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Parity-violating internucleon potential and strong-interaction enhancement*

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The $NN\pi$ and NNV vertices that enter the parity-violating internucleon potential are calculated in the Cabibbo and Weinberg-Salam models, using a mechanism whereby octet enhancement results from the short-distance behavior of the current-current product. A quark model is used to calculate the $NN\pi$ vertex, and for the NNV vertices, a modified factorization approach is proposed. The Cabibbo $NN\pi$ vertex is estimated to be an order of magnitude smaller than previous calculations had indicated and arguments against the previous method are given. In the Weinberg model the $NN\pi$ vertex is $A(N^0_-) = 1.3\sin^2\theta_W A(\Lambda^0_-)$, with only neutral currents contributing. In both models the NNV vertices have different SU(3) structure than previously found, and are enhanced. However, reasonable values of the enhancement parameters are not expected to be large enough to explain by themselves the large circular polarization measured in $n+p\to d+\gamma$.

I. INTRODUCTION

In addition to being studied at the high-energy frontier, theories of the weak interaction can be tested in low energy, parity-violating nuclear processes.[1] The study of such processes may prove invaluable to our knowledge of the fundamental interactions if the condsiderable experimental and theoretical difficulties can be overcome. Recent experiments, however, yield conflicting results. Studies of the decay of α decay of oxygen[2] and the γ-ray asymmetry[3] of ^{19}F are in rough agreement with the theoretical estimate, while the observation[4] of the circular polarization of γ rays in $n+p\to d+\gamma$ is several orders of magnitude greater than predicted by theory. The theoretical aspect is hampered by a lack of understanding of weak nonleptonic decays on the one hand, and by the difficulty with the nuclear physics involved on the other. It is possible to separate these two difficulties, so that, if we assume that the nuclear physics can be done correctly, the problem of finding the parity-violating internucleon interaction reduces to the evaluation of weak $NN\pi$ and NNV vertices. It is here that different weak-interaction theories yield different predictions. The purpose of this paper is to examine these vertices in the Weinberg-Salam model[5] and in the conventional Cabibbo model.

The approach here will be different from past attempts at the problem. In the Weinberg model it has been shown recently, by Gaillard and Lee[6] and Altarelli and Maiani,[7] that, for $\Delta S=1$ decays, the octet part of H_w is enhanced, and the 27-plet piece is suppressed by the short-distance behavior of the product of two currents. This provides a possible mechanism for the experimentally observed $\Delta I=\frac{1}{2}$ rule. Using similar techniques, the $\Delta S=0$ parity-violating Hamiltonian has been studied by Altarelli, Ellis, Maiani, and Petronzio.[8] The present paper is written within this framework of enhancement.

Another recent advance used is the construction of more realistic quark models.[9-11] Past approaches to the parity-violating problem have relied heavily on symmetry arguments. However, one of the results of this paper is that these arguments are suspect, and a dynamical approach is required.

The structure of the paper and the major results are as follows. In Sec. II the mechanism of enhancement is reviewed in light of its importance here. Section III is devoted to a general analysis of the $NN\pi$ vertex $A(N^0_-)$ in the Cabibbo theory. It is shown that within the model $A(N^0_-)$ is an order of magnitude smaller than most estimates. Since this result conflicts with the traditional derivations of $A(N^0_-)$, the latter is reviewed and shown to be inappropriate if octet enhancement occurs. In Sec. IV $A(N^0_-)$ is calculated within the Weinberg model. Only neutral currents contribute and we obtain

$$A(N^0_-)=1.3\sin^2\theta_W A(\Lambda^0_-).$$

The $NN\rho$, $NN\omega$, and $NN\phi$ vertices are evaluated in Sec. V for both the Weinberg and Cabibbo models. A modified factorization approach is used. The Weinberg model vertices are larger than past estimates of this quantity, due to enhancement factors. The Cabibbo model may also be enhanced; however, it lacks a detailed theory of enhancement. Section VI is devoted to a summary and discussion. The operators, written in terms of quark fields, that are used in the paper are written out in Appendix A. Finally, in Appendix B, the usual factorization approach for NNV vertices is examined and found to neglect certain important pieces. A modified factorization approach is presented and justified.

II. CALCULATIONAL FRAMEWORK

We work in a theory where quarks carry an internal quantum number called color and obey the usual fermion anticommutation relations. All observed particles are color singlets. For the Weinberg model it is necessary to specify that the strong interactions are mediated by massless, non-Abelian neutral gauge fields with the gauge group being SU(3) of color.

The hadronic weak currents in the Weinberg model are written in terms of quark fields. The charged currents are

$$J_\mu^{\Delta S=0}(x)=[\overline{\mathcal{P}}_i(x)\cos\theta_C-\overline{\mathcal{P}}_i'(x)\sin\theta_C]\gamma_\mu(1+\gamma_5)\mathfrak{N}_i(x),$$
$$J_\mu^{\Delta S=1}(x)=[\overline{\mathcal{P}}_i(x)\sin\theta_C+\overline{\mathcal{P}}_i'(x)\cos\theta_C]\gamma_\mu(1+\gamma_5)\lambda_i(x), \tag{1}$$

where i is a color index and is summed over, while $\mathcal{P}'$ is the charmed quark. This can be written as

$$J_\mu^+(x)=J_\mu^{\Delta S=0}(x)+J_\mu^{\Delta S=1}(x) =\overline{q}\gamma_\mu(1+\gamma_5)C_+q, \tag{2}$$

where in the basis $q=(\mathcal{P}',\mathcal{P},\mathfrak{N},\lambda)$

$$C_+=C_1+iC_2\begin{bmatrix}0&0&-\sin\theta_C&\cos\theta_C\\0&0&\cos\theta_C&\sin\theta_C\\0&0&0&0\\0&0&0&0\end{bmatrix}. \tag{3}$$

The hadronic neutral current is

$$J_\mu^0=\overline{q}[\gamma_\mu(1+\gamma_5)C_3-2\sin^2\theta_W\gamma_\mu Q]q =\overline{q}\{[C_3(1-2\sin^2\theta_W)-\tfrac{1}{3}\sin^2\theta_W]\gamma_\mu +C_3\gamma_\mu\gamma_5\}q. \tag{4}$$

Here θ_W is the Weinberg angle, Q is the charge matrix, and

$$C_3=\tfrac{1}{2}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix} \tag{5}$$

The second form in Eq. (4) follows for fractionally charged quarks.

The action of the charged (neutral) current is mediated by a charged (neutral) vector boson $W^\pm(Z^0)$, with the relationship $M_W=M_Z\cos\theta_W$. The theory is renormalizable[12] and asymptotically free.[13] The Cabibbo model contains only charged currents and can be obtained from the Weinberg theory by dropping all reference to neutral currents and to the charmed quark (but losing renormalizability in the process).

The mechanism for octet enhancement was originally suggested by Wilson.[14] The nonleptonic Hamiltonian in the Weinberg model can be written (neglecting possible Higgs-scalar exchange)

$$H_W(0)=\frac{G}{\sqrt{2}}M_W^2\int d^4x\,D_F(x,M_W)T[J_\mu^+(x)J^{-\mu}(0)] +\frac{G}{\sqrt{2}}M_Z^2\int d^4x\,D_F(x,M_Z)T[J_\mu^0(x)J^{0\mu}(0)]. \tag{6}$$

Since, for massive bosons, the vector-boson propagator $D_F(x,M)$ is sensitive primarily to the short-distance behavior of the currents, the time-ordered product can be expanded in a Wilson expansion:

$$T[J_\mu(x)J^\mu(0)]=\sum_i C_i(x)\mathcal{O}_i(0). \tag{7}$$

Here $\{\mathcal{O}_i\}$ is a complete set of operators carrying the appropriate quantum numbers, while $C_i(x)$ are c-number coefficients which contain the space-time dependence. It was Wilson's suggestion that the coefficient function for the SU(3) octet operator may be more singular than that of the 27-dimensional representation, thereby dominating H_w when the integration over x is performed. This has been verified and explicitly calculated for the Weinberg model in Refs. 6 and 7 ($\Delta S=1$ decays) and Ref. 8 ($\Delta S=0$, parity-violating Hamiltonian).

In asymptotically free theories the short-distance behavior can be calculated using perturbation theory. The operators $\{\mathcal{O}_i\}$ are decomposed in terms of multiplicatively renormalizable operators, with an associated dimension d_i. After the integration over the boson propagator, the weak Hamiltonian has the form[6]

$$H_W=\frac{G}{\sqrt{2}}\sum_i C_i(M)\mathcal{O}_i(0), \tag{8}$$

where

$$C_i(M)=C_i(1)\left(1+\frac{25g^2}{24\pi}\ln\frac{M}{m}\right)^{d_i} =C_i(1)c^{d_i}. \tag{9}$$

In this paper we fix constant $C_i(1)$ by requiring that as we turn off the strong interactions ($g^2\to 0$); H_w approaches the free-field limit. Since g^2, M, and m are all unknown, the factor in large parentheses can only be estimated to be of order 10. The various operators will be enhanced or suppressed depending on the sign and magnitude of d_i. The size of this effect appears too small to account completely for the $\Delta I=\frac{1}{2}$ rule, and other mechanisms may come into play.[6]

The group structure of the Hamiltonian can best be described in terms of SU(4). The product of two currents bilinear in quark fields gives rise to terms transforming as

$$\bar{4}\times 4\times \bar{4}\times 4 = 1 + 15 + 20 + 84 \qquad (10)$$

plus terms which drop out when we take a symmetric product. The SU(3) decomposition of the SU(4) multiplet is given in Table I, while the operators corresponding to the various SU(4) representations are written out in Appendix A.

For the Cabibbo theory we assume an enhancement mechanism similar in spirit, though not in detail, to that of the Weinberg model. The Hamiltonian will have the form

$$H_w = c_1 H_1 + c_1' H_1' + c_8 H_8 + c_8' H_8' + c_{27} H_{27}. \qquad (11)$$

The content of the H_i with definite SU(3) structure is given explicitly in Appendix A. Octet enhancement and the $\Delta I = \frac{1}{2}$ rule in hyperon and kaon decays can be accommodated by having c_8 (and maybe c_8') large, with $c_{27}/c_8 \approx \frac{1}{20}$.

III. THE CABIBBO MODEL $A(N_-^0)$

The Cabibbo model has been the standard picture for past calculations of parity-violating effects, although many other models have been considered.[1,25] The observation of neutral-current events with neutrinos on a hadron target[15] cannot be accounted for within the model. The Cabibbo theory must therefore be supplanted by a version incorporating neutral hadronic and leptonic currents, such as the Weinberg model. There are several reasons, however, why it is still worthwhile to consider the Cabibbo picture here. One is that it is instructive to compare the calculational method used here with past techniques. Another is that most models incorporating neutral currents retain the Cabibbo charged currents. The discussion here will then apply to the charged current product of those models.

We will attempt to calculate the pion vertex in a theory where baryons consist of three quarks, antisymmetric in color. The $\Delta S = 0$ parity-violating amplitude can be formed in the soft-pion limit as

TABLE I. Charm-conserving SU(3) submultiplets of SU(4) multiplets.

SU(4)	1	15	20	84
SU(3)	1	8	8	1+8+27

$$\lim_{q^\mu\to 0}\langle p\pi^-|H_w^{pv}|n\rangle = -\frac{i}{f_\pi}\langle p|[F_+^5, H_w^{pv}]|n\rangle = -\frac{i}{f_\pi}\langle p|[F_+, H_w^{pc}]|n\rangle, \qquad (12)$$

with $f_\pi \approx m_\pi$. As is well known, only the $\Delta I = 1$ piece of H_w can contribute in the soft-pion limit. The terms that are even under a Fierz rearrangement,

$$\bar{q}\gamma_\mu(1+\gamma_5)q'\bar{q}''\gamma^\mu(1+\gamma_5)q''' \to \bar{q}\gamma_\mu(1+\gamma_5)q'''\bar{q}''\gamma^\mu(1+\gamma_5)q' \qquad (13)$$

(that is, H_1', H_8', H_{27}), cannot contribute to the baryon-to-baryon matrix element in this framework, as first shown by Pati and Woo.[16] This means that only H_8 will contribute to $A(N_-^0)$. Here in the notation of Appendix A

$$H_{\text{eff}}^{pc} = [F_+, H_w^{pc}] = -c_8\frac{G}{2\sqrt{2}}\sin^2\theta\{:\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{N} - \bar{\mathcal{P}}\mathcal{N}\bar{\lambda}\lambda:\}. \qquad (14)$$

It is trivial to see that $A(N_-^0)$ vanishes in this model. This is because H_{eff}^{pc} contains the normal-ordered product of λ-quark fields, while the proton and neutron do not contain any λ quarks. This rather surprising result has been obtained before in other contexts.[17] Both here and in Körner's work,[17] this result depends primarily on the usual quark assignments for the nucleons. Here we also take into account strong-interaction enhancement effects via the short-distance behavior of the Wilson expansion. A crucial assumption in both methods is the neglect of possible quark pairs in the nucleons (i.e., the quark "sea").

It might be thought that the traditional estimate of $A(N_-^0)$, which appears to be based on more general techniques, contradicts this result and proves the inadequacy of the calculational framework. However, the reverse may be true. It is clearly worthwhile to review the traditional evaluation of $A(N_-^0)$ in view of the above result. If it is assumed that H_w transforms like an octet, then there is a sum rule relating the $\Delta S = 0$ process to $\Delta S = 1$ hyperon decays[18]:

$$A(N_-^0) = (\tfrac{2}{3})^{1/2}\tan\theta[2A(\Lambda_-^0) + A(\Xi_-^-)]. \qquad (15)$$

This can be derived using SU(3) in either physical limit, where $q_\pi^2 = m_\pi^2$, or in the soft-pion limit. A similar equation among $\Delta S = 1$ amplitudes, the Lee-Sugawara relation[19]

$$A(\Lambda_-^0) + 2A(\Xi_-^-) = \sqrt{3}A(\Sigma_0^+), \qquad (16)$$

appears to be well satisfied. It does not necessarily follow that Eq. (15) is also satisfied, since SU(3) symmetry breaking enters Eq. (15) and Eq. (16) differently, but one has in the past assumed that it would be accurate.

There is one problem that complicates the situation. In $A(N^0_-)$ we are concerned with the emission of a virtual pion, while in the $\Delta S=1$ amplitude the pion is on the mass shell. In general a $BB'\pi$ vertex can be decomposed into a commutator term, as in Eq. (12), and a residual term. The commutator term gives the vertex behavior in the soft-pion limit and is the only term we desire for the $NN\pi$ vertex. The residual term contains all amplitudes that vanish in the soft-pion limit such as the factorization diagrams of Fig. 1 or vector-meson pole terms. These are strictly speaking SU(3)-breaking effects since they depend on the mass difference of the two baryons, and must be accurately subtracted off from the physical hyperon amplitudes before using Eq. (15). Past estimates[20] have indicated that the contribution of the factorization term is about 15% of the $\Delta S=1$ amplitudes, and therefore small enough to neglect. However, if the octet piece of H_w is enhanced in a manner similar to that considered here, these estimates are too small and the factorization diagram's contribution to the residual term is large enough and uncertain enough to make Eq. (15) untrustworthy, as we shall now show.

In estimating the factorization contribution to hyperon decays within our model, we need the $\Delta S=1$ H_w written in terms of quark fields, given in Appendix A. The factorization term for $\Lambda\to p\pi^-$ is evaluated as

$$\langle p\pi^-|\bar{\mathcal{N}}_i\gamma_\mu(1+\gamma_5)\mathcal{P}_i\bar{\mathcal{P}}_j\gamma^\mu(1+\gamma_5)\lambda_j|\Lambda\rangle^{\text{pv}}_{\text{fact}} = \langle\pi^-|\bar{\mathcal{N}}_i\gamma_\mu\gamma_5\mathcal{P}_i|0\rangle\langle p|\bar{\mathcal{P}}_j\gamma^\mu\lambda_j|\Lambda\rangle. \quad (17)$$

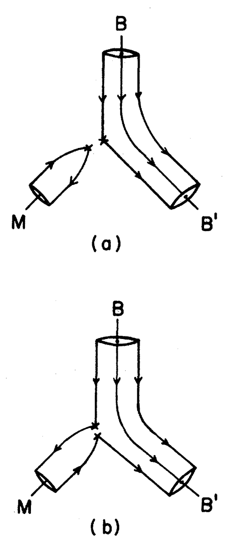

FIG. 1. The factorization diagram for $B\to B'M$. The x indicates the action of a current. (b) can be related to (a) by use of the Fierz rearrangement.

This is then known in terms of experimentally measured semileptonic parameters. Likewise, using the Fierz rearrangement

$$\begin{aligned}\langle p\pi^-|\bar{\mathcal{N}}_i\gamma_\mu(1+\gamma_5)\lambda_i\bar{\mathcal{P}}_j\gamma^\mu(1+\gamma_5)\mathcal{P}_j|\Lambda\rangle^{\text{pv}}_{\text{fact}} &= \langle\pi^-|\bar{\mathcal{N}}_i\gamma_\mu\gamma_5\mathcal{P}_j|0\rangle\langle p|\bar{\mathcal{P}}_j\gamma^\mu\lambda_i|\Lambda\rangle\\ &= \tfrac{1}{3}\langle\pi^-|\bar{\mathcal{N}}_i\gamma_\mu\gamma_5\mathcal{P}_i|0\rangle\langle p|\bar{\mathcal{P}}_j\gamma^\mu\lambda_j|\Lambda\rangle. \quad (18)\end{aligned}$$

The last step follows from the color structure of the particles. The factorization contribution is then

$$\frac{A(\Lambda^0_-)_{\text{fact}}}{A(\Lambda^0_-)} = (c_8+\tfrac{2}{5}c_8'+\tfrac{8}{5}c_{27})\times 0.05. \quad (19)$$

If the octet operator is enhanced sufficiently ($c_8\approx 10$ is reasonable), the factorization term will contribute more than the 15% obtained when considering the free field H_w ($c_i=1$). Clearly, when enhancement occurs it is ill advised to use the physical hyperon decay amplitudes in Eq. (15) to find $A(N^0_-)$ unless we can first subtract off such factorization contributions. Note also that the factorization terms satisfy the Lee-Sugawara relation to an acceptable degree, so that we cannot decide experimentally how much these terms contribute.

These conclusions can be supported from other viewpoints also. In any colored-quark model where the baryons consist of three quarks, we obtain for the hyperon decays in the soft-pion limit

$$\frac{A(\Lambda^0_-)}{A(\Xi^-_-)} = -\tfrac{1}{2}. \quad (20)$$

The two terms in Eq. (18) cancel exactly. Gronau,[21] in a pole model, has taken into account the K^*-pole contribution, which is the pole model's equivalent of the factorization diagram. He estimates its strength from experimental kaon decay, and from his numbers we obtain in the soft-pion limit

$$\frac{A(\Lambda^0_-)}{A(\Xi^-_-)} = -0.56, \quad (21)$$

so that the cancellation is nearly complete.[22] Finally, if one believes SU(4) symmetry currently in vogue, then the relation

$$\frac{A(\Lambda^0_-)}{A(\Xi^-_-)} = -\tfrac{1}{2} \quad (22)$$

again follows,[23] although this is merely suggestive since SU(4) is badly broken. The conclusion that can be drawn from this is that there may be adequate theoretical reason to abandon Eq. (15) as a derivation of $A(N^0_-)$ and to look for dynamical methods for calculating it.

We can obtain a nonzero value of $A(N_-^0)$ in our model if we abandon exact SU(2) symmetry. The factorization diagram will then enter, although at a much lower level than in hyperon decays because of the small neutron-proton mass differences. Using the same methods as above

$$A(N_-^0)_{\text{fact}} = \frac{G}{\sqrt{2}}\frac{A}{3}\left[\frac{c_1+c_1'}{3} + \left(c_8 + \frac{2c_8'}{5}\right)(\cos^2\theta - \tfrac{1}{3}) + \frac{c_{27}}{5}(8\cos^2\theta - 1)\right], \tag{23}$$

where

$$A = f_\pi(M_n - M_p)g_v = 1.9\times 10^{-4}M_p^{\,2}. \tag{24}$$

This amplitude is also isovector in character. Since we do not have a theory of the enhancement factors, we cannot evaluate this explicitly. However, a simple assumption, useful for an order of magnitude estimate, is to take $c_8 \approx 10$ and all other c_i small. This gives

$$A(N_-^0) = 0.28\times 10^{-8}. \tag{25}$$

This is an order of magnitude smaller than the usual estimate of $A(N_-^0)$. It is interesting to note that, using Gronau's model, a ρ-pole term gives essentially the same result.[22]

IV. $A(N_-^0)$ IN THE WEINBERG MODEL

To describe the pion vertex in the soft-pion limit we again use Eq. (12). The calculation in the Weinberg model simplifies considerably since we can eliminate many operators from $H_{\text{eff}}^{\text{pv}}$. The SU(4) singlet operators commute with F_+ and therefore vanish. $\mathcal{O}_{84}(C_i, C_i)$ vanishes via the Pati-Woo theorem. $\mathcal{O}_{20}(C_i, C_i)$ cannot contribute for the same reason that H_8 vanished in the Cabibbo model, that is, quark content of the nucleons. The only operators that can contribute belong to the SU(4) 15-dimensional representation, and are produced by the action of the neutral current.

We will calculate matrix elements by using theories developed to treat hadrons as extended objects composed of quarks. There are many available,[9-11] but for our final evaluation we will use the MIT bag model. The quark wave functions for this model are given in Ref. 11. The baryon-to-baryon matrix element for particles at rest is formed as

$$\langle p|\mathcal{O}_i(0)|n\rangle = \left\langle\alpha_p\right|\int d^3x\,\mathcal{O}_i(x)\left|\alpha_n\right\rangle. \tag{26}$$

Here $|\alpha_p\rangle$ is the spin-unitary spin state of the proton. Since $\mathcal{O}_i$ is written in terms of four quark fields, the integrand will contain x dependence in the quark wave functions.[24]

It is then easy to compute and compare $A(N_-^0)$ and $A(\Lambda_-^0)$. The result is

$$\frac{A(N_-^0)}{A(\Lambda_-^0)} = \frac{\sin^2\theta_W}{\sin\theta_C\cos\theta_C}\,\frac{4}{3\sqrt{6}}\,\frac{I}{I'}E(c), \tag{27}$$

where

$$E(c) = 0.17c^{0.38} + 0.93c^{-0.05} + 0.08c^{-0.61} - 0.19c^{-0.83}, \tag{28}$$

$$I = \int d^3x\,(u^4 - \tfrac{2}{3}u^2v^2 + v^4),\qquad I' = \int d^3x\,(u_{\mathcal{O}}^{\,2} + v_{\mathcal{O}}^{\,2})(u_{\mathcal{O}}u_\lambda + v_{\mathcal{O}}v_\lambda) \tag{29}$$

for a quark wave function of the form

$$\psi(x) = \begin{pmatrix} iu(r)\chi_m \\ v(r)\vec{\sigma}\cdot\hat{r}\,\chi_m \end{pmatrix} e^{-iEt}\;. \tag{30}$$

We have presented the results as a ratio in order to minimize the dependence on the enhancement factor and the precise shape of the wave functions. The enhancement factor $E(c)$ varies only slightly as we let c range over reasonable values and equals 1.22 for $c = 10$. The ratio I/I' is clearly equal to unity for any nonrelativistic model and for any relativistic theory[9] that uses two component wave functions for the quarks $[v(r)=0]$. The bag model contains both upper and lower components in the wave function, and numerical integration using the parameters of Ref. 11 yields

$$I/I' = 0.57. \tag{31}$$

The value of $A(\Lambda_-^0)$ that should be used in Eq. (27) is the amplitude in the soft-pion limit. In view of the comment in the last section we feel that to obtain this limit the factorization diagram should be subtracted off. There may be other factors which vanish in this limit, but we are not capable of estimating them. Equation (19) yields

$$\frac{A(\Lambda_-^0)_{\text{fact}}}{A(\Lambda_-^0)} = 0.22. \tag{32}$$

We therefore estimate the soft-pion limit by multiplying Eq. (27) by 0.78, yielding

$$A(N_-^0) = 1.3\sin^2\theta_W A(\Lambda_-^0). \tag{33}$$

V. THE ρ VERTEX

In the past the $NN\rho$ vertices have been evaluated in the factorization approximation.[25] This consists of

$$\langle p\rho^-|V_\mu^{(a)}A^{(b)\mu}|n\rangle \approx \langle\rho^-|V_\mu^{(a)}|0\rangle\langle p|A^{(b)\mu}|n\rangle. \tag{34}$$

However, if the currents are written in terms of quark fields, it is shown in Appendix B that this does not include all terms of comparable size. For example, the factorization approach would give different contributions for the two current products

$$\mathcal{O}_1 = \bar{\mathcal{P}}_i\gamma_\mu\gamma_5\mathcal{N}_i\bar{\mathcal{N}}_j\gamma_\mu\mathcal{P}_j + \bar{\mathcal{N}}_i\gamma_\mu\gamma_5\mathcal{P}_i\bar{\mathcal{P}}_j\gamma_\mu\mathcal{N}_j,$$

$$\mathcal{O}_2 = \bar{\mathcal{P}}_i\gamma_\mu\gamma_5\mathcal{P}_j\bar{\mathcal{N}}_j\gamma_\mu\mathcal{N}_i + \bar{\mathcal{N}}_i\gamma_\mu\gamma_5\mathcal{N}_j\bar{\mathcal{P}}_j\gamma_\mu\mathcal{P}_i \tag{35}$$

even though they are identical since they are related by a Feirz rearrangement. We therefore propose to evaluate the ρ vertex in the modified factorization approach called for by Appendix B.

Danilov[26] and Guberina, Missimer, and Tadic[27] have argued that, for conventional models (i.e., not gauge models), the $NN\rho$ amplitudes are in fact divergent. If this is true, the factorization result given here will be only one contribution to the total amplitude, with the other contribution coming from the properly cut off divergent term. Note, however, that this divergence will not occur in gauge models. It is related to operators of dimension 4 in the Wilson expansion, and Weinberg[28] has shown that in gauge theories these terms can be removed by mass and wave-function renormalization of the quark fields. The modified factorization approach therefore gives the Weinberg theory's major contribution to the ρ vertex.

The ρ amplitude can be expressed in general as

$$\langle N\rho^{(b)}|H_w|N^{(a)}\rangle = \epsilon_\mu^* M_\mu,$$

$$M_\mu = \bar{u}[\gamma_\mu\gamma_5 h_\rho(N_b^a) + \sigma_{\mu\nu}\gamma_5 q_\nu k_\rho(N_b^a) + iq_\mu\gamma_5 l_\rho(N_b^a)]u. \tag{36}$$

The terms containing k_ρ and l_ρ can be neglected as $q_\mu \to 0$. It is h_ρ that is calculated here. Expressed in terms of

$$h = \frac{G g_A m_\rho^2}{f_\rho} \tag{37}$$

the ρ vertex is

$$h_\rho(N_+^+) = -h_\rho(N_-^0) = A_0 + A_2,$$

$$h_\rho(N_0^+) = -\frac{A_0}{\sqrt{2}} + \sqrt{2}A_2 + A_1, \tag{38}$$

$$h_\rho(N_0^0) = \frac{A_0}{\sqrt{2}} - \sqrt{2}A_2 + A_1,$$

where in the Weinberg model

$$A_0 = -\frac{h}{9}\{c^{0.48}(3\cos^2\theta_C - 2\sin^2\theta_W) + 2c^{-0.24}[\cos^2\theta_C - \tfrac{2}{5}(1+\sin^2\theta_W)] + (0.3 - 0.2\sin^2\theta_W)(2.92c^{0.35} + 8.08c^{-0.40})\}, \tag{39}$$

$$A_1 = +\frac{\sqrt{2}h}{3}\sin^2\theta_W(0.05c^{0.85} - 0.03c^{0.43} + 0.21c^{-0.13} - 0.33c^{-0.35}), \tag{40}$$

$$A_2 = +\frac{4}{9}hc^{-0.24}(\sin^2\theta_C - 2\sin^2\theta_W), \tag{41}$$

whereas in the Cabibbo model

$$A_0 = -\frac{h}{3}(\tfrac{2}{5}c_8' + c_8)(\cos^2\theta_C - \tfrac{1}{3}) + \frac{c_1 + c_1'}{3} + \frac{c_{27}}{15}(4\cos^2\theta_C - 3)\Big], \tag{42}$$

$$A_1 = \frac{4h}{25\sqrt{2}}\sin^2\theta_C(c_8' - c_{27}), \tag{43}$$

$$A_2 = -h\tfrac{4}{9}c_{27}\cos^2\theta_C. \tag{44}$$

The ω and ϕ vertices can be done in the same fashion:

$$h_\omega(N_0^+) = B_0 + B_1,$$

$$h(N_0^0) = B_0 - B_1, \tag{45}$$

$$h_\phi(N_0^+) = C_0 + C_1,$$

$$h_\phi(N_0^0) = C_0 - C_1.$$

In the Weinberg model

$$B_0 = -\frac{h}{5\sqrt{2}}\{c^{0.48}(\cos^2\theta_C - \tfrac{2}{3}\sin^2\theta_W) - 2c^{-0.24}[\cos^2\theta_C - \tfrac{2}{5}(1+\sin^2\theta_W)] + \tfrac{1}{18}(3 - 2\sin^2\theta_W)(3.76c^{0.35} - 11.57c^{-0.40})\}, \tag{46}$$

$$B_1 = -\frac{2h}{3\sqrt{2}}\sin^2\theta_W(0.05c^{0.85} + 0.005c^{0.45} + 0.69c^{-0.13} + 0.42c^{-0.35}), \tag{47}$$

$$C_0=\frac{h}{10}\{c^{0.48}(\cos^2\theta_C-\tfrac{2}{3}\sin^2{}_W)-2c^{-0.24}[\cos^2\theta-\tfrac{2}{3}(1+\sin^2\theta_W)]+\tfrac{2}{18}(3-2\sin^2\theta_W)(-2.75c^{0.35}+3.36c^{-0.40})\}, \tag{48}$$

$$C_1=-\frac{h}{6}[c_1^{0.48}(1+\sin^2\theta_C-2\sin^2\theta_W)+2c^{-0.24}(\cos^2\theta_C-2\sin^2\theta_W)$$
$$+2\sin^2\theta_W(0.03c_1^{0.85}+0.13c_2^{0.43}+0.18c_3^{-0.13}+0.16_4^{-0.35})]. \tag{49}$$

In the Cabibbo model

$$B_0=-\frac{h}{5\sqrt{2}}\left[\left(c_8-\frac{6c_8'}{5}\right)(\cos^2\theta_C-\tfrac{1}{3})+\frac{c_1}{3}-c_1'-\frac{c_{27}}{5}(4\cos^2\theta_C-3)\right], \tag{50}$$

$$B_1=\frac{4}{15}\frac{h}{\sqrt{2}}\sin^2\theta_C(c_8'-c_{27}), \tag{51}$$

$$C_0=\frac{h}{5}\left[\frac{c_8}{2}(\cos^2\theta_C-\tfrac{1}{3})-\frac{c_1}{3}+\frac{c_1'}{3}-\frac{c_8'}{5}(\cos^2\theta_C-\tfrac{1}{3})-\frac{c_{27}}{5}(4\cos^2\theta_C-3)\right], \tag{52}$$

$$C_1=-\frac{h\sin^2\theta_C}{6}\left(c_8-\frac{2c_8'}{5}-\frac{8c_{27}}{5}\right). \tag{53}$$

In using h in the evaluation of ω and ϕ vertices we have implicitly assumed SU(3) symmetry for the vector meson-to-vacuum amplitudes. We have used the quark-model assignments of ω and ϕ (i.e., $\phi=\bar{\lambda}\lambda$) and use them in determining the $\langle B'|A_\mu|B\rangle$ amplitudes. These results are displayed in Table II (with $\sin\theta_C=0.235$, $\sin^2\theta_W=0.4$, and $c=10$), where it is obvious that the modified factorization approach yields different results from the usual method. The coefficients enhance the magnitude of the vertices, although, in the Weinberg model, not by enough to explain by itself the discrepancy between theory and experiment in $n+d\to p+\gamma$. Note that $\Delta I=2$ effects are suppressed by the coefficient functions. McKellar[30] has pointed out the apparent need for a large $\Delta I=2$ contribution if we are to understand $n+d\to p+\gamma$ without upsetting the approximate agreement with other experiments such as the α decay of oxygen.[2] This model cannot supply this contribution.

VI. SUMMARY

The parity-violating internucleon potential is the sum of π-, ρ-, ω-, and ϕ- exchange potentials. The potential due to π exhange is

$$V_\pi=\frac{g_{\pi NN}m_\pi^{-1/2}}{8\pi\sqrt{2}M_N}\times A(N_-^0)(\vec{\sigma}^{(1)}+\vec{\sigma}^{(2)})\cdot[\vec{p}_{12},\exp(-m_\pi r)/r]T_{12}^-, \tag{54}$$

where

$$T_{12}^-=\tau_+^{(1)}\tau_-^{(2)}-\tau_-^{(1)}\tau_+^{(2)},\quad \vec{p}_{12}=\vec{p}_1-\vec{p}_2. \tag{55}$$

The vector-meson potential is

TABLE II. The vector-meson vertex amplitudes, with $\sin\theta_C=0.23$, $\sin^2\theta_W=0.4$, in terms of $h=Gg_Am_\rho{}^2/f_\rho$.

	A_0	A_1	A_2	B_0	B_1	C_0	C_1
Cabibbo —previous result see Ref. 27	$-0.66h$	0	$-0.33h$	0	0	0	0
Weinberg —unenhanced ($c=1$)	$-0.60h$	$-0.20h$	$-0.36h$	$0.15h$	$-0.22h$	$0.02h$	$-0.06h$
Weinberg —enhanced ($c=10$)	$-1.03h$	$+0.05h$	$-0.19h$	$-0.33h$	$-0.20h$	$0.07h$	$-0.26h$
Cabibbo —unenhanced ($c_i=1$)	$-0.54h$	0	$-0.44h$	$0.14h$	0	$0.02h$	$0.04h$
Cabibbo —octet term only	$-0.22c_8h$	0	0	$-0.09c_8h$	0	$0.07c_8h$	$-0.01c_8h$

$$V_B = -\frac{f_B}{16\pi M_N}(\vec{\sigma}^{(1)}\cdot\{\vec{p}_{12}, \exp(-m_B r)/r\}H_{12}^B - \vec{\sigma}^{(2)}\cdot\{\vec{p}_{12}, \exp(-m_B r)/r\}H_{21}^B$$
$$-i(\vec{\sigma}^{(1)}\times\vec{\sigma}^{(2)})\cdot[\vec{p}_2, \exp(-m_B r)/r]\tilde{H}_{12}^B - i(\vec{\sigma}^{(2)}\times\vec{\sigma}^{(1)})\cdot[\vec{p}_1, \exp(-m_B r)/r]\tilde{H}_{21}^B) \qquad (56)$$

for $B=\rho, \omega, \phi$. The H_{ij}^B are isospin functions

$$H_{ij}^\rho = \sqrt{2}[-h_\rho(N_+^+)\tau_+^{(i)}\tau_-^{(j)} + h_\rho(N_-^0)\tau_-^{(i)}\tau_+^{(j)}] + \tfrac{1}{2}[h_\rho(N_0^+) - h_\rho(N_0^0)]\tau_3^{(i)}\tau_3^{(j)} + \tfrac{1}{2}[h_\rho(N_0^+) + h_\rho(N_0^0)]1^{(i)}\tau_3^{(j)}, \qquad (57)$$

$$H_{ij}^\omega = \tfrac{1}{2}[h_\omega(N_0^+) + h_\omega(N_0^0)]1^{(i)}1^{(j)} + \tfrac{1}{2}[h_\omega(N_0^+) - h_\omega(N_0^0)]\tau_3^{(i)}1^{(j)}, \qquad (58)$$

$$H_{ij}^\phi = \tfrac{1}{2}[h_\phi(N_0^+) + h_\phi(N_0^0)]1^{(i)}1^{(j)} + \tfrac{1}{2}[h_\phi(N_0^+) - h_\phi(N_0^0)]\tau_3^{(i)}1^{(j)}, \qquad (59)$$

$$\tilde{H}_{ij}^\rho = (1+\mu_p - \mu_n)H_{ij}^\rho, \qquad (60)$$

$$\tilde{H}_{ij}^\omega = H_{ij}^\omega, \qquad (61)$$

$$\tilde{H}_{ij}^\phi = H_{ij}^\phi. \qquad (62)$$

There are several general conclusions that follow from this paper. If the observed octet enhancement and 27-plet suppression occurs because of the short-distance behavior of the current product, then past methods of calculating the parity-violating amplitudes may not be valid. We have used a quark model to calculate the pion vertex $A(N_-^0)$ in the Weinberg theory and have argued that $A(N_-^0)$ in the Cabibbo model has been greatly overestimated. The Weinberg theory's $A(N_-^0)$ is of the same order of magnitude as past estimates of this quantity, so that the enhancement does not open up any new experimental realms. However, if further experimental work is done it will still be possible to distinguish between the various theories. For the ρ-exchange potential is was shown that a modified factorization approach is called for. The potential will contain $\Delta I = 0, 1, 2$ pieces, with the $\Delta I = 2$ pieces suppressed. Neutral ρ, ω, and ϕ exchange occurs even in the Cabibbo model. The amplitudes are enhanced over the old factorization results, although, in the Weinberg model, not by enough to explain the large discrepancy in $n+p\to d+\gamma$. However, in general a theoretical explanation of this process may be simple; we must take into account the operator enhancement due to the short-distance behavior.

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APPENDIX A: THE OPERATORS

For the Weinberg model we can take the operators and notation from Ref. 8. If M, N are arbitrary matrices in both SU(3) and color spaces, then a parity-violating four-quark operator has the form

$$\mathcal{O}(M,N) = \bar{q}\gamma_\mu\gamma_5 Mq\bar{q}\gamma_\mu Nq. \qquad (A1)$$

Likewise a parity-violating operator involving the gluon field can be formed:

$$\mathcal{O}(M) = \frac{1}{g}\nabla^\mu G_{\mu\nu}^A \bar{q}\gamma_\mu\gamma_5 M t^A q. \qquad (A2)$$

Here and later $\{t^A\}$ are the usual matrices in color space satisfying

$$[t^A, t^B] = if_{ABC}t^C. \qquad (A3)$$

The multiplicity renormalizable operators which can be formed are

$$\mathcal{O}_{20}(C_i, C_i) = \tfrac{1}{3}[\mathcal{O}(C_i, C_i) + \tfrac{1}{12}\mathcal{O}(1,1)] - \tfrac{1}{4}[\mathcal{O}(C_i t^A, C_i t^A) + \tfrac{1}{12}\mathcal{O}(t^A, t^A)], \qquad (A4)$$

$$\mathcal{O}_{84}(C_i, C_i) = \tfrac{2}{3}[\mathcal{O}(C_i, C_i) - \tfrac{1}{20}\mathcal{O}(1,1)] + \tfrac{1}{4}[\mathcal{O}(C_i t^A, C_i t^A) - \tfrac{1}{20}\mathcal{O}(t^A, t^A)], \qquad (A5)$$

$$\begin{bmatrix} \mathcal{O}_1^{15} \\ \mathcal{O}_2^{15} \\ \mathcal{O}_3^{15} \\ \mathcal{O}_4^{15} \\ \mathcal{O}_5^{15} \end{bmatrix} = \begin{bmatrix} 0.48 & 0.35 & -0.27 & -0.63 & -0.42 \\ 0.43 & -0.37 & 0.56 & -0.29 & -0.53 \\ -0.82 & 0.11 & 0.56 & -0.17 & 0.04 \\ 0.42 & 0.41 & 0.76 & -0.22 & 0.18 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathcal{O}(C_3, 1) \\ \mathcal{O}(C_3 t^A, t^A) \\ \mathcal{O}(1, C_3) \\ \mathcal{O}(t^A, {}_3t^A) \\ \mathcal{O}(C_3) - \mathcal{O}(C_3 t^A, t^A) \end{bmatrix}, \qquad (A6)$$

$$\begin{bmatrix}\mathcal{O}_1^1\\ \mathcal{O}_2^1\\ \mathcal{O}_3^1\end{bmatrix}=\begin{bmatrix}-0.76 & 0.42 & 0.49\\ 0.83 & 0.52 & 0.20\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathcal{O}(1,1)\\ \mathcal{O}(t^A,t^A)\\ \mathcal{O}(1)-\mathcal{O}(t^A,t^A)\end{bmatrix}. \tag{A7}$$

We fix the constants in the Wilson expansion by requiring that H_w reduce to the free-field limit when the enhancement factors are unity. The Weinberg-model Hamiltonian is then

$$\begin{aligned}H_w^{pv}=\sqrt{2}\,G\{&c_{20}[\mathcal{O}_{20}(C_1,C_1)+\mathcal{O}_{20}(C_2,C_2)+(1-2\sin^2\theta_W)\mathcal{O}_{20}(C_3,C_3)]\\ &+c_{84}[\mathcal{O}_{84}(C_1,C_1)+\mathcal{O}_{84}(C_2,C_2)+(1-2\sin^2\theta_W)\mathcal{O}_{84}(C_3,C_3)]\\ &+\tfrac{1}{180}(3-2\sin^2\theta_W)(6.02c_1^1\mathcal{O}_1^1+6.74c_2^1\mathcal{O}_2^1-4.3c_3^1\mathcal{O}_3^1)\\ &-\tfrac{1}{3}\sin^2\theta_W(0.18c_1^{15}\mathcal{O}_1^{15}+0.34c_2^{15}\mathcal{O}_2^{15}-0.76c_3^{15}\mathcal{O}_3^{15}+0.36c_4^{15}\mathcal{O}_4^{15}+0.22c_5^{15}\mathcal{O}_5^{15})\}.\end{aligned} \tag{A8}$$

The exponents d_i ($c_i \approx 10^{d_i}$) for the various operators are listed in Table III.

For the Cabibbo model we will explicitly list the operator that contributes to the $\Delta S=0$ and $\Delta S=1$ processes. For simplicity of notation we will leave off the Dirac and color indices. Hence

$$\bar{q}q\equiv\bar{q}_i\gamma_\mu(1+\gamma_5)q_i. \tag{A9}$$

The operators that enter the $\Delta S=0$ Cabibbo Hamiltonian are [the superscript is the SU(3) representation, the subscript is the isospin]

$$\tilde{\mathcal{O}}_1^8=:\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}-\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda-\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}+\bar{\mathcal{N}}\mathcal{N}\bar{\lambda}\lambda:,$$

$$\tilde{\mathcal{O}}_0^8=:2(\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\mathcal{N}-\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{P}}\mathcal{P})-(\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}-\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda+\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}-\bar{\mathcal{N}}\mathcal{N}\bar{\lambda}\lambda):,$$

$$\tilde{\mathcal{O}}_0^1=:\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{P}}\mathcal{P}-\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\mathcal{N}+\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda-\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}+\bar{\mathcal{N}}\mathcal{N}\bar{\lambda}\lambda-\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}:,$$

$$\mathcal{O}_2^{27}=:2(\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\mathcal{N}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{P}}\mathcal{P})-(\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{P}}\mathcal{P}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N}):,$$

$$\mathcal{O}_1^{27}=:\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{P}}\mathcal{P}-\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N}-2(\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}+\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda-\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}-\bar{\mathcal{N}}\mathcal{N}\bar{\lambda}\lambda):,$$

$$\mathcal{O}_0^{27}=:\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\mathcal{N}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{P}}\mathcal{P}+\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{P}}\mathcal{P}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N}+3\bar{\lambda}\lambda\bar{\lambda}\lambda-3(\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}+\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda+\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}+\bar{\lambda}\lambda\bar{\mathcal{N}}\mathcal{N}):,$$

$$\mathcal{O}_1^8=:2(\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{P}}\mathcal{P}-\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N})+\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}+\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda-\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}-\bar{\mathcal{N}}\mathcal{N}\bar{\lambda}\lambda:,$$

$$\mathcal{O}_0^8=:2(\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\mathcal{N}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{P}}\mathcal{P}+\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{P}}\mathcal{P}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N})-4\bar{\lambda}\lambda\bar{\lambda}\lambda-(\bar{\mathcal{P}}\lambda\bar{\lambda}\mathcal{P}+\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda+\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}+\bar{\lambda}\lambda\bar{\mathcal{N}}\mathcal{N}):,$$

$$\mathcal{O}_0^1=:\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\mathcal{N}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{P}}\mathcal{P}+\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N}+\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{P}}\mathcal{P}+\bar{\lambda}\lambda\bar{\lambda}\lambda+\bar{\lambda}\mathcal{P}\bar{\mathcal{P}}\lambda+\bar{\mathcal{P}}\mathcal{P}\bar{\lambda}\lambda+\bar{\mathcal{N}}\lambda\bar{\lambda}\mathcal{N}+\bar{\mathcal{N}}\mathcal{N}\bar{\lambda}\lambda:.$$

We can then form the $\Delta S=0$ Cabibbo Hamiltonian

$$H_w^{\Delta S=0}=c_1H^{(1')}+c_1'H^{(1)}+c_8H^{(8)}+c_8'H^{(8')}+c_{27}H^{(27)}, \tag{A10}$$

where

$$H^{(1)}=-\frac{G}{\sqrt{2}}\tfrac{1}{6}\tilde{\mathcal{O}}_0^1, \tag{A11}$$

$$H^{(1')}=\frac{G}{12\sqrt{2}}\tfrac{1}{6}\mathcal{O}_0^1, \tag{A12}$$

$$H^{(8)}=\frac{G}{\sqrt{2}}\left[\tfrac{1}{4}(\cos^2\theta_C-\tfrac{1}{3})\tilde{\mathcal{O}}_0^8+\frac{\sin^2\theta_C}{4}\tilde{\mathcal{O}}_1^8\right], \tag{A13}$$

$$H^{(8')}=\frac{G}{\sqrt{2}}\left[\tfrac{1}{20}(\cos^2\theta_C-\tfrac{1}{3})\mathcal{O}_0^8+\frac{\sin^2\theta_C}{20}\mathcal{O}_1^8\right], \tag{A14}$$

$$H^{(27)}=\frac{G}{\sqrt{2}}\left[\tfrac{1}{60}(4\cos^2\theta_C-3)\mathcal{O}_0^{27}-\frac{\sin^2\theta_C}{10}\mathcal{O}_1^{27}+\frac{\cos^2\theta_C}{6}\mathcal{O}_2^{27}\right]. \tag{A15}$$

Again when $c_i=1$ we recover the free-field limit.

The $\Delta S=1$ Hamiltonian is

$$H_w^{\Delta S=1}=c_8H_{\Delta S=1}^{(8)}+c_8'H_{\Delta S=1}^{(8')}+c_{27}H_{\Delta S=1}^{(27)}, \tag{A16}$$

with

$$H_{\Delta S=1}^{8}=\frac{G}{\sqrt{2}}\frac{\cos\theta_C\sin\theta_C}{2}\{:\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\lambda-\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{N}}\lambda:\}, \tag{A17}$$

$$H_{\Delta S=1}^{(8')}=\frac{G}{\sqrt{2}}\frac{\cos\theta_C\sin\theta_C}{10}\{:\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\lambda+\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{N}}\lambda+2\bar{\mathcal{N}}\lambda\bar{\mathcal{N}}\mathcal{N}+2\bar{\mathcal{N}}\lambda\bar{\lambda}\lambda:\}, \tag{A18}$$

$$H_{\Delta S=1}^{27}=\frac{G}{\sqrt{2}}\frac{\cos\theta_C\sin\theta_C}{15}\{\mathcal{O}_{1/2}^{(27)}+5\mathcal{O}_{3/2}^{(27)}\}, \tag{A19}$$

$$\begin{aligned}\mathcal{O}_{3/2}^{(27)}&=:\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\lambda+\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{N}}\lambda-\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\lambda:,\\ \mathcal{O}_{1/2}^{(27)}&=:\bar{\mathcal{N}}\mathcal{P}\bar{\mathcal{P}}\lambda+\bar{\mathcal{P}}\mathcal{P}\bar{\mathcal{N}}\lambda+2\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\lambda-3\bar{\mathcal{N}}\lambda\bar{\lambda}\lambda:.\end{aligned} \tag{A20}$$

APPENDIX B: THE MODIFIED FACTORIZATION APPROACH

In this section the modified factorization approach is presented and justified. We want to look at the $NN\rho$ amplitude for a parity-violating current product

$$A \equiv \langle N'\rho^{(a)}|:V_\mu^{(b)}(0)A^{(c)\mu}(0):|N\rangle. \quad (B1)$$

The LSZ reduction technique can be used to write this as

$$A = i\epsilon_\lambda^*(q^2+m_\rho^2) \times \int d^4x\, e^{-iq\cdot x}\langle N'|T(\rho_\lambda^{(a)}(x):V_\mu^{(b)}(0)A^{(c)\mu}(0):)|N\rangle \quad (B2)$$

plus possible seagull terms.[1] The ρ field can be replaced by the vector current by using the current-field identity[29] (CFI)

$$\rho_\lambda^{(a)}(x) = \frac{f_\rho}{m_\rho^2}V_\lambda^{(a)}(x). \quad (B3)$$

TABLE III. Exponents of the parity-violating operators. The operators are defined in Appendix A. The enhancement factor for operator $\mathcal{O}_i$ is $c_i = c^{d_i}$ with $c \approx 10$.

Operator	Exponent	Operator	Exponent
$\mathcal{O}_1^1$	0.35	$\mathcal{O}_1^{15}$	0.85
$\mathcal{O}_2^1$	−0.40	$\mathcal{O}_2^{15}$	0.43
$\mathcal{O}_3^1$	0.54	$\mathcal{O}_3^{15}$	−0.13
$\mathcal{O}_{20}(C_i,C_i)$	0.48	$\mathcal{O}_4^{15}$	−0.35
$\mathcal{O}_{84}(C_i,C_i)$	−0.24	$\mathcal{O}_5^{15}$	0.54

The time-ordered product of three currents must be evaluated. If we write the currents in terms of fundamental quark fields

$$V_\lambda^{(a)}(x) = \bar\psi(x)\gamma_\lambda\frac{\lambda^{(a)}}{2}\psi(x) \quad (B4)$$

then we can use Wick's theorem on the time-ordered product:

$$\begin{aligned}T(V_\lambda^{(a)}(x):V_\mu^{(b)}(0)A^{(c)\mu}(0):) =\; &:V_\lambda^{(a)}(x)V_\mu^{(b)}(0)A_\mu^{(c)}(0): + \langle 0|T(V_\lambda^{(a)}(x)V_\mu^{(b)}(0))|0\rangle A_\mu^{(c)}(0)\\ &- (\gamma_\mu)^{\alpha\beta}(\gamma^\mu\gamma_5)^{\gamma\delta}\frac{\lambda_{ij}^{(b)}}{2}\frac{\lambda_{kl}^{(c)}}{2}[\langle 0|T(V_\lambda^{(a)}:\bar\psi_{\alpha im}(0)\psi_{\delta ln}(0):)|0\rangle:\bar\psi_{\gamma kn}(0)\psi_{\beta jm}(0):\\ &+\langle 0|T(V_\lambda^{(a)}(x):\bar\psi_{\gamma kn}(0)\psi_{\beta jm}(0):)|0\rangle:\bar\psi_{\alpha im}(0)\psi_{\delta ln}(0):].\end{aligned} \quad (B5)$$

Here $\alpha,\beta,\gamma,\delta$ are Dirac indices, i,j,k,l are SU(3) indices, and m,n refer to color. The CFI and reduction can be done in reverse to obtain

$$\begin{aligned}A =\; &\langle\rho^{(a)}|V_\mu^{(b)}(0)|0\rangle\langle N'|A_\mu^{(c)}(0)|N\rangle\\ &-(\gamma_\mu)^{\alpha\beta}(\gamma^\mu\gamma_5)^{\gamma\delta}\frac{\lambda_{ij}^{(b)}}{2}\frac{\lambda_{kl}^c}{2}[\langle\rho^{(a)}|:\bar\psi_{\alpha im}(0)\psi_{\delta ln}(0):|0\rangle\langle N'|:\psi_{\gamma kn}(0)\psi_{\beta jm}(0):|N\rangle\\ &+\langle\rho^{(a)}|:\bar\psi_{\gamma kn}(0)\psi_{\beta jm}(0):|0\rangle\langle N'|:\bar\psi_{\alpha im}(0)\psi_{\delta ln}(0):|N\rangle]+\epsilon_\lambda^*R^\lambda,\end{aligned} \quad (B6)$$

where

$$R_\lambda = i(q^2+m_\rho^2)\frac{f_\rho}{m_\rho^2}\int d^4x\, e^{-iq\cdot x}\langle N'|:V_\lambda^{(a)}(x)V_\mu^{(b)}(0)A^{(c)\mu}(0):|N\rangle. \quad (B7)$$

The first term is what is normally called the factorization approximation and corresponds to Fig. 1(a). The second term is similar and will generally be of the same order of magnitude as the first term. It corresponds to Fig. 1(b), and is improperly neglected in the usual factorization approach. The third term will not contribute to h_ρ [see Eq. (36)] since $q_\lambda R^\lambda = 0$.

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