New relation between mesons and baryons*

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A new SU_{12} classification scheme for mesons and baryons is used in conjunction with a simple quark model and results in a striking correlation between the masses of the diquark subsystems in baryons and the masses of several known mesons. The new classification provides a simple scheme which involves a basic 36 multiplet for the pseudoscalar and vector mesons and a 112 multiplet for the baryons and antibaryons. The 36 multiplet of SU_{12} does not treat the η' as a singlet as in the conventional SU_6 scheme. The new scheme also has the advantages of providing a simple algebraic form of Zweig's rule for the decay of strange-quark-antiquark pairs. Absolute values of the magnetic moments of baryons are accurately reproduced and are found to be consistent with a model in which the effective quark masses are 338 MeV (nonstrange) and 465 MeV (strange).

I. INTRODUCTION

The possibility of calculating the spectroscopy of mesons and baryons from the same fundamental model has been examined by many workers,[1] but in recent years very little progress has been achieved.[2] Much of the difficulty stems from the assumptions usually made about the masses of quarks—either they are considered to be very heavy, with masses $m_q \gtrsim 5$ GeV, or very light, with $m_q \to 0$. In the first instance, the model runs into difficulties—particularly the prediction that quarks need very large anomalous g factors in order to explain the absolute values of magnetic moments for baryons. The very-light-quark models run into the technical mire of ultrarelativistic motions and an almost infinite number of possible dynamical equations for such hypothetical quarks. The purpose of the present work is to show that a medium-mass-quark model can be constructed with $m_q \sim 400$ MeV which avoids the problems of the heavy-quark models and at the same time provides a new and striking relationship between the masses (real part) of mesons ($q\bar{q}$) and the corresponding diquark (q^2) substructures of the baryons.

The new feature introduced here is the idea that particles and antiparticles can be regarded as spinors in a new space which we term K space. This idea is an extension of the usual isospin concept which allows particles of different charge but similar mass to be treated as "identical." Since the antiparticles have the same mass but opposite charges, baryon numbers, etc., then the introduction of a new spin $k=\frac{1}{2}$ with projections $k_z = \pm\frac{1}{2}$ allows us to treat particles and antiparticles on the same footing. It is worth noting that the continuous group of transformations in K-spin space are even more peculiar than those for isospin where superselection rules[3] are imposed to prevent the occurrence of states of mixed charge. In K space we require a superselection rule to forbid states of mixed baryon number and we also impose a boundary condition which forbids states with nonzero triality to occur as free particles. Such a representation can always be adopted; its merit, however, depends upon the usefulness of the quantum numbers associated with the representation.

In Sec. II the K-spin scheme for mesons is considered and leads to an SU_{12} classification of the 36 low-lying mesons with spin and parity 0^- and 1^-. This scheme differs from the conventional[4] "particle-hole" symmetry scheme of SU_6 obtained from

$$6 \times \bar{6} = 35 + 1$$

in that the η and η' mesons are differently assigned In the SU_{12} scheme the η' and η are assigned in an analogous manner to the ϕ and ω mesons and consequently the SU_{12} scheme with superselection rules provides a simpler explanation of the properties of the η' meson (e.g., mass and width) than the conventional SU_6 classification. Another feature of the SU_{12} scheme is that Zweig's rule[5] takes on a simple algebraic form and corresponds to the simultaneous requirements of conservation of G parity and K parity $[=(-1)^K]$.

The application of K spin to baryons as three quarks in Sec. III leads to the same results as the conventional SU_6 scheme. However, by using a "pair-interaction" quark model for baryons, the baryon masses are expressed (Sec. IV) entirely in terms of five parameters which involve the differences between diquark (q^2) masses and single-quark masses.

By associating the single-quark masses with the quark magnetic moments (Sec. V) and assuming a Dirac nature for their g factor, the absolute values of the baryon magnetic moments can be reproduced accurately. This allows the masses of the five diquark substructures contained in the 56 baryon multiplet to be determined in Sec. VI and compared with the known masses of appropriate mesons, i.e., mesons with the same SU_6 subgroup symmetry. The important relation [Eq. (6.3)] which is the central result of this paper is thereby shown to be remarkably well satisfied. Some possible interpretations of Eq. (6.3) (which states that q^2 and $q\bar{q}$ "masses" can be equated if the symmetry groups are identical) are given in Sec. VII. Possibilities for heavier particles than those discussed here are also suggested.

II. K SPIN AND MESON CLASSIFICATION

The concept of K spin is most easily introduced for the case of nonstrange quarks. The basic idea is to regard particles and antiparticles as spinor doublets in the new space. Table I shows the assignments for the $\mathfrak{N}$- and $\mathcal{P}$-type quarks. The four combinations of i_Z, k_{NZ} suggest a description of systems involving $\mathfrak{N}$- and $\mathcal{P}$-type quarks and $\mathfrak{N}$- and $\mathcal{P}$-type antiquarks in terms of SU_4. More generally, if we include Pauli spin, then the appropriate group is $SU_4 \otimes SU_2 \to SU_8$.

In the following we denote a member (particle or antiparticle) of the group by the letter χ. Mesons will be described by χ^2 only if the total K projection is zero, thus ensuring that χ^2 is a quark-antiquark ($q\bar{q}$) configuration. Requiring over-all antisymmetry for the χ^2 system means we are concerned with the antisymmetric Young diagram denoted here by $SU_8(1^2)$. For a Wigner supermultiplet we have the decomposition $SU_2 \otimes SU_2 \otimes SU_2$ corresponding to the three spaces with Casimir operators $\vec{S}, \vec{K}, \vec{I}$. Of course $\vec{S} = \vec{s}_1 + \vec{s}_2$, $\vec{K} = \vec{k}_1 + \vec{k}_2$, and $\vec{I} = \vec{i}_1 + \vec{i}_2$ for two-body systems. The resulting supermultiplet can be decomposed into vector and pseudoscalar multiplets, i.e., $S = 1$ yields $SU_4(1^2)$ and $S = 0$ gives $SU_4(2)$. These two SU_4 multiplets in K, I space contain the following elements:

$$SU_4(1^2) \to SU_2(1^2) \otimes SU_2(2) \to I = 0,\ K = 1 \to \omega,$$
$$\to SU_2(2) \otimes SU_2(1^2) \to I = 1,\ K = 0 \to \rho,$$
$$SU_4(2) \to SU_2(1^2) \otimes SU_2(1^2) \to I = 0,\ K = 0 \to \eta,$$
$$SU_2(2) \otimes SU_2(2) \to I = 1,\ K = 1 \to \pi.$$

The quantum numbers I, S determine the assignments of the low-lying mesons uniquely if they are assumed to only contain nonstrange quarks. The above classification introduces a new quantum number K which for χ^2 systems is either 0 or 1. For mesons we require $K_Z = 0$ to yield zero baryon number. States with $K_Z = \pm 1$ are presumed to be

TABLE I. Quantum numbers for nonstrange quarks and antiquarks.

Quark	i	i_Z	k	k_{NZ}	$Q = i_Z + \frac{1}{3}k_{NZ}$	$b = \frac{2}{3}k_{NZ}$
$\mathcal{P}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
$\mathfrak{N}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
$\bar{\mathcal{P}}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{3}$
$\bar{\mathfrak{N}}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$

physically forbidden as free particles because the baryon number would be nonintegral. This in turn means that while SU_8 (or SU_{12} below) provides a method of classifying states, the full angular momentum algebra associated with K spin will not always be appropriate. The subgroup of SU_{12} formed by the 20 nonstrange mesons (see below) involves only physically realizable free particles with definite K values and we expect the quantum number of K parity to be conserved within this subgroup. The situation here is analogous to the case of C parity for uncharged particle subgroups. The quantum number $(-)^K$ for nonstrange quark mesons clearly is conserved in the same manner as G parity. For example, the ρ meson has $K = 0$ and can only decay into an even number of π mesons because of the occurrence of Clebsch-Gordan coefficients in K space of the parity-conserving type, i.e. $C(k_1 k_2 K, 000)$, which vanishes unless $k_1 + k_2 + K =$ even. We see that G parity for the nonstrange mesons $(\pi, \eta, \rho, \omega)$ is simply given by

$$G = (-)^K = (-)^{I+S},$$

where the second equality is a direct consequence of the over-all antisymmetry imposed between quark and antiquark in the I, S, K spaces.

In order to fully classify the mesons using the K-spin approach, we need to assign the λ quark and $\bar{\lambda}$ antiquark the quantum numbers $k_\lambda = \frac{1}{2}$, $k_{\lambda Z} = \pm\frac{1}{2}$. However, owing to strangeness, the charge is given by $Q = i_Z - 2k_{\lambda Z}/3$ provided the strangeness is given by $\mathcal{S} = -2k_{\lambda Z}$. In this way the Gell-Mann–Nishijima relation

$$Q = I_Z + \tfrac{1}{2}(b + \mathcal{S})$$
$$= I_Z + \tfrac{1}{2}Y$$

is realized and using the above assignments we see that the hypercharges are given by

$$Y_\lambda = -4k_{\lambda Z}/3 = -Y_{\bar{\lambda}},$$
$$Y_{\mathfrak{N}} = Y_{\mathcal{P}} = 2k_{NZ}/3 = -Y_{\bar{\mathfrak{N}}} = -Y_{\bar{\mathcal{P}}}$$

as in the usual quark model.

Including the Pauli spin and zero isospin of the λ quark yields an over-all SU_4 description for the $\lambda\bar{\lambda}$ system. The complete supermultiplet must then involve twelve members (quarks + antiquarks) and the appropriate group[6] appears to be $SU_{12}(1^2)$. Decomposing such a group is not unique but a reasonable classification appears to be the decomposition into $SU_8 \otimes SU_4$, wherein as usual the SU_8 subgroup contains only $\mathfrak{N}$-$\mathcal{P}$ quarks and the SU_4 contains only strange quarks. We then have

$$SU_{12}(1^2) \to SU_8(1^2)$$
$$\to SU_8(1) \cdot SU_4(1)$$
$$\to SU_4(1^2).$$

The first subset contains the π, η, ρ, ω as described above. The group $SU_4(1^2)$ is easily decomposed and yields nonstrange mesons of the $\lambda\bar{\lambda}$ type, i.e., $SU_2(1^2) \otimes SU_2(2)$, corresponding to $K_\lambda = 1$, $S=0$, $I=0$, and $K_\lambda = 0$, $S=1$, $I=0$. These are to be identified with the η' and ϕ mesons (Table III). The decays of the η' and ϕ mesons into π, η, ω, ρ states are then forbidden if we require conservation of K parity $= (-1)^K$. The point is that G parity for the ϕ, η' is given by

$$G = (-)^{K_\lambda + 1}$$

so that simultaneous conservation of G parity and K parity is impossible for $\phi, \eta' \to \{\pi, \eta, \rho, \omega\}$. Zweig's rule for $\lambda\bar{\lambda}$ mesons is then rigorously satisfied as a parity selection rule in K space unless there is mixing between the $SU_8(1^2)$ and $SU_4(1^2)$ subgroups. Weak mixing would explain therefore the narrow widths of the ϕ and η' mesons.

The subgroup $SU_8(1) \cdot SU_4(1)$ contains all the strange mesons usually incorporated into the $SU_6(2, 1^4)$ description. In order to have strangeness and charge as good quantum numbers, it is necessary to use the *uncoupled* K-spin representation which yields the classification of Table II wherein $Y = (2k_{NZ} - 4k_{\lambda Z})/3$ as suggested above. If one attempts to couple the $k_N = \frac{1}{2}$ and $k_\lambda = \frac{1}{2}$ states to make a total K quantum number, it is not allowed in general because of superselection rules[3] concerning states of mixed charge and/or hypercharge. However, the selection rule for strangeness is not valid for weak interactions and it is possible to construct states of completely mixed strangeness provided no charge states are mixed. The K_L^0, K_S^0 mesons still have mixed K parity and consequently the decays of the ϕ meson into $K_L^0 K_S^0$ or K^+K^- are both allowed. The results for all $0^-, 1^-$ mesons are summarized in Table III and correspond to all the known (36) mesons of negative parity with masses below 1.2 GeV.

It can, of course, be asked what advantages the scheme of $SU_{12}(1^2)$ with the projection $K_Z = 0$ has over the conventional $SU_6(2, 1^4) + SU_6(1^6)$ classification. The major advantage appears to be the existence of a new quantum number K which yields a simple algebraic description of Zweig's rule. Another advantage is the occurrence of the η' in the same subgroup as the ϕ, whereas normally the 35 + 1 multiplets cause the η' (or η) to be treated as a special singlet. Finally, there is the advantage that a relationship between q^2 and $q\bar{q}$ states in the same SU_6 subgroup (isospin ⊗ spin) is possible as realized in Sec. VI.

TABLE II. Quantum-number assignments for strange mesons.

	i_Z	k_{NZ}	$k_{\lambda Z}$	Q	Y
K^+, K^{+*}	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	1
K^0, K^{0*}	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1
$\bar{K}^0, \bar{K}^{0*}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	−1
K^-, K^{-*}	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	−1	−1

III. K SPIN FOR BARYONS

If we require integral baryon numbers for physical systems, then for configurations of the type χ^3 we require that only q^3 or $\bar{q}^3$ be allowed. From a K-spin classification point of view, this means $K_Z = \pm\frac{3}{2}$ and $K = \frac{3}{2}$ are allowed but $K_Z = \pm\frac{1}{2}$, $K = \frac{1}{2}, \frac{3}{2}$

TABLE III. $SU_{12}(1^2)$ classification of mesons.

Meson	Subgroup	I^G	K	$Y = \mathcal{S}$
π	$SU_8(1^2)$	1^-	1	0
η	$SU_8(1^2)$	0^+	0	0
ρ	$SU_8(1^2)$	1^+	0	0
ω	$SU_8(1^2)$	0^-	1	0
K^+, K^{+*}	$SU_8(1) \cdot SU_4(1)$ [a]	$\frac{1}{2}$	0, 1	1
K^0, K^{0*}	$SU_8(1) \cdot SU_4(1)$ [a]	$\frac{1}{2}$	0, 1	1
$\bar{K}^0, \bar{K}^{0*}$	$SU_8(1) \cdot SU_4(1)$ [a]	$\frac{1}{2}$	0, 1	−1
K^-, K^{-*}	$SU_8(1) \cdot SU_4(1)$ [a]	$\frac{1}{2}$	0, 1	−1
K_S^0	$SU_8(1) \otimes SU_4(1)$ [b]	$\frac{1}{2}$	0, 1	1, −1
K_L^0	$SU_8(1) \otimes SU_4(1)$ [b]	$\frac{1}{2}$	0, 1	1, −1
η'	$SU_4(1^2)$	0^+	1	0
ϕ	$SU_4(1^2)$	0^-	0	0

[a] The notation $SU_8(1) \cdot SU_4(1)$ indicates schematically the usage of an uncoupled representation in K-spin space for the strange mesons.

[b] The coupling sign here simply denotes the usual linear combinations of K^0 and $\bar{K}^0$. Note K is not a good quantum number because K^0 and $\bar{K}^0$ have different I_Z values.

are not allowed as free particles. The baryons with configuration q^3 are therefore assumed to be described by *symmetric* $K=\frac{3}{2}$ states. Allowing for parastatistics, color, or internal orbital motions, then we assume the appropriate SU_{12} states for baryons are the completely symmetric states described by the Young diagrams $SU_{12}(3)$ for three "identical" particles. The $K=\frac{3}{2}$, $K_Z=\pm\frac{3}{2}$ projections of such an SU_{12} scheme mean that we can decompose the group uniquely as $SU_6\otimes SU_2$, where the SU_2 subgroup describes K space with a Young diagram (3) and the SU_6 group contains spin-isospin-hypercharge quantum numbers. Consequently baryons are describable by the conventional $q^3,\bar{q}^3$ configurations with the SU_6 group being decomposed either as $SU_4\otimes SU_2$ or $SU_3\otimes SU_2$. In the latter case we have[7]

$$SU_6(3)\rightarrow SU_3(3)\otimes SU_2(3)\rightarrow \text{decuplet}$$
$$\rightarrow SU_3(2,1)\otimes SU_2(2,1)\rightarrow \text{octet},$$

as is well known. The introduction of K spin for baryons appears to have no new results simply because the K symmetry is unique and symmetric. However, it leads to the possibility that the baryon masses can be related to the meson masses if the pair-interaction quark model is invoked. The point is that while we have argued that $K=1$, $K_Z=\pm 1$ states are not allowed as free particles because of nonintegral baryon number; nevertheless, these states can exist as subsystems in a baryon or antibaryon. Because the $K=1$, $K_Z=\pm 1$ configurations are part of the SU_{12} scheme which contains mesons, one might hope there is some relation between the meson states and the $K=1$ substates of the baryon systems. We now show that such a relation actually does exist with remarkable accuracy.

IV. QUARK MODEL, K SPIN, AND THE MESON-BARYON MASS RELATION

In the simplest model of the baryon, we assume a pair-interaction quark model described by a mass operator

$$H=H_1+H_2+H_3+V(12)+V(13)+V(23), \qquad (4.1)$$

in which H_i will yield the effective mass of a single quark. The model here assumes that there are no forces of the three-body type and the model corresponds closely with that of the nuclear shell model. Defining pair masses by $H_{ij}=H_i+H_j+V(ij)$ leads to a more useful (and possibly more general) form of H as

$$H=H_{12}+H_{13}+H_{23}-H_1-H_2-H_3. \qquad (4.2)$$

The masses of the baryons can therefore be described in terms of the differences between pair masses and single masses. Each state of the baryon multiplet $SU_6(3)$ has a unique set of quantum numbers so that, ignoring weak and electromagnetic effects, there is no mixing within the 56 multiplet. The masses of the baryon multiplet are therefore given by the expectation values of H, provided the model is assumed to be correct. Since the $\mathfrak{N}$, $\mathcal{P}$, and λ quarks need not have identical masses or interactions, there will be different results for the different pair and bare operators depending on the structure of the q^3 (or $\bar{q}^3$) configurations. In general the $|q^3,\alpha\rangle$ states ($\alpha=1,\ldots,56$) can be expanded using "fractional parentage" coefficients,

$$|q^3,\alpha\rangle=\sum_\beta \theta_\beta^\alpha\,|[q^2,\beta]\otimes[q_\beta]\alpha\rangle, \qquad (4.3)$$

as a series of states in which a definite labeled quark is extracted, leaving behind a correctly symmetrized pair of quarks in a definite quantum state β. The mass formula then becomes

$$M(q^3,\alpha)=M(\bar{q}^3,\alpha)$$
$$=\sum_\beta(\theta_\beta^\alpha)^2[\langle q^2,\beta|\textstyle\sum H_{ij}|q^2,\beta\rangle$$
$$-\langle q_\beta|\textstyle\sum H_i|q_\beta\rangle]. \qquad (4.4)$$

The θ_β^α coefficients can be determined algebraically for the 56 multiplet of SU_6 and lead to the results

A decuplet	B octet
$M(\Omega^-)=3m_1$	$M(N)=\frac{3}{2}(m_3+m_4)$
$M(\Xi(1530))=m_1+2m_2$	$M(\Lambda)=m_4+\frac{3}{2}m_2+\frac{1}{2}m_5$
$M(\Sigma(1385))=2m_2+m_3$	$M(\Sigma)=m_3+\frac{1}{2}m_2+\frac{3}{2}m_5$
$M(\Delta)=3m_3$	$M(\Xi)=m_1+\frac{1}{2}m_2+\frac{3}{2}m_5$

(4.5)

i.e., the eight baryon masses are determined in terms of five mass parameters m_i, $i=1,\ldots,5$. These five masses are given by

$$m_1=\langle\lambda^2,I=0,\ S=1|H_{12}|\lambda^2,I=0,\ S=1\rangle-\langle\lambda|H_1|\lambda\rangle,$$
$$m_2=\langle(\mathfrak{N}\lambda),I=\tfrac{1}{2},\ S=1|H_{12}|(\mathfrak{N}\lambda),I=\tfrac{1}{2},\ S=1\rangle$$
$$-\tfrac{1}{2}[\langle\mathfrak{N}|H_1|\mathfrak{N}\rangle+\langle\lambda|H_1|\lambda\rangle],$$
$$m_3=\langle\mathfrak{N}^2,I=1,\ S=1|H_{12}|\mathfrak{N}^2,I=1,\ S=1\rangle$$
$$-\langle\mathfrak{N}|H_1|\mathfrak{N}\rangle, \qquad (4.6)$$
$$m_4=\langle\mathfrak{N}^2,I=0,\ S=0|H_{12}|\mathfrak{N}^2,I=0,\ S=0\rangle$$
$$-\langle\mathfrak{N}|H_1|\mathfrak{N}\rangle,$$
$$m_5=\langle\mathfrak{N}\lambda,I=\tfrac{1}{2},\ S=0|H_{12}|\mathfrak{N}\lambda,I=\tfrac{1}{2},\ S=0\rangle$$
$$-\tfrac{1}{2}[\langle\mathfrak{N}|H_1|\mathfrak{N}\rangle+\langle\lambda|H_1|\lambda\rangle],$$

wherein we have used $\mathfrak{N}$ to denote either an $\mathfrak{N}$-type or $\mathcal{P}$-type quark since we have assumed the $\mathfrak{N},\mathcal{P}$ to have the same mass, whereas the strange quark is assumed to have a different mass.

It is interesting to note that five of the meson masses are given by similar matrix elements of a two-body operator, e.g., according to our classification and assuming annihilation channels mainly produce imaginary mass, then the real masses are

$$\begin{aligned} M(\phi)&=\langle\lambda\bar{\lambda},I=0,\ S=1|H_{12}|\lambda\bar{\lambda},I=0,S=1\rangle,\\ M(K^*)&=\langle\mathfrak{N}\bar{\lambda},I=\tfrac{1}{2},\ S=1|H_{12}|\mathfrak{N}\bar{\lambda},I=\tfrac{1}{2},\ S=1\rangle,\\ M(\rho)&=\langle\mathfrak{N}\bar{\mathfrak{N}},I=1,\ S=1|H_{12}|\mathfrak{N}\bar{\mathfrak{N}},I=1,\ S=1\rangle,\\ M(\eta)&=\langle\mathfrak{N}\bar{\mathfrak{N}},I=0,\ S=0|H_{12}|\mathfrak{N}\bar{\mathfrak{N}},I=0,\ S=0\rangle,\\ M(K)&=\langle\mathfrak{N}\bar{\lambda},I=\tfrac{1}{2},\ S=0|H_{12}|\mathfrak{N}\bar{\lambda},I=\tfrac{1}{2},\ S=0\rangle. \end{aligned} \quad (4.7)$$

Of course there is a possibility of mixing between the ϕ and ω or between the η and η'. The ϕ,ω states appear to be almost pure[7] (in accord with the classification above) but the η,η' mixing remains less certain. In determining the masses of the octet and decuplet from the above linear relationship a simple least-squares fitting was performed and yields the results $m_1=556.65$, $m_2=486.73$, $m_3=412.14$, $m_4=212.53$, and $m_5=351.13$ (all in MeV). The comparison of this five-parameter fit with the experimental values is shown in Table IV.

In this fitting the experimental masses are taken as average values for the isobaric multiplets. The differences between theory and experiment have an average error of $|\epsilon|=4$ MeV, which is of the order of electromagnetic shifts.

Three mass formulas[8] can be derived from the relation above, i.e.,

$$\begin{aligned} 3\Lambda+\Sigma-2(N+\Xi)&=2\Sigma^*-\Delta-\Xi^*,\\ \Xi^*-\Sigma^*&=\Xi-\Sigma\\ &=\tfrac{1}{3}(\Omega-\Delta). \end{aligned} \quad (4.8)$$

The first is the usual Gell-Mann–Okubo[9] mass formula for the octet if the right-hand side is zero, which, from the table above, is actually

TABLE IV. Baryon masses (MeV).

	Experiment	Theory
Ω^-	1672	1670
Ξ^*	1533	1530
Σ^*	1385	1386
Δ	1232	1236
Ξ	1318	1327
Σ	1192	1182
Λ	1115	1118
N	939	937

5 MeV. The left-hand side is equal to 23 MeV so that the theory here is consistent with the Gell-Mann–Okubo formula within the errors of electromagnetic shifts. The remaining relations are in agreement with previous mass relations[8] based on SU_6 and include the usual equal-spacing rules for the decuplet provided $2\Sigma^*=\Delta+\Xi^*$, which is the case experimentally. Theoretically we see that the relation $2\Sigma^*=\Delta+\Xi^*$ requires

$$m_1+m_3=2m_2,$$

or

$$\begin{aligned} &\langle\lambda^2,I=0,\ S=1|H_{12}|\lambda^2,\ I=0,\ S=1\rangle\\ &\quad+\langle\mathfrak{N}^2,I=1,\ S=1|H_{12}|\mathfrak{N}^2,I=1,\ S=1\rangle\\ &\qquad=2\langle(\mathfrak{N}\lambda),I=\tfrac{1}{2},\ S=1|H_{12}|(\mathfrak{N}\lambda),I=\tfrac{1}{2},S=1\rangle. \end{aligned} \quad (4.9)$$

This is to be contrasted with the well-known vector-meson mass relation

$$M(\phi)+M(\rho)=2M(K^*) \quad (4.10)$$

which holds to within 5 MeV. Consequently, we already might guess that meson masses and baryon subsystem matrix elements should be related. The difficulty with relating the five mass parameters $m_1,m_2,\ldots,m_5$ with the appropriate meson masses appears to be the lack of knowledge of the masses $\langle\lambda|H_1|\lambda\rangle$ and $\langle\mathfrak{N}|H_1|\mathfrak{N}\rangle$. To determine these we appeal to the data on magnetic moments for the baryons.

V. SINGLE-QUARK MASSES AND MAGNETIC MOMENTS OF BARYONS

If the quark model can be used to yield a mass formula, then it can be used to predict magnetic moments. Assuming no orbital contribution and the usual one-body "additive" form for baryons, the magnetic moment[10] is given by

$$\vec{\mu}=\frac{e\hbar}{2Mc}\left(\frac{2}{3}\frac{M}{m_{\mathcal{P}}}\vec{\sigma}_{\mathcal{P}}-\frac{1}{3}\frac{M}{m_{\mathfrak{N}}}\vec{\sigma}_{\mathfrak{N}}-\frac{1}{3}\frac{M}{m_\lambda}\vec{\sigma}_\lambda\right), \quad (5.1)$$

in which M is the mass of a proton, $m_{\mathfrak{N}}=m_{\mathcal{P}}$ is the effective mass of the nonstrange quark and $m_\lambda\neq m_{\mathfrak{N}}$ is the effective mass of the strange quark. Using the experimental values ($\mu_p=2.793$, $\mu_n=-1.913$, $\mu_\Lambda=-0.67$) we obtain $m_{\mathfrak{N}}=331\pm5$ MeV and $m_\lambda=467\pm46$ MeV.

In more conventional nonrelativistic quark models the quark masses are assumed to be $\gtrsim5$ GeV—mainly to justify the use of nonrelativistic quark models and the lack of observation of free quarks. Such heavy-quark models yield two incorrect results: (i) The absolute values of magnetic moments are predicted to be too small by a factor of about twenty; (ii) the magnetic moment of the Λ particle is predicted to be $-\frac{1}{3}\mu_p$, i.e., -0.93. The first difficulty is solved if one uses an "anomalous" g factor of $g \sim 20$ and the second if one uses $g_\lambda < g_{\mathfrak{N}}$ by an appropriate amount. However, there seems to be no justification for such an *ad hoc* approach. The light-quark model proposed here has the advantage of allowing the quarks to have normal g factors and to behave like simple Dirac particles.

Coupling the results from the magnetic moments and the five mass parameters together allows us to uniquely determine the masses of the q^2 subsystems in the 56 multiplet of baryons as shown in the next section.

VI. COMPARISON OF q^2 MASSES AND MESON MASSES

Using the five masses $m_1, \ldots, m_5$ determined from the least-squares fit to the baryon-mass spectrum and the two masses

$$m_n \equiv \langle \mathfrak{N} | H_1 | \mathfrak{N} \rangle = 331 \text{ MeV},$$

$$m_\lambda \equiv \langle \lambda | H_1 | \lambda \rangle = 467 \text{ MeV},$$

the resulting pair masses are found to be (in MeV)

$$M(\lambda^2, I=0,\ S=1) = 1024,$$

$$M(\mathfrak{N}\lambda, I=\tfrac{1}{2},\ S=1) = 886,$$

$$M(\mathfrak{N}^2, I=1,\ S=1) = 743,$$

$$M(\mathfrak{N}^2, I=0,\ S=0) = 543,$$

$$M(\mathfrak{N}\lambda, I=\tfrac{1}{2},\ S=0) = 749.$$

The first four of these results appear to agree very nicely with the real masses of the corresponding mesons. $\phi(1020)$, $K^*(892)$, $\rho(770)$, and $\eta(549)$, but the fifth does not agree with the $K(494)$. Actually the fifth result above should *not* be compared with the pseudoscalar meson. The important fallacy in such a comparison is that all the q^2 configurations belong to the group $SU_6(2)$ whereas only the ϕ, ρ, and η mesons belong to this same symmetry. As pointed out earlier in Sec. II, the K mesons in isospin space belong to an *uncoupled* K-spin scheme and this in turn means that the pseudoscalar K mesons belong to a *mixed* SU_6 symmetry (i.e., $SU_6(1^2)$ and $SU_6(2)$ in equal amounts).

In finding a solution to the fact that there appears to be no physical strange pseudoscalar mesons with definite K spin which are analogous to the q^2 configuration, it is important to note that isospin is not the appropriate subgroup to be considering. In considering symmetries for the $\mathfrak{N}\lambda$ pair of quarks we need to consider the subgroup for such pairs, i.e., U spin if "$\mathfrak{N}$" is the neutron quark and V spin if "$\mathfrak{N}$" = $\mathcal{P}$ is the proton quark.[11] For convenience we consider the U-spin situation wherein the $\mathfrak{N}\lambda$ pair has $U_Z = 0$. In order to have an overall $SU_6(2)$ symmetry, the $\mathfrak{N}\lambda$ pair belongs to the subgroup $SU_4(2)$ for U spin and Pauli spin. Consequently the appropriate q^2 quantum numbers for the $\mathfrak{N}\lambda$ pairs are $U=1$, $S=1$, $K=1$ and $U=0$, $S=0$, $K=1$, with mass values of 886 and 749 MeV, respectively. If we construct $U=1$, $S=1$ and $U=0$, $S=0$ mesons out of the analogous quark-antiquark pairs (i.e., $\mathfrak{N}\bar{\mathfrak{N}}$ or $\lambda\bar{\lambda}$ since $\mathfrak{N} \to \bar{\lambda}$ and $\lambda \to \bar{\mathfrak{N}}$ under k-lowering operations), then we find the $U=1$, $S=1$ state involves either the linear mass relation

$$M(\mathfrak{N}\bar{\mathfrak{N}}, \lambda\bar{\lambda}, U=1,\ S=1) = \tfrac{1}{4}M(\rho) + \tfrac{1}{4}M(\omega) + \tfrac{1}{2}M(\phi) \tag{6.1}$$

or the equivalent quadratic form. The result is

$$M(\mathfrak{N}\bar{\mathfrak{N}}, \lambda\bar{\lambda}, U=1,\ S=1) \approx 900 \pm 10 \text{ MeV}$$

for linear or quadratic assumptions. This nicely agrees with the K^* mass of 892 MeV which is the $U=1$, $S=1$, $U_Z=1$ member of the same SU_4 U-spin meson multiplet.

The $U=0$, $S=0$, $K=0$ "meson" can also be constructed and we find similarly

$$M(\mathfrak{N}\bar{\mathfrak{N}}, \lambda\bar{\lambda}, U=0,\ S=0) = \tfrac{1}{4}M(\pi) + \tfrac{1}{4}M(\eta) + \tfrac{1}{2}M(\eta'). \tag{6.2}$$

In this case the mass is sensitive to the assumption about a linear versus a quadratic form. For linear we find a result of 650 MeV and for the quadratic situation 734 MeV. As is expected for pseudoscalar mesons the quadratic form[1] appears to agree very closely with its q^2 "analog" $M(\mathfrak{N}\lambda, U=0,\ S=0)$ with a derived mass of 749 MeV.

Although we have obtained a remarkable correlation between the q^2 masses and their $q\bar{q}$ counterparts we have had to rely on a somewhat inaccurate value for the Λ magnetic moment so that the λ-quark mass is not well determined. If we invert the philosophy and *assume* the relation

$$M(q^2, SU_6(2), \beta) = M(q\bar{q}, SU_6(2), \beta) \tag{6.3}$$

(i.e., that q^2 masses and the real part of $q\bar{q}$ masses should be equal provided they have the *same* SU_6 symmetry quantum numbers β appropriate to the given configurations), then we can perform a least-squares fit on the four known meson masses using the two parameters $m_{\mathfrak{N}} \equiv \langle \mathfrak{N} | H_1 | \mathfrak{N} \rangle$ and $m_\lambda \equiv \langle \lambda | H_1 | \lambda \rangle$ to yield the four mass values $m_1, \ldots, m_4$. The results are shown in Table V corresponding to a least-squares fit to the known ϕ, K^*, ρ, and η masses (all in MeV).

The results not in parentheses require $m_{\mathfrak{N}} = 347$ MeV and $m_\lambda = 463$ MeV, which agree closely with the earlier estimates of 331 MeV and 467 MeV from the magnetic moments. Changing the ρ mass to 750 MeV provides a slightly better

TABLE V. Least-squares fits to the four known meson masses. Numbers in parentheses correspond to the results obtained using a mass of 750 MeV for the ρ meson.

Meson mass (expt)	Predicted mass
1020	1020 (1022)
892	892 (888)
770 (750)	759 (750)
549	560 (551)

result, as shown in parentheses, and yields $m_{\mathfrak{N}} = 338$ MeV and $m_\lambda = 465$ MeV. Clearly the results are stable to within a few MeV and the meson masses are now reproduced to within the errors expected from electromagnetic or annihilation shifts. With the above scheme of fitting, the fifth meson is predicted to have a mass of 756 and 752 MeV corresponding to choosing the ρ meson mass as 770 or 750, respectively. The agreement with the estimate of 734 MeV (or 650 MeV if linear mass relations are used) is still reasonably good. Using the above results [when $\rho(750)$ is assumed] for the single-quark masses allows us to predict the magnetic moments of the baryons as shown in Table VI.

Also shown in Table VI are the predicted magnetic moments corresponding to the massive-quark model[12] where $m_q \geq 5$ GeV means $m_\Lambda = m_{\mathfrak{N}}$ (to within 2%) and anomalous magnetic g factors, $g_q = 2.79 m_q/M \geq 15$ are used. The only accurate measurements are the first three entries and although the present model is closer to the observed value for the Σ^+ it is apparently less accurate for the Ξ^- value. Unfortunately, in both cases the errors are too large to make any conclusions. It is interesting to note that the predicted magnetic moments for $\Sigma^0(1385)$ and $\Xi^0(1530)$ are not zero in the present model, but a measurement of such values appears to be unfeasible with current technology. One hopes that the values for the Σ^+ and Ξ^- will be measured more accurately in the not-too-distant future.

The central result of the paper is contained in Eq. (6.3) which appears to be remarkably consistent with all the data. The effective masses of the quarks appear to be $m_{\mathfrak{N}} = 338 \pm 10$ MeV and $m_\lambda = 465 \pm 10$ MeV, where the errors arise to some extent depending upon the method of analysis used.

VII. CONCLUSIONS

The major results obtained here are perhaps somewhat surprising and indeed lead us to reconsider the dynamical structure of the mesons and baryons. It should be emphasized that except for the assumption of a mass equation for baryons which ignores "three-body" interactions, there are really no additional assumptions to those often accepted in the field. The simplest interpretation of the present results is that the mesons are roughly diatomic molecules with very stiff spring constants and the baryons are the corresponding triatomic molecules with the *same* spring constants between the diatomic subsystems. The point here is that *the addition of a third quark does not appear to have any strong interaction or "polarization" effect on the two-quark subsystem.* Ironically enough, however, the single- and double-quark systems cannot apparently exist as free particles (clearly with the light masses determined above they could energetically emerge as decay products) because the third quark is required in order to yield integral values of the quantum numbers. An obvious possibility is that the quarks represent topologically connected spatial volumes which contain continuously connected distributions of mass, charge, spin, etc. The integer nature of baryon number, charge, etc. could therefore correspond to continuity requirements for the corresponding distributions in such topological structures.

The important result expressed by Eq. (6.3) suggests that the conventional field-theory ideology about particles interacting via mesons needs reexamining. It has been argued from relatively general field-theory concepts that the strong interactions between a particle and a particle will *not* be the same as that between a particle and an antiparticle. For gravitational interactions the

TABLE VI. Baryon magnetic moments (nuclear magnetons).

Particle	Old quark model	This model	Experiment
n	−1.86	−1.85	−1.913
p	+2.79	+2.78	+2.793
Λ	−0.93	−0.67	−0.67 ± 0.06
Σ^+	+2.79	+2.69	+2.62 ± 0.41
Σ^0	+0.93	+0.84	···
Σ^-	−0.93	−1.01	(0.8 → −1.6)
Ξ^0	−1.86	−1.51	···
Ξ^-	−0.93	−0.59	−1.93 ± 0.75
Δ^{++}	+5.58	+5.55	···
Δ^+	+2.79	+2.78	···
Δ^0	0	0	···
Δ^-	−2.79	−2.78	···
$\Sigma^+(1385)$	+2.79	+3.03	···
$\Sigma^0(1385)$	0	+0.26	···
$\Sigma^-(1385)$	−2.79	−2.52	···
$\Xi^0(1530)$	0	+0.51	···
$\Xi^-(1530)$	−2.79	−2.27	···
Ω^-	−2.79	−2.01	···

Newtonian interaction is the same for the q^2 and $q\bar{q}$ systems. For electrostatics the interactions have opposite signs. For strong interactions it is argued that there are many contributions corresponding to the type and number of mesons transferred from one particle to the other. However, it is apparently generally accepted[13] that the individual meson contributions have the same sign for q^2 and $q\bar{q}$ systems if the G parity of the transferred mesons is positive, and opposite sign of the G parity is negative. In general, this leads to the idea that particle-particle interactions will be different from particle-antiparticle interactions. The results obtained here for quarks contradict such a general conclusion and therefore cast doubt on the conventional mesonic theory of strong interactions. A relatively simple change in the basic assumptions about meson exchanges would be to regard the "mesonic charge" of particles and antiparticles as spinors rather than as simple scalars. Such an idea coincides closely with the K-spin ideology which automatically uses the spinorial representation. Of course it is also possible that meson-exchange theory has nothing at all to do with quark-antiquark interactions.

There is one result obtained above which is possibly disturbing, namely, it is the $K=0$, $K_Z=0$ mesons which are being compared with $K=1$, $K_Z=\pm 1$, q^2 or $\bar{q}^2$ subsystems, i.e., they have opposite K symmetry because of the assumptions made about the SU_{12} symmetries for mesons (over-all antisymmetry) and baryon (over-all symmetry). This could be rectified by adopting an $SU_{12}(2)$ classification for mesons. However, if K parity is to be conserved, then one finds decays like $\rho \rightarrow 2\pi$ would be forbidden because with over-all symmetry the ρ meson would have $K=1$ and the π meson $K=0$. It should also be mentioned that the $q\bar{q}$ system has opposite intrinsic parity to the q^2 subsystem, so the concept of a multiplet is already somewhat unusual.

Finally, it is interesting to speculate on the structure of mesons and baryons at higher masses than those considered here. The possibility of $q^2\bar{q}^2$ structures with definite SU_{12} symmetry such as $SU_{12}(1^4)$ are being considered, as well as more exotic combinations which were abandoned in the earlier quark models.[1] If the present relation between q^2 and $q\bar{q}$ is accepted, then it appears possible that the ideas expressed earlier[14] about the relation between nucleon-nucleon and nucleon-antinucleon interactions may well be true. This in turn leads to the possibility of very heavy mesons as extended structures with quarks clustering into baryons and antibaryons in a way similar to the baryonic structure of all nuclei. One possibility is that the ψ (or J) mesons recently discovered are quark-antiquark structures of the type $q^5\bar{q}^5$ and $q^6\bar{q}^6$, a suggestion which has already been discussed elsewhere.[14]

A second possibility is that the "molecular-type" structures undergo vibrations and rotations to produce the various excited states. The apparent rigidity of the q^2 subsystems suggests the lowest excitations will be rotational. A simple calculation for meson rotations shows that the problem is extremely relativistic and therefore considerable care will need to be exercised in developing such "collective" quark models. Again since the $q\bar{q}$ bond is apparently very stiff, vibrations will occur at energies much larger than the rotational energies. Such states could possibly be related to the ψ, ψ', etc. mesons but clearly much more work is needed to lay a proper foundation for such a theory before these speculations can be turned into useful results.

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[1]A good review is contained in the book *The Quark Model* by J. J. J. Kokkedee (Benjamin, New York, 1969).

[2]This is evident from the fact that the Particle Data Group did not see any need for a new issue of "Review of Particle Properties" this year.

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[5]G. Zweig (unpublished).

[6]The SU_6 and SU_{12} schemes used here differ from those of earlier workers [see R. Delbourgo, A. Salam, and J. Strathdee, Phys. Rev. 138, B420 (1965); M. A. B. Bĝg and A. Pais, Phys. Rev. Lett. 14, 267 (1965)] in that the antiparticles are treated like particles rather than by the conventional conjugate symmetry for a "hole" state. We shall not concern ourselves here with the problems which arise in a relativistic approach to the problem of SU_6 symmetry.

[7]See, for example, *Lie Groups for Pedestrians*, by H. J. Lipkin (North-Holland, Amsterdam, 1966).

[8]Similar results have been obtained earlier by P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Lett. 22, 208 (1966).

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[11]For a more detailed discussion of U and V spins, see Ref. 7.

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