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Smearing method in the quark model*

E. C. Poggio, H. R. Quinn,† and S. Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
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We propose that perturbation theory in a gauge field theory of quarks and gluons can be used to calculate physical mass-shell processes, provided that both the data and the theory are smeared over a suitable energy range. This procedure is explored in detail for the process of electron-positron annihilation into hadrons. We show that a smearing range of 3 GeV^2 in the squared center-of-mass energy should be adequate to allow the use of lowest-order perturbation theory. The smeared data are compared with theory for a variety of models. It appears to be difficult to fit the present data with theory, unless there is a new charged lepton or quark with mass in the region 2 to 3 GeV.

I. INTRODUCTION

It appears likely that the strong interactions are governed by an asymptotically free gauge field theory involving elementary quark and gluon fields. This field theory can be used to calculate a number of quantities of physical interest, including the hadronic contribution to vacuum polarization for large specelike momenta and the asymptotic values of the moments of electroproduction structure functions.[1] However, direct application of quark perturbation theory to calculate quantities in the physical region is clearly not justifiable, even at high energy. Specifically, in every order perturbation theory predicts production of quarks and gluons, whereas we expect that in fact the only free particles that can be produced are color singlet bound states of quarks and gluons—that is, hadrons.

The problem of demonstrating the phenomenon of quark trapping in a gauge field theory is as difficult as it is interesting. This paper proposes a method by which, even without a thorough understanding of quark trapping, it is possible to use the quark-gluon perturbation theory to obtain detailed information about mass-shell matrix elements at high energy.

Suppose for example that we wish to calculate the cross section $\sigma(s)$ for the process $e^+e^- \to$ hadrons at a center-of-mass energy $\sqrt{s}$. We consider energies high enough so that

$$\alpha_s \equiv g^2(s)/4\pi \ll 1\,, \tag{1}$$

where $g^2(s)$ is the gauge coupling constant defined by renormalizing at a Euclidean point with momentum scale $\sqrt{s}$. As indicated above, even at such energies it would be vain to hope that the quark-gluon field theory could be used to calculate $\sigma(s)$ directly; perturbation theory predicts thresholds for quark-antiquark and multigluon production processes, which are believed to be absent in nature, while $\sigma(s)$ does exhibit multihadron thresholds, which are certainly not present in perturbation theory.

We suggest instead that perturbation theory should be used to calculate a certain *smeared* cross section. It is convenient to express this in terms of the familiar ratio

$$R(s) \equiv \frac{\sigma(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)} = \frac{12\pi s\sigma(s)}{e^4}\,. \tag{2}$$

The smeared ratio is defined as

$$\bar{R}(s,\Delta) = \frac{\Delta}{\pi}\int_0^\infty \frac{ds' R(s')}{(s'-s)^2+\Delta^2}\,. \tag{3}$$

The integral averages out both the quark-gluon thresholds in the theoretical cross section and the hadronic thresholds and resonances in perturbation theory.[2] We shall argue that as long as Δ is sufficiently large, we can calculate $\bar{R}(s,\Delta)$, using some number N_Δ of terms of perturbation theory, with an error which is smaller than the N_Δth term by a factor of order α_s. We would, of course, like to take Δ as small as possible in order to extract as much information as possible from the data. However, N_Δ decreases with decreasing Δ; if we try to take an average that is too fine-grained, then N_Δ may become 1 or 0, which means that perturbation theory could only be used in lowest order, or not at all.

We do not have a rigorous justification for this procedure. However, it is supported by a simple argument, which seems to us to be quite persuasive. The cross-section ratio $R(s)$ is related to a suitably normalized vacuum-polarization amplitude $\Pi(z)$ by the familiar dispersion relation

$$\Pi(z) = \frac{1}{\pi}\int_0^\infty \frac{R(s)}{z-s}ds\,. \tag{4}$$

The smeared ratio may therefore be written as

$$2i\bar{R}(s,\Delta) = \Pi(s+i\Delta) - \Pi(s-i\Delta)\,. \tag{5}$$

Now, the reason we expect perturbation theory to fail for $R(s)$, even at energies sufficiently high so that α_s is small, is that infrared singularities will produce strong forces among subsets of quarks and gluons, forces strong enough to bind them into hadrons in the final state of the e^+e^- annihilation process. But in calculating $\Pi(s\pm i\Delta)$ with $\Delta\neq 0$, the imaginary part of the photon four-momentum will flow through the internal lines of the diagram,[3] preventing the vanishing of any propagator denominators, and thus providing a natural infrared cutoff. This cutoff may break down in high orders of perturbation theory, because there are intermediate states with so many quarks and gluons that each carries only a small share of the complex photon momentum. However, up to this order, perturbation theory can be used to calculate $\Pi(s\pm i\Delta)$, and hence $\bar{R}(s,\Delta)$.

The essential point for practical applications is to know how large a value of Δ is needed to permit the use of a given order of perturbation theory. Avoiding altogether the problem of the details of quark trapping, we first analyze this question in terms of the failures of perturbation theory which are well understood.[4] We argue that it is precisely these well-understood features which arise for the largest values of Δ, and that by avoiding these we are also avoiding the infrared logarithmic singularities which presumably build up to give the quark trapping. Because QED (quantum electrodynamics) is a more familiar theory, in which the problem of bound states already exists, though in a much less severe form, we will first discuss the application of our method to QED. We then move on to color gauge theories, pointing out a few of the differences. Finally, we use our method to compare theoretical and experimental results for electron-positron annihilation.

II. ANALYSIS OF THE BREAKDOWN OF PERTURBATION THEORY IN QED

The breakdown of perturbation theory associated with the existence of positronium is well understood,[5] so we begin by discussing that case. Near the threshold for the production of an electron-positron pair, diagrams such as those of Fig. 1 contribute to the perturbation expansion of $\Pi(s)$ terms of order $(\alpha/v)^n$, where

$$v=\left(\frac{s-4m^2}{s}\right)^{1/2}. \tag{6}$$

The quantity v is the fraction of the velocity of light with which the electron and positron move in the center-of-mass frame. As $s\to 4m^2$, v becomes small compared to α and the $(\alpha/v)^n$ terms become singular, thus causing a breakdown of perturbation

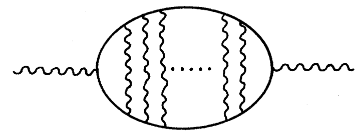

FIG. 1. Diagram contributing $(\alpha/v)^n$ singularities in QED.

theory. As is well known, these terms can be summed, yielding a sequence of poles at the energies of the positronium bound states

$$M_n(\alpha)=2m-\frac{\alpha^2 m}{4n^2}, \tag{7}$$

where here $\alpha=e^2/4\pi$ is the electromagnetic fine-structure constant. Corrections to the positions of these poles can be calculated by including diagrams other than Fig. 1. General diagrams, for example those of Fig. 2, can produce contributions of the form $(\alpha/v)^n(\alpha\ln v)^m$, which are also singular as $v\to 0$. Rather than attempting to calculate all such terms, we replace s with $z=s+i\Delta$, and we choose Δ large enough to render these terms harmless. We now turn to an examination of how this can be assured.

The amplitude Π is a function of the variables α, z, and m^2. We know that this function has branch points in z at the multiparticle thresholds given by

$$s^j(\alpha)=\left[\sum_n N_n^j M_n(\alpha)\right]^2 \tag{8}$$

for any set of integers N_n^j such that the state j with N_n^j particles of type n has the quantum numbers of a single photon. For any complex $z=s+i\Delta$ there is some choice of complex $\alpha=\alpha_j(z)$ for each j, which puts the threshold at z:

$$z=s^j[\alpha_j(z)]. \tag{9}$$

The perturbation expansion for $\Pi(z)$ in terms of α has a singularity at $\alpha=\alpha_j(z)$, and will therefore break down unless[6]

$$|\alpha|\leq\min_j|\alpha_j(z)|. \tag{10}$$

Thus the requirement that we impose on Δ is that Eq. (10) be satisfied for $z=s\pm i\Delta$, where the minimum is taken for all states j which contribute up

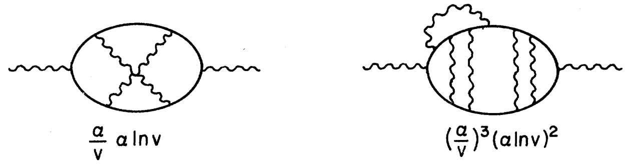

FIG. 2. Diagrams contributing $(\alpha/v)^n(\alpha\ln v)^m$ singularities in QED.

to a desired order of perturbation theory.

Consider, for example, the state j consisting of N positronium atoms in their ground state. Using Eqs. (7)–(9) we find, to lowest order,

$$\alpha_j^2(z) = \frac{z - 4N^2m^2}{m^2N^2}. \qquad (11)$$

Near this threshold, Eq. (10) is satisfied for the physical α and complex $z = s \pm i\Delta$, provided

$$\Delta^2 \geq N^4m^4\alpha^4 - (s - 4N^2m^2)^2 . \qquad (12)$$

The region excluded by this choice is shown in Fig. 3. Avoiding this region clearly also eliminates any problems from N excited positronium atoms.

The fact that there are no singularities of $\Pi(z)$ on the real axis other than these normal thresholds follows from Kinoshita's theorem[7]: Each Feynman graph for $\Pi(z)$ is free of infrared divergences, despite the infrared divergences for individual intermediate states. This is not to say that we believe that the perturbation series actually converges on the real axis between thresholds, but only that in this region we can use QED in the usual way to obtain accurate numerical results.

We also note that production of N electron-positron pairs requires at least $2(N-1)$ factors of α [beyond the over-all factor that multiplies $\Pi(z)$]. Thus N bound-state thresholds affect only the $[2(N-1)]$th order of perturbation theory. If we wish to calculate only to low order in α, we can ignore their existence altogether and choose Δ so that we avoid only the first few thresholds in s. This means that in QED, effectively only the e^+e^- threshold is relevant for most purposes. The value of Δ near that threshold can be taken to be small simply because both α and m are small.

III. NON-ABELIAN GAUGE THEORIES

The analysis of non-Abelian gauge theories proceeds for the most part very much like that of QED. We postulate an effective Lagrangian in which there are parameters corresponding to quark masses, as well as a gauge coupling constant $g(\mu^2)$. These parameters depend logarithmically on the choice of renormalization point. A renormalization-group analysis of Π for s large compared to all masses shows that it is, in this region, only a function of $\alpha_s = g^2(s)/4\pi$. For s in the region of interest to us it depends also on the masses, or rather on the dimensionless quantity s/m^2. By choosing to write Π in terms of α_s we eliminate the possibility that the perturbation expansion for Π in terms of α breaks down because of large logarithms of the form $\ln s/\mu^2$. (Any renormalization mass of order s would be satisfactory for this purpose.) We renormalize the quark masses so that to any order in perturbation theory the quark propagator has its singularity at m_q^2. The physical meaning of this parameter will be discussed briefly in the following section.

Let us consider first a world in which there are only heavy quarks, of mass m. (By "heavy" we mean such that $\alpha_{4m^2} \ll 1$.) The analysis of this situation is similar to that of QED. Once again Π is a function of two independent variables α_s and s/m^2. The perturbation expansion contains terms of the form $(\alpha_s/v)^n$ from diagrams of Fig. 1, which are the most severe singularities in the neighborhood of any quark-antiquark threshold. There are once again also contributions of the form $(\alpha \ln v)^m(\alpha/v)^n$ from diagrams such as those of Fig. 2 and also from the characteristically non-Abelian diagrams such as those of Fig. 4. The effect of these terms on the position of the bound-state pole and the nature of the effective binding potential is presumably more dramatic in this case than in QED, but that does not affect our analysis. As long as $|v|$ is kept large enough that both α/v and $\alpha \ln v$ terms are controlled, the perturbation series may be used to estimate $\Pi(s \pm i\Delta)$.

The α/v terms can be summed to yield the leading approximation for the hadron masses in terms of quark masses, which is just the Coulomb binding formula of Eq. (7). Because α_s is actually not so small in the region we wish to study (namely, near the new resonances), we allow for the possibility that $\alpha \ln v$ corrections may increase the binding of these poles by as much as a factor of 2.[8] Following the arguments of Eqs. (8)–(12) then

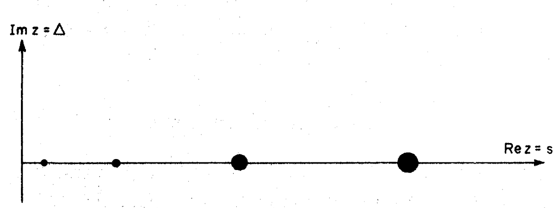

FIG. 3. The excluded region of the s plane in QED. The picture is drawn for $\alpha = 0.1$ because the excluded areas with the physical value of α are too small to draw reasonably until one reaches very-high-order thresholds.

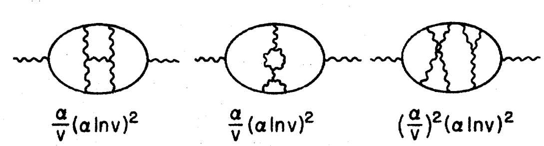

FIG. 4. Typically non-Abelian graphs giving $(\alpha/v)^n(\alpha \ln v)^m$ corrections. Each graph is labeled by the relevant values of n and m.

gives

$$\Delta^2 \geq 16N^4m^4\alpha^4 - (s - 4N^2m^2)^2 \tag{13}$$

for the region $s \approx 4N^2m^2$, in order to avoid singularities from thresholds for hadron states composed of N quark-antiquark pairs. Once again it costs $2(N-1)$ powers of α_s to produce such pairs, so for low-order calculations only low N thresholds are relevant.

The picture is only slightly more complicated when there are also light quarks in the theory. We cannot discuss the region near the few-light-quark thresholds as α_s is too large in that region. However, the region near the heavy-quark-antiquark threshold can be treated. Shortly above that threshold is the threshold for one heavy quark and its antiquark, plus one light quark and its antiquark. These may bind in pairs in either of two ways, but the lowest bound state, and hence that which causes the smallest α_j, is produced when each quark binds with its own antiparticle. The Coulomb binding formula then is

$$s(\text{threshold}) = \left(2m_l + 2m_h - \frac{\alpha^2(s)m_h}{4} - \frac{\alpha^2(s)m_l}{4}\right)^2 . \tag{14}$$

We argue that perturbation theory will provide a good estimate for Π near this threshold provided

$$\Delta^2 \geq 16(m_h+m_l)^4\alpha^4 - [s - 4(m_h+m_l)^2]^2 , \tag{15}$$

again doubling the binding to allow for shifting by higher-order corrections.

Contributions from higher numbers of quarks and antiquarks can be treated similarly, giving the general result

$$\Delta^2 \geq 16(Nm_h+N'm_l)^4\alpha^4 - [s - 4(Nm_h+N'm_l)^2]^2 \tag{16}$$

near the threshold for hadronic states containing N heavy- and N' light-quark-antiquark pairs. The generalization to more than two types of quark is obvious. Equation (16) yields a quite different picture to that of Fig. 3. If α is large enough so that $\alpha_s^2 m_h^2 > m_l^2$, the circles for various k values overlap, producing the excluded region

$$|\Delta| \geq s\alpha_s^2 \tag{17}$$

if one wishes to calculate to high orders in perturbation theory.

However, for practical purposes we are more interested in the value for Δ necessary to allow a calculation, say, through order α_s. The threshold singularities to this order are fully accounted for by choosing Δ to avoid the single-quark-antiquark threshold only. However, to assume that above and below this circle we can take $\Delta \to 0$ appears to us to be somewhat suspect. For any value of s on the real axis we know that the perturbation theory predicts quark and gluon production but physical amplitudes are (by assumption) due only to hadron production. This is quite different from the QED case, where in the region away from thresholds the cross section is in fact dominated by free electron-positron production and the bound-state contributions are only significant near threshold. We assume that, in non-Abelian theories, a nonzero Δ is also required in the region between thresholds to provide a cutoff for the singularities related to quark binding. The question that remains is: What value of Δ is sufficient for this purpose? We cannot answer this question without some idea of the behavior of the singularities. However, we have argued that the Coulombic threshold singularities already discussed are the most severe (being a linear rather than a logarithmic problem). Therefore we choose to make our comparison of data and theory using a constant value of Δ, large enough to satisfy Eq. (13) at the $N=1$ (one quark, one antiquark) threshold, and we assume this will satisfactorily protect us from infrared logarithms, if any exist in the remaining region. Specifically, taking $\alpha_s \approx 0.5$ and $m \approx 1.73$ GeV (values which appear reasonable in the light of our numerical analysis) Eq. (13) gives

$$\Delta \geq 4m^2\alpha_s^2 \approx 3 \text{ GeV}^2 . \tag{18}$$

We will adopt a value $\Delta = 3$ GeV^2 throughout our numerical analysis, though this Δ may be somewhat too small very close to thresholds.

We are not able to extend our analysis much below the region of interest aroung the new resonances, because α_s quickly becomes too large. Hence the choice of a constant Δ also provides a reasonable cutoff for the lowest s values that are included in our analysis. We find that even this presumably generous choice of Δ allows us to make a more sensitive comparison of models and data than has previously been justifiable. Of course, since the smearing in the threshold region is wide compared to the structure in this region, this analysis does not allow us to make any precise statements about resonance widths or positions.

IV. NUMERICAL ANALYSIS

The comparison of quark models with data can now be made by calculating the quantity $\bar{R}(s, \Delta)$ for a variety of models and parameters and comparing with the smeared integral of the data from SPEAR.[9] We will choose $\Delta = 3$ GeV^2 throughout. In order to limit our sensitivity to the unmeasured region $s > 60$ GeV^2 and to the low-s region where we cannot reliably calculate, we present results only for the range $8 \leq s \leq 37$ GeV^2.

We have treated data and theory alike in the low-s region, arbitrarily ignoring any contribution from s less than 4 GeV^2. At the high energy end the empirical curves displayed are calculated assuming that the data will continue to be flat at $R = 5.25$ from 60 GeV^2 to infinity. With our choice of Δ, the *total* contribution from this region is about 5% at the highest-s points shown, and the difference between the contribution assuming $R = 5.25$ in that region and, for example, assuming $R = 3.33$ in the same region is about 1%.

Figure 5(a) shows the SPEAR data for R [excluding the J (ψ) and ψ' resonances]. The solid line represents a crude "eyeball" fit to the data that produce the curve shown in Fig. 5(b) and are labeled "data" on Figs. 6–18. We have tested the sensitivity of this curve to changes in the eyeball approximation. The only feature which changes significantly is the appearance of the "second step" in the region $22 \leq s \leq 28$ GeV^2. It seems impossible to eliminate this feature altogether with a reasonable approximation to the data, though it can be flattened or shifted somewhat from the version shown. For example, the dotted line on Fig. 5(a) produces the smeared curve shown in Fig. 5(c). With the published errors on the data we feel that our crude treatment of it is reasonable. We cannot definitely state whether the second-step feature we find in our smeared curves is real. If it is real, it is very interesting, for we shall see that *it indicates that there must be some new threshold in this region, either an additional heavy lepton or a further quark.*[10] Any improvement of the data in the region $W = 4.5$ to $W = 6$ GeV would be helpful in testing this conclusion. We have chosen to compare models to the curve shown in Fig. 5(b). The parameters we obtain, especially the mass of the heavy quark or lepton are sensitive to the approximations we have made, but our general conclusions are not.

The most severe problem in comparing models

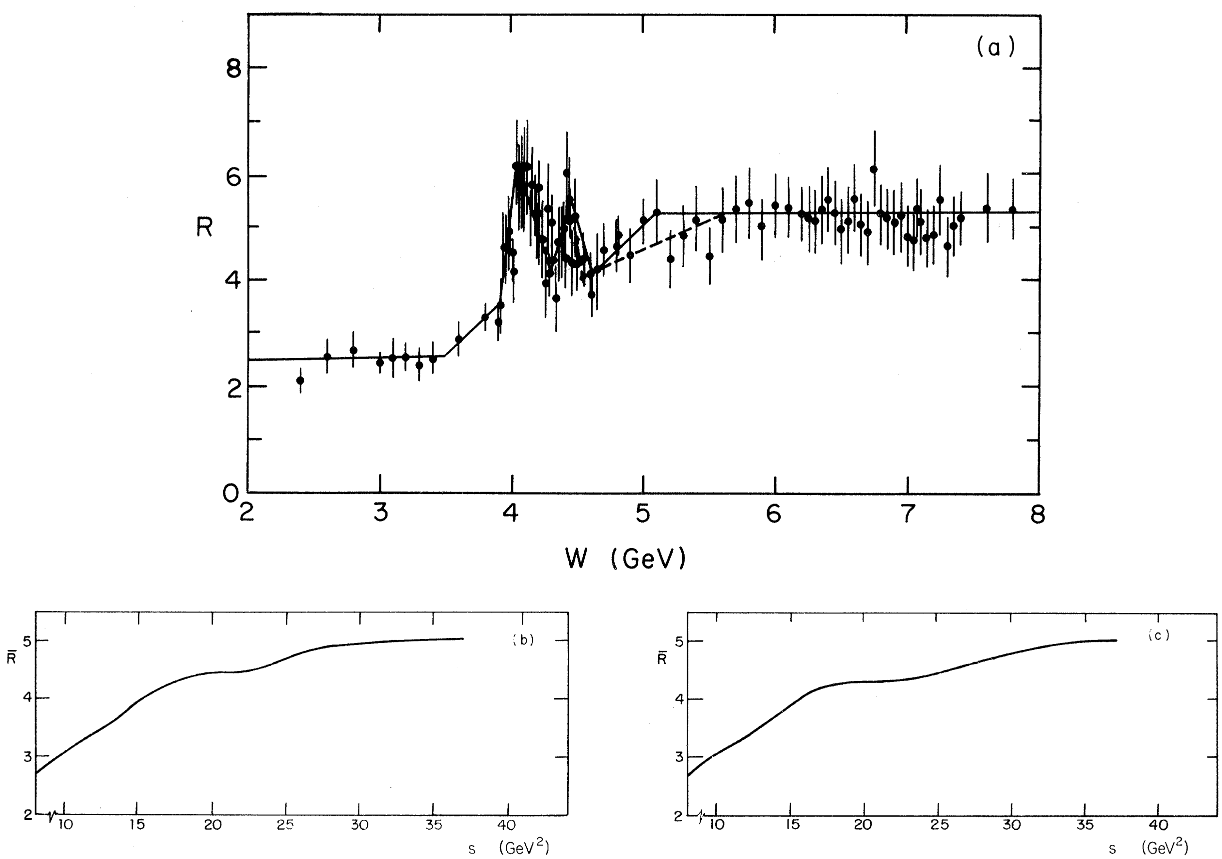

FIG. 5. (a) SPEAR data for $R(s)$. The solid line represents the eyeball fit used in obtaining the curve $\bar{R}$ shown as "data" in Figs. 6–18. The dashed line is an alternate fit included to indicate the possible variations in $\bar{R}$ due to our crude fitting procedure. (b) $\bar{R}(s, \Delta)$ for $\Delta = 3$ GeV^2 generated using solid line in Fig. 5(a). (c) $\bar{R}(s, \Delta)$ for $\Delta = 3$ GeV^2 generated using dotted line in Fig. 5(a).

with this data is that of estimating the possible error in the experimental $\bar{R}(s,\Delta)$. The quoted uncertainty in the data includes both systematic and statistical contributions and amounts to about $\pm 10\%$ in R over most of the range of s. The contribution of statistical errors is certainly reduced by our smearing procedure since with $\Delta = 3$ GeV we average together many points (though not with equal weights). If the quoted errors were entirely statistical, then they would produce an uncertainty of about $\pm 3\%$ in $\bar{R}$. If, on the other hand, the principal contribution to the error is a systematic one, representing an over-all normalization error, then smearing will not reduce this uncertainty. However, the features that are most model-dependent, such as the shoulder discussed above, are not affected by such an over-all normalization change, so that some conclusions can be drawn which are independent of this uncertainty. Some of the tentative nature of our conclusions could perhaps be removed by a calculation of the experimental $\bar{R}(s, \Delta = 3\ \text{GeV}^2)$ from presently existing data including full knowledge of the errors and eliminating our crude eyeball fit—we urge that this should be done by the experimentalists at SPEAR themselves.

Our theoretical curves are calculated in an SU(3) color gauge theory, with various numbers of quarks q and heavy leptons l. The coupling constant in this theory is given by[11]

$$\alpha_s = 12\pi\left[33\ln\left(\frac{s}{\Lambda^2}\right) - 2\sum_q \ln\left(\frac{s+5m_q^2}{\Lambda^2+5m_q^2}\right)\right]^{-1}, \tag{19}$$

with Λ an adjustable parameter. We calculate $R(s)$ through order α_s, using the approximate form given by Schwinger[5] for positronium, suitably modified for color gauge theories.[4] This gives

$$R_{\text{theor}}(s) = \tfrac{3}{2}\sum_q v_q(3-v_q^2)Q_q^2[1+\tfrac{4}{3}\alpha_s f(v_q)] + \tfrac{1}{2}\sum_l v_l(3-v_l^2), \tag{20}$$

where Q_q is the charge of quark q, v_q and v_l are given as before by

$$v \equiv \left(\frac{s-4m^2}{s}\right)^{1/2} \tag{21}$$

and

$$f(v) \equiv \frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right). \tag{22}$$

We include in Eq. (20) any leptons heavier than the muon, because the experimental procedure used at SPEAR would include production of such leptons as part of the "hadronic" cross section.[12]

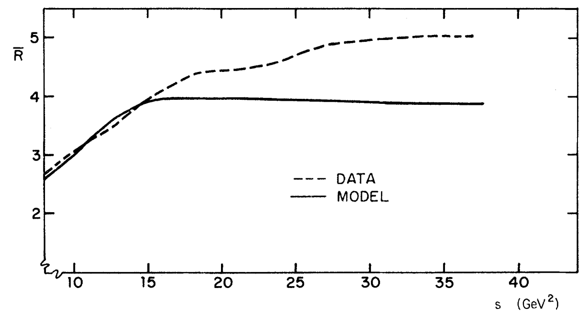

FIG. 6. Comparison of four-quark model with the data.

The theoretical $R(s)$ is smeared with the same value $\Delta = 3\ \text{GeV}^2$ as for the empirical curves.

There is some uncertainty in the value of the strong-interaction parameter Λ. In particular, the problem mentioned above of an uncertainty in the over-all normalization of the data can to some extent be obviated by allowing a free choice of Λ. However, any model which can only be matched to the data by choosing a large α_s in the region $s = 9-30\ \text{GeV}^2$ is to some extent unsatisfactory, as in such a model the approximate scaling of deep-inelastic scattering data in the region $q^2 \gtrsim 1\ \text{GeV}^2$ is apparently fortuitous. Since the explanation of this scaling is one of the principal virtues of asymptotically free theories,[1] we regard as suspect any model which requires $\Lambda^2 \gtrsim 0.6\ \text{GeV}^2$.

Figures 6 through 17 show the comparison of various models with the data. The models are chosen as a representative selection of those appearing in the literature.[13] The masses of possible new heavy quarks and leptons are adjusted in each model to obtain a reasonable fit to the data, except that in some models we put a heavy lepton at 1.7 GeV as suggested by observations of $\mu - e$ production.[14] We have made no attempt to do a thorough search for best fit parameters, as we feel it is not warranted by the present state of the data.

Figure 6 shows that the naive quark-charge-

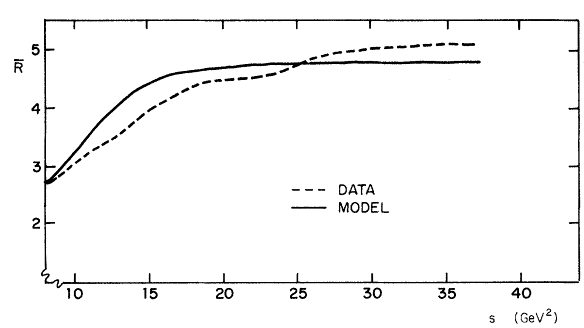

FIG. 7. Model with four quark flavors and one heavy lepton at $m_l = 1.7$ GeV.

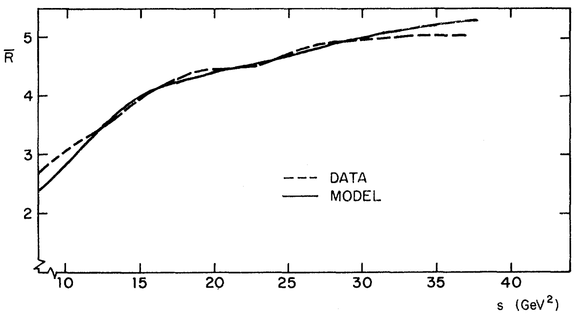

FIG. 8. Model with four quark flavors and two heavy leptons of masses 1.7 and 2.5 GeV.

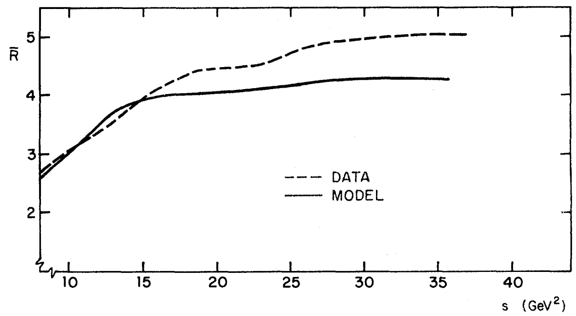

FIG. 10. Model with five quark flavors, charge $\frac{1}{3}$ for additional quark, and nondegenerate fourth and fifth flavors.

counting statement, that color SU(3) models with four quark flavors cannot fit the high-s data, is indeed warranted. Even with the unreasonably large $\Lambda^2 = 1.2$ GeV^2 this model falls far short of matching the data. Adding a single heavy lepton helps considerably in the region of s below 24 GeV^2, as is shown in Fig. 7, but the model still falls significantly below the smeared data curve in the high-s region. Increasing Λ^2 to 1.2 increases the value of R at $s = 37$ GeV^2 by about 0.1, but it also makes the prediction significantly too high in the region $s < 20$ GeV^2. Varying the lepton mass parameter up to values as high as 2.2 GeV does not produce a satisfactory fit.

We therefore conclude that *either there is more than one heavy lepton*[12] *in the region* $1.5 \leq m_L \leq 3$ GeV *or that there is more than one quark threshold in this region*. Figure 8 shows a reasonable fit to present data obtained assuming four quark flavors and two heavy leptons with masses 1.7 and 2.5 GeV. Five quark flavors alone, choosing $\frac{1}{3}$ for the charge of the additional quark, do not lead to a satisfactory fit, as is shown in Figs. 9 and 10. Figures 11 and 12 show five quark models with no additional leptons and a charge of $\frac{2}{3}$ for the fifth quark flavor. If the fourth and fifth quarks are degenerate, this provides a possible fit to the data with a very small effective coupling $\alpha(9) = 0.22$.

A model with five quark flavors, charge $\frac{1}{3}$ for the additional quark, and one heavy lepton gives a reasonable fit. Figures 13–15 show various parameter choices in such models. With the present state of the data we regard all these choices as equally probable, though the parameters of Fig. 14 give the nicest fit. It will be very interesting to see if this remains true with improved data. The parameter choice of Fig. 15 is perhaps unattractive in the light of other experimental suggestions[14] that there is a heavy lepton of mass about 1.7 GeV.

Finally in Figs. 16 and 17 we show the results for a model with six quarks of mass less than 3 GeV, where both the fifth and sixth quark flavors carry charge $\frac{1}{3}$. The model including one heavy lepton (Fig. 16) gives a satisfactory fit with small effective coupling.

For easy reference we present in Table I the parameters used for each of the curves presented in Figs. 6–17. All calculations used charges $\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ for the first four quark flavors, a mass of 0.35 GeV for the light quarks, and 1.66 GeV for the fourth quark mass. The curves are effectively insensitive to the light-quark mass pa-

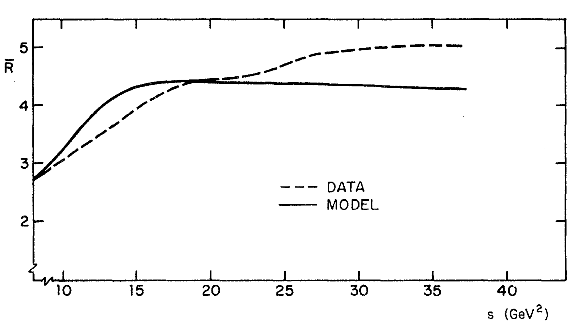

FIG. 9. Model with five quark flavors, charge $\frac{1}{3}$ for additional quark, and degenerate fourth and fifth flavors.

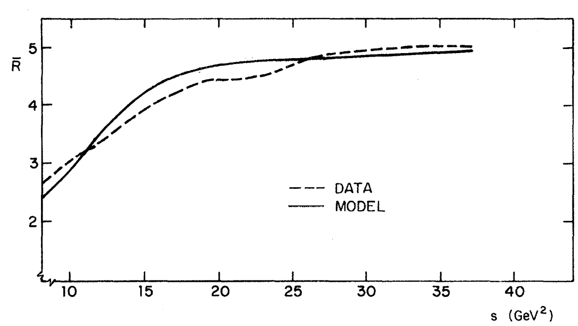

FIG. 11. Model with five quark flavors, charge $\frac{2}{3}$ for additional quark, and degenerate fourth and fifth flavors.

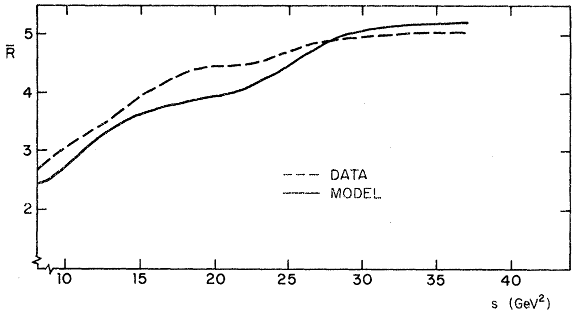

FIG. 12. Model with five quark flavors, charge $\frac{2}{3}$ for additional quark, and nondegenerate fourth and fifth flavors.

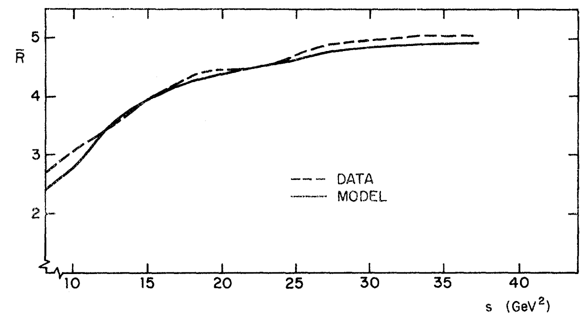

FIG. 14. The model of Fig. 10 with an additional heavy lepton, $m_l = 1.7$ GeV.

rameter provided it is less than about 0.5 GeV, so we do not attach any significance to this number.

Figure 18 shows how our prediction changes as we change the heavy-quark mass for the model of Fig. 14. The effect is similar in any model. The value of Δ used may be a little small in the region of s values sensitive to m_q. With this in mind, it appears that a mass of 1.66 GeV provides a good fit to the data, whereas a mass as high as 1.8 GeV does not.

These remarks raise the question of the meaning of this mass parameter. In our treatment the mass parameter is most closely identified with the threshold for production of particles containing the new quarks. However, the model is smeared in such a way that the distinction between contributions from quark-antiquark bound states and actual production of particles carrying new quantum numbers is washed out. Hence we do not feel that any specific meaning can be attached to the precise value of the heavy-quark mass parameter that we use.

V. SUMMARY AND CONCLUSIONS

We have argued that quark-gluon perturbation theory can be used to predict suitably defined averages of physical quantities. These averages can be calculated without encountering the breakdown of perturbation theory associated with the binding of quarks and gluons in physical color singlet particles. In particular, for the process $e^+e^- \to$ hadrons we find that the smeared average of the data

$$\bar{R}(s,\Delta) = \frac{\Delta}{\pi}\int \frac{ds' R(s')}{(s-s')^2+\Delta^2} \tag{23}$$

can be calculated in perturbation theory for moderate values of Δ.

Comparing with presently available data we find that SU(3) color gauge models with only four flavors of quarks of mass less than 2.7 GeV are not satisfactory, even when an additional heavy-lepton contribution is included. Also ruled out are models with a fifth quark flavor carrying charge $\frac{2}{3}$, unless that fifth flavor is essentially degenerate with the fourth and the effective gauge coupling is very small by $s = 9$ GeV^2. Preferred models are those with four quark flavors and two heavy leptons in the region $1.5 < m_L < 2.7$ GeV, and those with five or six flavors of quarks of mass less than 2.7 GeV, charge $\frac{1}{3}$ for the additional heavy quarks, and one heavy charged lepton. The one common feature of the preferred models

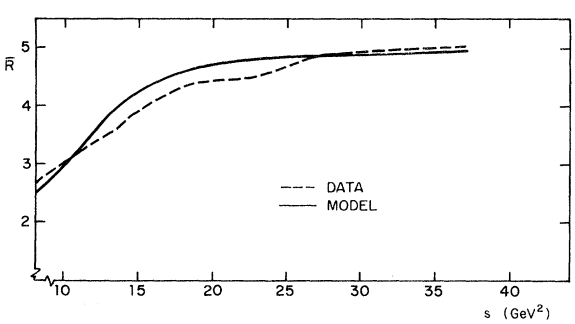

FIG. 13. The model of Fig. 9 with an additional heavy lepton, $m_l = 1.7$ GeV.

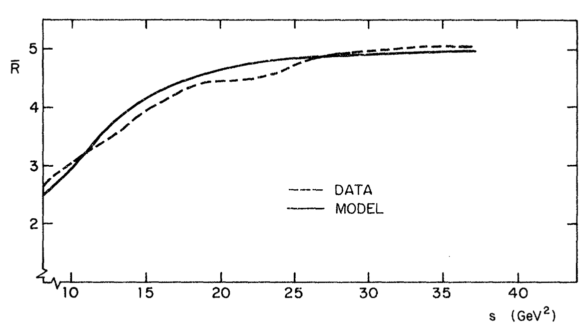

FIG. 15. The model of Fig. 9 with an additional heavy lepton, $m_l = 2.0$ GeV.

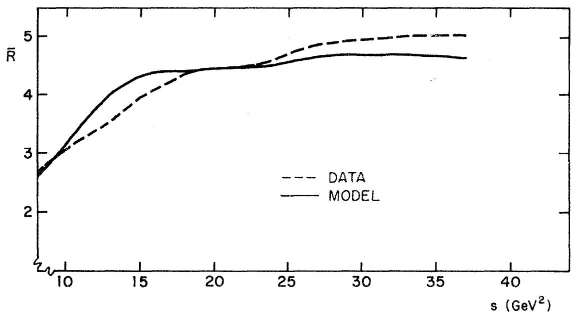

FIG. 16. The model of Fig. 9 with an additional nondegenerate $\frac{1}{3}$-charge quark flavor, $m_q = 2.5$ GeV.

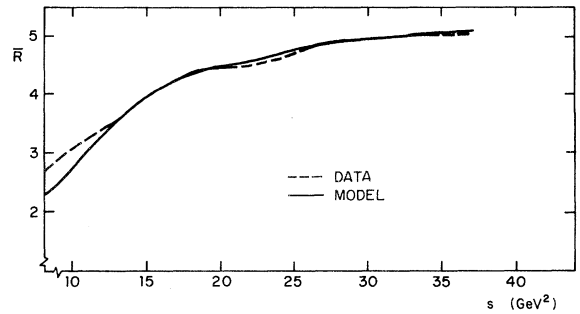

FIG. 17. The model of Fig. 16 with an additional heavy lepton, $m_l = 1.7$ GeV.

is that they need a new particle—quark or lepton—between 2 and 3 GeV to account for the "shoulder" in the smeared data at $s \approx 25$ GeV. The value of the mass for this additional particle is rather sensitive to our approximation to the data, but the need for such a second threshold appears to persist. The present state of the data is not such that we can be sure of this conclusion, but a reduction of the experimental errors in this region could possibly answer this question before long.

It is principally this feature of sensitivity to thresholds in the data which makes our technique an improvement over previous dispersion treatments, in which e^+e^- annihilation data were compared with quark perturbation calculations in the spacelike region.[15] Such treatments cannot possibly distinguish between a threshold at $s = 11$ GeV^2 and one at around $s = 25$ GeV^2, whereas these are quite distinct in our analysis.

Furthermore, our analysis is less sensitive to the unmeasured region $s > 60$ GeV^2 than are these previous dispersion treatments. Let us assume for the sake of comparison that R is flat at a value R_∞ throughout this region. The contribution

TABLE I. Model parameters. Summary of parameter values for Figs. 6 through 17. All these curves were calculated using charges $-\frac{1}{3}$, $\frac{2}{3}$, $-\frac{1}{3}$, and $\frac{2}{3}$ and masses 0.35 GeV for the first three quark flavors and 1.66 GeV for the fourth. The parameters m_q and Λ shown in the table were chosen to achieve a reasonable fit if possible.

Fig. No.	No. of quark flavors	m_q (GeV)	$\lvert Q_q \rvert$	No. of heavy leptons	M_l (GeV)	Λ^2 (GeV^2)	α (9 GeV^2)	Comments
6	4	···	···	0	···	1.2	0.65	α too large and R is still too small at high s
7	4	···	···	1	1.7	1.2	0.65	Same as above
8	4	···	···	2	1.7, 2.5	0.3	0.39	Not bad
9	5	1.66	$\frac{1}{3}$	0	···	1.2	0.66	As for 6
10	5	2.5	$\frac{1}{3}$	0	···	1.2	0.66	As for 6
11	5	1.66	$\frac{2}{3}$	0	···	0.025	0.22	Not bad; rather small α
12	5	2.5	$\frac{2}{3}$	0	···	0.5	0.46	Wrong shape
13	5	1.66	$\frac{1}{3}$	1	1.7	0.3	0.39	Not bad
14	5	2.5	$\frac{1}{3}$	1	1.7	0.3	0.39	Looks best
15	5	1.66	$\frac{1}{3}$	1	2.0	0.6	0.49	Not bad
16	6	1.66, 2.5	$\frac{1}{3}, \frac{1}{3}$	0	···	1.2	0.67	As for 6
17	6	1.66, 2.5	$\frac{1}{3}, \frac{1}{3}$	1	1.7	0.05	0.25	Rather like 14; extra $\frac{1}{3}$-charge quark has little effect except to decrease required coupling

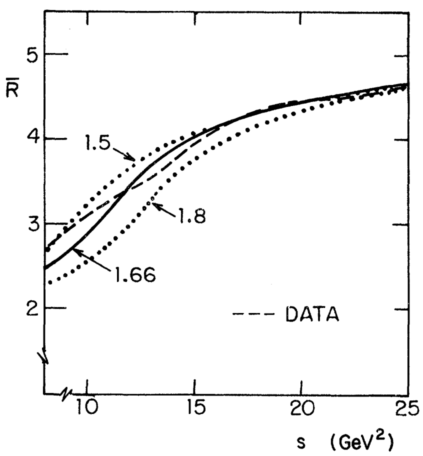

FIG. 18. Sensitivity of $\bar{R}(s,\Delta)$ to mass of fourth quark. The curves are calculated for the model of Fig. 14 varying only the quark mass parameter as shown.

of such an R to our $\bar{R}(s,\Delta)$ is

$$\frac{R_\infty}{\pi}\left[\frac{\pi}{2}-\tan^{-1}\left(\frac{60-s}{\Delta}\right)\right],$$

which gives $0.04R_\infty$ for the highest-s value used in our calculations. Making the same analysis for the quantity calculated in the Euclidean region at a value $s=-Q^2$ shows that the contribution of a flat $R=R_\infty$ from 60 GeV^2 to ∞ is $Q^2R_\infty/(60+Q^2)$ or approximately $0.15R_\infty$ at $Q^2=10$ GeV^2. This shows that our treatment is considerably less sensitive to contributions from that region and therefore to the assumptions we have made about the behavior of the data in that region. Also, the contribution of some new threshold at $s=s_0>60$ GeV^2 to the quantities we have calculated is negligible. Such higher thresholds cannot destroy the validity of our comparison of models with the present data.

It seems likely that there are other inclusive quantities that can similarly be predicted by smeared quark-gluon perturbation calculations without solving the problem of quark trapping. However, to calculate quantities relevant to specific final states requires a further understanding of that problem.

ACKNOWLEDGMENTS

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†Alfred Sloan Foundation Fellow.

[1]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, *ibid.* 30, 1343 (1973); Phys. Rev. D 8, 3497 (1973); T. A. Appelquist and H. Georgi, *ibid.* 8, 4000 (1973); A. Zee, *ibid.* 8, 4038 (1973).

[2]An analogous procedure was employed for similar reasons some time ago in the theory of nuclear reactions. Neutron cross sections typically exhibit many close narrow resonances, but the data, suitably smeared over many resonances, are well represented by a complex potential model. See F. L. Friedman and V. F. Weisskopf, in *Niels Bohr and the Development of Physics,* edited by W. Pauli (McGraw-Hill, New York, 1955), p. 147 ff., and references quoted therein.

[3]This statement can be made more precise by writing the vacuum polarization amplitude $\Pi(z)$ as an integral over Feynman parameters. The integral can be analytically continued from the negative real axis to any complex z value without distorting the contours of integration of the Feynman parameters away from the real axis, so that the imaginary part of z manifestly provides an infrared cutoff for the integral. See R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge Univ. Press, New York, 1966), pp. 84–85; T. T. Wu, Phys. Rev. 123, 678 (1961). Alternatively, it is possible to extend $\Pi(z)$ to complex z by writing it as an old-fashioned perturbation series; the imaginary part of z prevents the vanishing of any energy denominator.

[4]T. W. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); Phys. Rev. D 12, 1404 (1975). Much of our discussion parallels this work.

[5]J. Schwinger, *Particles, Sources, and Fields,* Vol. II (Addison-Wesley, New York, 1973), Chap. 5-4. See also G. Källén and A. Sabry, K. Dansk. Vidensk. Selsk. Mat.-Fys. Medd. 29, No. 17 (1955).

[6]See, e.g., S. Weinberg, Phys. Rev. 131, 440 (1963); 133, B232 (1964).

[7]T. Kinoshita, J. Math. Phys. 3, 650 (1962); see also T. D. Lee and M. Nauenberg, Phys. Rev. 133, B1544 (1964).

[8]The value of Δ we choose corresponds to $\alpha/v \simeq 1$ and $\alpha \ln v \simeq \alpha \ln\alpha$. For the range of values of α we find in reasonable models this gives $\alpha \ln v$ as large as 0.6. It is clear that in this approximation the neglected $\alpha \ln v$ terms may be almost as important in their contribution to the binding as the α/v terms which are summed to give the Coulomb formula. We allow for this by choosing Δ large enough that even if the Coulomb binding is increased by a factor of 2 by the corrections, our treatment remains valid. We believe this is a generous estimate. We remark also that setting α/v less than 1 is sufficient to render higher-order terms $(\alpha/v)^n$ negligible, as these terms appear with coefficients which decrease with increasing n. Specifically the series

begins $1+x/2+x^2/12+\cdots$, where $x=4\pi\alpha/3v$ (Ref. 5).

[9]Talk given by R. Schwitters at the International Symposium on Photon and Lepton Interactions at High Energies, Stanford, 1975 (unpublished).

[10]The introduction of a new quark at $m_q \simeq 2.5$ GeV raises the problem that no very narrow resonances like the J (ψ) have been observed in the neighborhood of 5 GeV. We believe that a charge $\frac{1}{3}$ quark is compatible with the data, since the resonance contribution from such a quark will be decreased by a factor of 4 compared to the fourth-quark resonances. Furthermore, any mixing of this state with any excited fourth-quark states with which it is nearly degenerate will presumably broaden the peak. This is a question which merits further study, but we nevertheless feel that it is quite reasonable, at this time, to introduce a quark in this region.

[11]The term $\ln[(5m_q{}^2+q^2)/(5m_q{}^2+\Lambda^2)]$ in our equation for α_s is an approximation to the integral

$$6\int_0^1 dx\,x(1-x)\ln\left[\frac{q^2x(1-x)+m_q{}^2}{\Lambda^2x(1-x)+m_q{}^2}\right],$$

which is the quark contribution to the renormalization-group β function. Note that when $q^2 \gg m_q{}^2$, $\Lambda^2 \gg m_q{}^2$, one recovers the standard light-quark result for α_s. This result has been independently derived by De Rújula and Georgi (see Ref. 15).

[12]The procedure for identifying "hadronic" events at SPEAR includes noncolinear $\mu^+\mu^-$ and e^+e^- pairs. Because of the missing neutrinos the leptonic decays of a pair of heavy leptons will produce such noncolinear pairs and therefore will be included in the quantity R as defined by SPEAR.

[13]For a review of models see the invited talk given by M. Barnett at the Sixth Hawaii Topical Conference on Particle Physics, Univ. of Hawaii, Honolulu, 1975 (unpublished); Phys. Rev. D 13, 671 (1976).

[14]G. Feldman, talk given at the International Symposium on Photon and Lepton Interactions at High Energies, Stanford University, 1975 (unpublished); invited talk at the 1975 Seattle Meeting of Division of Particles and Fields of the American Physical Society (unpublished).

[15]S. L. Adler, Phys. Rev. D 10, 3714 (1974); A. De Rújula and H. Georgi, *ibid.* 13, 1296 (1976).