# Analysis of *CP* violation through phase angles in weak charged currents in a relativistic quark model\*

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We test models of *CP* violation through phase angles in the weak currents of the type of Glashow for  $K \rightarrow 2\pi$  decays. The analysis, based on a relativistic quark model for mesons, shows that the phases are bound to be very small (~ 10<sup>-4</sup>).

#### I. INTRODUCTION

A year after the discovery<sup>1</sup> of the decay  $K_L \rightarrow 2\pi$ it was proposed that the CP violation could be due to a phase difference between the vector and the axial-vector Cabibbo currents.<sup>2</sup> Ten years later, the validity of these proposals is still uncertain, owing to the difficulty of computing their predictions. In fact, only estimates based on symmetry considerations have been done.<sup>3</sup> Here we present a detailed numerical analysis based on explicit Bethe-Salpeter (BS) amplitudes for the mesons.<sup>4</sup> which have been shown to reproduce accurately their decay rates,<sup>5</sup> as well as to imply the symmetry results of CP violation in the "milliweak" theory with charged currents.<sup>6</sup> In Sec. II we present the CP-violation model and the main phenomenology. In Sec. III we compute the  $K \rightarrow 2\pi$ amplitudes and in Sec. IV we discuss the determination of the *CP*-violating phases and the results of the model.

#### II. CP-VIOLATION MODELS AND PHENOMENOLOGY

There have been proposed several models<sup>2</sup> in which the *CP* violation is attributed to phase angles between the weak vector and axial-vector charged currents. In the Glashow<sup>2</sup> model the phases are added in such a way as to have a weak charged current of V - A type

$$J = \cos\theta_{C} (V + e^{i\phi}A)_{1+i2} + \sin\theta_{C} (V + e^{i\omega}A)_{4+i5}$$
(2.1)

instead of the standard Cabibbo current ( $\phi = \omega = 0$ ).

In a model proposed by two of the authors,<sup>2</sup> the universal weak current was obtained through a

generalization of the Cabibbo rotation of the isospin axial-vector current so as to include also a rotation around the other neutral  $F_6$  direction in the unitary space. For the vector part the usual Cabibbo rotation is kept, but for the axial-vector part of the current, a rotation  $\exp^{(2i\theta_A F_7 \tau^{2i}\phi_A F_6)}$  is performed. Then the *CP*-violation effects one gets for the nonleptonic weak Hamiltonian are due only to a "small" presence of the  $F_6$  rotation in the axial-vector current. In this model, the weak charged current of V - A type is

$$J = \cos\theta_C (V + A)_{1+i2} + \sin\theta_C (V + e^{i\omega}A)_{4+i5}, \qquad (2.2)$$

where

$$e^{i\omega} = \frac{\theta_A + i\phi_A}{\theta_C}.$$
 (2.3)

The second model, which leads to the current (2.2), can of course be formally considered as a particular case of (2.1), where no *CP* violation is allowed in the  $\Delta S=0$  current as suggested by  $\beta$  decay. However, in this model the phase angle  $\omega$  has a physical meaning in terms of the Cabibbo-type rotations.

As usual the weak nonleptonic Hamiltonian is of the current  $\times\, current$  form

$$\mathcal{H}_{\rm NL} = \frac{G}{2\sqrt{2}} \{J, J^{\dagger}\}.$$
 (2.4)

In order to connect the experimental *CP*-violating parameters with the phases  $\phi$  and  $\omega$  let us state a few remarks about the usual *CP* phenomenological analysis, and fix our phases and conventions.

The neutral kaons of strangeness  $S = \pm 1$ ,  $K^0$  and  $\overline{K}^0$ , are defined as

$$\overline{K}^{0} \rangle = CP | K^{0} \rangle. \tag{2.5}$$

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The neutral-kaon states with a definite lifetime  $K_{s, L}$  are given in terms of the strangeness  $\pm 1$  basis, using *CPT* invariance, as

$$|K_{S,L}\rangle = \frac{1}{[2(1+|\epsilon|^2)]^{1/2}} [\langle |K^0\rangle \pm |\overline{K}^0\rangle) + \epsilon \langle |K^0\rangle \mp |\overline{K}^0\rangle]. \quad (2.6)$$

The *CP* impurity in the corresponding  $|K_s\rangle$  or  $|K_L\rangle$  state is given by the complex parameter  $\epsilon$ , whose real part measures its lack of orthogonality,

$$\langle K_s | K_L \rangle = \frac{2 \operatorname{Re} \epsilon}{(1 + |\epsilon|^2)}, \qquad (2.7)$$

and is expected to be of the order of magnitude of 10<sup>-3</sup>. In fact Re  $\epsilon$  can be experimentally measured, because the charge asymmetry parameter  $\delta_L$  for the semileptonic decays of the neutral kaon can be written as

$$\delta_L = \frac{N_* - N_-}{N_* + N_-} = 2 \operatorname{Re} \epsilon$$
(2.8)

if one assumes that the  $\Delta S = -\Delta Q$  amplitudes are small compared to the  $\Delta Q = \Delta S$  amplitudes (its ratio, *x*, is consistent with zero; Rex = -0.003 $\pm 0.027$ , Im $x = -0.005 \pm 0.038$ ). We shall use therefore Re $\epsilon$  as an experimental input, its value<sup>7</sup> being

$$2 \operatorname{Re} \epsilon = (3.34 \pm 0.12) \times 10^{-3}$$
. (2.9)

The experimental parameters we shall use are

$$\eta_{ij} = \frac{A(K_L \to \pi^i \pi^j)}{A(K_S \to \pi^i \pi^j)} = \frac{A_{ij}^L}{A_{ij}^S} = \frac{(A_{ij} - \overline{A}_{ij}) + \epsilon(A_{ij} + \overline{A}_{ij})}{(A_{ij} + \overline{A}_{ij}) + \epsilon(A_{ij} - \overline{A}_{ij})},$$
(2.10)

where

$$A_{ij} \equiv A(K^{0} \rightarrow \pi^{i}\pi^{j}), \quad \overline{A}_{ij} \equiv A(\overline{K}^{0} \rightarrow \pi^{i}\pi^{j})$$

and (i, j) are either (+, -) or (0, 0).

The current experimental values for  $\eta_{**}$  and  $\eta_{\rm oo}$  as given in Ref. 7 are

$$\begin{aligned} &|\eta_{+-}| = (2.30 \pm 0.035) \times 10^{-3}, \\ &|\eta_{00}/\eta_{+-}| = 1.013 \pm 0.046, \\ &\phi_{+-} = 49.4^{\circ} \pm 1.7^{\circ}, \\ &\phi_{00} - \phi_{+-} = 4^{\circ} \pm 13^{\circ}. \end{aligned}$$
(2.11)

The strong interaction of the final  $\pi\pi$  system is taken into account, as usual, by the phase shifts in the isospin channels. The phase shifts can be determined from the experimental decay rates of  $K_s \rightarrow 2\pi$  and  $K^* \rightarrow \pi^*\pi^0$  and can also be extracted

from the  $\pi\pi$  phase-shift analysis. We shall use our result of Ref. 8,  $\delta_2 - \delta_0 = 53^\circ$ , which is inside the error of a recent determination in the same spirit which includes radiative corrections,<sup>9</sup> and which is the only result compatible with the phaseshift analysis, summarized in Ref. 9. We do not use the Wu-Yang<sup>10</sup> gauge, but rather let<sup>11,8</sup>

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$$\begin{aligned} \langle 2\pi, T = \mathbf{0} | \mathcal{H}_{\mathrm{NL}} | K_{1}^{0} \rangle &= a_{0} e^{i \delta_{0}}, \\ \langle 2\pi, T = \mathbf{0} | \mathcal{H}_{\mathrm{NL}} | K_{2}^{0} \rangle &= i a_{0} \gamma e^{i \delta_{0}} \end{aligned}$$

$$(2.12)$$

instead of the more standard former choice  $\gamma = 0$ . The parameters  $a_0$  and  $\gamma$  are real numbers because of *CPT*.  $K_1^0$  ( $K_2^0$ ) is the even (odd) *CP* eigenstate. Note also that the *CP* impurity parameter in the Wu-Yang gauge,  $\epsilon'$ , can be expressed as

$$\epsilon' = \epsilon_0, \tag{2.13}$$

whereas in our phase convention the corresponding impurity parameter, denoted by  $\epsilon$  [see Eq. (2.6)], is given by

$$\epsilon = \epsilon_0 - i\gamma, \tag{2.14}$$

where  $\epsilon_0$  is the ratio

$$\epsilon_{0} = \frac{\langle 2\pi, T = 0 | \Im \mathcal{C}_{\mathrm{NL}} | K_{L} \rangle}{\langle 2\pi, T = 0 | \Im \mathcal{C}_{\mathrm{NL}} | K_{S} \rangle}.$$
(2.15)

The relation between the impurities in both gauges is

$$\epsilon' = \epsilon + i\gamma_{\circ} \tag{2.16}$$

Note also that transforming the  $K^0$  and  $\overline{K}^0$  states as

$$K^{0} \rightarrow (K^{0})' = e^{i\xi}K^{0} \simeq (1+i\xi)K^{0},$$
  

$$\overline{K}^{0} \rightarrow (\overline{K}^{0})' = e^{-i\xi}\overline{K}^{0} \simeq (1-i\xi)\overline{K}^{0}$$
(2.17)

(in going from one phase convention to another), one obtains for the infinitesimal  $\xi$  the value

$$\xi = \gamma = \frac{\langle 2\pi, T=0 | \mathcal{B}_{\mathrm{NL}} | K_{2}^{0} \rangle}{\langle 2\pi, T=0 | \mathcal{B}_{\mathrm{NL}} | K_{1}^{0} \rangle}, \qquad (2.18)$$

keeping the same value for  $\epsilon_0$  in both gauges. The expressions above then give the connection between the two gauges.

### **III. COMPUTATION OF THE AMPLITUDES**

For the dynamical computation of the relevant meson decays we apply the work of Böhm, Joos, and Krammer, extended to include symmetry breaking in a simple way. The mesons are considered as quark-antiquark bound states, represented by Bethe-Salpeter (BS) amplitudes  $\chi(r, P)$ , where r is the relative momentum and P the bound-state momentum, which satisfy the (BS) equation

$$S^{-1}(P_1)\chi(r,P)\overline{S}^{-1}(P_2) = i \int d^4r' K(r,r',P)\chi(r',P).$$
(3.1)

The basic assumptions for the former BS equation are large quark masses (m > 1 GeV), unrenormalized propagators  $S^{-1} = \gamma \cdot P - m$ , and a convolutiontype, energy-independent kernel. From the requirement that the spectrum derived from Eq. (3.1) should reproduce the nonrelativistic one, the spin structure turns out to be pseudoscalar + vector - scalar.<sup>12</sup> The equation is solved in the Wickrotated form, for a smooth kernel approximated by an oscillator form, as an expansion in  $(M/m)^2$  of the bound-state zero-mass solution. The (BS) amplitudes for the pseudoscalar mesons are

$$\chi_{0-}(P,r) = \frac{4}{\sqrt{3\beta}} \left( 1 + \frac{P}{M} \right) \gamma_5 \exp\left(-\frac{r^2}{2\sqrt{\beta}}\right) |q\overline{q}\rangle.$$
(3.2)

 $2\sqrt{\beta} = 1 \text{ GeV}^2$  is obtained from the meson level spacing and  $|q\bar{q}\rangle$  is the SU(3) quark content of the mesons. For the SU(3) breaking in weak meson decays, it has been shown<sup>5</sup> recently that it can be explained by an additional term

$$\delta u(K) (K - K')^{\mu} s(K').$$
 (3.3)

The coefficient  $\delta$  of the SU(3) symmetry-breaking term is proportional to the mass difference between the strange and nonstrange quarks.  $\delta$  vanishes in the limit of exact SU(3) symmetry, which implies that (3.3) is a current of the first kind. The numerical value of  $\delta$  which modifies the bare Cabibbo current in hadronic decays can be fixed from the experimental<sup>13</sup> decay rate  $\Gamma(K^* \rightarrow \pi^* \pi^0)$ =  $1.12 \times 10^{-17}$  GeV, which yields

$$\delta = -0.144 \text{ GeV}^{-1}. \tag{3.4}$$

We would like to remark that the subsequent analysis is rather weakly dependent on the numerical value of  $\delta$  since only ratios of amplitudes enter.

The amplitudes for  $K_s \rightarrow \pi^+\pi^-$  and  $K_s \rightarrow \pi^0\pi^0$  are then given by the traces of the diagrams shown in Figs. 1(a) and 1(c) and Figs. 1(b) and 1(c), respectively, which are

$$g_{a} = -e^{i\phi} \left[ f_{K} (m_{K}^{2} - m_{\pi}^{2}) + \frac{16\delta\sqrt{\beta}}{3\sqrt{3}\pi} m_{\pi}^{2} \right] \exp\left(\frac{m_{\pi}^{2}}{16\sqrt{\beta}}\right) \frac{G}{\sqrt{2}} \sin\theta_{C} \cos\theta_{C} , \qquad (3.5)$$

$$g_{b} = \left[\frac{e^{i\phi} + e^{-i\omega}}{2} f_{K}(m_{K}^{2} - m_{\pi}^{2}) - \frac{16\sqrt{\beta}M}{3\sqrt{3}\pi} (e^{i\phi} - e^{-i\omega}) + \frac{4\sqrt{\beta}\delta}{3\sqrt{3}\pi} \left(2\sqrt{\beta} - \frac{5m_{K}^{2}}{8} + \frac{m_{\pi}^{2}}{16}\right) e^{i\phi}\right] \exp\left(\frac{m_{\pi}^{2}}{16\sqrt{\beta}}\right) \frac{G}{\sqrt{2}} \sin\theta_{C} \cos\theta_{C}$$

(3.6)

$$g_{c} = \left\{ \frac{16M\sqrt{\beta}}{3\sqrt{3}\pi} \left( e^{i\phi} - e^{-i\omega} \right) + \frac{\delta\sqrt{\beta}}{3\sqrt{3}\pi} \left[ \frac{3}{4} \left( m_{K}^{2} - 4m_{\pi}^{2} \right) + 8\sqrt{\beta} \right] e^{i\phi} \right\} \exp\left( \frac{m_{K}^{2}}{16\sqrt{\beta}} \right) \frac{G}{\sqrt{2}} \sin\theta_{c} \cos\theta_{c} \,. \tag{3.7}$$

The quark model predicts the existence of a  $T = 0 \pi \pi$  resonance ( $\epsilon$  meson), with (BS) amplitude

$$\chi_{0+}(P,r) = \frac{4\pi\sqrt{2}}{3\beta^{3/4}} \left[ \not\!\!\!/ - \frac{\not\!\!\!\!/ (P \cdot r)}{m^2} + \frac{\not\!\!\!\!/ r}{M} - \frac{P \cdot r}{M} + \frac{r^2}{M} - \frac{(P \cdot q)^2}{Mm^2} \right] \exp\left(-\frac{r^2}{2\sqrt{\beta}}\right) |q \,\overline{q} \,\rangle.$$
(3.8)

Experimentally there is in fact indication of the existence of such a (broad) resonance in the 600-700 MeV mass region, but with a huge uncertainty. We need such a resonance to account for the total  $K_s \rightarrow 2\pi$  width and the values which best reproduce this width and the  $K_s - K_L$  mass difference turns out to be  $m_e \sim \Gamma_e = 600$  MeV. The contribution from the diagram shown in Fig. 1(d) is given by

$$g_{d} = \left[\frac{16\beta^{5/4}}{3\sqrt{3}^{2}} \left(1 - \frac{m_{K}^{2}}{4m_{e}^{2}}\right) e^{i\phi} \delta - \frac{4}{M} \left(e^{i\phi} - e^{-i\omega}\right)\right] \frac{g_{e\pi\pi}}{m_{K}^{2} - m_{e}^{2}} \frac{G}{\sqrt{2}} \sin\theta_{C} \cos\theta_{C} , \qquad (3.9)$$

where a real propagator  $1/(m_e^2 - m_K^2)$  has been used to satisfy *CTP* and unitarity.

The physical amplitudes for the decays  $K_s \rightarrow \pi^* \pi^$ and  $K_s \rightarrow \pi^0 \pi^0$  are

$$\begin{split} A_{\star-} = & \left(\frac{2}{3}a_{\star-} - \frac{\sqrt{2}}{3}a_{00}\right)e^{i\delta_0} + \frac{1}{3}(a_{\star-} + \sqrt{2}a_{00})e^{i\delta_2}, \\ & (3.10) \\ A_{00} = & \left(\frac{\sqrt{2}}{3}a_{\star-} + \frac{a_{00}}{3}\right)e^{i\delta_0} + \frac{1}{\sqrt{3}}(a_{\star-} - \sqrt{2}a_{00})e^{i\delta_2}, \end{split}$$

where  $a_{\star}$  and  $a_{00}$  are the "bare" amplitudes determined from Eqs. (3.5), (3.6), (3.7), and (3.9):

$$a_{+-} = (g_a + g_b + g_d),$$
  
$$a_{00} = \frac{1}{\sqrt{2}} (g_b - g_c - g_d).$$
 (3.11)

Their numerical expressions in terms of the phase angles  $\phi$  and  $\omega$  are (in keV)

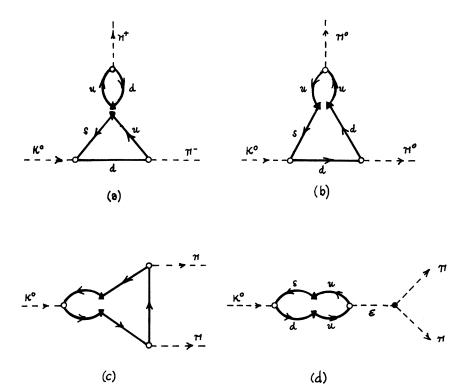


FIG. 1. Diagrams for the decay  $K \rightarrow \pi\pi$ . Solid lines represent the quarks u, d, and s and dashed lines the mesons. Circles stand for BS amplitudes and triangles for the currents.

$$A_{+-} = [(-1.63 - i1.13)e^{i\phi} + (1.35 + i0.77)e^{-i\omega}],$$

$$(3.12)$$

$$A_{--} = [(0.50 - i0.80)e^{i\phi} + (-0.68 + i0.54)e^{-i\omega}].$$

## IV. APPLICATION TO THE PHASE-ANGLES *CP*-VIOLATION MODELS

The amplitudes for the  $K \rightarrow 2\pi$  decays of the preceding section have been cast in the form

$$A_{+-} = A[(m+in)e^{i\phi} - (c+id)e^{-i\omega}], \qquad (4.1a)$$

$$\overline{A}_{\star\star} = A[(m+in)e^{-i\phi} - (c+id)e^{i\omega}], \qquad (4.1b)$$

$$A_{00} = A[(m_0 + in_0)e^{i\phi} - (c_0 + id_0)e^{-i\omega}], \qquad (4.1c)$$

$$\overline{A}_{00} = A[(m_0 + in_0)e^{-i\phi} - (c_0 + id_0)e^{i\omega}], \qquad (4.1d)$$

where, as we said before,  $\phi$  and  $\omega$  are the *CP*-violating phases of the Glashow-type models and  $m, \ldots, d_0$  are given by (3.12).

In order to fix the *CP*-violating theoretical parameters  $\phi$  and  $\omega$  we shall express the amplitudes  $A(K_S \rightarrow \pi^i \pi^j)$  and  $A(K_L \rightarrow \pi^i \pi^j)$  and the related experimental parameters  $\eta_{ij}$  as a handy function of  $\phi$ ,  $\omega$ , and  $\epsilon$ . Then we use the best measured

quantities  $|\eta_{\star-}|$ ,  $\Phi_{\star-}$ , and Re $\epsilon$  to get a system of equations which fixes the phases  $\phi$  and  $\omega$  and Im $\epsilon$ , and with these values the remaining experimental parameter  $\eta_{00}$  is predicted.

The experimental parameter  $\eta_{ij}$  is given, according to (2.10), by

$$\eta_{\bullet\bullet} = \frac{(\epsilon \cos\phi + i \sin\phi) - (\alpha + i\beta)(\epsilon \cos\omega - i \sin\omega)}{(\cos\phi + i\epsilon \sin\phi) - (\alpha + i\beta)(\cos\omega - i\epsilon \sin\omega)},$$
(4.2)

where

$$\alpha + i\beta = \frac{c + id}{m + in} = 0.78 + i0.07.$$
(4.3)

For  $\eta_{00}$  one gets the same expression with  $\alpha_0$  and  $\beta_0$  instead of  $\alpha$  and  $\beta$ , given by

$$\alpha_0 + i\beta_0 = \frac{c_0 + id_0}{m_0 + in_0} = 0.87 + i0.38.$$
 (4.4)

Neglecting second-order terms in the CP-violating phases, we have from (4.2)

$$\epsilon = \eta_{*-} - i \frac{\phi + Z\omega}{1 - Z},\tag{4.5}$$

where  $Z = \alpha + i\beta$ .

The real and imaginary parts of Eq. (4.5) lead to

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(4.6)

$$(\phi + \omega) = \frac{(1 - \alpha)^2 + \beta^2}{\beta} (\operatorname{Re} \epsilon - \operatorname{Re} \eta_{+-})$$

and

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$$\operatorname{Im} \epsilon - \omega = \operatorname{Im} \eta_{+-} - \frac{1-\alpha}{\beta} (\operatorname{Re} \epsilon - \operatorname{Re} \eta_{+-}).$$

Any conclusion based on values of  $\operatorname{Re}\epsilon$  and  $\operatorname{Re}\eta_{+-}$ is of course independent of the phase convention, whereas the value of  $\operatorname{Im}\epsilon$  is gauge-dependent. Since we are considering *CP* violation in the weak Hamiltonian, the most natural convention is not the one of Wu and Yang,<sup>10</sup> where the amplitude  $\langle 2\pi, T=0 | \mathcal{H}_{NL} | K_2^0 \rangle$  is set equal to zero, reducing in fact all *CP* violation to the mass matrix. We want instead in our analysis a nonzero value for the former *CP*-violating amplitude.

One can see that  $\phi + \omega$  is proportional to the difference (Re $\epsilon$  - Re $\eta_{\star}$ ), where the proportionality factor, given by the values of  $\alpha$  and  $\beta$  quoted in (4.3), is 0.77. The experimental value<sup>8</sup> of

$$(\operatorname{Re}\epsilon - \operatorname{Re}\eta_{\star}) = (1.7 \pm 0.8) \times 10^{-4}$$

has a 50% uncertainty, so that we have at least the same indeterminacy for the sum of the phases, which would be

$$\phi + \omega = (-1.3 \pm 0.6) \times 10^{-4}. \tag{4.7}$$

The phases are then nearly opposite, but as long as  $\operatorname{Re} \epsilon \neq \operatorname{Re} \eta_{+-}$ , the exact relation  $\phi = -\omega$  is impossible. In fact, the conclusion that  $\phi + \omega = 0$  is excluded if  $\operatorname{Re} \epsilon \neq \operatorname{Re} \eta_{+-}$  holds for any current-current model. Should experiments confirm that  $\operatorname{Re} \eta_{+-}$ =  $\operatorname{Re} \epsilon$ , then  $\phi + \omega = 0$  necessarily, and then our Eq. (4.5) would become simply

$$\eta_{+-} = \eta_{00} = \epsilon - i\omega. \tag{4.8}$$

In this case one can illustrate again the difference between our phase convention and the Wu-Yang gauge. In our case, using Eq. (2.14) and  $\epsilon_0 \simeq \frac{1}{3}(2\eta_{\star-} + \eta_{00})$ , one gets

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$$\eta_{+-} = \eta_{00} = \epsilon_0 - i(\gamma + \omega) = \epsilon_0, \qquad (4.9)$$

i.e.,

$$\gamma = -\omega, \qquad (4.10)$$

whereas in the Wu-Yang case

$$\eta_{+-} = \eta_{00} = \epsilon_0 = \epsilon', \qquad (4.11)$$

where  $\epsilon_0$  and  $\gamma$  were defined in Eqs. (2.15) and (2.12), respectively, and where  $\epsilon'$  is the *CP* impurity parameter in the Wu-Yang gauge. So the real parts of  $\eta_{+-}$  are given by the same expression in both gauges,

$$\operatorname{Re} \eta_{\star-} = \operatorname{Re} \epsilon = \operatorname{Re} \epsilon_0 \quad (\text{in our gauge}),$$
$$\operatorname{Re} \eta_{\star-} = \operatorname{Re} \epsilon' = \operatorname{Re} \epsilon_0 \quad (\text{in the Wu-Yang gauge})$$

whereas the imaginary part of  $\eta_{+-}$  is expressed by different expression, but which give of course the same value

$$\operatorname{Im}\eta_{+-} = \operatorname{Im}\epsilon - \omega = \operatorname{Im}\epsilon_0$$
 (in our gauge),

 $\operatorname{Im} \eta_{+-} = \operatorname{Im} \epsilon' = \operatorname{Im} \epsilon_0$  (in the Wu-Yang gauge).

On the other hand, the situations  $\phi = \omega$ ,  $\phi = 0$ and  $\omega \neq 0$ , and  $\omega = 0$  and  $\phi \neq 0$  are allowed with  $\phi = \omega = -6.6 \times 10^{-5}$ ,  $\omega = -1.3 \times 10^{-4}$ , and  $\phi = -1.3 \times 10^{-4}$ , respectively. In these cases the equations can be solved exactly, and the results of the model are, for all three cases,

Im 
$$\epsilon = 2.23 \times 10^{-3}$$
,  
Re  $\eta_{00} = 2.04 \times 10^{-3}$ , (4.12)

 $\phi_{00} = 46.4^{\circ},$ 

yielding for  $|\eta_{00}/\eta_{\star-}|$  the value 1.28, which does not rule out the model in view of the large errors involved, as shown in (2.11).

Finally, we remark that a recent theoretical analysis<sup>14</sup> of the  $K^{\circ}$  meson self-energy matrix gives a value for  $\phi + \omega$  one order of magnitude smaller than in Eq. (4.7).

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