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Nonet ansatz and hadronic decays of ψ and ψ' *

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A phenomenological model for normal hadronic decays of ψ and ψ' has been proposed, where these decays are assumed to proceed via SU(3) vector currents. We predict an approximate universal decay ratio of ψ and ψ' into any normal hadronic channels. A potential inconsistency of the quark-line rule has been pointed out. If we remedy this defect suitably, then the nonet ansatz can explain the puzzling ratio $\Gamma(\psi\to\phi\pi^+\pi^-)/\Gamma(\psi\to\omega\pi^+\pi^-) = 0.20 \pm 0.10$ in a simple way. Our model predicts also many relations which can be easily tested experimentally.

I. NORMAL DECAYS OF ψ AND ψ'

One year after the discovery of ψ (3.1 GeV) and ψ' (3.7 GeV), it is now fairly well established[1,2] that they are hadrons with $J^P = 1^-$ and $I^{CG} = 0^{--}$. Many hadronic decay modes have been identified[3] with a strong indication that ψ (3.1 GeV) is dominantly a singlet under SU(3).

In this note we shall make a phenomenological analysis of normal (i.e., noncharmed) hadronic decay modes of ψ and ψ' by means of a specific model. Especially, we can explain a puzzlingly large experimental ratio of[3]

$$\Gamma(\psi\to\phi\,\pi^+\pi^-)/\Gamma(\psi\to\omega\pi^+\pi^-)=0.20\pm 0.10\,,$$

as we shall see in Sec. III.

Let $\psi_\mu(x)$ and $\psi'_\mu(x)$ be vector field operators representing ψ and ψ'. If we are only interested in calculation of decay rates of ψ and ψ' into normal SU(3) hadrons, then we can describe the virtual electromagnetic interaction in terms of the following effective Hamiltonian:

$$H_1(x)=j_\mu^{\text{em}}(x)\left[f\psi_\mu(x)+f'\psi'_\mu(x)\right], \tag{1.1}$$

where $j_\mu^{\text{em}}(x)$ is the electromagnetic current of normal hadrons. If the leptonic decays of ψ and ψ' into a lepton pair $l\bar{l}$ (l being either electron or muon) are entirely due to the elctromagnetic interaction, then their decay rates are computed by

$$\Gamma(\psi\to l\bar{l}\,)=\frac{f^2}{12\pi}M\,,$$
$$\Gamma(\psi'\to l\bar{l}\,)=\frac{(f')^2}{12\pi}M', \tag{1.2}$$

in terms of the same coupling constants f and f' introduced in (1.1). Here, M and M' are masses of ψ and ψ', respectively, and we neglected small lepton masses. Experimentally, we know[3]

$$\Gamma(\psi\to l\bar{l}\,)=4.8\pm 0.6\text{ keV}\,,$$
$$\Gamma(\psi'\to l\bar{l}\,)=2.2\pm 0.3\text{ keV}\,,$$

so that we compute

$$f^2/4\pi=(4.65\pm 0.58)\times 10^{-6}\,,$$
$$(f')^2/4\pi=(1.79\pm 0.24)\times 10^{-6}\,. \tag{1.3}$$

On the other hand, the interaction (1.1) is known[1] to be inadequate to account for all hadronic decays of ψ and ψ' and we must have an additional nonelectromagnetic interaction. Since its explicit form is unknown, we will write the interaction as

$$H_2(x)=gJ_\mu(x)\psi_\mu(x)+g'J'_\mu(x)\psi'_\mu(x) \tag{1.4}$$

in analogy to (1.1), where $J_\mu(x)$ and $J'_\mu(x)$ are some normal neutral hadronic currents with negative charge-conjugation parity and without carrying any hypercharge (i.e., $Y=0$). They are likely to be isoscalar,[1] but we need not assume this for a while.

Now, our first important assumption is that these new two currents are really identical, i.e., $J'_\mu(x)\equiv J_\mu(x)$. This may imply that the nonelectromagnetic decay mechanism of ψ and ψ' owes its origin to a single common cause. At any rate, adding (1.1) and (1.4), the normal hadronic decay Hamiltonians of ψ and ψ' are assumed to be

$$H_\psi(x)=\left[gJ_\mu(x)+fj_\mu^{\text{em}}(x)\right]\psi_\mu(x)\,,$$
$$H_{\psi'}(x)=\left[g'J_\mu(x)+f'j_\mu^{\text{em}}(x)\right]\psi'_\mu(x)\,. \tag{1.5}$$

Now, suppose for a moment that the new interaction is much stronger than the electromagentic interaction. Then, neglecting the latter, the ratio of decay rates $\Gamma(\psi\to n)$ and $\Gamma(\psi'\to n)$ of ψ and ψ' into a channel "n" consisting only of normal hadrons will satisfy a relation

$$\Gamma(\psi'\to n)/\Gamma(\psi\to n)\simeq(g'/g)^2(\Omega'/\Omega)\,, \tag{1.6}$$

where Ω' and Ω are effective phase volumes of two decay modes. Actually, the electromagnetic term is by no means negligible in comparison to the $J_\mu(x)$ term as we shall see in the next section. However, the relation (1.6) will still be correct even in the presence of the electromagnetic terms, if g and g' approximately satisfy a relation

$$f'/g' \simeq f/g \tag{1.7}$$

both in sign and in magnitude. Hereafter, we shall assume this to be the case. Then, setting

$$R(n) = \frac{\Gamma(\psi' \to n)}{\Gamma(\psi \to n)} \frac{\Gamma(\psi \to \text{all})}{\Gamma(\psi' \to \text{all})}, \tag{1.8}$$

the relation (1.6) will be rewritten as

$$R(n) \simeq \left(\frac{g'}{g}\right)^2 \left(\frac{\Omega'}{\Omega}\right) \frac{\Gamma(\psi \to \text{all})}{\Gamma(\psi' \to \text{all})}. \tag{1.9}$$

This also holds for the case $n = l\bar{l}$ because of (1.7). Since we expect $\Omega' \approx \Omega$ in view of a rather small mass difference between M and M', the right-hand side of (1.9) should be roughly independent of a specific channel "n." From (1.3) and (1.7), we estimate that the common value for $R(n)$ with $\Omega' = \Omega$ will be given by

$$R(n) \simeq (0.12^{+0.13}_{-0.06}), \tag{1.10}$$

where we used the experimental values[3] of $\Gamma(\psi \to \text{all}) = 69 \pm 15$ keV and $\Gamma(\psi' \to \text{all}) = 225 \pm 56$ keV. These should be compared to the experimental values[3] of

$$R(K^+K^-\pi^-\pi^+) \simeq 0.12^{+0.12}_{-0.04},$$

$$R(\pi^+\pi^+\pi^-\pi^-\pi^0) \simeq 0.09^{+0.08}_{-0.05},$$

$$R(p\bar{p}) = 0.19^{+0.16}_{-0.11},$$

$$R(l\bar{l}) = 0.14^{+0.05}_{-0.04},$$

$$R(\pi^0\rho^0) < 0.23^{+0.07}_{-0.04},$$

so that the general agreement is satisfactory. If the present idea is correct, then we would roughly expect to have

$$\frac{\Gamma(\psi' \to \text{all normal hadrons})}{\Gamma(\psi \to \text{all normal hadrons})} \simeq \left(\frac{g'}{g}\right)^2 \simeq 0.38^{+0.11}_{-0.09}, \tag{1.11}$$

from which we estimate

$$\frac{\Gamma(\psi' \to \text{all normal hadrons})}{\Gamma(\psi' \to \text{all})} \simeq (10 \sim 15\%).$$

Together with known decay rates for $\psi' \to \psi + \text{any}$ and $\psi' \to l\bar{l}$, this can account for roughly 70% of the total decay rate of ψ'. Presumably,[4] the remaining unaccountable 30% or so is due to new decay channels such as

$$\psi'(3.7 \text{ GeV}) \to \eta_c(2.8 \text{ GeV}) + \text{pions or } \eta,$$

$$\psi'(3.7 \text{ GeV}) \to \psi_c(3.5 \text{ GeV}) + \gamma,$$

where η_c (2.8 GeV) and ψ_c (3.5 GeV) are newly discovered charmonium states.[5,6]

Next, we will study the SU(2) and SU(3) properties of the current $J_\mu(x)$. First of all, it is likely to be isoscalar with negative G parity. Then, it cannot contribute for the decay rate $\Gamma(\psi \to n)$ whenever the final state n consists of even numbers of pions with positive G parity. In that case, irrespective of the validity of (1.7), we should have

$$\begin{aligned} &\Gamma(\psi \to G = +1)/\Gamma(\psi \to \mu\bar{\mu}) \\ &\qquad = \sigma(e\bar{e} \to G = +1)/\sigma(e\bar{e} \to \mu\bar{\mu}), \\ &\Gamma(\psi' \to G = +1)/\Gamma(\psi' \to \mu\bar{\mu}) \\ &\qquad = \sigma(e\bar{e} \to G = +1)/\sigma(e\bar{e} \to \mu\bar{\mu}) \end{aligned} \tag{1.12}$$

for any such state n with $G = +1$ where $\sigma(e\bar{e} \to G = +1)$ and $\sigma(e\bar{e} \to \mu\bar{\mu})$ should be measured just off the resonance energy at $t = M^2$ or $t = M'^2$. Such a relation has been experimentally tested by Feldman and Perl[1] for $n = 2\pi^+2\pi^-$ and $3\pi^+3\pi^-$ states at $t = 9$ GeV^2, and they found that it is well satisfied within experimental errors.

Next, suppose that SU(3) is a good symmetry and that $J_\mu(x)$ is a U-spin scalar. Since the electromagnetic current is also a U-spin scalar, we can immediately derive relations such as

$$\Gamma(\psi \to \pi^-\rho^+) = \Gamma(\psi \to K^-K^{+*}), \tag{1.13}$$

$$\Gamma(\psi \to p\bar{p}) = \Gamma(\psi \to \Sigma^+\bar{\Sigma}^+), \tag{1.14}$$

$$\Gamma(\psi \to \pi^-\bar{K}^0K^{+*}) = \Gamma(\psi \to K^-K^0\rho^+) \tag{1.15}$$

as well as triangular inequalities

$$|(\Gamma_1)^{1/2} - 2(\Gamma_2)^{1/2}| \leq (3\Gamma_3)^{1/2} \leq (\Gamma_1)^{1/2} + 2(\Gamma_2)^{1/2}, \tag{1.16}$$

$$|2(\Gamma_1)^{1/2} - (\Gamma_2)^{1/2}| \leq (3\Gamma_4)^{1/2} \leq 2(\Gamma_1)^{1/2} + (\Gamma_2)^{1/2}. \tag{1.17}$$

Here, we have set

$$\begin{aligned} \Gamma_1 &= \Gamma(\psi \to \pi^+\pi^-\pi^0), \\ \Gamma_2 &= \Gamma(\psi \to K^+K^-\pi^0), \\ \Gamma_3 &= \Gamma(\psi \to \pi^+\pi^-\eta), \\ \Gamma_4 &= \Gamma(\psi \to K^+K^-\eta), \end{aligned} \tag{1.18}$$

$$\Gamma_1 + \Gamma_3 = \Gamma_2 + \Gamma_4, \tag{1.18$'$}$$

and we have neglected all mass differences among SU(3) multiplets. The same inequalities (1.16) and (1.17) will hold also if we replace π^+ and K^+ there by ρ^+ and K^{+*}, respectively. We may test (1.13) against the experimental value[3] of

$$\frac{\Gamma(\psi \to \pi^-\rho^+)}{\Gamma(\psi \to K^-K^{+*})} = \frac{0.43 \pm 0.10}{0.15 \pm 0.03},$$

which is far from the exact ratio 1. Even if we take into account the difference of phase volumes of two decay modes, it is quite difficult to reconcile this value to (1.13), as we shall see in the next section. We then must conclude that the new current $J_\mu(x)$ cannot be a U-spin scalar. However, it is wiser to wait for more experimental information. Also, we do not know how accurate U-spin invariance could be in the elctromagnetic interaction. We should test its validity for the corresponding U-spin relations such as

$$\sigma(e\bar{e} \to \pi^-\rho^+) = \sigma(e\bar{e} \to K^-K^{+*}), \qquad \sigma(e\bar{e} \to \pi^-\bar{K}^0K^{+*}) = \sigma(e\bar{e} \to K^-K^0\rho^+) \tag{1.19}$$

before we are able to make a final judgment on this important question. These relations are valid for any arbitrary incident energy of electron. The triangular relations (1.16) and (1.17) are also valid for $e\bar{e}$ reactions, if we replace the symbols ψ and Γ there by $e\bar{e}$ and σ, respectively.

Another application of the U-spin invariance (or more weakly, the Weyl reflection W_{23} interchanging quarks q_2 and q_3) is for $\psi \to PPV$ decays. Assuming the usual ideal nonet mixing[7] between ϕ and ω, we can easily derive the following relations among matrix elements of various decay modes:

$$\sqrt{2}\,M(\psi\to\pi^-\pi^+\phi) = M(\psi\to K^-K^+\omega) - M(\psi\to K^-K^+\rho^0), \tag{1.20}$$

$$\sqrt{2}\,M(\psi\to K^-K^+\phi) = M(\psi\to\pi^-\pi^+\omega) - M(\psi\to\pi^-\pi^+\rho^0), \tag{1.21}$$

$$M(\psi\to\pi^-\pi^+\rho^0) + M(\psi\to\pi^-\pi^+\omega) = M(\psi\to K^-K^+\rho^0) + M(\psi\to K^-K^+\omega). \tag{1.22}$$

From these, we can obtain corresponding triangular inequalities. The same relations also hold for $M(e\bar{e}\to PPV)$ if we replace ψ by $e\bar{e}$.

In ending this section, we briefly comment on a recent work by Hara,[8] who assumes that $J_\mu(x)$ is a SU(3) scalar and the elctromagnetic interaction is negligible in comparison to $J_\mu(x)$. Moreover, he assumes that the decay $\psi\to PPP$ can be calculable from a *local* effective Hamiltonian of a form[9]

$$H'(x) = G\epsilon_{\mu\nu\alpha\beta}\,\psi_\mu(x)\,\mathrm{Tr}[\partial_\nu P(x)\partial_\alpha P(x)\partial_\beta P(x)]. \tag{1.23}$$

Then he finds

$$\Gamma_1:\Gamma_2:\Gamma_3:\Gamma_4 = 4:1:0:3 \tag{1.24}$$

for decay rates of (1.18). This relation is stronger than the triangular inequalities (1.16) and (1.17). However, as we shall see in the next section, the electromagnetic interaction is in general not negligible so that the result (1.24) is probably a very rough approximation.[9]

II. DYNAMICAL MODEL

In the preceding section, we derived various kinematical relations. However, it is not possible to quantitatively compute each decay rate of ψ, since explicit form of the new current $J_\mu(x)$ was not specified there. To guess its precise form is quite difficult at this time. For example, if the nonelectromagnetic interaction should result from exchange of three gluons,[4,10] its form will be very complicated although it will give a predominantly SU(3) singlet interaction. Alternatively, if it results from the mass-mixing problem[11-14] in diagonalizing SU(4) mass matrices, then we may expect that $J_\mu(x)$ is dominantly a linear combination of vector field operators representing ω, ρ_0, and ϕ mesons. In this case, $J_\mu(x)$ will be again a mixture of SU(3) singlet and octet. Although the SU(3) singlet component in $J_\mu(x)$ is expected to be dominant, the precise statement is difficult to make. Indeed, if we change numerical mass values of ρ^0 meson slightly by a few standard deviations, then the SU(3) octet component could be[12,14] larger than the singlet one. At any rate, since ω, ρ_0, and ϕ mesons are intimately related to hadronic vector currents by the vector-dominance model,[15] we make an *Ansatz* here that $J_\mu(x)$ has a form

$$J_\mu(x) = \alpha_0 j_\mu^{(0)}(x) + \alpha_8 j_\mu^{(8)}(x) \tag{2.1}$$

in terms of the usual nonet of SU(3) vector currents $j_\mu^{(\alpha)}(x)$ ($\alpha = 0,1,2,\ldots,8$), where α_0 and α_8 are some constants. If we assume the usual SU(3) quark model, then $j_\mu^{(\alpha)}(x)$ is given by

$$j_\mu^{(\alpha)}(x) = \frac{i}{2}\,\bar{q}(x)\gamma_\mu\lambda_\alpha q(x), \tag{2.2}$$

where λ_α ($\alpha = 0,1,2,\ldots,8$) are the usual 3×3 matrices. Also, the electromagnetic current $j_\mu^{\mathrm{em}}(x)$ will be written as

$$j_\mu^{\mathrm{em}}(x) = j_\mu^{(3)}(x) + \frac{1}{\sqrt{3}}\,j_\mu^{(8)}(x) + \beta j_\mu^{(0)}(x). \tag{2.3}$$

So far, the possible existence of an SU(3) singlet current in the normal hadronic electromagnetic current is not clear[16] and we assume $\beta = 0$ hereafter unless otherwise stated.

Although we do not know whether (2.1) must be regarded as a mere good approximation or if it could be exact, we assume hereafter that it is the exact expression for our problem or at least a working hypothesis, especially in view of no simple alternative model. Our Hamiltonian (1.5) will then be written as

$$H_\psi(x) = [g_0 j_\mu^{(0)}(x) + g_8 j_\mu^{(8)}(x) + f j_\mu^{(3)}(x)]\,\psi_\mu(x) \equiv j_\mu(x)\,\psi_\mu(x), \tag{2.4}$$

where g_0 and g_8 are defined by

$$g_0=\alpha_0 g+\beta f\,,\qquad g_8=\alpha_8 g+\frac{1}{\sqrt{3}}f\,. \tag{2.5}$$

The most attractive choice would be to set $\beta=\alpha_8=0$ so that we have

$$g_8=\frac{1}{\sqrt{3}}f\,. \tag{2.6}$$

However, we shall often consider the more general form (2.4) by the reason that the choice $\alpha_8=0$ leads to the validity of (1.13) which appears to conflict with the present experimental data.

Now, let us first compute the inclusive normal hadronic decay rate of ψ by means of (2.4), regarding it to be exact. We need the numerical value of

$$\Delta_{\alpha\beta}(t)=\int \alpha^4 x\, e^{iqx}\langle 0\,|\,j_\mu^{(\alpha)}(x)j_\mu^{(\beta)}(0)\,|\,0\rangle\,,\quad t=-q^2 \tag{2.7}$$

at $t=M^2$. Since the ψ mass M is fairly large, we may appeal to the idea of asymptotic nonet symmetry[17] or the quark-parton model.[18] Then we expect to have

$$\Delta_{\alpha\beta}(t)=\delta_{\alpha\beta}\,\Delta(t),\quad t>>1 \tag{2.8}$$

for all α, $\beta=0, 1, 2, \ldots, 8$. Setting

$$R=\frac{\sigma(e\bar{e}\to\text{all normal hadrons})}{\sigma(e\bar{e}\to\mu\bar{\mu})}\,, \tag{2.9}$$

we find

$$R=\frac{8\pi}{3t}\,\Delta(t)$$

so that we compute

$$\Gamma(\psi\to\text{all normal hadrons})=\frac{1}{16\pi}\left[(g_0)^2+(g_8)^2+f^2\right]MR\,. \tag{2.10}$$

Assuming the experimental value of $R=2.5$ around this energy range[1] $t=M^2$, and using the estimate (1.3), this leads to

$$\frac{1}{4\pi}\left[(g_0)^2+(g_8)^2\right]=(2.58\pm0.78)\times10^{-5}\,, \tag{2.11}$$

where we assume that all decay rates of ψ into channels other than those of the leptonic pair $l\bar{l}$ and of normal hadronic ones are negligible. If we assume (2.6), i.e., $g_8=(1/\sqrt{3})f$, then (2.11) gives

$$\frac{1}{4\pi}(g_0)^2=(2.43\pm0.80)\times10^{-5}\,, \tag{2.12}$$

$$|\,g_0/f\,|=2.29\pm0.52\,. \tag{2.13}$$

Note that f is by no means small in comparison to g_0 and any interference between g_0 and f terms affects the decay rates considerably, depending upon the relative signs of f and g_0.

Next, let us consider a decay of ψ into odd numbers of pions and suppose that the third quark q_3 does not participate at all for this decay mode. Then, we can replace $j_\mu^{(0)}(x)$ by $\sqrt{2}\,j_\mu^{(8)}(x)$ for such a decay mode. We remark that this assumption can be justified on the basis of either the quark-parton picture[18] or the quark-line rule[19] or the nonet *Ansatz*.[7] Since the isovector current $j_\mu^{(3)}(x)$ does not contribute for decays of ψ into odd numbers of pions by G parity, we find the following analog of (1.12):

$$\Gamma(\psi\to\text{odd pions})=\frac{1}{4\pi}(\sqrt{2}\,g_0+g_8)^2\,\frac{\sigma(e\bar{e}\to\text{odd pions})}{\sigma(e\bar{e}\to\mu\bar{\mu})}M \tag{2.14}$$

for any final state containing odd numbers of pions. The cross sections $\sigma(e\bar{e}\to\text{odd pions})$ and $\sigma(e\bar{e}\to\mu\bar{\mu})$ should be measured just off the resonance energy $t=M^2$. We also note that (2.14) holds for total as well as differential decay rates. Hence, we predict the same energy spectrum for pions in both $\Gamma(\psi\to\text{pions})$ and $\sigma(e\bar{e}\to\text{pions})$ around $t\simeq M^2$. If $\sigma(e\bar{e}\to\text{odd pions})$ are experimentally measured, then we can determine g_0 and g_8 separately by means of (2.11) and (2.14). However, so far any definite experimental information on $\sigma(e\bar{e}\to\text{odd pions})$ does not seem to be available. Here, we assume the estimate (2.12) and (2.13) with $g_8=(1/\sqrt{3})f$. Then, (2.14) gives

$$\frac{\Gamma(\psi\to\text{odd pions})}{\Gamma(\psi\to\text{all})}=(3.28^{+0.11}_{-0.26})\,\frac{\sigma(e\bar{e}\to\text{odd pions})}{\sigma(e\bar{e}\to\mu\bar{\mu})} \tag{2.15}$$

for the solution $g_0/f=2.29\pm0.52$, and

$$\frac{\Gamma(\psi\to\text{odd pions})}{\Gamma(\psi\to\text{all})}=(1.60^{+0.29}_{-0.42})\,\frac{\sigma(e\bar{e}\to\text{odd pions})}{\sigma(e\bar{e}\to\mu\bar{\mu})} \tag{2.16}$$

for the negative solution $g_0/f=-2.29\pm0.52$. Note that (2.16) is one-half as large as (2.15). Conversely, from the known decay rates[3] of $\Gamma(\psi\to\pi\rho)$ and $\Gamma(\psi\to\pi\pi\omega)$, we predict

$$\frac{\sigma(e\bar{e}\to\pi^+\pi^-\omega)}{\sigma(e\bar{e}\to\mu\bar{\mu})}=(2.4^{+0.9}_{-0.7})\times10^{-3}\,,\qquad \sigma(e\bar{e}\to\pi^-\rho^+)\simeq\tfrac{1}{2}\sigma(e\bar{e}\to\pi^+\pi^-\omega) \tag{2.17}$$

near the resonance region $t\simeq M^2$, where we have used $g_0/f=+2.29\pm0.52$. If we had used another solution $g_0/f=-2.29\pm0.52$, then the predicted value in (2.17) would increase by a factor of 2. The cross section (2.17) is, unfortunately, very small. More generally, we can compute the ratio of decay rates of ψ into odd numbers of pions against even numbers of pions from (1.12) and (2.14) to be

$$\frac{\Gamma(\psi\to\text{odd pions})}{\Gamma(\psi\to\text{even pions})}=3\left(\frac{\sqrt{2}\,g_0+g_8}{f}\right)^2\frac{\sigma(e\bar{e}\to\text{odd pions})}{\sigma(e\bar{e}\to\text{even pions})}\,. \tag{2.18}$$

Again using the estimate (2.13), this gives

$$\frac{\Gamma(\psi\to\text{odd pions})}{\Gamma(\psi\to\text{even pions})}=(43.7^{+18.5}_{-15.2})\frac{\sigma(e\bar{e}\to\text{odd pions})}{\sigma(e\bar{e}\to\text{even pions})} \tag{2.19}$$

for the solution $g_0/f=+2.29\pm0.52$, and

$$\frac{\Gamma(\psi\to\text{odd pions})}{\Gamma(\psi\to\text{even pions})}=(21.2^{+13.4}_{-10.1})\frac{\sigma(e\bar{e}\to\text{odd pions})}{\sigma(e\bar{e}\to\text{even pions})} \tag{2.20}$$

for $g_0/f=-2.29\pm0.52$. Hence, assuming a rough guess $\sigma(e\bar{e}\to\text{odd pions})\approx\frac{1}{3}\sigma(e\bar{e}\to\text{even pions})$, this implies that $\Gamma(\psi\to\text{odd pions})$ will be roughly 10 times larger than $\Gamma(\psi\to\text{even pions})$ in agreement with the experiment.[3]

Next, we shall show that we can estimate the absolute decay rate of $\Gamma(\psi\to\pi\rho)$ as follows. Assuming again that the third quark q_3 does not contribute at all to the decay, we can replace $j_\mu^{(0)}(x)$ by $\sqrt{2}\,j_\mu^{(8)}(x)$ so that the decay matrix element is proportional to

$$(4k_0k_0'V^2)^{1/2}\langle\pi^-(k')\rho^+(k)|j_\mu^{(8)}(0)|0\rangle=\sqrt{3}\,F(t)\epsilon_{\mu\rho\lambda\tau}\bar{\epsilon}_\rho(k)k_\lambda k_\tau'\,, \tag{2.21}$$

where $\epsilon_\rho(k)$ is the polarization vector of the ρ meson and $F(t)$ with $t=-(k+k')^2$ is the form factor of the vertex function. Then, we compute

$$\begin{aligned}\Gamma(\psi\to\pi^+\rho^-)&=\Gamma(\psi\to\pi^-\rho^+)\\&=\Gamma(\psi\to\pi^0\rho^0)\\&=\frac{1}{4\pi}(\sqrt{2}\,g_0+g_8)^2k^3|F(M^2)|^2\,.\end{aligned} \tag{2.22}$$

Here, k is the magnitude of the final pion momentum and $F(t)$ must be measured at $t=M^2$. Also, the value of $F(t)$ at $t=0$ can be calculated from the formula

$$\Gamma(\rho^-\to\pi^-\gamma)=\frac{e^2}{12\pi}q^3|F(0)|^2 \tag{2.23}$$

for radiative decay of the ρ meson, where q is the photon energy. Experimentally,[20] it is known to be

$$\Gamma(\rho^-\to\pi^-\gamma)=35\pm10\text{ keV} \tag{2.24}$$

so that $F(0)$ is calculable from (2.23). In order to estimate $F(M^2)$, we assume the standard vector dominance form[21] of

$$F(t)=F(0)\frac{m_\omega{}^2}{m_\omega{}^2-t}\,, \tag{2.25}$$

where m_ω is the mass of the ω meson. Moreover, again assuming $g_8=(1/\sqrt{3})f$ with (2.12) and (2.13), we can now compute the absolute decay rate without introducing any additional free parameters to be

$$\Gamma(\psi\to\pi^-\rho^+)=\begin{cases}(0.27^{+0.28}_{-0.16})\text{ keV, for } g_0/f=2.29\pm0.52\\(0.13^{+0.18}_{-0.09})\text{ keV, for } g_0/f=-2.29\pm0.52,\end{cases} \tag{2.26}$$

which should be compared to the experimental value[3] of $\Gamma(\psi\to\pi^-\rho^+)=0.43\pm0.10$ keV. The agreement is satisfactory in view of the uncertainty of the form factor $F(t)$.

Next, let us investigate the more general decay $\psi\to VP$, where V is a vector nonet and P is the psuedoscalar octet. If we assume the nonet *Ansatz*[7] or quark-line rule,[19] then the effective interaction current responsible for the decay is proportional to

$$\mathrm{Tr}\,(jVP)+\mathrm{Tr}\,(jPV) \tag{2.27}$$

because of the charge-conjugation invariance where j is a spurion matrix representing the nonet current $j_\mu^{(\alpha)}(x)$ appearing in (2.4). More explicitly, (2.27) implies that we can set

$$(4k_0k_0'V^2)^{1/2}\langle V^{(\alpha)}(k)P^{(\beta)}(k')|j_\mu^{(\gamma)}(0)|0\rangle=3d_{\alpha\beta\gamma}F(t)\epsilon_{\mu\nu\lambda\tau}\bar{\epsilon}_\nu(k)k_\lambda k_\tau' \tag{2.28}$$

for all $\alpha,\beta,\gamma=0,1,2,\ldots,8$ where $F(t)$ is the same form factor appearing in (2.21). Then, taking into account the difference of the kinematical factor k^3 as in (2.22), we compute

$$\frac{\Gamma(\psi\to K^-K^{+*})}{\Gamma(\psi\to\pi^-\rho^+)}=\left(\frac{2\sqrt{2}\,g_0-g_8+\sqrt{3}\,f}{2\sqrt{2}\,g_0+2g_8}\right)^2\times0.85, \tag{2.29}$$

$$\frac{\Gamma(\psi\to\bar{K}^0K^{0*})}{\Gamma(\psi\to\pi^-\rho^+)}=\left(\frac{2\sqrt{2}\,g_0-g_8-\sqrt{3}\,f}{2\sqrt{2}\,g_0+2g_8}\right)^2\times0.85, \tag{2.30}$$

$$\frac{\Gamma(\psi\to\eta\phi)}{\Gamma(\psi\to\pi^-\rho^+)}=\frac{4}{3}\left(\frac{g_0-\sqrt{2}\,g_8}{\sqrt{2}\,g_0+g_8}\right)^2\times0.75, \tag{2.31}$$

$$\frac{\Gamma(\psi\to\eta\omega)}{\Gamma(\psi\to\pi^-\rho^+)}=\frac{1}{3}\times0.89=0.30, \tag{2.32}$$

$$\frac{\Gamma(\psi\to\eta\rho^0)}{\Gamma(\psi\to\pi^-\rho^+)}=\frac{0.89}{3}\frac{\Gamma(\psi\to\pi^0\omega)}{\Gamma(\psi\to\pi^-\rho^+)}=\left(\frac{f}{\sqrt{2}\,g_0+g_8}\right)^2\times0.89, \tag{2.33}$$

$$\Gamma(\psi\to\pi^0\phi)=0. \tag{2.34}$$

Here, we assumed the ideal mixing[7] of ω and φ and neglected small η-η' mixing. Note that (2.32) for $\Gamma(\psi\to\eta\omega)/\Gamma(\psi\to\pi^-\rho^+)$ is independent of g_0, g_8, and f so that it constitutes an immediate test of our *Ansätze*. If we choose $g_8=(1/\sqrt{3})f$ with (2.13), then (2.29) leads to

$$\Gamma(\psi\to K^-K^{+*})/\Gamma(\psi\to\pi^-\rho^+)=0.85, \tag{2.35}$$

which is independent of g_0/f and is equal to the ratio of the kinematical k^3 factors of two decay modes. On the other hand, (2.30) gives

$$\frac{\Gamma(\psi\to\overline{K}_0K^{0*})}{\Gamma(\psi\to\pi^-\rho^+)}=\begin{cases}0.25^{+0.08}_{-0.09} & \text{for } g_0/f=2.29\pm0.52\\ 2.32^{+0.75}_{-0.38} & \text{for } g_0/f=-2.29\pm0.52.\end{cases} \tag{2.36}$$

These values must be compared to experimental values[3] of

$$\begin{aligned}\Gamma(\psi\to\pi^-\rho^+)/\Gamma(\psi\to\text{all})&=0.42\pm0.10,\\ \Gamma(\psi\to K^-K^{+*})/\Gamma(\psi\to\text{all})&=0.16\pm0.03,\\ \Gamma(\psi\to\overline{K}^0K^{0*})/\Gamma(\psi\to\text{all})&=0.12\pm0.02.\end{aligned} \tag{2.37}$$

The result (2.35), which is actually the U-spin relation (1.13), is rather badly satisfied as we noted in the previous section. However, the relation (2.36) is in good agreement with (2.37), if we choose g_0/f to be positive. The negative sign for g_0/f appears to give too large a value in (2.36) to be consistent with the experiment. Note the difference of nearly a factor of 10 for two cases.

If we take the experimental value for $\Gamma(\psi\to K^-K^{+*})$ seriously, then we have to give up the simple *Ansatz* $g_8=(1/\sqrt{3})f$. But we can compute g_0, g_8, and f separately from (2.29), (2.30), and (2.37) to be

$$\begin{aligned}g_8/g_0&=0.47\pm0.22,\\ \frac{f}{\sqrt{2}\,g_0+g_8}&=0.052\pm0.156,\end{aligned} \tag{2.38}$$

which requires a rather large SU(3) octet component in $J_\mu(x)$.

Another trouble is that the nonet *Ansatz* (2.28) as well as the simple quark-model calculation[22] predicts[7]

$$\begin{aligned}\frac{\Gamma(\rho^-\to\pi^-\gamma)}{\Gamma(\omega\to\pi^0\gamma)}&=\frac{1}{9}\times0.94=0.105,\\ \frac{\Gamma(K^{0*}\to K^0\gamma)}{\Gamma(\omega\to\pi^0\gamma)}&=\frac{4}{9}\times0.53=0.235,\end{aligned} \tag{2.39}$$

both of which are three times larger than the experimental values.[20,23] However, the ratio

$$\Gamma(\rho^-\to\pi^-\gamma)/\Gamma(K^{0*}\to K^0\gamma)=0.45 \tag{2.40}$$

is in good agreement with the experiments.[23] This is analogous to the situation in which the calculated ratio $\Gamma(\psi\to\pi^-\rho^+)/\Gamma(\psi\to\overline{K}^0K^{0*})$ agrees with the experiment. Unfortunately, we cannot offer any good resolution on this puzzling fact. We should note that if we had used the experimental value of $\Gamma(\omega\to\pi^0\gamma)$ rather than $\Gamma(\rho^-\to\pi^-\gamma)$ as input, then our previous estimate (2.26) would increase by a factor of 3 because of this discrepancy. One way to dissolve this dilemma is to directly measure $\sigma(e\overline{e}\to\omega\pi^0)$, $\sigma(e\overline{e}\to\pi^-\rho^+)$, $\sigma(e\overline{e}\to K^-K^{+*})$, and $\sigma(e\overline{e}\to\overline{K}^0K^{0*})$ at any energy which is not necessary at $t=M^2$.

We can compute the decay rate $\Gamma(\psi\to p\overline{p})$ in an analogous way. On the basis of (2.4), it is straightforward to find

$$\Gamma(\psi\to p\overline{p})=\frac{1}{12\pi}\left[1-\left(\frac{2m}{M}\right)^2\right]^{1/2}\times\left((F_M)^2+\frac{2m^2}{M^2}(F_E)^2\right)M, \tag{2.41}$$

where m is the proton mass and F_M and F_E are magnetic and electric form factors of the protons, with respect to the total ψ current $j_\mu=g_0\,j_\mu^{(0)}+g_8\,j_\mu^{(8)}+f j_\mu^{(3)}$. If we assume as before that the third quark q_3 does not participate in the decay, then we can replace $j_\mu^{(0)}$ by $\sqrt{2}\,j_\mu^{(8)}$. In that case, we can compute F_M and F_E in terms of the conventional electromagnetic form factors G_M and G_E of the nucleon by

$$\begin{aligned}F_M&=\frac{\sqrt{3}}{2}(\sqrt{2}\,g_0+g_8)(G_M^{(p)}+G_M^{(n)})+\frac{f}{2}(G_M^{(p)}-G_M^{(n)}),\\ F_E&=\frac{\sqrt{3}}{2}(\sqrt{2}\,g_0+g_8)(G_E^{(p)}+G_E^{(n)})+\frac{f}{2}(G_E^{(p)}-G_E^{(n)}).\end{aligned} \tag{2.42}$$

These form factors must be evaluated at the timelike value of $t=M^2$. If we assume the usual dipole forms

$$\begin{aligned}G_E^{(p)}(t)&=G_M^{(p)}(t)/\mu_p\\ &=G_M^{(n)}(t)/\mu_n\\ &=\left(1-\frac{t}{0.71}\right)^{-2},\\ G_E^{(n)}(t)&=0\end{aligned} \tag{2.43}$$

even at the very timelike region of $t=M^2$, then we can estimate $\Gamma(\psi\to p\overline{p})$ again with *Ansatz* $g_8=(1/\sqrt{3})f$. Choosing $g_0/f=+2.29$ and neglecting all experimental errors, this gives

$$\Gamma(\psi\to p\overline{p})/\Gamma(\psi\to\text{all})\simeq0.55\times10^{-4}.$$

This should be compared to the experimental value[3] of

$$\Gamma(\psi\to p\overline{p})/\Gamma(\psi\to\text{all})=(2.1\pm0.4)\times10^{-3}$$

so that the calculated value is forty times smaller.

However, it is well known[24,25] that the usual scaling-dipole form (2.43) cannot be valid in the timelike t, because at $t=4m^2$, we must have a kinematical constraint equation

$$G_M^{(p)}(4m^2)=G_E^{(p)}(4m^2),$$
$$G_M^{(n)}(4m^2)=G_E^{(n)}(4m^2) \tag{2.44}$$

for both proton and neutron form factors. There is also some indication[24,25] that $G_E^{(p)}$ and $G_M^{(p)}$ are much larger than the dipole form (2.43) in the timelike value around $t\simeq 4$ GeV^2. Therefore, we should not take the discrepancy seriously. Somehow, the electromagnetic form factors of the baryons appear to behave very differently from those of bosons.[21]

Let us comment briefly on $\Gamma(\psi\to\Lambda\bar{\Lambda})$, whose form factors are now given by

$$F_M^{(\Lambda)}=\left(\frac{3}{2}\right)^{1/2}(G_M^{(p)}+G_M^{(n)})g_0+\frac{\sqrt{3}}{2}G_M^{(n)}g_8,$$
$$F_E^{(\Lambda)}=\left(\frac{3}{2}\right)^{1/2}(G_E^{(p)}+G_E^{(n)})g_0+\frac{\sqrt{3}}{2}G_E^{(n)}g_8, \tag{2.45}$$

if we use the SU(3) symmetry as well as (2.42). Unless precise forms of $G_M^{(p)}$, $G_E^{(p)}$, $G_M^{(n)}$, and $G_E^{(n)}$ are known at $t=M^2$, we cannot compute $\Gamma(\psi\to\Lambda\bar{\Lambda})$. Since the electromagnetic current is not negligible in our model, we cannot resort to a simple SU(3) result[3] of $\Gamma(\psi\to\Lambda\bar{\Lambda})\simeq\Gamma(\psi\to p\bar{p})$.

We note that the isospin invariance demands

$$2M(\psi\to\Sigma^0\bar{\Sigma}^0)=M(\psi\to\Sigma^+\bar{\Sigma}^+)+M(\psi\to\Sigma^-\bar{\Sigma}^-) \tag{2.46}$$

for decay matrix elements of $\psi\to\Sigma\bar{\Sigma}$. Moreover, if we have $g_8=(1/\sqrt{3})f$, then our Hamiltonian is U-spin scalar and we have U-spin results such as

$$\begin{aligned}
&M(\psi\to p\bar{p})=M(\psi\to\Sigma^+\bar{\Sigma}^+),\\
&M(\psi\to n\bar{n})=M(\psi\to\Xi^0\bar{\Xi}^0),\\
&M(\psi\to\Sigma^-\bar{\Sigma}^-)=M(\psi\to\Xi^-\bar{\Xi}^-),\\
&M(\psi\to\Lambda\bar{\Lambda})=\tfrac{1}{3}[M(\psi\to\Sigma^0\bar{\Sigma}^0)+2M(\psi\to n\bar{n})],\\
&M(\psi\to\Lambda\bar{\Sigma}^0)=M(\psi\to\Sigma^0\bar{\Lambda})\\
&\qquad=\sqrt{3}[M(\psi\to\Lambda\bar{\Lambda})-M(\psi\to n\bar{n})].
\end{aligned} \tag{2.47}$$

It is likely that $\Gamma(\psi\to\Sigma^0\bar{\Lambda})+\Gamma(\psi\to\Lambda\bar{\Sigma}^0)$ is not negligible in comparison to $\Gamma(\psi\to\Lambda\bar{\Lambda})$ in our model.

Finally, we compute

$$\Gamma(\psi\to\pi^+\pi^-)=\frac{f^2}{4\pi}\frac{M}{12}\left[1-\left(\frac{2m_\pi}{M}\right)^2\right]^{3/2}|F_\pi(M^2)|^2, \tag{2.48}$$

where $F_\pi(t)$ with $F_\pi(0)=1$ is the electromagnetic form factor of the pion. Using the experimental value of Bollini *et al.*[21] at $t=9$ GeV^2

$$F_\pi(9)=0.129^{+0.064}_{-0.129},$$

we compute

$$\frac{\Gamma(\psi\to\pi^+\pi^-)}{\Gamma(\psi\to\text{all})}=(2.85^{+6.33}_{-2.85})\times10^{-4}. \tag{2.49}$$

III. ($\psi\to\phi\pi^+\pi^-$ AND $\omega\pi^+\pi^-$)

Experimentally, we know[3] $\Gamma(\psi\to\phi\pi^+\pi^-)/\Gamma(\psi\to\omega\pi^+\pi^-)=0.2\pm0.10$. Since $\Gamma(\psi\to\phi\pi^+\pi^-)$ is doubly forbidden by SU(4) quark-line rule, this is in general regarded[3,4] as the most conspicuous failure of the quark-line rule. However, as will become clear shortly, the weaker nonet *Ansatz* is capable of explaining the experimental ratio quantitatively in good agreement. First of all, we should emphasize the difference between the nonet *Ansatz*[7] and the quark-line rule[19] for more than three-body processes, although they are identical with each other for the three-body process. This fact will become clearer when we go into detail. Let V be a SU(3) vector nonet, and P be either a pseudoscalar SU(3) octet or nonet. We shall discuss decay modes

$$\psi\to VP_1P_2,$$

where P_1 and P_2 designate two pseudoscalar octet or nonets with four momenta k_1 and k_2, respectively. Then, our Hamiltonian (2.4) indicates that we can effectively replace ψ in the decay by an SU(3) nonet of currents $j_\mu^{(\alpha)}(x)$. Hence, suppressing all Lorentz indices, the most general decay matrix elements consistent with the SU(3) invariance and with the charge-conjugation invariance are a linear combination of the following seventeen terms:

$$S_1=\mathrm{Tr}(jVP_1P_2)+\mathrm{Tr}(jVP_2P_1)+\mathrm{Tr}(jP_1P_2V)+\mathrm{Tr}(jP_2P_1V)+\mathrm{Tr}(jP_1VP_2)+\mathrm{Tr}(jP_2VP_1), \tag{3.1}$$

$$S_2=\mathrm{Tr}(jVP_1P_2)+\mathrm{Tr}(jVP_2P_1)+\mathrm{Tr}(jP_1P_2V)+\mathrm{Tr}(jP_2P_1V)-2\,\mathrm{Tr}(jP_1VP_2)-2\,\mathrm{Tr}(jP_2VP_1), \tag{3.2}$$

$$S_3=\mathrm{Tr}(jV)\mathrm{Tr}(P_1P_2), \tag{3.3}$$

$$S_4=\mathrm{Tr}(jP_1)\mathrm{Tr}(VP_2)+\mathrm{Tr}(jP_2)\mathrm{Tr}(VP_1), \tag{3.4}$$

$$A_1=\mathrm{Tr}(jVP_1P_2)+\mathrm{Tr}(jP_2P_1V)-\mathrm{Tr}(jVP_2P_1)-\mathrm{Tr}(jP_1P_2V), \tag{3.5}$$

$$A_2=\mathrm{Tr}(jP_1)\mathrm{Tr}(VP_2)-\mathrm{Tr}(jP_2)\mathrm{Tr}(VP_1), \tag{3.6}$$

$$S_5=(\mathrm{Tr}V)[\mathrm{Tr}(jP_1P_2)+\mathrm{Tr}(jP_2P_1)], \tag{3.7}$$

$$S_6=(\mathrm{Tr}j)[\mathrm{Tr}(VP_1P_2)+\mathrm{Tr}(VP_2P_1)], \tag{3.8}$$

$$S_7=(\mathrm{Tr}P_1)[\mathrm{Tr}(jVP_2)+\mathrm{Tr}(jP_2V)]+(\mathrm{Tr}P_2)[\mathrm{Tr}(jVP_1)+\mathrm{Tr}(jP_1V)], \tag{3.9}$$

$$A_3=(\mathrm{Tr}P_1)[\mathrm{Tr}(jVP_2)+\mathrm{Tr}(jP_2V)]-(\mathrm{Tr}P_2)[\mathrm{Tr}(jVP_1)+\mathrm{Tr}(jP_1V)], \tag{3.10}$$

$$S_8 = \mathrm{Tr}(jV)(\mathrm{Tr}P_1)(\mathrm{Tr}P_2), \quad (3.11)$$

$$S_9 = (\mathrm{Tr}V)[(\mathrm{Tr}P_1)\mathrm{Tr}(jP_2) + (\mathrm{Tr}P_2)\mathrm{Tr}(jP_1)], \quad (3.12)$$

$$A_4 = (\mathrm{Tr}V)[(\mathrm{Tr}P_1)\mathrm{Tr}(jP_2) - (\mathrm{Tr}P_2)\mathrm{Tr}(jP_1)], \quad (3.13)$$

$$S_{10} = (\mathrm{Tr}j)[(\mathrm{Tr}P_1)\mathrm{Tr}(VP_2) + (\mathrm{Tr}P_2)\mathrm{Tr}(VP_1)], \quad (3.14)$$

$$A_5 = (\mathrm{Tr}\,j)[(\mathrm{Tr}P_1)\mathrm{Tr}(VP_2) - (\mathrm{Tr}P_2)\mathrm{Tr}(VP_1)], \quad (3.15)$$

$$S_{11} = (\mathrm{Tr}j)(\mathrm{Tr}V)\mathrm{Tr}(P_1P_2), \quad (3.16)$$

$$S_{12} = (\mathrm{Tr}j)(\mathrm{Tr}V)(\mathrm{Tr}P_1)(\mathrm{Tr}P_2). \quad (3.17)$$

In the above, j represents a 3×3 spurion matrix corresponding to a linear combination

$$j = g_0 j^{(0)} + g_8 j^{(8)} + f j^{(3)}. \quad (3.18)$$

Also, S_j and A_j stand for symmetric and antisymmetric combinations, respectively, of two mesons P_1 and P_2. Because of the Bose statistics, all S_j must be multiplied by symmetric wave functions of momenta k_1 and k_2, of two mesons, while all A_j must be multiplied by antisymmetric wave functions of k_1 and k_2.

The nonet *Ansatz* demands that all terms involving $\mathrm{Tr}V$ or $\mathrm{Tr}j$, or $\mathrm{Tr}P_1$, or $\mathrm{Tr}P_2$ should not be taken into account. Hence, we need not consider any terms other than S_1, S_2, S_3, S_4, A_1, and A_2. Moreover, the quark-line rule implies that we have to discard all disconnected quark-line diagrams. In our notation, this is equivalent to stating that only three terms, S_1, S_2, and A_1, need be considered. Note that S_3, S_4, and A_2 correspond to quark-line diagrams which are simply disconnected, while terms S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, A_3, A_4, and A_5 are highly disconnected.

If we are now only interested in the decay matrix elements of $\psi \to \phi\pi^+\pi^-$ and $\psi \to \omega\pi^+\pi^-$, then we can easily check that terms corresponding to S_2, A_1, S_4, and A_2 do not contribute at all for these decays. Therefore, we need consider only two terms S_1 and S_3 for the nonet *Ansatz*, while S_1 alone is responsible for the decays in the stronger quark-line rule. The troublesome term is precisely S_1, which allows $\psi \to \omega\pi^+\pi^-$ but forbids $\psi \to \phi\pi^+\pi^-$. Thus, this gives difficulty for the quark-line rule. We argue that the presence of the term S_1 is inconsistent with the original spirit of both the nonet *Ansatz* and the quark-line rule. The reason is as follows. Because of an SU(3) identity relation,[26] we can prove an identity

$$S_1 = S_3 + S_4 + S_5 + S_6 + S_7 - S_8 - S_9 - S_{10} - S_{11} + S_{12}. \quad (3.19)$$

Bringing all negative terms S_8, S_9, S_{10}, and S_{11} into the left-hand side, (3.19) is restated to imply that a sum of all possible terms containing an even number of traces is equal to a sum of those having an odd number of traces for products involving four 3×3 matrices j, V, P_1, and P_2. At any rate, S_1 in (3.1) is a sum of terms representing all connected quark-line diagrams, while all terms in the right-hand side of (3.19) correspond to disconnected quark diagrams. Therefore, (3.19) implies that a sum of all connected diagrams is equal to a sum of all disconnected diagrams with appropriate multiplicative signs. This is inconsistent with the idea of a simple quark-line rule, unless we demand the special combination S_1 to give no contribution at all for the decay mode $\psi \to VPP$. Then, we have to consider the singly disconnected diagram S_3 to account for $\psi \to \omega\pi^+\pi^-$ and $\psi \to \phi\pi^+\pi^-$ since other terms S_2, A_1, S_4, and A_2 do not give any contribution at all for these decays. Hence, the discussion will reduce to that of the nonet *Ansatz*. The situation is somewhat reminiscent of the first forbidden transition in atomic or nuclear physics. However, the above argument is perhaps a little oversimplifying a more complicated situation. In the level of the nonet *Ansatz*, (3.19) now implies that a special combination $S_1 - S_3 - S_4$ which is allowed by the nonet rule is now equal to a sum of all other terms which are forbidden by the same rule. Unfortunately, this is all we can say. But taking a guide from the quark-line rule, we assume that six most general linearly independent components consistent with the nonet rule are $S_1 - S_3 - S_4$, S_2, A_1, $S_3 + S_4$, $2S_3 - S_4$, and A_2 in order of their importance. But since the first combination $S_1 - S_3 - S_4$ should not appear by the reason we mentioned, the decay interaction for $\psi \to VP_1P_2$ must be written as a linear combination of S_2, A_1, $S_3 + S_4$, $2S_3 - S_4$, and A_2. Especially for $\psi \to \pi^-\pi^+\omega$ and $\psi \to \pi^-\pi^+\phi$ decays, only S_3 alone can give nonzero contributions so that their decay rates can be uniquely determined by S_3. If we accept this philosophy, then we compute

$$\frac{\Gamma(\psi \to \phi\pi^+\pi^-)}{\Gamma(\psi \to \omega\pi^+\pi^-)} = 0.78\left(\frac{g_0 - \sqrt{2}\,g_8}{\sqrt{2}\,g_0 + g_8}\right)^2, \quad (3.20)$$

where the extra factor 0.78 in the right-hand side represents the ratio of available phase volumes for two decay modes. Assuming $g_8 = (1/\sqrt{3})f$ with (2.13), this gives

$$\frac{\Gamma(\psi \to \phi\pi^+\pi^-)}{\Gamma(\psi \to \omega\pi^+\pi^-)} = \begin{cases} 0.12 \pm 0.04 & \text{for } g_0/f = 2.29 \pm 0.52, \\ 1.06^{+0.35}_{-0.17} & \text{for } g_0/f = -2.29 \pm 0.52. \end{cases} \quad (3.21)$$

The experimental value of 0.20 ± 0.10 is nicely in accord with the case $g_0/f > 0$, again. We note that if our argument is correct, then we should also find

$$\frac{\sigma(e\bar{e} \to \phi\pi^+\pi^-)}{\sigma(e\bar{e} \to \omega\pi^+\pi^-)} = 2K(t) \quad (3.22)$$

by exactly the same reasoning. Here K stands for a correction factor due to the difference of the phase volumes for two reactions and is given by

$$K(t)=\frac{t^2-m_\phi^4-2tm_\phi^2\ln(t/m_\phi^2)}{t^2-m_\omega^4-2tm_\omega^2\ln(t/m_\omega^2)} \tag{3.23}$$

if we neglect the small pion mass.

We should emphasize the fact that this relation should be valid not only near $t\approx M^2$ but also for any arbitrary values of t which are sufficiently large. The experimental test of (3.22) is crucial for our hypothesis. At $t=M^2$, we can compute $\sigma(e\bar{e}\to\omega\pi^+\pi^-)$ from (2.17) and

$$\sigma(e\bar{e}\to\mu\bar{\mu})=\frac{4\pi}{3t}\left(\frac{e^2}{4\pi}\right)^2 \tag{3.24}$$

to be

$$\sigma(e\bar{e}\to\omega\pi^+\pi^-)=(2.2^{+0.8}_{-0.6})\times10^{-2}\text{ nb}. \tag{3.25}$$

This is a bit small but it will not be impossible to measure experimentally $\sigma(e\bar{e}\to\pi^+\pi^-\omega)$ and $\sigma(e\bar{e}\to\pi^+\pi^-\phi)$ at slightly lower energies, say at $t=5$ GeV^2 since the cross sections are expected then to be much larger than (3.25). Another possible test of (3.22) would be the Primakoff effect. Then, analogous to (3.22), we expect to have

$$\frac{\sigma_C(\pi^\pm N\to\pi^\pm\phi N)}{\sigma_C(\pi^\pm N\to\pi^\pm\omega N)}\simeq2 \tag{3.26}$$

for any nuclear target N in the forward diffraction dissociation region where the Coulomb excitation is, we hope, dominant.

When we combine (3.20) with the U-spin results (1.20) and (1.22), then we find

$$\frac{2M(\psi\to K^-K^+\rho^0)-M(\psi\to\pi^-\pi^+\rho^0)}{M(\psi\to\pi^+\pi^-\omega)}=\frac{3f}{\sqrt{6}\,g_0+f}$$
$$=0.45^{+0.11}_{-0.07}, \tag{3.27}$$

which may be experimentally tested, when data for $\Gamma(\psi\to K^-K^+\rho^0)$ and $\Gamma(\psi\to\pi^+\pi^-\rho^0)$ will become available. Because of G-parity, $M(\psi\to\pi^-\pi^+\rho^0)$ is of the order f so that (3.27) forces $M(\psi\to K^-K^+\rho^0)$ to be proportional to $f=\sqrt{3}\,g_8$ rather than g_0 itself. Therefore, we expect that both $\Gamma(\psi\to\pi^-\pi^+\rho^0)$ and $\Gamma(\psi\to K^-K^+\rho^0)$ are of the same order and smaller by a factor of 10 in comparison to $\Gamma(\psi\to\pi^-\pi^+\omega)$. This is in accord with the experiment if we replace ρ^0 and ω by $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$, respectively. Also, assuming $g_8=(1/\sqrt{3})f$, we can show the validity of

$$M(\psi\to\omega K^0\bar{K}^0)=M(\psi\to\omega\pi^+\pi^-),$$
$$M(\psi\to\phi K^0\bar{K}^0)=M(\psi\to\phi\pi^+\pi^-), \tag{3.28}$$

as well as

$$M(e\bar{e}\to\rho^0K^0\bar{K}^0)=3M(e\bar{e}\to\omega\pi^+\pi^-)=3M(e\bar{e}\to\omega K^0\bar{K}^0)=-\frac{3}{\sqrt{2}}M(e\bar{e}\to\phi K^0\bar{K}^0),$$
$$M(e\bar{e}\to\phi K^+\bar{K}^-)=-\sqrt{2}\,M(e\bar{e}\to\omega K^+K^-). \tag{3.29}$$

These will be additional checks of the validity of our *Ansatz*.

We should note that within the framework of the asymptotically free gluon theory, $\psi\to\pi^+\pi^-\phi$ will require[27] four-gluon exchange in contrast to the three-gluon-exchange mechanism for $\psi\to\pi^+\pi^-\omega$. However, as has been emphasized by Harari,[27] both could give a similar rate due to a dynamical consideration. However, we do not expect to have a relation such as (3.22) for this case, so that the validity of (3.22) will distinguish our model from the gluon model.

In ending this section, we briefly comment on the SU(4) theory. So far we discussed our model within the framework of SU(3), since ψ and ψ' are effectively replaced in terms of the SU(3) currents $J_\mu(x)$ and $j_\mu^{em}(x)$. Therefore, the validity of SU(4) symmetry is largely irrelevant for our discussion. But because of this fact, we cannot perhaps discuss the radiative decays of ψ and ψ' since the SU(4) charmed component of the electromagnetic current is expected to be important for such decay modes. Then, we have to consider the full SU(4) symmetry. Although we could adjust our method for the case of SU(4), some new complications may arise. Especially, we have to modify our criteria of the self-consistency of the quark-line rule discussed in this section since the identity (3.19) is correct only for 3×3 matrices. The details for these points will be treated elsewhere.

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[1]G. J. Feldman and M. L. Perl, Phys. Rep. 19C, 233 (1975). In accordance with this reference, we call the new particles ψ and ψ' rather than J and J', since the latter may be confused with the spin notation as well as the expression for currents. Also, t in our paper refers to the square of the invariant mass of $e\bar{e}$ system.

[2]V. Lüth *et al.*, Phys. Rev. Lett. 35, 1124 (1975).

[3]G. Abrams, rapporteur's talk at the International Symposium on Lepton and Photon Interactions at High Energy Physics, Stanford, 1975 (unpublished). Hereafter, we shall abbreviate this conference simply as the Stanford Conference.

[4]H. Harari, rapporteur's talk at the Stanford Conference, 1975 (unpublished).

[5]B. Wiik, rapporteur's talk at the Stanford Conference, 1975 (unpublished).

[6]W. Braunschweig *et al.*, Phys. Lett. 57B, 407 (1975); G. J. Feldman *et al.*, Phys. Rev. Lett. 35, 821 (1975); W. Tanenbaum *et al.*, *ibid.* 35, 1323 (1975).

[7]S. Okubo, Phys. Lett. 5, 165 (1963).

[8]Y. Hara, University of Tsukuba Report No. UTHEP-4, 1975 (unpublished).

[9]If the electromagnetic interaction is taken into account, then the interaction Hamiltonian is no longer unique so that the relation (1.24) is no longer valid.

[10]A. De Rújula and S. L. Glashow, Phys. Rev. Lett. 34, 46 (1975).

[11]Z. Maki, T. Maskawa, and Z. Umemura, Prog. Theor. Phys. 47, 1682 (1972); Z. Maki *et al.*, *ibid.* 54, 823 (1975).

[12]V. S. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D 11, 2572 (1975).

[13]Y. Hara, Prog. Theor. Phys. 54, 230 (1975).

[14]E. Takasugi and S. Oneda, Phys. Rev. Lett. 34, 988 (1975).

[15]E.g., J. J. Sakurai, in *Laws of Hadronic Matter*, proceedings of the 1973 International Summer School "Ettore Majorana," Erice, Italy, edited by A. Zichichi (Academic, New York, 1975).

[16]E.g., see S. Okubo, in *Symmetries and Quark Models*, edited by R. Chand (Gordon and Breach, New York, 1970).

[17]T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 18, 761 (1967); S. Okubo, in *Proceedings of the 1967 International Conference on Particles and Fields*, edited by C. R. Hagen, G. Guralnik, and V. S. Mathur (Wiley, New York, 1967).

[18]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972); J. Kogut and L. Susskind, Phys. Rep. 8C, 76 (1973).

[19]G. Zweig, 1964 (unpublished); J. Iizuka, Prog. Theor. Phys. Suppl. 37-38, 21 (1966).

[20]B. Gobbi *et al.*, Phys. Rev. Lett. 33, 1450 (1974).

[21]Note that the electromagnetic form factor of the pion in the timelike t can be excellently fit by the Gournaris-Sakurai formula which is a refinement of such a simple pole model. See, e.g., D. Benaksas *et al.*, Phys. Lett. 39B, 289 (1972); M. Bernardini *et al.*, *ibid.* 46B, 261 (1973); D. Bollini *et al.*, CERN report, 1975 (unpublished).

[22]C. Becchi and G. Morpurgo, Phys. Rev. 140, B687 (1965); A. Bramôn and M. Greco, Phys. Lett. 48B, 137 (1974).

[23]W. C. Carithers, P. Mühleman, D. Underwood, and D. G. Ryan, Phys. Rev. Lett. 35, 349 (1975).

[24]M. Gourdin, Phys. Rep. 11C, 30 (1974).

[25]C. Bemporad, rapporteur's talk at the Stanford Conference, 1975 (unpublished).

[26]S. Okubo, Prog. Theor. Phys. 27, 949 (1962); see Lemma II of the Appendix.

[27]H. Harari, Weizmann Institute of Science Report No. WIS-75/40, Ph., 1975 (unpublished); see also Ref. 4.