Symmetry breaking for vector couplings

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The decay width of about 5 keV observed for $\psi \rightarrow e\bar{e}$ implies that the SU(4) symmetry for the vector-meson-photon coupling constant γ_V is broken, and that the quantity $\gamma_V m_V^{-1/2}$ (where m_V is the mass of the vector meson V) obeys the symmetry. Using this experimental result in conjunction with vector dominance and broken SU(3), the author calculates the decay width for $\rho \rightarrow \pi\pi$, and obtains $\Gamma(\rho \rightarrow \pi\pi) = 142$ MeV, in improved agreement with experiment.

The inclusion of the newly discovered $\psi(3.1)$ resonance in an SU(4) vector-meson 16-plet would require a close look at the pattern of symmetry breaking. It is clear that since the mass of the ψ is much larger than that of any of the other vector mesons in the multiplet, the mass symmetry is badly broken. This leads to the question of the extent to which coupling constants obey the symmetry. For example, consider the coupling constant γ_V of the photon to the vector meson V, defined by

$$\langle V|J_\mu^{\mathrm{em}}|0\rangle = (m_V{}^2/\gamma_V)\epsilon_\mu, \tag{1}$$

where J_μ^{em} is the electromagnetic current, ϵ_μ is the polarization vector of the vector meson, and m_V is the mass of the vector meson. If the coupling constants γ_V obey the symmetry of the SU(4) group, that is, they satisfy

$$\gamma_\rho{}^2/4\pi : \gamma_\phi{}^2/4\pi : \gamma_\omega{}^2/4\pi : \gamma_\psi{}^2/4\pi = \tfrac{1}{9} : \tfrac{1}{2} : 1 : \tfrac{1}{8}, \tag{2}$$

the leptonic decay width predicted for the ψ, assumed to be a $c\bar{c}$ state, would be about 16.4 keV,[1] much larger than the experimental value of about 4.8 keV,[2] showing that the symmetry is broken significantly. In fact, as pointed out by Yennie,[3] the ratios in (2) hold for the experimental widths ($\propto \gamma_V{}^{-2} m_V$) rather than for the coupling constants. Thus, it is the quantities $\gamma_V{}^2/m_V$ which are SU(4)-symmetric. The modified realtions, supported by this experimental evidence, would have an effect on other theoretical predictions such as electromagnetic form factors and suppression of photoproduction of the ψ.[1,4]

In this paper we support this symmetry-breaking rule by showing that it improves the decay width of the ρ into 2π calculated by using vector-meson dominance for the coupling of the photon to two pseudoscalar mesons and a broken SU(3) model, along the lines of Ref. 5. Using experimental values of $\Gamma(K^* \rightarrow K\pi)$ and $\Gamma(\phi \rightarrow K\bar{K})$, we obtain $\Gamma(\rho \rightarrow \pi\pi) = 142$ MeV, in better agreement with experiment. [Use of SU(3)-symmetric coupling constants γ_V gives $\Gamma(\rho \rightarrow \pi\pi) = 173$ MeV.]

Assuming SU(3) symmetry breaking through the eighth component of the octet, we can write the coupling of a vector meson to two pseudoscalar mesons as[6]

$$\mathcal{H}_I \sim i[a_1 \mathrm{Tr}(V^\mu[M, \partial_\mu M]) + a_2 \mathrm{Tr}(V^\mu(M\lambda_8\partial_\mu M - \partial_\mu M\lambda_8 M)) + a_3 \mathrm{Tr}(V^\mu\{\lambda_8, [M, \partial_\mu M]\}) + b_1 V_0^\mu \mathrm{Tr}(M\lambda_8\partial_\mu M - \partial_\mu M\lambda_8 M)], \tag{3}$$

where V^μ and M are the vector and pseudoscalar octet matrices, V_0^μ is the vector singlet, λ_8 is the eighth traceless Hermitian 3×3 generator matrix of SU(3), and a_1, a_2, a_3, and b_1 are real parameters. The electromagnetic current can be written as

$$J_\mu^{\mathrm{em}} = V_{8\mu} + \frac{1}{\sqrt{3}} V_{0\mu}. \tag{4}$$

Now, taking the coupling of the electromagnetic current to two identical charged pseudoscalar mesons at zero-momentum transfer to be a universal constant, and the corresponding coupling to uncharged mesons as zero, we get the following relations by considering the electromagnetic coupling of the π^+, K^+, and K^0, and using vector-meson dominance:

$$f = A[2^{1/2}a_1 + (\tfrac{2}{3})^{1/2}a_2 + 2(\tfrac{2}{3})^{1/2}a_3], \tag{5}$$

$$f = A[2^{-1/2}a_1 - (\tfrac{2}{3})^{1/2}a_2 + (\tfrac{2}{3})^{1/2}a_3] + B[(\tfrac{3}{2})^{1/2}a_1 - 2^{1/2}a_3] - C(3^{1/2}b_1), \tag{6}$$

$$0 = A[-2^{-1/2}a_1 + (\tfrac{2}{3})^{1/2}a_2 - (\tfrac{2}{3})^{1/2}a_3] + B[(\tfrac{3}{2})^{1/2}a_1 - 2^{1/2}a_3] - C(3^{1/2}b_1), \tag{7}$$

where f is a constant proportional to the charge e,

$$A = m_\rho^{-1/2} = 0.036\ \mathrm{MeV}^{-1/2},$$

$$B = 3^{-1/2}(\cos^2\theta/m_\phi^{1/2} + \sin^2\theta/m_\omega^{1/2})$$

$$= 0.0184\ \mathrm{MeV}^{-1/2}, \tag{8}$$

$$C = 3^{-1/2}\sin\theta\cos\theta(1/m_\omega^{1/2} - 1/m_\phi^{1/2})$$

$$= 0.00098\ \mathrm{MeV}^{-1/2},$$

θ is the ω-ϕ mixing angle, which is taken to be 40°, and where use is made of the relations

$$\gamma_\rho m_\rho^{-1/2} : \gamma_\phi m_\phi^{-1/2} : \gamma_\omega m_\omega^{-1/2} = \frac{1}{3^{1/2}} : -\frac{1}{\cos\theta} : \frac{1}{\sin\theta}. \tag{9}$$

The other pseudoscalar mesons in the octet give no new relations. The elimination of f from (5) and (6) gives, with the help of (7), the relations

$$a_2 = 0 \tag{10}$$

and

$$A[-2^{-1/2}a_1 - (\tfrac{2}{3})^{1/2}a_3] + B[(\tfrac{3}{2})^{1/2}a_1 - 2^{1/2}a_3] = 3^{1/2}Cb_1, \tag{11}$$

where the latter relation leaves two independent parameters. These are determined using the experimental widths of the decays $K^* \to K\pi$ and $\phi \to K\bar{K}$. Using the interaction (3), these are

$$\Gamma(K^* \to K\pi) = (0.0384)[3(2^{-1/2}a_1 - 6^{-1/2}a_3)^2]$$

$$= 49.8\ \mathrm{MeV}, \tag{12}$$

$$\Gamma(\phi \to K\bar{K}) = (0.0420)\{-\cos\theta[(\tfrac{3}{2})^{1/2}a_1 - 2^{1/2}a_3] + \sin\theta(-3^{1/2}b_1)\}^2$$

$$= 3.4\ \mathrm{MeV}, \tag{13}$$

where we have used the fact that $a_2 = 0$, and where the scales of a_1, a_3, and b_1 are rechosen so that

$$\Gamma(\rho \to \pi\pi) = [2^{1/2}a_1 + 2(\tfrac{2}{3})^{1/2}a_3]^2. \tag{14}$$

Solving relations (11), (12), and (13) for the parameters and substituting them in (14), we obtain $\Gamma(\rho \to \pi\pi) = 142$ MeV, in good agreement with the experimental value of 150 ± 10 MeV. This should be compared with the corresponding value of 173 MeV obtained by assuming SU(3)-symmetric couplings. [This value is obtained from (12) and (14) by setting $a_3 = 0$.]

This result supports the hypothesis that the quantities $\gamma_V m_V^{-1/2}$ satisfy the SU(3) relations (9), which is necessary for explaining the rather small width for $\psi \to e\bar{e}$. It should be noted that the above treatment is not strictly complete, since one should extend it to SU(4) to include the ψ. However, the results would not change appreciably, because the coupling of the ψ to uncharmed mesons is experimentally found to be quite small, and hence our calculation is justified.

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