quartet model. In this case, the details of the discussion in this article apply to the $\mathrm{SU}(3)''$-color singlet states. On the other hand, this type of mass analysis remains relevant, though not in detail, even if additional heavy quarks[12-14] are found to be necessary, because this analysis preserves successful SU(6) results and accepts the heavy-quark explanation, based on the Okubo-Zweig-Iizuka rule, of the narrowness of the new particles. A single charmed quark is certainly the simplest of such heavy-quark models.

Lastly, we emphasize the basic contrast between (a) the present spectra and mass analyses in which the $c\bar{c}$ purity of the ψ and ψ' is given greatest importance, and (b) various previous analyses[15] in which broken SU(4) is treated in analogy with broken SU(3) so as to derive mass relations and mass mixing angles and to predict specific mass values from existing data, but where $\psi\sim c\bar{c}+\epsilon(p\bar{p}+n\bar{n})+\delta\lambda\bar{\lambda}$ results with $\epsilon,\delta\neq0$.

We first discuss the mesons in Secs. II and III, and then the baryons in Sec. IV.

II. SPIN-UNITARY-SPIN SPLITTING OF THE MESON $\underline{64}$ SUPERMULTIPLET OF SU(8)

We will make use of the well-known fact[16] that the breaking of an approximate symmetry group can be simply expressed in terms of a "chain" of successively smaller subgroups which are valid to an increasingly better approximation. The prime example is the chain $\mathrm{SU}(6)\to\mathrm{SU}(3)\times\mathrm{SU}(2)_{S_q}$, where S_q stands for the spin of the noncharmed quarks, with $SU(3) \to SU(2)_I \times U(1)_Y$ in SU(6) theory. Here, for instance, the hypercharge operator, which breaks the SU(3) symmetry, is conserved at the level of the smaller $SU(2)_I \times U(1)_Y$ subgroup. The eigenvalues of commuting sets of generators in the chain provide quantum numbers with which to label, in practice uniquely, the states in the irreducible representations of the initial approximate symmetry group. Often two or more chains are relevant physically, and then superposition effects occur such that the physical resonances are eigenstates of neither chain. In the preceding example, there is also the chain $SU(6) \to SU(4)_{\mathfrak{N},\mathcal{P}} \times SU(2)_{S_\lambda} \times U(1)_Y$ with $SU(4)_{\mathfrak{N},\mathcal{P}} \to SU(2)_I \times SU(2)_{S_{\mathfrak{N},\mathcal{P}}}$, where $S_{\mathfrak{N},\mathcal{P}}$ stands for the spin of the $\mathfrak{N}$- *and* $\mathcal{P}$-type quarks, whose existence is announced by the "mixing" of the $I = Y = 0$ pairs of s-wave meson states, the ϕ-ω and η-η'.

In the SU(8) theory there are two analogous reduction chains, the "SU(6) chain"

$$SU(8) \to SU(6) \times SU(2)_{S_c} \times U(1)_{Y_c}, \tag{1}$$

where the SU(6) subgroup is that discussed above; it acts on the $\mathcal{P}$-, $\mathfrak{N}$-, and λ-type quarks. The other chain is

$$SU(8) \to SU(4) \times SU(2)_S. \tag{2}$$

The SU(4) subgroup here acts on the $\mathcal{P}$-, $\mathfrak{N}$-, λ-, and c-type quarks; it does not involve spin and is not to be confused with the $SU(4)_{\mathfrak{N},\mathcal{P}}$ subgroup of SU(6) theory. It has the further reduction

$$SU(4) \to SU(3) \times U(1)_{Y_c},$$

where SU(3) is the usual group for the $\mathcal{P}$-, $\mathfrak{N}$-, and λ-type quarks. This second chain we will call the "SU(4) chain."

The physical resonances, as we noted, need not be eigenstates of either chain so we will, first, consider the meson eigenstates in each chain separately. The ground state of an s-wave, Fermi quark-antiquark pair with negative parity and spin $J = S = 0, 1$ is the reducible $\underline{64}$ of SU(8)

$$\underline{8} \times \underline{8}^* = \underline{64} = \underline{1} + \underline{63}.$$

In the "SU(6) chain," the direct sum

$$\underline{64} = [1,1]_0^0 + [35,1]_0^0 + [6,2^*]_1^{-1} + [6^*,2]_1^1 + [1,1]_2^0 + [1,3]_2^0$$

with the notation $[\dim SU(6), \dim SU(2)_{S_c}]_{n_c}^{Y_c}$, where n_c is the total number of charmed quarks plus charmed antiquarks. Mesons associated with the first two representations contain no charmed quarks and are the familiar ones from the SU(6) theory. Continuing the chain, there next are the further reductions, $SU(6) \to SU(3) \times SU(2)_{S_q}$ with the notation $(\dim SU(3), \dim SU(2)_{S_q})$

$$\underline{1} = (1,1),$$
$$\underline{35} = (8,1) + (1,3) + (8,3),$$
$$\underline{6} = (3,2),$$
$$\underline{6}^* = (3^*, 2^*),$$

and similarly for the other SU(6) subchain $SU(6) \to SU(4)_{\mathfrak{N},\mathcal{P}} \times SU(2)_{S_\lambda} \times U(1)_Y$ with the notation $(\dim SU(4)_{\mathfrak{N},\mathcal{P}}, SU(2)_{S_\lambda})_{n_\lambda}^Y$

$$\underline{1} = (1,1)_0^0,$$
$$\underline{35} = (15,1)_0^0 + (4^*,2)_1^{-1} + (4,2^*)_1^1 + (1,1)_2^0 + (1,3)_2^0,$$
$$\underline{6} = (4,1)_0^0 + (1,2)_1^{-1},$$
$$\underline{6}^* = (4^*,1)_0^0 + (1,2^*)_1^1.$$

The $SU(6) \to SU(3) \times SU(2)_{S_q}$ subchain yields, for the $n_c \neq 0$ states, after recoupling the spins by $\vec{S} = \vec{S}_q + \vec{S}_c$,

$$[6,2^*]_1^{-1} = (3,2)1^- + (3,2)0^-,$$
$$[6^*,2]_1^1 = (3^*,2^*)1^- + (3^*,2^*)0^-,$$
$$[1,1]_2^0 = (1,1)0^-,$$
$$[1,3]_2^0 = (1,1)1^-,$$

with the notation $(\dim SU(3), \dim SU(2)_{S_q})J^P$, $J = S$ for the $\underline{64}$ representation. The $(3^*,2^*)1^-$ consists of the isospin singlet $F^{*+} = (\bar{\lambda}c)^+$ and a doublet $D^{*+} = (\bar{\mathfrak{N}}c)^+$ and $D^{*0} = (\bar{\mathcal{P}}c)^0$. The $(3^*,2^*)0^-$ consists of a singlet $F^+ = (\bar{\lambda}c)^+$ and a doublet $D^+ = (\bar{\mathfrak{N}}c)^+$ and $D^0 = (\bar{\mathcal{P}}c)^0$. The $[6,2^*]_1^{-1}$ contains their antiparticles. The $[1,3]_2^0$ is the $J^P = 1^-$ isospin singlet $\phi_c^0 = (\bar{c}c)^0$, and the $[1,1]_2^0$ is the 0^- singlet $\eta_c^0 = (\bar{c}c)^0$.

On the other hand, for the "SU(4) chain" under $SU(8) \to SU(4) \times SU(2)_S$ these SU(8) representations decompose into

$$1 = \{1,1\},$$
$$63 = \{15,1\} + \{1,3\} + \{15,3\},$$

with the notation $\{\dim SU(4), \dim SU(2)_S\}$. Then, under $SU(4) \to SU(3) \times U(1)_{Y_c}$ the SU(4) representations decompose into

$$\underline{1} = 1_2^0,$$
$$\underline{15} = 1_0^0 + 8_0^0 + 3_1^1 + 3_1^{*-1},$$

with the notation $\dim SU(3)_{n_c}^{Y_c}$. The corresponding wave functions can be easily written down; we only note that in this chain the eigenstates are superpositions of $\omega_8 - \phi_1 - \psi_c$, and of $\eta_8 - \eta_1 - \eta_c$.

For several reasons, we will assume that the $SU(6) \times SU(2)_{S_c} \times U(1)_{Y_c}$ subgroup of SU(8) and the chain associated with it are of major importance for the breaking of SU(8) for mesons and baryons.

First and foremost, the striking narrowness of the $\psi(3.1)$ and the $\psi'(3.7)$ suggests that they are pure $c\bar{c}$ states due to some new dynamical invariance principle, for example, n_c is exactly conserved in the strong interactions responsible for the mass spectra. In particular, the J/ψ will be identified with the $[1,3]^0_2$ irreducible representation of the $SU(6)\times SU(2)_{S_c}\times U(1)_{Y_c}$ subgroup of SU(8). Note that decay modes such as $\psi\to 5\pi$ can go, for instance, via unitarity corrections rather than from mixings of the quark content of the ψ and ψ'. This[17] has been pointed out in the context of Okubo-Zweig-Iizuka–rule suppressions, e.g., $\phi\to K\bar{K}$ and $K\bar{K}\to 3\pi$ both have connected duality diagrams so $\phi\to 3\pi$, with a hairpin diagram, can go via unitarity correlations which are difficult to distinguish from ϕ being other than an eigenstate of the $SU(6)\to SU(4)_{\mathcal{N},\mathcal{P}}\times SU(2)_{S_\lambda}\times U(1)_Y$ chain. However, in the case of the ϕ resonance, the mass spectrum (see Appendix B) indicates that the latter $\phi\sim\lambda\bar{\lambda}+\epsilon(\mathcal{P}\bar{\mathcal{P}}+\mathcal{N}\bar{\mathcal{N}})$, $\epsilon\neq 0$, indeed occurs. Second, the lowest mesons can be identified in the $\underline{64}$ of SU(8) with the $\underline{1}$ and $\underline{35}$ of SU(6), and this $\underline{1}+\underline{35}$ can be identified with the $[1+35,1]^0_0$ representation of the $SU(6)\times SU(2)_{S_c}\times U(1)_{Y_c}$ subgroup of SU(8). Also, as discussed in Sec. IV, the lowest baryons can be similarly identified in the $\underline{120}$ of SU(8) with the $\underline{56}$ of SU(6), and this $\underline{56}$ can be identified with the $[56,1]^0_0$ irreducible representation of the SU(6) $\times SU(2)_{S_c}\times U(1)_{Y_c}$ subgroup of SU(8).

Thus, in our derivation of the mass-splitting operators for the s-wave meson $\underline{64}=\underline{1}+\underline{63}$ and baryon $\underline{120}$ representations of SU(8), we will assume that the operator (i) commutes with the Casimir operators of the SU(6), $SU(2)_{S_c}$, and $U(1)_{Y_c}$ subgroups of this chain, and is invariant under rotations. We want the derivation to be a direct extension of SU(6) analyses in the symmetric quark model used to rederive[1] the Gürsey-Radicati result, used to study the first excited baryon multiplet, the $(70,1^-)_1$ in the notation (dim SU(6), $L^P)_N$ with N the number of orbital or radial quanta excited in harmonic-oscillator shells, and used[3] to treat uniformly all of the baryon multiplets with $N=0$, 1, or 2 harmonic-oscillator excitation quanta. Hence, we will assume that the mass-splitting operator (ii) contains only one- and two-body operators,[18] and that it (iii) transforms like a linear combination of three types of terms which transform, respectively, as a singlet, as the hypercharge operator under SU(3), and as the hypercharm operator under SU(4). For one-body operators, this transformation assumption is equivalent to mass splitting between the nonstrange and strange quarks, and to an independent mass splitting between the noncharmed and charmed quarks.

III. MESON MASS OPERATOR AND INDEPENDENT MASS FORMULAS

We use a formalism in terms of the generators of SU(8) to derive the ground state $SU(6)\times SU(2)_{S_c}\times U(1)_{Y_c}$ meson and baryon mass-splitting operators. The 63 generators of SU(8), I^{Ns}_{Mr} with $M=1,2,3,4$ or $\mathcal{P},\mathcal{N},\lambda,c$ for SU(4) and $r=1,2$ or $\uparrow,\downarrow$ for $SU(2)_S$, are constructed from Fermi creation and annihilation operators for s-wave quarks and antiquarks in the charmed symmetric quark model in Appendix A. The SU(8) commutation relations, which can be easily computed from Eq. (A1), are

$$[I^{Ns}_{Mr},I^{N's'}_{M'r'}]=\delta^{Ns}_{M'r'}I^{N's'}_{Mr}-\delta^{N's'}_{Mr}I^{Ns}_{M'r'}. \tag{3}$$

The charm operator C with eigenvalue 1 (–1) for a c quark (antiquark) and 0 for $\mathcal{P},\mathcal{N},\lambda$ quarks and their antiquarks is not a linear combination of generators of SU(8) so we introduce the operator

$$Y_c\equiv C-\tfrac{3}{4}B=I^{c\uparrow}_{c\uparrow}+I^{c\downarrow}_{c\downarrow} \tag{4}$$

with B the baryon number operator. This relation is the analog of $Y=S+B=\mathcal{G}^{\lambda\uparrow}_{\lambda\uparrow}+\mathcal{G}^{\lambda\downarrow}_{\lambda\downarrow}$ which relates the the hypercharge SU(3) generator and strangeness operator for the $\mathcal{P}$, $\mathcal{N}$, and λ quarks. The generators of SU(6) and its subgroups will be denoted by script letters to distinguish them from SU(8) generators. Since the c quark has $B=\frac{1}{3}$, $S=0$, the phenomenological extension to include the c quark is $Y=\mathcal{G}^{\lambda\uparrow}_{\lambda\uparrow}+\mathcal{G}^{\lambda\downarrow}_{\lambda\downarrow}=B+S-\frac{1}{3}C$.

In Sec. II, we discussed the relevant reduction chains which occur in SU(8). Generators[19] for the subgroups in these chains are tabulated in Table I.

TABLE I. Generators of SU(8) subgroups.

Subgroup	Generators
SU(4)	$I^N_M=I^{Ns}_{Ms}$; $M=1,2,3,4$
$SU(2)_S$	$S^s_r=I^{Ms}_{Mr}=(S_c+\mathcal{S})^s_r$; $r=1,2$
SU(3)	$\mathcal{G}^{q'}_q=I^{q'}_q-\frac{1}{3}\delta^{q'}_qI^{p'}_p=I^{q'}_q+\frac{1}{3}\delta^{q'}_qY_c$; $q=1,2,3$
$U(1)_{Y_c}$	$Y_c=I^c_c=-I^p_p$
SU(6)	$\mathcal{G}^{q's}_{qr}=I^{q's}_{qr}+\frac{1}{6}\delta^{q'}_q\delta^s_rY_c$
$SU(2)_{S_c}$	$(S_c)^s_r=I^{cs}_{cr}-\frac{1}{2}\delta^s_rY_c$
$SU(2)_{S_q}$	$(\mathcal{S})^s_r=I^{qs}_{qr}+\frac{1}{2}\delta^s_rY_c=(\mathcal{S}_{\mathcal{N},\mathcal{P}}+\mathcal{S}_\lambda)^s_r$
$SU(2)_I$	$(\mathcal{G}_{\mathcal{N},\mathcal{P}})^n_m=\mathcal{G}^n_m-\frac{1}{2}\delta^n_mY$; $m=1,2$
$U(1)_Y$	$Y=-\mathcal{G}^3_3=\mathcal{G}^m_m$
$SU(4)_{\mathcal{N},\mathcal{P}}$	$(\mathcal{G}_{\mathcal{N},\mathcal{P}})^{ns}_{mr}=\mathcal{G}^{ns}_{mr}-\frac{1}{4}\delta^n_m\delta^s_rY$
$SU(2)_{S_{\mathcal{N},\mathcal{P}}}$	$(\mathcal{S}_{\mathcal{N},\mathcal{P}})^s_r=\mathcal{G}^{ms}_{mr}-\frac{1}{2}\delta^s_rY$
$SU(2)_{S_\lambda}$	$(\mathcal{S}_\lambda)^s_r=\mathcal{G}^{3s}_{3r}+\frac{1}{2}\delta^s_rY$

The tensor operators in the mass formula will be expressed in terms of the Casimir operators for the various subgroups. Casimir operators needed are

$$C_2^{(8)}=\tfrac{1}{2}[I_{Mr}^{Ns},I_{Ns}^{Mr}]_+,\quad C_2^{(4)}=\tfrac{1}{2}[I_M^N,I_N^M]_+,$$

$$C_2^{(2)}(S)=\tfrac{1}{2}[S_r^s,S_s^r]_+=2S(S+1),$$

$$C_2^{(6)}=\tfrac{1}{2}[\mathcal{G}_{qr}^{q's},\mathcal{G}_{q's}^{qr}]_+=\tfrac{1}{2}[I_{qr}^{q's},I_{q's}^{qr}]_+-\tfrac{1}{6}Y_c^{\,2},$$

$$C_2^{(3)}=\tfrac{1}{2}[\mathcal{G}_q^{q'},\mathcal{G}_{q'}^{q}]_+=\tfrac{1}{2}[I_q^{q'},I_{q'}^{q}]_+-\tfrac{1}{3}Y_c^{\,2},$$

$$C_2^{(2)}(I)=\mathcal{G}_m^n\mathcal{G}_n^m-\tfrac{1}{2}Y^2=2I(I+1),$$

$$C_2^{(4)}(\mathfrak{N},\mathcal{P})=\mathcal{G}_{mr}^{ns}\mathcal{G}_{ns}^{mr}-\tfrac{1}{4}Y^2,$$

and those for the several SU(2) subgroups describing the spins of particular sets of quarks. All these Casimir operators can be expressed as bilinear terms in the SU(8) generators of the form $[X,Y]_+$.

We can now derive[20] the mass-splitting operator for the s-wave meson $\underline{64}=\underline{1}+\underline{63}$. Our assumptions require the mass operator be a quadratic polynomial in the generators of SU(8), commute with $C_2^{(6)}$, $C_2^{(2)}(S_c)$, and $Y_c^{\,2}$, be invariant under rotations, and transform like a linear combination of three types of terms which, respectively, transform as a singlet, as Y under SU(3), and as Y_c under SU(4). For mesons, the mass operator must be invariant under charge conjugation. We group the 63 generators of SU(8) into seven types: $I_q^{q'}$, I_c^c, I_q^c and I_c^q, I_{qr}^{qs}, I_{cr}^{cs}, $I_{qr}^{q's}$, I_{qr}^{cs} and I_{cr}^{qs}. Modulo pieces to make them traceless, $I_q^{q'}$ are the generators of SU(3); I_c^c is the generator of $U(1)_{Y_c}$; I_q^c and its adjoint, I_c^q, are generators of SU(4) not contained in the SU(3) and $U(1)_{Y_c}$ subalgebras, etc.

The most general term linear in the generators and a scalar under rotations must be a linear combination of the generators of SU(4). Generators $I_q^{q'}$ and I_c^c are clearly admissible. We next consider the linear combination of the remaining generators $\mathcal{O}=a_c^qI_q^c+b_q^cI_c^q$. Using the identity

$$[[X,Y]_+,Z]_-+[[Y,Z]_+,X]_-+[[Z,X]_+,Y]_-=0 \qquad (5)$$

and Eq. (1),

$$\tfrac{1}{2}[[I_c^c,I_c^c]_+,a_c^qI_q^c+b_q^cI_c^q]_-=-a_c^q[I_c^c,I_q^c]_++b_q^c[I_c^c,I_c^q]_+.$$

Since symmetrized expressions which have different numbers of different types of generators are linearly independent, the conditions that this commutator vanish are

$$a_c^q=b_q^c=0 \quad (\forall\, q). \qquad (6)$$

Thus n_λ and n_c are the only admissible one-body terms which satisfy the transformation requirement and which are invariant under charge conjugation.

For bilinear terms in the generators of SU(8), invariance under rotations implies that there are two classes of terms that can be considered separately: those constructed from $I_q^{q'}$, I_c^c, I_q^c and I_c^q, and those constructed from I_{qr}^{qs}, I_{cr}^{cs}, $I_{qr}^{q's}$, I_{qr}^{cs} and I_{cr}^{qs}. We write the terms quadratic in the generators in the form $[X,Y]_+$ so that they are linearly independent of the terms linear in the generators. The most general term in $I_q^{q'}$, I_c^c, and I_q^c and its adjoint is a linear combination of the six expressions of the form $[X,Y]_+$. Of these, only $[I_q^{q'},I_p^c]_+$ when commuted with $\frac{1}{2}[I_c^c,I_c^c]_+$ yields combinations of the form

$$[I_c^c,[I_q^{q'},I_p^c]_+]_+$$

so these can be considered separately. The vanishing of

$$\tfrac{1}{2}[[I_c^c,I_c^c]_+,a_{qq'p}[I_q^{q'},I_p^c]_+]_-=-a_{qq'p}[I_c^c,[I_q^{q'},I_p^c]_+]_+$$

implies that

$$a_{qq'p}=0 \quad (\forall\, q,q',p). \qquad (7)$$

The adjoint $[I_{q'}^q,I_c^p]_+$ is also eliminated. The same argument with $I_q^{q'}\to I_c^c$ eliminates terms of the form $[I_c^c,I_q^c]_+$ and its adjoint. Similarly, the terms $[I_q^c,I_{q'}^c]_+$ and their adjoint are excluded. Under commutation with

$$\tfrac{1}{2}[I_{cr}^{cs},I_{cs}^{cr}]_+,$$

only

$$[I_q^c,I_c^{q'}]_+$$

leads to combinations of

$$[I_q^c,[I_{cs}^{cr},I_{cr}^{q's}]_+]_+$$

and its adjoint; so, this term can be considered separately. The vanishing of

$$\tfrac{1}{2}[[I_{cr}^{cs},I_{cs}^{cr}]_+,a_{qq'}[I_q^c,I_c^{q'}]_+]_- =a_{qq'}\{[I_q^c,[I_{cs}^{cr},I_{cr}^{q's}]_+]_+-[I_c^{q'},[I_{cr}^{cs},I_{qs}^{cr}]_+]_+\}$$

implies that

$$a_{qq'}=0 \quad (\forall\, q,q'). \qquad (8)$$

This then leaves as admissible terms

$$[I_q^{q'},I_p^{p'}]_+,\ [I_c^c,I_c^c]_+,\ \text{and } [I_q^{q'},I_c^c]_+.$$

Linear combinations of these lead to

$$[\mathcal{G}_q^p,\mathcal{G}_p^q]_+,$$

which transforms as a singlet,

$$y_q^{q'}[\mathcal{G}_{q'}^p,\mathcal{G}_p^q]_+,$$

which transforms as Y, and

$$c_M^N[I_N^0,I_0^M]_+-\tfrac{1}{4}[I_N^0,I_0^N]_+,$$

which transforms as Y_c and a singlet. Here the numerical diagonal matrices $y_q^{q'}=\mathrm{diag}(\frac{1}{3},\frac{1}{3},-\frac{2}{3})$ and $c_M^N=\mathrm{diag}(-\frac{1}{4},-\frac{1}{4},-\frac{1}{4},\frac{3}{4})$ have been used.

Next, we discuss the second class of bilinear terms: those formed from generators transforming as vectors under rotations. These terms must also commute with the Casimir operators $C_2^{(6)}, C_2^{(2)}(S_c)$ and with $Y_c{}^2$ so the candidates are the rotation-vector analogs of the surviving first-class terms, plus those constructed with $(S_c)_r^s$ and $(\mathcal{S})_r^s$. These analogs are

$$[I_{qr}^{q's}, I_{ps}^{p'r}]_+, [I_{cr}^{cs} I_{cs}^{cr}]_+, \text{ and } [I_{qr}^{q's}, I_{cs}^{cr}]_+.$$

Since the admissible terms linear in the generators are $I_q^{q'}$ and I_c^c, the only candidates constructed from $(S_c)_r^s$ and $(\mathcal{S})_r^s$ are

$$[(S_c)_r^s, (S_c)_s^r]_+, \ [(\mathcal{S})_r^s, (\mathcal{S})_s^r]_+ \ [(S_c)_r^s, I_{cs}^{cr}]_+,$$

and

$$[(\mathcal{S})_r^s, I_{qs}^{q'r}]_+.$$

Commutation with $Y_c{}^2$, $C_2(S_c)$, and $C_2^{(6)}$ shows that all these terms enter. The additional linear combinations of bilinear terms, satisfying the transformation requirement, are

$$[\mathcal{G}_{qr}^{ps}, \mathcal{G}_{ps}^{qr}]_+, \ y_q^{q'}[\mathcal{G}_{q'r}^{ps}, \mathcal{G}_{ps}^{qr}]_+, \ c_M^N[I_{Ns}^{Or}, I_{Or}^{Ns}]_+$$

$$-\tfrac{1}{4}[I_{Ns}^{Or}, I_{Or}^{Ns}]_+, \ [S_r^s, S_s^r]_+,$$

$$y_q^p[\mathcal{G}_{ps}^{qr}, \mathcal{S}_r^s]_+, \text{ and } c_M^N[I_{Ns}^{Mr}, S_r^s]_+.$$

Finally, we collect the admissible terms and obtain the mass formula for the s-wave meson $\underline{64} = \underline{1} + \underline{63}$. The terms, together with their form in terms of quantum numbers, are tabulated in Table II. These give the following twelve-parameter mass formula:

$$M = m_0 + m_1 n_\lambda + m_2 C_2^{(6)} + m_2 2S(S+1) + m_4 C_2^{(3)} + m_5[I(I+1) - \tfrac{1}{4}Y^2] + m_6[2S_\lambda(S_\lambda+1) - C_2^{(4)}(\mathfrak{N}, \mathcal{P}) + \tfrac{1}{4}Y^2]$$
$$+ m_7[2S_{\mathfrak{N},\mathcal{P}}(S_{\mathfrak{N},\mathcal{P}}+1) - 2S_\lambda(S_\lambda+1) - \tfrac{1}{3}2S_q(S_q+1)] + m_8 n_c + m_9 Y_c{}^2 + m_{10} 2S_c(S_c+1) + m_{11} 2S_q(S_q+1). \tag{9}$$

This mass formula predicts the following independent equalities:

$$F^* - D^* = F - D; \tag{10}$$

the strange-nonstrange mass differences for the pseudoscalar and vector SU(3) triplets, and antitriplets, are equal; and from the observed mass splittings of the states in the meson $\underline{36}$ of SU(6), the magnitude of this mass difference

$$F - D = 76 \text{ MeV linear mass formula}$$
$$(= 0.074 \text{ GeV}^2 \text{ quadratic mass formula}). \tag{11}$$

TABLE II. Independent terms contributing to mass-splitting operator.

Form in terms of quantum numbers	In form of generators
2 Linear Terms:	
n_λ	$Y(q) - Y(\bar{q})$
n_c	$Y_c(q) - Y_c(\bar{q})$
9 Bilinear Terms:	
$C_2^{(6)}$	$\tfrac{1}{2}[\mathcal{G}_{qr}^{ps}, \mathcal{G}_{ps}^{qr}]_+$
$2S(S+1)$	$\tfrac{1}{2}[S_r^s, S_s^r]_+$
$C_2^{(3)}$	$\tfrac{1}{2}[\mathcal{G}_q^p, \mathcal{G}_p^q]_+$
$I(I+1) - \tfrac{1}{4}Y^2 - \tfrac{1}{6}C_2^{(3)}$	$\tfrac{1}{2}y_q^{q'}[\mathcal{G}_{q'}^p, \mathcal{G}_p^q]_+$
$2S_\lambda(S_\lambda+1) - C_2^{(4)}(\mathfrak{N}, \mathcal{P}) + \tfrac{1}{4}Y^2 + \tfrac{1}{3}C_2^{(6)}$	$-y_q^{q'}[\mathcal{G}_{q'r}^{ps}, \mathcal{G}_{ps}^{qr}]_+$
$S_{\mathfrak{N},\mathcal{P}}(S_{\mathfrak{N},\mathcal{P}}+1) - S_\lambda(S_\lambda+1) - \tfrac{1}{3}S_q(S_q+1)$	$\tfrac{1}{2}[\mathcal{S}_r^s, y_q^p \mathcal{G}_{ps}^{qr}]_+$
$\tfrac{2}{3}Y_c{}^2 - C_2^{(3)}$	$c_M^N[I_N^O, I_O^M]_+ - \tfrac{1}{4}[I_N^O, I_O^N]_+$
$2S_c(S_c+1) + \tfrac{1}{3}Y_c{}^2 - C_2^{(6)}$	$c_M^N[I_{Ns}^{Or}, I_{Or}^{Ns}]_+ - \tfrac{1}{4}[I_{Ns}^{Or}, I_{Or}^{Ns}]_+$
$S_q(S_q+1) - S_c(S_c+1) - \tfrac{1}{2}S(S+1)$	$-\tfrac{1}{2}[S_r^s, c_M^N I_{Ns}^{Mr}]_+$

The numerical reduced coefficients as determined by the $\underline{36}$ are given in Appendix B. The quadratic formula for a D mass of 2 GeV implies an almost degenerate F mass of 2.02 GeV. Equation (10), as well as Eq. (11), is an SU(8) prediction, since the four states are in the $[6, 2^*]_1^{-1}$ multiplet of SU(6) $\times \mathrm{SU}(2)_{S_c} \times \mathrm{U}(1)_{Y_c}$ which involves recoupling the charmed and noncharmed quark spins.

Note that in Eq. (9) the SU(6)-breaking terms enter with the same coefficients for each SU(6) multiplet. This is the same as for the coefficients of the SU(3)-breaking terms in the Gürsey-Radicati formula and in the SU(8) baryon mass formula obtained below, Eq. (13). Here Eq. (9) mixes the $\underline{1}$ and $\underline{63}$ of SU(8) only as a consequence of the standard $\underline{1}$ and $\underline{35}$ mixing of η_1 and η_8 in the SU(6) theory. Relative to the $\mathrm{SU}(6) \to \mathrm{SU}(4)_{\mathfrak{N},\mathcal{P}} \times \mathrm{SU}(2)_{S_\lambda}$ chain, the $\eta - \eta'$ mixing is due to the $C_2^{(6)}$ and $C_2^{(3)}$ terms; the breaking of ideal ϕ-ω mixing is due to the $C_2^{(3)}$ term, i.e., it is solely responsible for ϕ not being a pure $\lambda\bar{\lambda}$ state. The choice $M_\omega = M_\rho$ requires, in addition to $C_2^{(3)}$ being absent, the absence of $[I(I+1) - \frac{1}{4}Y^2]$. From the viewpoint of an SU(3) singlet-octet system, n_λ mixes the singlet and octet whereas $[I(I+1) - \frac{1}{4}Y^2]$ only breaks the octet; however, the m_6 and m_7 terms also mix the states and break the octet. If systematic use of SU(6) is used to classify the terms, as in the SU(6) irreducible tensor approach, the m_5, m_6, and m_7 terms arise[16,21] from SU(6) tensors ${}^{35}T_{35}^{8,1}$, ${}^{35}T_{189}^{8,1}$, and ${}^{35}T_{405}^{8,1}$. Irreducible tensor operators are labeled ${}^{m}T_{\dim \mathrm{SU}(6)}^{\dim \mathrm{SU}(3), \dim \mathrm{SU}(2)_S}$, where m specifies the SU(6) state of q and/or $\bar{q}$. Experience[2,3] with baryon levels and past confusions[22] over inadequate meson-mass operators indicate that such operators with the larger SU(6) representations should *not* be excluded, but should be retained as has been done here. This means that only mixing angles can be predicted for the s-wave mesons of the $\underline{36}$ of SU(6); however, these predictions alone are significant for decay tests.

IV. MASS OPERATOR AND INDEPENDENT MASS FORMULAS FOR THE BARYON $\underline{120}$ SUPERMULTIPLET

For completeness, we first discuss the "SU(4) chain" reduction of the totally symmetric three-particle representation of SU(8), the $\underline{120}$, in which we place the baryons. As in the symmetric quark model, to be consistent with the spin and statistics theorem, we assume that there exists an SU(3)″-color degree of freedom and that the states in the $\underline{120}$ are in the totally antisymmetric three-particle representation of SU(3)″-color, the singlet. The direct sum

$$\underline{120} = \{20_s, 4\} + \{20_m, 2\}$$

in the $\mathrm{SU}(8) \to \mathrm{SU}(4) \times \mathrm{SU}(2)_S$ chain with the notation $\{\dim \mathrm{SU}(4), \dim \mathrm{SU}(2)_S\}$, where a permutation-symmetry subscript, s = symmetric and m = mixed, suffices to distinguish the two twenty-plet Young diagrams. Under $\mathrm{SU}(4) \to \mathrm{SU}(3) \times \mathrm{U}(1)_{Y_c}$,

$$20_m = 8_0 + 6_1 + 3_1^* + 3_2,$$

$$20_s = 10_0 + 6_1 + 3_2 + 1_3,$$

with the notation $\dim \mathrm{SU}(3)_{n_c}$. Note that $n_c = C$, i.e., n_c has the charm eigenvalue, for states containing no antiquarks. While the SU(3) decuplet and octet are obtained by this reduction, their physical relation as submultiplets of the $\underline{56}$ of SU(6) is not made manifest by the SU(4) reduction chain. Hence, we return to the SU(6) chain.

The relevant reductions of the $\underline{120}$ under the $\mathrm{SU}(8) \to \mathrm{SU}(6) \times \mathrm{SU}(2)_{S_c} \times \mathrm{U}(1)_{Y_c}$ chain are

$$\underline{120} = [56, 1]_0 + [21, 2]_1 + [6, 3]_2 + [1, 4]_3$$

with the notation, as before, of $[\dim \mathrm{SU}(6), \dim \mathrm{SU}(2)_{S_c}]_{n_c}$ and then under $\mathrm{SU}(6) \to \mathrm{SU}(3) \times \mathrm{SU}(2)_{S_q}$

$$\underline{56} = (10, 4) + (8, 2)$$

$$\underline{21} = (6, 3) + (3^*, 1),$$

$$\underline{6} = (3, 2),$$

$$\underline{1} = (1, 1),$$

with the notation $(\dim \mathrm{SU}(6), \dim \mathrm{SU}(2)_{S_q})$. In order to obtain the physical states with $n_c \neq 0$, the spin of the charmed and noncharmed quarks must be recoupled, i.e., $\vec{S} = \vec{S}_q + \vec{S}_c$. This yields

$$[21, 1]_1 = (6, 3)\tfrac{3}{2}^+ + (6, 3)\tfrac{1}{2}^+ + (3^*, 1)\tfrac{1}{2}^+,$$

$$[6, 3]_2 = (3, 2)\tfrac{3}{2}^+ + (3, 2)\tfrac{1}{2}^+,$$

$$[1, 4]_3 = (1, 1)\tfrac{3}{2}^+,$$

with the notation $(\dim \mathrm{SU}(3), \dim \mathrm{SU}(2)_{S_q})J^P$, $S = J$ for the $\underline{120}$ representation. It is a straightforward exercise to tabulate the wave functions for the thirteen charmed states, and from their composition in terms of $\mathcal{P}$-, $\mathfrak{N}$-, λ-, and c-type quarks to read off their respective I, B, Y, and C quantum numbers. The (6, 3) consists of an isospin singlet $(\lambda\lambda c)^0$, a doublet $(\mathfrak{N}\lambda c)_S^0$ and $(\mathcal{P}\lambda c)_S^+$, and a triplet $(\mathfrak{N}\mathfrak{N}c)^0$, $(\mathfrak{N}\mathcal{P}c)_S^+$, and $(\mathcal{P}\mathcal{P}c)^{++}$. The $(3^*, 1)$ consists of a doublet $(\mathfrak{N}\lambda c)_A^0$ and $(\mathcal{P}\lambda c)_A^+$, and a singlet $(\mathfrak{N}\mathcal{P}c)_A^+$. The (3, 2) consists of a singlet $(\lambda cc)^+$ and a doublet $(\mathfrak{N}cc)^+$ and $(\mathcal{P}cc)^{++}$. The S and A subscripts denote the permutation symmetry of the two-particle combination of $\mathcal{P}$, $\mathfrak{N}$, λ quarks.

For the 120 representation of SU(8) there exists the identity

$$2S_c(S_c+1)=C+\tfrac{1}{2}C^2. \tag{12}$$

Thus, by the derivation of Sec. III, the most general $SU(6)\times SU(2)_{S_c}\times U(1)_{Y_c}$ mass formula for the 120 which satisfies our conditions has eleven parameters and is

$$\begin{aligned} M={}&m_0+m_1Y+m_2C_2^{(3)}+m_3[I(I+1)-\tfrac{1}{4}Y^2]+m_4C_2^{(6)}+m_5 2S(S+1)+m_6Y_c+m_7Y_c^{\,2}\\ &+m_8[2S_\lambda(S_\lambda+1)-C_2^{(4)}(\mathfrak{N},\mathcal{P})+\tfrac{1}{4}Y^2]+m_9[2S_{\mathfrak{N},\mathcal{P}}(S_{\mathfrak{N},\mathcal{P}}+1)-2S_\lambda(S_\lambda+1)]+m_{10}2S_q(S_q+1). \end{aligned} \tag{13}$$

On the 56 of SU(6) only the first four terms, the Gürsey-Radicati formula, are independent. Here, as in SU(6) theory, for baryons the two-body dominance assumption has led to a significant simplification.

For the 56 of SU(6) the Gürsey-Radicati formula yields four independent sum rules. For the thirteen additional states in the 120 of SU(8), Eq. (13) predicts the following six new independent equalities:

$$[(\mathcal{P}\mathcal{P}c)-(\mathcal{P}\lambda c)_S]_{(6,3)1/2^+}=[(\mathcal{P}\lambda c)_S-(\lambda\lambda c)]_{(6,3)1/2^+}, \tag{14}$$

$$[(\mathcal{P}\mathcal{P}c)-(\mathcal{P}\lambda c)_S]_{(6,3)3/2^+}=[(\mathcal{P}\lambda c)_S-(\lambda\lambda c)]_{(6,3)3/2^+}, \tag{15}$$

$$[(\mathcal{P}\mathcal{P}c)-(\mathcal{P}\lambda c)_S]_{(6,3)1/2^+}=[\text{same}]_{(6,3)3/2^+}, \tag{16}$$

$$[(\mathcal{P}cc)-(\lambda cc)]_{(3,2)1/2^+}=[\text{same}]_{(3,2)3/2^+}, \tag{17}$$

$$(\mathcal{P}cc)_{(3,2)3/2^+}-(\mathcal{P}cc)_{(3,2)1/2^+}=(\mathcal{P}\mathcal{P}c)_{(6,3)3/2^+}-(\mathcal{P}\mathcal{P}c)_{(6,3)1/2^+}, \tag{18}$$

$$\begin{aligned} 4[(\mathcal{P}cc)-(\lambda cc)]_{(3,2)1/2^+}-3[(\mathcal{P}\mathcal{P}c)-(\mathcal{P}\lambda c)_S]_{(6,3)1/2^+}-\tfrac{1}{5}[(\mathfrak{N}\mathcal{P}c)_A-(\mathcal{P}\lambda c)_A]_{(3^*,1)1/2^+}&=\tfrac{4}{5}[N-\tfrac{1}{4}(\Lambda+3\Sigma)]\\ &=-188\text{ MeV}, \end{aligned} \tag{19}$$

where the subscripts denote (dim SU(3), dim $SU(2)_{S_q}$)J^P. Equalities (14) and (15) specify equal spacing for both of the two SU(3) sextets, Eq. (16) specifies that this spacing is also common, Eq. (17) specifies a common spacing for the two SU(3) triplets, Eq. (18) specifies the same separation between iostopic spin multiplets in the $J^P=\frac{3}{2}^+$ and $\frac{1}{2}^+$ levels for the triplets as for the sextets, and Eq. (19) specifies a relation between the splitting of the antitriplet and those of the other charm levels and the nucleon octet.

Equations (14) and (15) are SU(3) equal-spacing statements. Both Eq. (16) and (17) are SU(8) results because recoupling of $\vec{S}_q$ and $\vec{S}_c$ is involved, and clearly Eqs. (18) and (19) are SU(8) results.

V. SUMMARY

We again emphasize, from the point of view of future ψ spectroscopy, that the mass relations derived in this article preserve $c\bar{c}$ purity of the $J/\psi(3.1)$ resonance. We studied the charmed symmetric quark model for mesons and baryons using approximate $SU(6)\times SU(2)_{S_c}\times U(1)_{Y_c}$ symmetry with breaking in the Y and Y_c directions in order to resolve mass degeneracies among resonances in the same submultiplets. To reduce the number of possible mass formulas for baryons, we assumed that one- and two-body contributions to the mass-splitting operator dominate. For the six meson levels containing charmed quarks, we predicted two new independent mass relations. For the corresponding thirteen new baryon levels, we predicted six new mass relations. In an appendix, for reference relative to previous SU(6) symmetric-quark-model mass analyses, we gave the reduced numerical coefficients as determined by the meson 36 of SU(6).

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APPENDIX A: SECOND-QUANTIZED FORMALISM

We introduce a set of Fermi creation and annihilation operators[1] for the s-wave quarks and antiquarks, $a_{Mr}^{q''\dagger}$, $a_{q''}^{Ns}$, $b_{q''}^{Mr\dagger}$, and $b_{Ns}^{q''}$ where $M=1,2,3,4$ for SU(4), $r=1,2$ for $SU(2)_{\mathcal{S}}$, and $q''=1,2,3$ for SU(3)″ color. The spin and SU(4) indices for the antiquark operators can be grouped together since complex-conjugate representations in SU(2) are unitarily equivalent to the original ones. The generators of SU(8) constructed in terms of these are

$$I_{Mr}^{Ns}=I(q)_{Mr}^{Ns}+I(\bar{q})_{Mr}^{Ns}, \tag{A1}$$

with

$$\begin{aligned} I(q)_{Mr}^{Ns}&=a_{Mr}^{q''\dagger}a_{q''}^{Ns}-\tfrac{1}{8}\delta_{Mr}^{Ns}a_{Ot}^{q''\dagger}a_{q''}^{Ot},\\ I(\bar{q})_{Mr}^{Ns}&=-(b_{q''}^{Ns\dagger}b_{Mr}^{q''}-\tfrac{1}{8}\delta_{Mr}^{Ns}b_{q''}^{Ot\dagger}b_{Ot}^{q''}). \end{aligned} \tag{A2}$$

From these expressions, $Y_c(q)=\frac{1}{4}(-N_q+3N_c)$ and $\overline{Y}_c(\overline{q})=-\frac{1}{4}(-N_{\overline{q}}+3N_{\overline{c}})$. These lead in the s-wave meson mass-splitting operator, for a system composed of a fixed number of quarks and antiquarks, to a single charge-conjugation invariant term,

$$n_c=N_c+N_{\overline{c}}=[Y_c(q)-\overline{Y}_c(\overline{q})]+\tfrac{1}{4}(N+\overline{N}). \qquad \text{(A3)}$$

This is the number operator for the total number of charmed quarks and antiquarks.

From the other linearly independent terms in the $SU(6)\times SU(2)_{S_c}\times U(1)_{Y_c}$ meson and baryon mass-splitting operators in the text, this second quantized formalism can also be used to extract specific dynamical parameters characterizing single quarks and the two-body interquark forces. Such an explicit interpretation in the three-quartet model of the forces responsible for the observed hadronic mass splittings brings these mass operators in closer contact with more basic quantum-field-theory approaches to quark dynamics, for example, gauge fields on a lattice and the bag model.

APPENDIX B: NUMERICAL COEFFICIENTS AS DETERMINED BY MESON $\underline{36}$ OF SU(6)

On the s-wave meson $\underline{36}$ multiplet of SU(6) the terms in the meson mass formula derived in the text, Eq. (9), reduce to the first eight terms. For each term a normalization factor $\mathfrak{N}_i=(T^i_{\max}-T^i_{\min}+1)^{-1}$ is introduced so as to treat them in a comparable manner. It is in order of the terms in Eq. (9), 1, $\frac{1}{3}$, $\frac{1}{13}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{5}$, $\frac{1}{13}$, and $\frac{1}{9}$. The reduced coefficients, $M_i=m_i/\mathfrak{N}_i$, for the linear (quadratic) mass formula as determined from the experimental data[23] are 1.000, −0.489, −1.853, 0.594, 1.800, −0.411, 0.593, and 0.530 (0.975, −0.353, −2.364, 0.697, 1.967, −0.710, −0.176, and 0.054) in units of GeV (GeV^2). The last term does not contribute significantly to the quadratic mass formula. Otherwise, for a simultaneous treatment of $J^P=0^-$ and 1^- states it does not seem possible to reduce the number of terms *a priori*, for example, by abstracting rules from the nearness of mesons to eigenstates of the $SU(6)\rightarrow SU(4)_{\mathfrak{N},\mathcal{P}}\times SU(2)_{S_\lambda}$ chain. Note that for both the linear and quadratic formulas, the two terms with largest reduced coefficients are $C_2^{(6)}$ and $C_2^{(3)}$ which are the operators responsible for $\eta-\eta'$ and ϕ-ω mixing of the associated eigenstates of the $SU(6)\rightarrow SU(4)_N\times SU(2)_{S_\lambda}$ chain.

The ideal mixing angle is

$$\theta_{SU(4)_{\mathfrak{N},\mathcal{P}}}=\tan^{-1}(1/\sqrt{2})=35°16'$$

to be compared with the empirical mixing angles $\theta_V=37°27'$ (linear), $39°59'$ (quadratic) and $\theta_P=-24°$ (linear), $-10°33'$ (quadratic) as determined from $\sin^2\theta_V=[\phi-\frac{1}{3}(4K^*-\rho)]/(\phi-\omega)$, etc.

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