Some predictions of the Dashen–Gell-Mann program concerning the asymptotic behavior of the form factors and the threshold behavior of the scaling functions νW_2. II. Diquark model for baryons

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Baryons are viewed as the two-body bound states, composed of a quark and a new pointlike object which can be regarded as two quarks glued together (diquark). The spin of the diquark can be either zero or one, and its charge is simply the sum of charges of the individual quarks that form the diquark. From the group-theoretical point of view the diquarks are regarded as spinors from the (reducible) representation $\underline{6}\times\underline{6}$ of the SU(6) group. We discuss the scaling properties of the diquark model and make a number of predictions concerning the scaling behavior of the invariant structure functions of the proton and the neutron target. We stress that the umbilical cord with the current algebra can easily be broken, and that most predictions of the diquark model can be obtained without reference to the Dashen–Gell-Mann program.

DIFFICULTIES WITH THE ORDINARY QUARK MODEL

In the quark model of Gell-Mann and Zweig, one deals with the three elementary pointlike objects, characterized by spin-parity $\frac{1}{2}^+$ and fractional charges $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$. Mesons are bound states of $q\bar{q}$ (quark-antiquark) type, while baryons are three-body bound states of qqq type. This model has been remarkably successful in providing an acceptable explanation for almost all aspects of particle phenomenology such as mass spectra, magnetic moments, electromagnetic and strong decay rates, scaling, etc. Recently, however, several problems have appeared, including the difficulties in understanding, within the boundaries of the usual symmetric qqq configuration for the nucleons, why the ratio of the scaling functions $(\nu W_2)_{\rm neutron}/(\nu W_2)_{\rm proton}$ drops so low (~0.29) at the threshold $\xi\to 1$ and, equally disturbing, why the predicted SU(6) baryonic multiplets $\underline{20}$ are still not seen. Both difficulties can be easily resolved within the framework of the diquark model, apparently without the penalty of having to explain embarrassing disagreements with experiment elsewhere. This initial success of the diquark model, together with the obvious advantage from the computational point of view of dealing with two bodies instead of three, encourages us to consider the possibility of the quark-diquark configuration in the nucleon quite seriously.

DIQUARK MODEL FOR THE BARYONS

In the ordinary quark model, baryons are regarded as the bound states of three quarks with two relative coordinates as the expressions of the internal space-time degrees of freedom. We adopt here a somewhat modified point of view according to which two quarks in the baryon are glued together (diquark), leaving only one relative coordinate free.[1] We remain faithful to the idea that quarks are equipped with spin $\frac{1}{2}$, and that it is meaningful and profitable to regard them as different states of the fundamental 6-dimensional spinorial representation of SU(6) group (which mixes spin and unitary spin degrees of freedom). Using the obvious notation we can put this statement in the succinct form by writing

$$\text{quark} = \begin{bmatrix} \mathcal{P}^\uparrow \\ \mathfrak{N}^\uparrow \\ \lambda^\uparrow \\ \mathcal{P}^\downarrow \\ \mathfrak{N}^\downarrow \\ \lambda^\downarrow \end{bmatrix}.$$

(The electric charges of $\mathcal{P}$, $\mathfrak{N}$, and λ quarks are respectively $\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$.) With the assumption that the quarks carry SU(6) degrees of freedom, the essence of the group-theoretical aspect of the diquark picture can be expressed in the following two equations:

$$\underline{6}\times\underline{6}=\underline{21}+\underline{15}$$

and

$$\underline{21}\times\underline{6}=\underline{56}+\underline{70}\,.$$

The antisymmetric representation $\underline{15}$ leads to the controversial $\underline{20}$ and another $\underline{70}$ via

$$\underline{15}\times\underline{6}=\underline{20}+\underline{70}\,.$$

Since the baryonic multiplet $\underline{20}$ has not been observed thus far, it is conceivable that the diquarks from $\underline{15}$ do not couple to quarks. This is the diquark model explanation for the absence of $\underline{20}$.[2] No statement of similar simplicity can be made within the framework of the ordinary quark model.

The SU(3) content of $\underline{21}$ is reflected in the symbolic statement

$$\underline{21}=\{6\}\times 3+\{\bar{3}\}\times 1\,,$$

where the curly brackets enclose the irreducible representations of SU(3) and the factors 3 and 1 refer to the diquark spin degrees of freedom. In other words, there is an SU(3) sextet of axial-vector diquarks and an SU(3) triplet of scalar diquarks. We will denote by α the mixing angle between these two irreducible representations of SU(3). The presence of α, of course, means breaking of SU(6) symmetry. However, the SU(6) symmetry can be restored by setting α equal to $\pi/4$, meaning that in this case $\{6\}$ and $\{\bar{3}\}$ will enter in $\underline{21}$ with the same weight.

The new SU(6) basis for baryons is formed from the tensor products of the diquark and the remaining quark states. In particular, for the proton and the neutron wave functions, we will have

$$\begin{aligned}
|\mathrm{proton}^{\uparrow}\rangle &= \frac{1}{\sqrt{18}}\left[\left(2d_{11}^{\uparrow}\mathcal{N}^{\downarrow}-\sqrt{2}\,d_{11}^{\rightarrow}\mathcal{N}^{\uparrow}-\sqrt{2}\,d_{10}^{\uparrow}\mathcal{P}^{\downarrow}+d_{10}^{\rightarrow}\mathcal{P}^{\uparrow}\right)\sin\alpha+3t_{00}\mathcal{P}^{\uparrow}\cos\alpha\right],\\
|\mathrm{proton}^{\downarrow}\rangle &= \frac{1}{\sqrt{18}}\left[\left(2d_{11}^{\downarrow}\mathcal{N}^{\uparrow}-\sqrt{2}\,d_{11}^{\rightarrow}\mathcal{N}^{\downarrow}-\sqrt{2}\,d_{1-1}^{\downarrow}\mathcal{P}^{\uparrow}+d_{10}^{\rightarrow}\mathcal{P}^{\downarrow}\right)\sin\alpha+3t_{00}\mathcal{P}^{\downarrow}\cos\alpha\right],\\
|\mathrm{neutron}^{\uparrow}\rangle &= \frac{1}{\sqrt{18}}\left[\left(-2d_{1-1}^{\uparrow}\mathcal{P}^{\downarrow}+\sqrt{2}\,d_{1-1}^{\rightarrow}\mathcal{P}^{\uparrow}+\sqrt{2}\,d_{10}^{\uparrow}\mathcal{N}^{\downarrow}-d_{10}^{\rightarrow}\mathcal{N}^{\uparrow}\right)\sin\alpha+3t_{00}\mathcal{N}^{\uparrow}\cos\alpha\right],\\
|\mathrm{neutron}^{\downarrow}\rangle &= \frac{1}{\sqrt{18}}\left[\left(-2d_{1-1}^{\downarrow}\mathcal{P}^{\uparrow}+\sqrt{2}\,d_{1-1}^{\rightarrow}\mathcal{P}^{\downarrow}+\sqrt{2}\,d_{10}^{\downarrow}\mathcal{N}^{\uparrow}-d_{10}^{\rightarrow}\mathcal{N}^{\downarrow}\right)\sin\alpha+3t_{00}\mathcal{N}^{\downarrow}\cos\alpha\right],
\end{aligned} \tag{1}$$

where

$$d_{11}^{\uparrow}=\mathcal{P}^{\uparrow}\mathcal{P}^{\uparrow}\,,\quad d_{10}^{\uparrow}=\frac{1}{\sqrt{2}}(\mathcal{P}^{\uparrow}\mathcal{N}^{\uparrow}+\mathcal{N}^{\uparrow}\mathcal{P}^{\uparrow})\,,\quad d_{1-1}^{\uparrow}=\mathcal{N}^{\uparrow}\mathcal{N}^{\uparrow}\,,$$

$$d_{11}^{\rightarrow}=\frac{1}{\sqrt{2}}(\mathcal{P}^{\uparrow}\mathcal{P}^{\downarrow}+\mathcal{P}^{\downarrow}\mathcal{P}^{\uparrow})\,,\quad d_{10}^{\rightarrow}=\tfrac{1}{2}[(\mathcal{P}^{\uparrow}\mathcal{N}^{\downarrow}+\mathcal{N}^{\downarrow}\mathcal{P}^{\uparrow})+(\mathcal{P}^{\downarrow}\mathcal{N}^{\uparrow}+\mathcal{N}^{\uparrow}\mathcal{P}^{\downarrow})]\,,\quad d_{1-1}^{\rightarrow}=\frac{1}{\sqrt{2}}(\mathcal{N}^{\uparrow}\mathcal{N}^{\downarrow}+\mathcal{N}^{\downarrow}\mathcal{N}^{\uparrow})$$

$$d_{11}^{\downarrow}=\mathcal{P}^{\downarrow}\mathcal{P}^{\downarrow}\,,\quad d_{10}^{\downarrow}=\frac{1}{\sqrt{2}}(\mathcal{P}^{\downarrow}\mathcal{N}^{\downarrow}+\mathcal{N}^{\downarrow}\mathcal{P}^{\downarrow})\,,\quad d_{1-1}^{\downarrow}=\mathcal{N}^{\downarrow}\mathcal{N}^{\downarrow}\,,$$

and

$$t_{00}=\tfrac{1}{2}[(\mathcal{P}^{\uparrow}\mathcal{N}^{\downarrow}+\mathcal{N}^{\downarrow}\mathcal{P}^{\uparrow})-(\mathcal{P}^{\downarrow}\mathcal{N}^{\uparrow}+\mathcal{N}^{\uparrow}\mathcal{P}^{\downarrow})]\,.$$

Note that the electric charges of diquark states are

$$\langle d_{11}|\,\mathrm{charge}\,|d_{11}\rangle=\tfrac{4}{3}\,,$$

$$\langle d_{10}|\,\mathrm{charge}\,|d_{10}\rangle=\tfrac{1}{3}\,,$$

$$\langle d_{1-1}|\,\mathrm{charge}\,|d_{1-1}\rangle=-\tfrac{2}{3}\,,$$

and

$$\langle t_{00}|\,\mathrm{charge}\,|t_{00}\rangle=\tfrac{1}{3}\,.$$

There exist, of course, other diquark states that do not appear in the proton or the neutron wave functions. They can be found in the wave functions of Λ, Σ, and Ξ baryons, or in the wave functions of the $\frac{3}{2}^{+}$ baryonic decouplet, and the wave functions of baryons from the SU(6) representation $\underline{70}$.

The subscripts of d and t states refer to the isotopic spin T and the third component T_3 of the isotopic spin. For example, d_{10} stands for the $T=1$, $T_3=0$ axial-vector diquark state, t_{00} stands for the $T=0$, $T_3=0$ scalar diquark state, etc. The arrows attached to d_{10} and $d_{1\pm1}$ states stand for three possible directions of spin 1, while the spin of the t_{00} state is zero and therefore there is no need for a special superscript. Note that in the nucleonic wave function only $T=0$ and $T=1$ diquarks participate. It so happens that the $T=0$ diquarks also carry spin 0 (scalar diquarks), while the $T=1$ diquarks are equipped with spin 1 (vector diquarks).

Many interesting physical questions can be answered solely from the knowledge of the Clebsch-Gordan coefficients that appear in (1). The calculations are elementary. We give a small sample of examples that involve averaging over the quark and the diquark helicities:

probability that the proton is found in the ($\mathcal{P}$-quark, isoscalar diquark) state $=\frac{1}{2}\cos^2\alpha\langle t_{00}\mathcal{P}\,|\,t_{00}\mathcal{P}\rangle$,

probability that the proton is found in the ($\mathcal{P}$-quark, isovector diquark) state $=\frac{1}{6}\sin^2\alpha\langle d_{10}\mathcal{P}\,|\,d_{10}\mathcal{P}\rangle$,

probability that the proton is found in the ($\mathcal{N}$-quark, isovector diquark) state $=\frac{1}{3}\sin^2\alpha\langle d_{11}\mathcal{N}\,|\,d_{11}\mathcal{N}\rangle$,

etc.

Note that the squares of the individual components of the nucleonic wave function that appear on the right-hand sides of these equalities are functions of the space-time coordinates. In the scaling region this dependence upon the space-time is replaced by the dependence upon the scaling variable ξ. We will make use of the given set of examples later on, when we discuss the rates of specific electroproduction processes that depend in a crucial way upon the SU(3) degrees of freedom in the nucleonic wave functions.

Baryonic currents in the diquark picture that we have described in the preceding article involve only one relative coordinate. They read

$$F_i(\vec{q}_\perp)=\frac{\omega_i}{2}\exp(i\vec{q}_\perp\cdot\vec{X}_\perp/2)+\frac{\lambda_i}{2}\exp(-i\vec{q}_\perp\cdot\vec{X}_\perp/2)\,,$$

where $\lambda_i/2$ are the 3×3 Gell-Mann matrices acting on the single quark components of the baryonic wave function, while $\omega_i/2$ are the 9×9 matrices in the space of diquarks whose SU(3) section is reducible and split into the direct sum of $\{6\}$ and $\{\overline{3}\}$. In the present article we allow for more freedom by considering the most general case of two bodies, i.e.,

$$F_i(\vec{q}_\perp)=\frac{\omega_i}{2}\exp(i\vec{q}_\perp\cdot\vec{X}_{\perp,1})+\frac{\lambda_i}{2}\exp(i\vec{q}_\perp\cdot\vec{X}_{\perp,2})\,. \qquad (2)$$

We believe that the novelty of having to deal with two space-time coordinates, instead of one, will not upset appreciably the validity of our basic conjecture, namely that the Dashen–Gell-Mann program can be trusted for large values of the momentum transfer and/or for values of the scaling variable close to the threshold $\xi=1$. On the other hand, by using the most general expression for the two-body currents, we gain considerable leeway when comparing the predictions of the model with the experimental data.

At this point it should be emphasized that most of the structure of the diquark model can stand without the scaffolding of the Dashen–Gell-Mann program, although reference to the latter will become desirable when specific model calculations are being contemplated.

The currents of the most general type (2) give rise to the four different independent spin-averaged components of the nucleonic scaling function νW_2, corresponding to the four different spin-averaged components of the nucleonic wave function in the configuration space. These components differ in the isospin number of the participating diquark, which is either 0 or 1, and in the manner in which the scattering takes place, i.e., whether the quark or the diquark piece of the electromagnetic current is engaged in the scattering process (see Fig. 1). We denote the described four components of the nucleonic scaling function by $f_{0,1}^{q,d}(\xi)$, and state the normalization conditions

$$\int_0^1 f_{0,1}^{q,d}(\xi)d\xi=1\,.$$

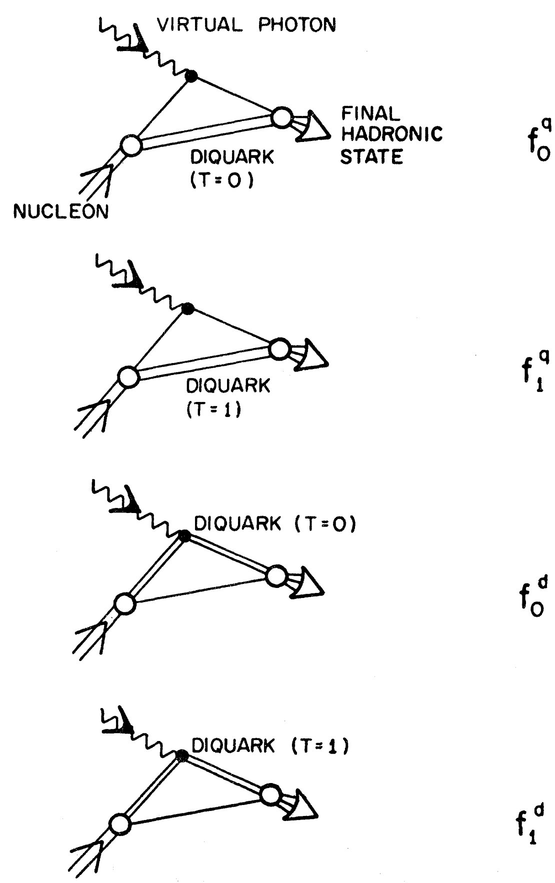

FIG. 1. Graphic representation for the scaling components $f_{0,1}^{q,d}(\xi)$ of the nucleonic structure functions $(1/\xi)(\nu W_2)$ in the diquark model.

Among other things, these normalization conditions guarantee that the positive-definite quantities $f_{0,1}^{q,d}$ will be significantly different from zero on at least one sector of the $0 \le \xi \le 1$ interval. In particular, it is impossible for a diquark to play always the role of a spectator. For some values of ξ the electrons will necessarily scatter on diquarks as well.

The SU(6) content of the nucleonic wave function in the diquark model is given by (1). It leads to

$$\begin{aligned} \frac{1}{\xi}(\nu W_2)_{\text{proton}} &= \tfrac{4}{18}\cos^2\alpha f_0^q(\xi) + \tfrac{1}{18}\cos^2\alpha f_0^d(\xi) \\ &\quad + \tfrac{2}{18}\sin^2\alpha f_1^q(\xi) + \tfrac{11}{18}\sin^2\alpha f_1^d(\xi)\,, \\ \frac{1}{\xi}(\nu W_2)_{\text{neutron}} &= \tfrac{1}{18}\cos^2\alpha f_0^q(\xi) + \tfrac{1}{18}\cos^2\alpha f_0^d(\xi) \\ &\quad + \tfrac{3}{18}\sin^2\alpha f_1^q(\xi) + \tfrac{3}{18}\sin^2\alpha f_1^d(\xi)\,. \end{aligned} \tag{3}$$

The integrals over the proton and neutron structure functions

$$I_{\text{proton},\text{neutron}} = \int_0^1 \frac{1}{\xi}(\nu W_2)_{\text{proton,neutron}}\, d\xi$$

are equal to

$$I_{\text{proton}} = \tfrac{5}{18} + \tfrac{8}{18}\sin^2\alpha\,, \quad I_{\text{neutron}} = \tfrac{2}{18} + \tfrac{4}{18}\sin^2\alpha\,.$$

Note that

$$I_{\text{proton}} + I_{\text{neutron}} = \tfrac{7}{18} + \tfrac{2}{3}\sin^2\alpha \ge \tfrac{7}{18}\,,$$

which represents a modification of the well-known Bjorken lower bound $I_{\text{proton}} + I_{\text{neutron}} \ge \frac{1}{2}$. To get a rough estimate of the value of the phenomenological mixing parameter α, we consider the experimental data for the difference

$$\Delta I = I_{\text{proton}} - I_{\text{neutron}}\,.$$

In the usual quark model with fractional charges, the theoretical value of ΔI is $\frac{1}{3}$. Experimentally, $\Delta I_{\text{exp}} = 0.2 \pm (?)$. The diquark model gives

$$\Delta I(\alpha) = \tfrac{3}{18} + \tfrac{4}{18}\sin^2\alpha\,,$$

a number which lies in the interval

$$0.167\big|_{\alpha=0} \le \Delta I(\alpha) \le 0.389\big|_{\alpha=\pi/2}\,,$$

$$\Delta I_{\text{exp}} = 0.2 \pm (?)\,.$$

We note that the experimental data favor a small value for α, and definitely rule out the values above $\alpha = \pi/4$, say. (Compare the ease with which the diquark model accommodates the experimental value for ΔI with the difficulties of the ordinary quark model in this area.) In a search for possible inconsistencies in the described diquark model for baryons, we have made rough calculations of the mass spectra, the magnetic moments for the $\frac{1}{2}^+$ baryonic octet, and the $|g_A/g_V|$ ratio for the neutron. A comparison with experiment favors in all cases a small value for α, giving assurances that the diquark idea, as far as we can tell at present, is not self-contradictory.

Nevertheless, from a quantitative point of view the situation is less gratifying, since the discrepancies between various predicted values of α are large, ranging all the way from $\alpha = 0$ to $\alpha = \pi/4$. This instability of the parameter α should be interpreted as an early warning against taking an overly optimistic attitude toward the diquark model. Although it is already clear that the diquark picture is capable of answering in at least qualitative fashion a considerable number of important physical questions, it appears unlikely that the diquark model will develop into a complete physical theory in the sense that all dynamical issues will eventually get resolved in the outlined narrow framework. Presumably the nucleon spends a fraction of its time in the usual 3-quark configuration, and a sizable fraction of its time in the quark-diquark configuration. On the phenomenological level this ambivalent behavior of the nucleon can be accounted for by introducing the associated probabilities P_q and P_d, $0 \le P_q$, $P_d \le 1$, $P_q + P_d = 1$, and by calculating the quantities of physical interest according to the following rule of averaging:

$$\langle A\rangle = P_q\langle A\rangle_{3\text{-quark configuration}} + P_d\langle A\rangle_{\text{diquark configuration}}\,.$$

In the present article we do not consider further this more general framework, nor do we attempt to go deeper into the diquark model by trying to answer the more fundamental question of why the quarks spend a good fraction of their time glued together in pairs.

Let us now return to the four functions $f_{0,1}^{q,d}(\xi)$ about which we know at this point very little. To get a rough idea of the behavior of the individual $f_{0,1}^{q,d}(\xi)$ components on the interval $0 \le \xi \le 1$, we consider the sum rules for the quantities

$$J_{\text{proton,neutron}} = \int_0^1 (\nu W_2)_{\text{proton,neutron}}\, d\xi\,.$$

Experimentally, these quantities are known with considerable accuracy, namely

$$J_{\text{proton}} = 0.15 \pm 0.01\,, \quad J_{\text{neutron}} = 0.11 \pm 0.01\,.$$

From (3) we obtain

$$\begin{aligned} \frac{\cos^2\alpha}{18}(4Q_0 + D_0) + \frac{\sin^2\alpha}{18}(2Q_1 + 11D_1) &= J_{\text{proton}}\,, \\ \frac{\cos^2\alpha}{18}(Q_0 + D_0) + \frac{\sin^2\alpha}{18}(3Q_1 + 3D_1) &= J_{\text{neutron}}\,, \end{aligned} \tag{4}$$

where the four entities $Q_{0,1}$ and $D_{0,1}$ are the first moments of the corresponding $f_{0,1}^{q,d}$ functions, i.e.,

$$Q_{0,1}=\int_0^1 \xi f_{0,1}^{q}(\xi)d\xi$$

and

$$D_{0,1}=\int_0^1 \xi f_{0,1}^{d}(\xi)d\xi\,.$$

Note that all four entities $Q_{0,1}$ and $D_{0,1}$ are positive-definite, and furthermore limited to the interval [0,1],

$$0\le Q_{0,1}\le 1\,,\qquad 0\le D_{0,1}\le 1\,. \tag{5}$$

The inequalities (5) place a severe constraint on the type of solutions that one can expect from the system of equations (4). Nonetheless, for α small but finite, the system (4) still has a variety of solutions from which we must select the one which does not contradict the observed behavior of the ratio

$$R(\xi\mid\alpha)=\frac{(\nu W_2)_{\rm neutron}}{(\nu W_2)_{\rm proton}}=\frac{\cos^2\alpha[f_0^q(\xi)+f_0^d(\xi)]+\sin^2\alpha[3f_1^q(\xi)+3f_1^d(\xi)]}{\cos^2\alpha[4f_0^q(\xi)+f_0^d(\xi)]+\sin^2\alpha[2f_1^q(\xi)+11f_1^d(\xi)]} \tag{6}$$

on the interval $0\le\xi\le1$. Specifically, we assume that

$$11D_1\ll 2Q_1\,,\quad D_0\ll Q_0\,,$$

and

$$4J_{\rm neutron}-J_{\rm proton}\le\tfrac{5}{9}\sin^2\alpha\le\tfrac{5}{9}-(3J_{\rm proton}-2J_{\rm neutron})\,.$$

This last condition serves to assure us that the constraints (5) will indeed be satisfied as well. Note the implicit assumption

$$J_{\rm proton}+J_{\rm neutron}\le\tfrac{5}{18}\,,$$

which represents a tight upper bound, almost saturated by the experimental numbers $J_{\rm proton}\simeq0.15$ and $J_{\rm neutron}\simeq0.11$.

Under the existing conditions, the solution of the system (4) is given by

$$Q_0=\tfrac{9}{5}(3J_{\rm proton}-2J_{\rm neutron})/\cos^2\alpha$$

and

$$Q_1=\tfrac{9}{5}(4J_{\rm neutron}-J_{\rm proton})/\sin^2\alpha\,.$$

In the limits $\xi\to0$ and $\xi\to1$, respectively, we have

$$R(\xi\mid\alpha)=\frac{f_0^d(\xi)+3\tan^2\alpha f_1^d(\xi)}{f_0^d(\xi)+11\tan^2\alpha f_1^d(\xi)}\quad(\xi\to0)$$

and

$$R(\xi\mid\alpha)=\frac{f_0^q(\xi)+3\tan^2\alpha f_1^q(\xi)}{4f_0^q(\xi)+2\tan^2\alpha f_1^q(\xi)}\quad(\xi\to1)\,.$$

We have derived these expressions on the basis of the assumption that the $f_{0,1}^q$ functions are large at the threshold $\xi\to1$, while the functions $f_{0,1}^d$ are large for small values of ξ, $\xi\to0$. This conjecture about the behavior of the $f_{0,1}^{q,d}$ functions on the interval $0\le\xi\le1$ is based on the earlier estimate of the magnitude of their first moments $Q_{0,1}$ and $D_{0,1}$. Of course, a knowledge of the first moment by itself does not provide us with sufficient information to determine the exact shape of the associated function, but in the implicit presence of some additional assumptions about the "reasonable" behavior of the function in question, it is possible to make the conclusion that we made earlier, i.e., that the function will be "large" in the area whose location is determined by the value of its first moment. A better idea about the exact shape of a function can be obtained by considering higher moments as well.

The domain of variation of the ratio $R(\xi\mid\alpha)$ is rather restricted. It is not difficult to verify that under the most general circumstances

$$\tfrac{1}{4}\le R(\xi\mid\alpha)\le\tfrac{3}{2}\,,$$

irrespective of the value of the parameter α. For comparison we mention that the SU(3)-symmetric quark-parton model predicts[3]

$$\tfrac{1}{4}\le R\le 3\,,$$

while the SU(2)-symmetric quark-parton model results in[3]

$$\tfrac{1}{4}\le R\le 4\,.$$

Experimental data[4] show no indication that any one of these three different constraints on the ratio R is in imminent danger of being violated. In the case the threshold behavior of the functions f_0^q and f_1^q is the same, i.e.,

$$\lim_{\xi\to1}\frac{f_0^q(\xi)}{f_1^q(\xi)}=1\,,$$

we have

$$R(1\mid\alpha)=\frac{1+3\tan^2\alpha}{4+2\tan^2\alpha}\,.$$

Similar reasoning about the behavior of the functions f_0^d and f_1^d for small values of ξ leads to

$$R(0\mid\alpha)=\frac{1+2\tan^2\alpha}{1+11\tan^2\alpha}\,.$$

For sufficiently small values of α these numbers can be made consistent with the available data.

Notice, again, the ease with which the diquark model accommodates the scaling data at $\xi \to 1$, as opposed to the difficulties of the symmetric quark model in this area. In the symmetric quark model the scaling functions of $\mathcal{P}$ and $\mathcal{N}$ quarks in the nucleon should be the same, and one cannot escape the prediction

$$R(1 \mid \alpha) = \frac{2 \times (\text{charge of } \mathcal{N} \text{ quark})^2 + (\text{charge of } \mathcal{P} \text{ quark})^2}{2 \times (\text{charge of } \mathcal{P} \text{ quark})^2 + (\text{charge of } \mathcal{N} \text{ quark})^2} = \tfrac{2}{3}.$$

This prediction is in flagrant disagreement with the existing data.

OTHER PREDICTIONS OF THE DIQUARK MODEL

At the threshold $\xi \to 1$ the dominant contribution to the wave function of either proton or neutron target comes from the $f^q_{0,1}$ components; the $f^d_{0,1}$ components are small at the threshold. This has definite experimental consequences. Consider the exclusive processes

$$e^- + p \to e^- + \Sigma^0 + K^+$$

and

$$e^- + p \to e^- + \Lambda^0 + K^+ ,$$

and let us limit our attention to those events in which the K^+ meson is produced in the direction of the momentum of the virtual photon: $\vec{q} = \vec{k}'_{el} - \vec{k}_{el}$. For such events it was observed[5] that the ratio of the differential cross sections

$$A = \frac{d\sigma(e^- + p \to e^- + \Sigma^0 + K^+)}{d\sigma(e^- + p \to e^- + \Lambda^0 + K^+)}$$

rapidly decreases with an increase in the momentum transfer variable. For a fixed value of the missing mass, this means that the ratio A tends to zero when the scaling variable ξ becomes larger $[\xi = -q^2/2M_p\nu = (1-(M^2-M_p^2)/q^2)^{-1}]$. Now, if the K^+ mesons are emitted in the forward direction, i.e., in the direction of the virtual photon, we can envisage the described processes to proceed as illustrated in Fig. 2. In other words, a single $\lambda\bar{\lambda}$ quark pair is created from the vacuum, $\bar{\lambda}$ combines with one of the $\mathcal{P}$ quarks of the proton to form the K^+ meson, while λ joins the remaining two quarks in the proton to produce either Λ^0 (isospin 0) or Σ^0 (isospin 1), depending on whether these two quarks were caught in the isospin-0 or isospin-1 state (recall that the isospin of the λ quark is 0).

On the basis of this line of reasoning, we can make the following prediction:

$$A(\xi \mid \alpha) = \frac{\text{probability that the proton is found in the } (\mathcal{P}\text{-quark, isovector diquark) state}}{\text{probability that the proton is found in the } (\mathcal{P}\text{-quark, isoscalar diquark) state}} = \frac{\tan^2\alpha}{3}\,\frac{f^q_1(\xi)}{f^q_0(\xi)},$$

a result which we obtained by looking directly at the Clebsch-Gordan coefficients in the proton wave function (1), and taking into account the probabilistic interpretation of the quantities $f^q_{0,1}(\xi)$.

A similar method can be used to arrive at the prediction

$$B = \frac{d\sigma(e^- + p \to e^- + \Sigma^+ + K^0)}{d\sigma(e^- + p \to e^- + \Sigma^0 + K^+)} = \frac{\text{probability that the proton is found in the } (\mathcal{N}\text{-quark, isovector diquark) state}}{\text{probability that the proton is found in the } (\mathcal{P}\text{-quark, isovector diquark) state}} = 2,$$

a prediction which should be valid everywhere, i.e., irrespective of whether K mesons are produced in the forward direction or not.

The obtained formula for the ratio A can be expressed in terms of the quantity $R(\xi \mid \alpha)$, discussed previously. For ξ sufficiently close to the threshold, we obtain

$$A(\xi \mid \alpha) = \frac{4R(\xi \mid \alpha) - 1}{9 - 6R(\xi \mid \alpha)} \quad (\xi \to 1). \tag{7}$$

Since $R(\xi \mid \alpha)$ is known experimentally over most of the interval $0 \le \xi \le 1$, we have in (7) a specific prediction which can be, and should be, tested experimentally. The presently available data for A lie in the interval $0.1 \lesssim \xi \lesssim 0.25$, which region of ξ axis is presumably too far removed from the threshold to provide a sensible test of (7). In fact, the covered sector of the ξ axis is better suited for checking another prediction of the diquark model, namely

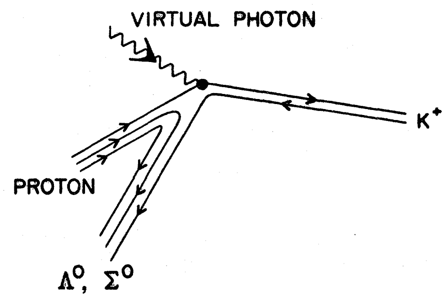

FIG. 2. Electroproduction of K^+ mesons in the "forward" direction, as envisaged in the described diquark model of baryons.

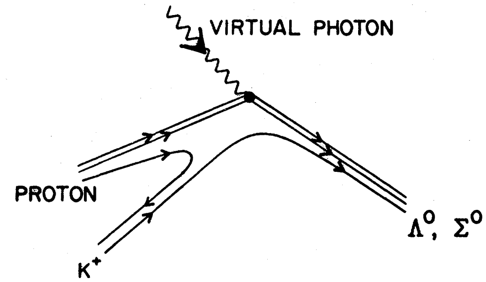

FIG. 3. Electroproduction of K^+ mesons in the "backward" direction, as envisaged in the described diquark model of baryons.

$$A'(\xi|\alpha)=\frac{\tan^2\alpha}{3}\,\frac{f_1^d(\xi)}{f_0^d(\xi)}$$
$$=\cdots$$
$$=\frac{1-R(\xi|\alpha)}{24-33(1-R(\xi|\alpha))}\quad(\xi\to 0),$$

where A' is the same ratio of the cross sections as A, but referring this time to the events in which it is Σ^0 and Λ^0 that are produced in the direction of the current, instead of K^+'s (Fig. 3).

Within the framework of the diquark model there exists room for a potentially large number of predictions of the type that we have discussed in this section. Some of them have been reproduced already in somewhat different form by Nachtmann[6] and by Cleymans and Close.[7] However, these authors do not work with diquarks, but talk instead about the "sea of wee partons,"[6] or about the "core" or the "shell" of the nucleon.[7] Some of their assumptions come very close in spirit to the assumptions that we have made in the described diquark model of baryons, but the basic underlying philosophy is different, and the degree of similarity among the three different approaches to the threshold behavior of the nucleonic scaling functions remains to be clarified.

SPIN-1 PARTONS

The basis of the parton model is the observation that the electroproduction structure functions for free pointlike spin-0 or spin-$\frac{1}{2}$ objects scale. This property of spins 0 and $\frac{1}{2}$ is not shared by the particles of higher spin. Since the diquark model deals with spin 1, aside from spins 0 and $\frac{1}{2}$, the issue of scaling must be approached with care.

Consider the most general current matrix element between two spin-1 particle states of the same (nonvanishing) mass,

$$\langle ip'|\,j_\mu(0)|\,jp\rangle=\frac{1}{(4p_0'p_0)^{1/2}}\{[\epsilon^i\cdot\epsilon^j F_1(q^2)+\epsilon^i\cdot q\,\epsilon^j\cdot q\,F_3(q^2)](p'+p)_\mu+F_2(q^2)(\epsilon^j\cdot q\,\epsilon^i-\epsilon^i\cdot q\,\epsilon^j)_\mu\}.$$

q is the four-momentum transfer $q=p'-p$, $q^2\le 0$, ϵ^i, and ϵ^j are the components of the polarization four-vectors of the particles in question, and $F_1(q^2)$, $F_2(q^2)$, and $F_3(q^2)$ are the three form factors, analogous to the two form factors in the spin-$\frac{1}{2}$ case, and the one form factor in the case of particles with spin 0. Note that $F_1(0)=1$, which is the consequence of the normalization condition for the particle states in question.

Starting from this most general expression for the current matrix element of spin-1 particles, it is a straightforward procedure to derive the expressions for the corresponding spin-averaged invariant structure functions W_1 and W_2. They read ($\nu=p\cdot q/M$, M is the mass of the particles in question)

$$2MW_1(q^2,\nu)=\frac{1}{3M}\,F_2{}^2(q^2)(-q^2)\left(1-\frac{q^2}{4M^2}\right)\delta\left(\nu+\frac{q^2}{2M}\right)$$

and

$$W_2(q^2,\nu)=\left\{\frac{1}{3}\left[F_1(q^2)\left(1-\frac{q^2}{2M^2}\right)+F_2(q^2)\frac{q^2}{2M^2}+F_3(q^2)q^2\left(1-\frac{q^2}{4M^2}\right)\right]^2+\frac{2}{3}\left[F_1{}^2(q^2)-F_2{}^2(q^2)\frac{q^2}{4M^2}\right]\right\}\delta\left(\nu+\frac{q^2}{2M}\right).$$

For the pointlike particles the form factors must be constant. The minimal deviation from scaling is achieved by setting the form factor F_3 to zero, and by making F_1 and F_2 equal, i.e., $F_1 = F_2 = 1$. In this case the violations of scaling are only in the first order in q^2, i.e., $(\xi = -q^2/2M\nu)$,

$$-2M\xi W_1 = \left[\tfrac{1}{3} - \left(1 - \frac{q^2}{6M^2}\right)\right]\xi\delta(1-\xi)$$

and

$$\nu W_2 = \left(1 - \frac{q^2}{6M^2}\right)\xi\delta(1-\xi)\,.$$

An interesting fact is that the combination

$$\nu W_2 - 2M\xi W_1 = \tfrac{1}{3}\,\xi\delta(1-\xi)$$

does scale, although the individual structure functions W_1 and νW_2 do not. The described violations of scaling will be small as long as $-q^2/6M^2 \ll 1$. If we allow for the possibility that the spin-1 diquarks possess an internal structure, reflected in the nontrivial decreasing form factors F_1, F_2, and F_3, the violations of scaling due to the spin-1 character of these objects can always be neutralized by a suitably chosen decreasing internal form factor. For example, the choice $F_3 = 0$ and $F_1 = F_2 = (1 - q^2/6M^2)^{-1/2}$ results in perfect scaling for νW_2, although it does not provide a similar cure for W_1.

The described spin-induced violations of scaling will have the following effect on the $f_{0,1}^{q,d}$ components of the nucleonic νW_2 functions. The $f_{0,1}^{q}$ components and the f_0^d component will remain unaffected, since the scattering takes place on the objects with spins $\frac{1}{2}$ and 0, respectively. The f_1^d component will acquire the scale-violating factor $1 - q^2/6M^2$, i.e.,

$$f_1^d(\xi) \to \left(1 - \frac{q^2}{6M^2}\right) f_1^d(\xi)\,.$$

We recall that this "minimal" violation of scaling is achieved for the $F_1 = F_2 = 1$, $F_3 = 0$ choice for the internal form factors of the (axial-) vector diquarks-partons.

If the degrees of freedom expressed in $F_i(q^2)$ are partially preserved by allowing that F_1 and F_2 be functions of q^2, constrained by the requirement $F_1 = F_2 = f_1^d(q^2)$ and $F_3 = 0$, we will have

$$f_1^d(\xi) \to \left(1 - \frac{q^2}{6M^2}\right) f_1^d(q^2) f_1^d(\xi)\,.$$

Similarly, the quark and the scalar diquark components of the nucleonic νW_2 functions can also be supplied by a small internal form factor, as illustrated by

$$f_{0,1}^q(\xi) \to f_{0,1}^q(q^2) f_{0,1}^q(\xi)$$

and

$$f_0^d(\xi) \to f_0^d(q^2) f_0^d(\xi)\,.$$

Of course, it is up to experiment to tell us whether the introduced internal form factors are indeed different from unity, betraying a nontrivial internal structure of the objects that were initially introduced as "elementary."

VIOLATIONS OF SCALING

Recent data[8] show unmistakable signs of small deviations from scaling for large values of $-q^2$. For small fixed values of the scaling variable ξ, $\xi \lesssim 0.25$, there is an upward shift of data points for νW_2, while in the interval $0.25 \lesssim \xi \leqslant 1$ the trend of data is reversed, i.e., for a fixed ξ νW_2 slowly decreases as $-q^2$ becomes larger. This curious behavior of scaling data can find a qualitative explanation in the diquark model. (It cannot be accounted for in the quark model.) The focal point of the argument is the already described violations of scaling due to the spin-1 character of vector diquarks. These kinematical, spin-induced violations of scaling are in the opposite direction from the violations of scaling that one would expect from a possible internal structure of the constituent.[9] A competition between these two effects can in principle explain what is observed experimentally.

As we have seen earlier, for small values of ξ the electrons tend to scatter mostly on diquarks, while for larger values of ξ they land most of the time on quarks. A hypothetical internal form factor of quarks explains the downward trend of scaling data for $0.25 \lesssim \xi$, while in order to explain the violations of scaling for $\xi \lesssim 0.25$, one must assume that the scale-violating term $1 - q^2/6M_V{}^2$ that multiplies the vector diquark component of νW_2 prevails over a small intrinsic form factor. Unfortunately, quantitative estimates are difficult to make since ignorance about the exact value of the mass of vector diquarks is compounded by our ignorance concerning their possible internal form factor.

OUTLOOK

We would like, once again, to caution against too optimistic a view on the ability of the presented diquark model to produce everywhere reliable quantitative predictions. There are indications that the nucleon spends about 50% of its time in the usual 3-quark configuration ($P_q \simeq \frac{1}{2}$), and 50% of the time in the quark-diquark configuration. To get more reliable quantitative estimates in the scaling region, one should perhaps start with the phenomenological scaling functions

$$\frac{1}{\xi}(\nu W_2)^{\text{nucleon}} = P_q \frac{1}{\xi}(\nu W_2)^{\text{nucleon}}_{3\text{-quark configuration}} + P_d \frac{1}{\xi}(\nu W_2)^{\text{nucleon}}_{\text{diquark configuration}},$$

where

$$\frac{1}{\xi}(\nu W_2)^{\text{proton}}_{3\text{-quark configuration}} = \tfrac{8}{9} f_{\mathcal{P}\ \text{quark}}(\xi) + \tfrac{1}{9} f_{\mathcal{N}\ \text{quark}}(\xi),$$

$$\frac{1}{\xi}(\nu W_2)^{\text{neutron}}_{3\text{-quark configuration}} = \tfrac{2}{9} f_{\mathcal{P}\ \text{quark}}(\xi) + \tfrac{4}{9} f_{\mathcal{N}\ \text{quark}}(\xi),$$

and

$$\frac{1}{\xi}(\nu W_2)^{\text{proton}}_{\text{diquark configuration}} = \tfrac{4}{18}\cos^2\alpha f_0^q(\xi) + \tfrac{1}{18}\cos^2\alpha f_0^d(\xi) + \tfrac{2}{18}\sin^2\alpha f_1^q(\xi) + \tfrac{11}{18}\sin^2\alpha f_1^d(\xi),$$

$$\frac{1}{\xi}(\nu W_2)^{\text{neutron}}_{\text{diquark configuration}} = \tfrac{1}{18}\cos^2\alpha f_0^q(\xi) + \tfrac{1}{18}\cos^2\alpha f_0^d(\xi) + \tfrac{3}{18}\sin^2\alpha f_1^q(\xi) + \tfrac{3}{18}\sin^2\alpha f_1^d(\xi)$$

[all $f(\xi)$ functions are normalized to unity]. In this representation for the nucleonic structure functions we deal with the three real parameters P_q, P_d, and α, and with the six *a priori* unknown scaling functions $f_{\mathcal{P}\ \text{quark}}(\xi)$, $f_{\mathcal{N}\ \text{quark}}(\xi)$, and $f_{0,1}^{q,d}(\xi)$. These three parameters and the functions $f(\xi)$ should be determined by comparing the predictions of the model with the scaling data for the proton and the neutron spin-averaged targets. After all these numerous degrees of freedom are satisfied, we still hope to have some predictive power left. In particular, we should be able to make definite predictions about the rates of various exclusive channels in electroproduction experiments, an area of considerable experimental and theoretical interest where the knowledge of the target wave function plays an important role. Other areas where the predictions of the diquark model could be tested include the deep-inelastic neutrino-nucleon scattering, the physics of high $p_\perp$ events, the Drell-Yan process $p+p \to (\mu^+\mu^-)$ + hadrons, the deep-inelastic scattering of electrons on polarized targets, etc.

DIQUARK MODEL—SYNOPSIS

Group-theoretical aspects

The quarks are regarded as spinors from the fundamental 6-dimensional representation of the SU(6) group that mixes spin and the unitary spin degrees of freedom. Their quantum numbers are

	$\mathcal{P}$	$\mathcal{N}$	λ
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0
electric charge	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
hypercharge	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
baryon number	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$.

The diquarks are the spinors from the (reducible) representation $\underline{6}\times\underline{6}$ of the SU(6) group. There are in total 36 diquark states which fall into the two irreducible representations of the SU(6) group, namely $\underline{15}$ and $\underline{21}$ ($\underline{6}\times\underline{6}=\underline{15}+\underline{21}$). The SU(3) content of these irreducible representations is

$$\underline{21} = \{6\}\times 3 + \{\overline{3}\}\times 1$$

and

$$\underline{15} = \{\overline{3}\}\times 3 + \{6\}\times 1,$$

where the curly brackets enclose the irreducible representations of SU(3), while the numbers outside the brackets count the degrees of freedom associated with the ordinary spin. In other words, in $\underline{21}$ we find an SU(3) sextet of axial-vector diquarks and a triplet of scalar diquarks, while in $\underline{15}$ the situation is reversed, i.e., we have a sextet of scalar diquarks and a triplet of axial-vector diquarks. The SU(3) representation $\{\overline{3}\}$ consists of an isotopic singlet with hypercharge $\frac{2}{3}$ and electric charge $\frac{1}{3}$, and an isotopic doublet with hypercharge $-\frac{1}{3}$ and electric charges $-\frac{2}{3}$ and $\frac{1}{3}$, respectively.

The SU(3) representation $\{6\}$ contains an isotopic singlet with hypercharge $-\frac{4}{3}$ and electric charge $-\frac{2}{3}$, an isotopic doublet with hypercharge $-\frac{1}{3}$ and electric charges $-\frac{2}{3}$ and $\frac{1}{3}$, and finally an isotopic triplet with hypercharge $\frac{2}{3}$ and electric charges $-\frac{2}{3}$, $\frac{1}{3}$, and $\frac{4}{3}$.

Since the isospin of nucleons is $\frac{1}{2}$, and no λ quarks are present, we expect to find in nucleons only the diquarks with isospins 0 and 1. The isospin-0 diquark in the nucleon comes from the SU(3) representation $\{\overline{3}\}$, while the isospin-1 diquark is supplied by $\{6\}$. (There is another isospin-0 diquark from $\{6\}$, but this one does not show its presence in the nucleonic wave function, because it carries hypercharge $-\frac{4}{3}$ which cannot be neutralized by an accompanying quark to yield the required nucleonic hypercharge 1.) Consequently, in the case of nucleons we find an accidental match between the ordinary spin and the isospin of the diquarks, a convenient circumstance from the "mnemonic" point of view (scalar diquarks are also isoscalar, and vector diquarks are also isovector).

Dynamical assumptions

Diquarks and quarks are both regarded as pointlike objects until proved otherwise. Baryons are bound states of the quark-diquark type. The well-established SU(6) baryonic multiplets $\underline{56}$ and $\underline{70}$ of the lowest mass are built from the diquarks supplied by the irreducible representation $\underline{21}$, i.e., $\underline{6}\times\underline{21}=\underline{56}+\underline{70}$. In the case of low-lying baryons the relative orbital angular momentum between the quarks and the diquarks is zero. Nonzero orbital angular momenta may be associated with the baryons that lie higher in mass. The other diquark irreducible representation $\underline{15}$ leads to the baryonic multiplets $\underline{20}$ and $\underline{70}$ via $\underline{6}\times\underline{15}=\underline{20}+\underline{70}$. Since the experimental status of $\underline{20}$ is uncertain at present, it is not clear whether the diquarks from $\underline{15}$ do indeed couple to quarks.

The two bases, one in the reducible space $\underline{6}\times\underline{21}$, and the other one in the sum of the irreducible spaces $\underline{56}$ and $\underline{70}$, are connected by the known matrix of the Clebsch-Gordan coefficients.

In the case of nucleons these Clebsch-Gordan coefficients are given by

$$|\text{proton}^{\uparrow}\rangle=\frac{1}{\sqrt{18}}[(2d_{11}^{\uparrow}\mathfrak{N}^{\downarrow}-\sqrt{2}\,d_{11}^{\rightarrow}\mathfrak{N}^{\uparrow}-\sqrt{2}\,d_{10}^{\uparrow}\mathcal{P}^{\downarrow}+d_{10}^{\rightarrow}\mathcal{P}^{\uparrow})\sin\alpha+3t_{00}\mathcal{P}^{\uparrow}\cos\alpha],$$

$$|\text{neutron}^{\uparrow}\rangle=\frac{1}{\sqrt{18}}[(-2d_{1-1}^{\uparrow}\mathcal{P}^{\downarrow}+\sqrt{2}\,d_{1-1}^{\rightarrow}\mathcal{P}^{\uparrow}+\sqrt{2}\,d_{10}^{\uparrow}\mathfrak{N}^{\downarrow}-d_{10}^{\rightarrow}\mathfrak{N}^{\uparrow})\sin\alpha+3t_{00}\mathfrak{N}^{\uparrow}\cos\alpha],$$

and similarly for the "spin down" states $|\text{proton}^{\downarrow}\rangle$ and $|\text{neutron}^{\downarrow}\rangle$.

α is a phenomenological parameter that breaks the SU(6) symmetry and mixes the SU(3) representations $\{6\}$ and $\{\overline{3}\}$. The SU(6) symmetry can be restored by setting $\alpha=\pi/4$. d_{1-1}, d_{10}, and d_{11} are the components of the isospin-1 diquark, while t_{00} is the isospin-0 diquark from $\{\overline{3}\}$.

The spin-averaged invariant electroproduction structure functions of the nucleon in the scaling region are obtained from the expression

$$\frac{1}{\xi}(\nu W_2)_{\text{nucleon}}=\langle\text{nucleon}|(\text{charge of quark})^2F_{\text{quark}}(\xi)+(\text{charge of diquark})^2F_{\text{diquark}}(\xi)|\text{nucleon}\rangle,$$

where $F_{\text{quark}}(\xi)$ and $F_{\text{diquark}}(\xi)$ are the pieces of the electromagnetic current operator that act on the individual quark and diquark components of the nucleonic wave function. The coefficients in

$$\frac{1}{\xi}(\nu W_2)_{\text{proton}}=\tfrac{4}{18}\cos^2\alpha f_0^q(\xi)+\tfrac{1}{18}\cos^2\alpha f_0^d(\xi)+\tfrac{2}{18}\sin^2\alpha f_1^q(\xi)+\tfrac{11}{18}\sin^2\alpha f_1^d(\xi)$$

and

$$\frac{1}{\xi}(\nu W_2)_{\text{neutron}}=\tfrac{1}{18}\cos^2\alpha f_0^q(\xi)+\tfrac{1}{18}\cos^2\alpha f_0^d(\xi)+\tfrac{3}{18}\sin^2\alpha f_1^q(\xi)+\tfrac{3}{18}\sin^2\alpha f_1^d(\xi)$$

are therefore the result of the combination of the squares of the quark and the diquark charges with the values of the Clebsch-Gordan coefficients in the nucleonic wave function in the diquark model.

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The concept of diquark in the role of parton is due to Professor Bjorken, and the present work should be regarded merely as an attempt to understand and implement as faithfully as possible his own ideas on the subject. It is entirely my own fault if I have failed in this undertaking.

[1]The basic idea of the diquark model, i.e., that two of the three quarks in the baryon are glued together, has been explored before. A sample of representative literature is given as follows: S. Ono, Prog. Theor. Phys. 48, 964 (1972); 49, 573 (1973); 50, 589 (1973); M. Ida and R. Kobayashi, *ibid.* 36, 846 (1966); D. B. Lichtenberg and L. J. Tassie, Phys. Rev. 155, 1601 (1967); D. B. Lichtenberg, Nuovo Cimento 49A, 435 (1967); P. D. De Souza and D. B. Lichtenberg, Phys. Rev. 161, 1513 (1967); D. B. Lichtenberg, L. J. Tassie, and P. C. Keleman, *ibid.* 167, 1535 (1968); J. Carroll, D. B. Lichtenberg, and J. Franklin, *ibid.* 174, 1681 (1968); D. B. Lichtenberg, *ibid.* 178, 2197 (1969). As far as we know, the scaling properties of the diquark model (diquarks in the role of partons) have not as yet been systematically investigated.

[2]In fact, it is not clear at present whether $\underline{20}$ is indeed absent, or has not as yet been discovered. From the experimental point of view the problems with $\underline{20}$ stem from the fact that $\underline{20}$ does not couple to the usual baryonic target-mesonic projectiles system $\underline{56}\times\underline{35}$ ($\underline{56}\times\underline{35}=\underline{56}+\underline{70}+\underline{1134}+\underline{700}$). Therefore, in order to observe $\underline{20}$, one must first produce the $\underline{70}$ from $\underline{56}\times\underline{35}$, and then look for $\underline{20}$ in $\underline{70}\times\underline{35}$. Experimentally, this is not an easy procedure.

[3]O. Nachtmann, Nucl. Phys. B38, 397 (1972).

[4]A. Bodek *et al.*, Phys. Lett. 51B, 417 (1974).

[5]C. J. Bebek *et al.*, Phys. Rev. Lett. 32, 21 (1974).

[6]O. Nachtmann, Nucl. Phys. B74, 422 (1974).

[7]J. Cleymans and F. E. Close, Nucl. Phys. B85, 429 (1975).

[8]E. M. Riordan *et al.*, contributed paper to the International Symposium on Lepton and Photon Interactions, Stanford, California, 1975, Report No. SLAC-PUB-1634 (unpublished).

[9]M. S. Chanowitz and S. D. Drell, Phys. Rev. Lett. 30, 807 (1973); Phys. Rev. D 9, 2078 (1974); K. Matumoto, Prog. Theor. Phys. 47, 1795 (1972).