Color gluon models and ψ-lepton couplings

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It is shown that if the ψ particles are interpreted as gluons or hadronic gauge fields of one scheme or another of a renormalizable field theory, they must be directly coupled to charged leptons as a result of ψ-γ mixing and consequent nonmultiplicative renormalization. Among many possible models a $U(3)' \times SU(4)$ color quark-gluon model is considered in some detail. Only two narrow $J^P = 1^-$ e^+e^- resonances find a *natural* place in the model, and it is suggested that $\psi(3.1)$ is the ninth color-singlet gluon whereas $\psi(3.7)$ and other broader states, if any, may be $c\bar{c}$ states.

I. INTRODUCTION

Recent experiments on e^+e^- annihilation to hadrons[1] have dramatically revealed new phenomena, sparking off various speculations.[2] We shall show in this paper that if the newly discovered ψ particles are interpreted as fundamental hadronic spin-1 fields, such as, for example, gluons or gauge fields (of one scheme or another) of a renormalizable Lagrangian field theory, charged leptons must possess a direct coupling to these renormalized neutral vector fields and, hence, a (tiny) hadronic radius. This would, in turn, necessarily imply a breakdown of quantum electrodynamics at a corresponding (short) distance. However, unlike in the theories of Pati and Salam[3] and Georgi and Glashow,[4] lepton and baryon numbers remain exactly conserved in these theories and the hadronic form factor of charged leptons is not an input but an inevitable consequence of γ–vector meson mixing and consequent nonmultiplicative renormalization.

A $U(3)' \times SU(4)$ color gluon model has been considered. Only two narrow $J^P = 1^-$ e^+e^- resonances find a *natural* place in the model (without invoking angular or radial excitations), and it is argued that $\psi(3.1)$ may be the ninth color singlet gluon, while $\psi(3.7)$ and other broader states, if there are any, may be $c\bar{c}$ states.

The plan of the paper is as follows. We outline the general argument in Sec. II. The $U(3)' \times SU(4)$ color gluon model is considered in Sec. III. Sec. IV contains some concluding remarks.

II. GENERAL REMARKS

It has been shown before that in a conventional renormalizable quantum field theory in which the photon can mix with other neutral massive vector-meson fields in an electromagnetic gauge-invariant manner, a direct coupling of charged leptons ($e^\pm$ and $\mu^\pm$ but not ν's) to the renormalized neutral massive vector-meson fields is inevitable.[5] The essential point is that multiplicative renormalization does not suffice to remove all infinities in such models, and the renormalized neutral spin-1 fields are, in general, linear combinations of the unrenormalized ones. Thus, even if charged leptons were, to start with, coupled to the unrenormalized photon field A_λ^0 only, this mixing of the spin-1 fields would induce a direct coupling of charged leptons to the renormalized massive vector-meson fields. Let us consider theories of the generalized Kroll-Lee-Zumino type,[6]

$$\mathcal{L} = e_0(\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda \mu + J_\lambda^{\text{had}})A_\lambda^0 + g_0(J_\lambda^{\text{had}})^\alpha B_\lambda^{0\alpha} + \cdots , \tag{1}$$

where

$$J_\lambda^{\text{had}} = \sum_\alpha r_\alpha (J_\lambda^{\text{had}})^\alpha$$

is the hadronic electromagnetic current, α is a (collective) internal-symmetry index that runs over n appropriate values corresponding to the internal hadronic symmetry group under consideration, r_α are appropriate numerical coefficients, $B_\lambda^{0\alpha}$ are the corresponding set of n unrenormalized hadronic neutral vector-meson fields, and the dots represent kinetic energy and other interaction terms. Now, since the photon can mix with the hadronic neutral vector-meson fields through their common source currents $(J_\lambda^{\text{had}})^\alpha$, one requires counterterms of the form $C_\alpha A_{\mu\nu} B_{\mu\nu}^\alpha$ to remove all the infinities from the off-diagonal elements of the $(n+1)$-by-$(n+1)$ spin-1 propagator matrix.[7] Such counterterms cannot be generated unless the unrenormalized spin-1 fields are linear combinations of the renormalized ones; naive multiplicative renormalization obviously does not suffice. In general, one must have

$$A_\mu^0 = R_{11} A_\mu + \sum_\alpha R_{1\alpha} B_\mu^\alpha , \tag{2a}$$

$$B_\mu^{0\alpha} = R_{\alpha 1} A_\mu + \sum_\beta R_{\alpha\beta} B_\mu^\beta . \tag{2b}$$

However, symmetry requirements may force some of the elements of the wave-function renormalization matrix R_{ij} either to be zero or to be related. In particular, a vanishing photon mass (or electromagnetic gauge invariance) requires that $R_{\alpha 1} = 0$. One therefore obtains

$$\begin{aligned}\mathcal{L} &= (\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda \mu + J_\lambda^{\text{had}})\left(e_0 R_{11} A_\lambda + e_0 \sum_\alpha R_{1\alpha} B_\lambda^\alpha\right) + \cdots \\ &= (\bar{e}\gamma_\lambda e + \bar{\mu}\gamma_\lambda \mu + J_\lambda^{\text{had}})\left(eA_\lambda + \sum_\alpha g_{\alpha l} B_\lambda^\alpha\right) + \cdots ,\end{aligned} \tag{3}$$

where $e = e_0 R_{11}$ is the renormalized electric charge and $g_{\alpha l} = e_0 R_{1\alpha}$ are the renormalized vector-meson–lepton coupling constants. Since renormalizability requires $R_{1\alpha} \neq 0$, it follows that $g_{\alpha l} \neq 0$. Notice also that muon-electron universality is naturally preserved in these theories. However, $g_{\alpha l}$ are nonvanishing but arbitrary parameters in conventional renormalizable quantum field theories of the type under consideration, and they have to be determined from experiments. Nonvanishing but small $g_{\alpha l}$, however, inevitably imply a breakdown of QED at sufficiently short distances. This comes about through a modification of the photon propagator. If we consider lepton-lepton scattering, then the matrix element (apart from the leptonic currents and Lorentz indices) is given by

$$M = e^2 D_{11} + \sum_\alpha g_{\alpha l}{}^2 D_{\alpha\alpha} + 2e \sum_\alpha g_{\alpha l} D_{1\alpha} , \tag{4}$$

where D_{11} and $D_{\alpha\alpha}$ denote the exact renormalized photon and (massive) vector-meson propagators, respectively, and $D_{1\alpha}$ are the appropriate off-diagonal elements of the renormalized spin-1 propagator matrix. Under certain reasonable smoothness assumptions[5] the form factor is then given by

$$F(q^2) \simeq 1 - q^2 \sum_\alpha \frac{1}{(e^2/g_{\alpha l}{}^2) m^2(B^\alpha)} . \tag{5}$$

One therefore obtains

$$(\Lambda_+^\alpha)^2 = (e^2/g_{\alpha l}{}^2) m^2(B^\alpha) , \tag{6}$$

where Λ_+^α is the conventional (positive metric) cutoff in QED. It is therefore clear that if $g_{\alpha l} \ll e$, then for a given Λ_+^α the hadronic vector meson B_λ^α need not be very massive. On the other hand, if $g_{\alpha l} \gg e$, then B_λ^α must be sufficiently massive. The B-γ mixing can therefore induce strong B-lepton couplings, but the validity of QED would require in such a case that the B_λ^α be sufficiently massive.

III. U(3)′×SU(4) COLOR GLUON MODEL

Having outlined the general theory we shall now consider a specific quark-gluon model in this section. It is well known that asymptotically free theories imply Bjorken scaling to within computable logarithmic deviations.[8] In order to have asymptotic freedom it has become customary to invoke the color octet quark-gluon model.[9] Color is assumed to be an exact symmetry, and it is the color SU(3)′ symmetry which is gauged. The octet of colored Yang-Mills gauge gluons are asymptotically free and also provide a possible mechanism for the permanent confinement of all colored states through infrared slavery.[4] However, since the octet of colored gluons cannot couple with the electromagnetic current, which is a color singlet, we have to look for a color-singlet neutral gluon. This would require an extension of the color gauge group from SU(3)′ to at least U(3)′. The B-γ mixing can then take place (through quark loops) if ordinary SU(3) is broken. The SU(3) breaking can occur in two ways in such a model. If the ninth gluon (B_μ) is assumed to be an SU(3) singlet, SU(3) breaking can be put in by hand in the quark mass matrix as in Ref. 8. On the other hand, the ninth gluon can in general transform like $(\lambda_0 + a\lambda_3 + b\lambda_8)$ and be itself responsible for a dynamical breaking of SU(3). We leave this choice open at the moment. We emphasize, however, that in any case B-γ mixing can occur only through SU(3) breaking in such a model. However, one can also introduce charmed quarks C_i into the model by having three quartets of fractionally charged quarks with different colors.[10,4] B-γ mixing can then occur through the common source $\sum_i \bar{C}_i \gamma_\mu C_i$ even in the limit of exact SU(3) symmetry. Thus a simple renormalizable field theory in which a hadronic form factor of charged leptons arises naturally but lepton and baryon numbers are exactly conserved is the color U(3)′ × SU(4) quark-gluon model

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\text{YM}} + \text{Tr}\bar{\psi}\left\{i\gamma_\mu\left(\partial_\mu - ig\sum_{A=0}^{8} \tfrac{1}{2}\chi_A B_{\mu A}\right)\psi\right\} \\ + \text{quark mass term} + \cdots ,\end{aligned} \tag{7}$$

where ψ is the (3×4) quark matrix and χ_A are the color analogs of the Gell-Mann λ matrices.

That it is possible to preserve, though somewhat arbitrarily, the asymptotic freedom of the quark-gluon interaction in a U(3)′ gauge model can be seen from the following argument. We assume as in (7) that all nine gluons couple universally to quarks. Though the U(3)′ symmetry does not necessarily imply it, universality is an attractive though additional hypothesis.[11] Suppose then that the renormalized coupling constant g of the color octet gluons is related to this universal coupling constant g_0 by $g = Zg_0$, while the renormalized coupling g' of the ninth gluon to quarks is related to it by $g' = Z'g_0$, where $Z = 1 + k \ln \Lambda$ and $Z' = 1 + k' \ln \Lambda$ are appropriate cutoff-dependent renormalization constants (k and k' are computable numbers in perturbation theory). Since $(\partial/\partial \ln\Lambda)(g/g') = 0$ it follows that $(\partial/\partial \ln \Lambda)(Z/Z') = 0$. This implies $k = k'$, so that $Z/Z' = 1$ and $g = g'$. However, the ninth gluon does not interact with either the octet of colored gluons or itself, so it can contribute to coupling-constant renormalization only through its coupling to quarks. Simple perturbation theory therefore indicates $Z \neq Z'$. But this cannot be if the theory is to be renormalizable. So one concludes that either a U(3)$'$ or, in general, a U(N) gauge theory with a single coupling constant is nonrenormalizable, or the single coupling constant must be taken to imply that U(N) is really a subgroup of a larger purely non-Abelian group. In the latter case, the (hidden) larger symmetry group will automatically ensure the existence of the required Ward identity $Z = Z'$ and also asymptotic freedom.

Whether the asymptotic freedom is lost on introducing electromagnetism depends on whether the electromagnetic interaction is itself asymptotically free, because as already pointed out earlier multiplicative renormalization does not hold any more and the gluon-lepton coupling g_{Bl} has the same asymptotic behavior as e. Now, our model is, of course, consistent with any one of a variety of unified gauge models of weak and electromagnetic interactions. Although the Salam-Weinberg model[12] is not asymptotically free, the Georgi-Glashow[13] model may be, since its group structure is O(3), i.e., purely non-Abelian. We believe that only future developments will throw more light on this question.

If one does not require universality, the U(3)$'$ theory is not asymptotically free. However, one can then use Bjorken scaling to set an upper limit on the coupling constant of the ninth gluon to quarks, which is a free parameter. *We therefore emphasize that what we need require our model to satisfy at this stage is approximate Bjorken scaling rather than asymptotic freedom.*

We have not yet specified how the ninth gluon acquires the required mass. A spontaneous breaking of U(3)$'$ down to SU(3)$'\times$ U(1)$'$, in addition to that contributed by electromagnetism, can easily be arranged by adding to (7) a piece similar to the Abelian Higgs model. It is also well known that the mass of an Abelian gauge boson coupled to a conserved source does not spoil renormalizability, nor is it necessarily in conflict with gauge invariance. One may also adopt the point of view that the local U(3)$'$ symmetry remains exact for the couplings and is broken only by the mass term of the ninth gluon. We shall therefore prefer to leave the precise mechanism unspecified so as not to prejudice the issue.

We are now in a position to discuss the relevance of our U(3)$'\times$SU(4) model to the newly discovered narrow resonances in e^+e^- annihilation. We speculate that $\psi(3.1)$ is the color singlet ninth gluon, while $\psi(3.7)$ and the other broader states (if any) may be $c\bar{c}$ states.[2] In the following we shall argue in support of our conjecture, and shall also make a few predictions.

The $\psi(3105)$ seems to have the right quantum numbers $J^{PC} = 1^{--}$. But the most puzzling thing about it has been its extremely narrow width. It is not impossible that the ninth gluon couples to charmed quarks only, i.e., transforms like λ_{15} under SU(4); it would then mimic a $c_i\bar{c}_i$ state, and if $m(c_i) \geq 1.6$ GeV, it would be stable according to the "hairpin rule." But even if such an empirical rule were correct, a $c_i\bar{c}_i$ state would be expected to have a typical width of at least 1 or 2 MeV, and one would still require an additional suppression factor of about[14] α. We rather suspect that the clue to the narrow width of $\psi(3105)$ lies in something else, namely, its being a color singlet gluon. It is well known that even if the fundamental quark-gluon coupling is weak ($\sim e$), the vicious infrared instability of a color gluon theory can lead to strong phenomenological interactions among hadrons.[4] Now the ninth gluon, being a color singlet, has no incurable infrared divergences associated with it, and moreover it can be massive as already discussed. Furthermore, it can have the same universal coupling to quarks as the octet of colored gluons, and this is what one has to assume to have an asymptotically free U(3)$'$ theory. Its coupling to hadrons is then very similar to that of the photon (which also couples to hadrons through quarks). If, on the other hand, one does not assume universality, one can argue that approximate Bjorken scaling implies that the coupling constant of the ninth gluon to quarks, which is a free parameter in the theory, must be rather small ($\approx e$). In either case there is a hint of a possible and interesting correlation between approximate Bjorken scaling on the one hand and the narrow width of the ninth gluon on the other. Thus *infrared slavery and/or Bjorken scaling offer a clue to the mystery of the narrow width of $\psi(3.1)$ if it is the ninth gluon.*

Let us then identify $\psi(3.1)$ with the ninth color-singlet gluon B and assume that its coupling g_ψ to quarks (and therefore also hadrons) is[4] e, i.e., $g_\psi{}^2/4\pi = \alpha$. Although B can in general transform like $(a\lambda_0 + b\lambda_3 + c\lambda_8 + d\lambda_{15})$, let us further assume that $a \approx b \approx c \approx 0$, so that B mimics a $c_i\bar{c}_i$ state. The usual "hairpin rule" would then predict a suppression of the hadronic decays of B by a factor of α, and the observed narrow width of $\psi(3.1)$ would be explained.

An elegant feature of the model is that the universality of the coupling of B to charged leptons is a *natural* consequence of the universality of electric charge. The model therefore predicts that $\Gamma(\psi\to\bar{e}e)=f\,\Gamma(\psi\to\bar{\mu}\mu)$, where f is a pure phase-space factor, which is consistent with experiments.

Another specific consequence of $\psi\equiv B$ is that *it does not couple to neutrinos*.[15] This can be tested by accurately measuring the total and partial widths of ψ.

Since $g_\psi{}^2/4\pi=\alpha$, the model predicts that the photoproduction cross section of $\psi(3.1)$ on hadrons should be smaller by a factor α compared to that of other vector mesons such as ρ, ω, and ϕ. The latest data indicate that such is indeed the case.[16] Furthermore, one would also for the same reason be naturally led to expect that

$$\frac{\Gamma(\psi(3.7)\to\psi(3.1)+\pi^+\pi^-)}{\Gamma(\rho'\to\rho+\pi^+\pi^-)}\simeq\alpha(m_{\rho'}/m_\psi),$$

which is also of the right order of magnitude.

Finally, if $\psi(3.1)$ is really a fundamental field like the ninth gluon (or a colored gluon[17]) and not a composite $c\bar{c}$ state, then it follows from our previous discussion that it must be directly but very weakly coupled to leptons. One can in principle test this by looking for the photoproduction of $\psi(3.1)$ on leptons. Since $\Gamma(\psi\to\bar{e}e)\simeq 5$ keV, one obtains

$$g_{\psi l}{}^2/4\pi=3\Gamma(\psi\to\bar{e}e)/m_\psi\simeq5\times10^{-6}, \qquad (8)$$

which implies via Eq. (6) that $\Lambda_+\approx100$ GeV; the latest experiments[18] indicate that $\Lambda_+\gtrsim35$ GeV.

IV. CONCLUSION

The main purpose of the present paper has been to show that if the ψ particles are fundamental hadronic vector-meson fields like, for example, gluons or gauge fields of one scheme or another of a renormalizable field theory, their mixing with the photon then necessarily induces a hadronic form factor of charged leptons and hence a breakdown of QED at sufficiently small distances. This is shown to be an inevitable consequence of the nonmultiplicative renormalizability of such theories. Among many models in which this would be true, a $U(3)'\times SU(4)$ color gluon model has been discussed in detail. One virtue of this model is that it naturally accommodates only two narrow $J^P=1^-$ e^+e^- resonances without invoking angular or radial excitations. Arguments have been given to support the speculation that $\psi(3.1)$ is the ninth gluon whereas $\psi(3.7)$ and other broader states, if there are any, may be $c\bar{c}$ composites. All simple and straightforward consequences of the model are in good qualitative agreement with experiments. It should be noted that the model possesses the full richness of a charmed spectroscopy and, *in addition,* a color-singlet gluon. One unaesthetic feature of the model, however, is that it does not treat all ψ particles on the same footing. On the other hand, there is no compelling reason either to definitely rule out the theoretical possibility of a ninth gluon that can appear as a narrow e^+e^- resonance around 3 GeV.

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