Method for model-independent determination of the pion form factor

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The extrapolation method of Cutkosky and Deo is generalized by the use of biorthogonal polynomial expansions of "amplitudes" which are the sum of functions having different but known analytic structures. The method is used to determine the pion form factor from electroproduction without using dispersion relations. An analysis of the data of the Cambridge Electron Accelerator group at $K^2 = 0.354$, 0.426, 0.451, and 0.396 GeV^2 is carried out as an illustration, and we get fairly unique extrapolated values for F_π at the first three values of K^2.

I. INTRODUCTION

It has been pointed out by Kellett and Verzegnassi[1] and Dombey and Read[2] that there is inherent ambiguity in the determination of the pion form factor from model-dependent analyses of electroproduction data. The earlier authors[1] claim that the subtraction constant required in one of the dispersion relations involves an unknown function of t and K^2 which introduces an ambiguity in the determination of the pion form factor. Dombey and Read approach the problem from the standpoint of PCAC (partially conserved axial-vector current) and the current-algebra treatment of pion photoproduction and electroproduction. They point out that the theory, in general, involves terms depending on the axial-vector form factor and these should be accounted for by the theoretical models. This axial-vector form factor has been identified[2] with the coupling of the seagull term coming from the pseudovector interaction[3] scheme between pions and nucleons.

These difficulties do not arise if one takes recourse to a model-independent extrapolation procedure. Attempts in this direction have not yet been successful. Frazer[4] initiated the idea of extrapolation in 1959. The experimental data were scanty and he concluded that though in principle the Chew-Low extrapolation could determine the pion form factor from electroproduction, the data available were not accurate enough. Devenish and Lyth[5] performing the Chew-Low extrapolation on electroproduction data find that different polynomial fits, which are equally likely in the physical region, differ considerably when continued to the pion pole. They conclude that this method cannot by itself give a determination of F_π but if F_π is constrained to have the form $F_\pi=1/(1+K^2/m^2)$ then m^2 is found to lie in the range $0.3 < m^2 < 0.55$ GeV^2. However, the Chew-Low method does not exploit analyticity to the fullest extent. The optimal extrapolation technique devised by Cutkosky and Deo[6,7] and Ciulli[8] exploits analyticity maximally as a result of which the predicted value of an analytic function at some point has the maximum stability with respect to the errors in the input information. This technique has successfully been applied to the determination of ΛpK (see Refs. 6 and 9) and πNN (see Refs. 6 and 10) coupling constants. Recently, Nerciu *et al.*, Raszillier *et al.*, and also Kellett and Verzegnassi[11] have made an attempt to apply the optimal extrapolation technique to the determination of the pion form factor from charged pion electroproduction. They find little indication of a preferred value for F_π within the range of $0 \leq F_\pi \leq 2$ and suggest that the knowledge of some additional constraint or information in the unphysical region will be necessary.

In general the virtual photoproduction cross section can be written as [see Eq. (A5) of the Appendix]

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_T}{d\Omega} + \frac{d\sigma_0}{d\Omega} + \cos 2\phi \sin^2\theta\, T + [\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2} \cos\phi \sin\theta\, S\,, \tag{1}$$

where $d\sigma_T/d\Omega$, $d\sigma_0/d\Omega$, T, and S are functions of s, t, and u containing the poles and cuts. $d\sigma/d\Omega$ is an example of an analytic function which is a sum of analytic functions having different cut structures. In any analytic extrapolation procedure, the presence of the terms $\cos\phi \sin\theta$ and $\cos 2\phi \sin^2\theta$ has to be taken seriously. Kellett and Verzegnassi, in their analysis, have not accounted for these terms. Since a fixed $\sin\theta$ term gives a branch cut in the $\cos\theta$ plane, any polynomial fit will not converge. Frazer[4] and Devenish and Lyth[5] have emphasized that this additional singularity must be removed in some way before an extrapolation is attempted. This provided us with the motivation for reexamining the possibility of applying optimal polynomial extrapolation techniques to the electroproduction data with proper accounting of the terms proportional to $\sin\theta$ in the cross section.

In Sec. II we give the kinematics and the essentials of the method, followed in Sec. III by a discussion of the results. An appendix at the end gives the formulas used for calculating the virtual photoproduction cross section.

II. KINEMATICS AND THE METHOD OF EXTRAPOLATION

We are considering the virtual photoproduction reaction

$$\gamma^{\text{virtual}}(K)+p(P_1)\to\pi^+(Q)+n(P_2). \qquad (2)$$

The four-momentum of each particle is indicated in parentheses, and $P_1^2=P_2^2=-M^2$, $Q^2=-m_\pi^2$. The Mandelstam variables are defined as

$$\begin{aligned} s&=-(K+P_1)^2=-(Q+P_2)^2,\\ t&=-(K-Q)^2=-(P_2-P_1)^2,\\ u&=-(K-P_2)^2=-(Q-P_1)^2. \end{aligned} \qquad (3)$$

In the center-of-mass system, define

$$\begin{aligned} K&=(\vec{k},k_0),\\ P_1&=(-\vec{k},E_1),\\ Q&=(\vec{q},q_0),\\ P_2&=(-\vec{q},E_2). \end{aligned} \qquad (4)$$

θ, the c.m. angle between the pion and the virtual photon, is given by

$$\begin{aligned}\cos\theta&=\frac{t-m_\pi^2+K^2+2k_0q_0}{2|\vec{k}||\vec{q}|}\\ &=\frac{-(u-M^2+K^2+2k_0E_2)}{2|\vec{k}||\vec{q}|}. \end{aligned}\qquad (5)$$

In the complex $\cos\theta$ plane there are the π^+ exchange pole in the t channel and the neutron exchange pole in the u channel. The left- and right-hand cuts start from the values of $\cos\theta$ obtained by putting $u=(M+m_\pi^2)^2$ and $t=(2m_\pi)^2$, respectively, in Eq. (5). The expression for the differential cross section [Eq. (A5) of the appendix] reveals an additional singularity. There is a term proportional to $\sin\theta$ which introduces branch points at $\cos\theta=\pm1$. Frazer eliminated this undesirable $\sin\theta$ by defining a "symmetrized" cross section

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle_{\text{sym}}=\frac{d\sigma}{d\Omega}(\theta)+\frac{d\sigma}{d\Omega}(-\theta). \qquad (6)$$

An equivalent definition is

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle_{\text{sym}}=\frac{d\sigma}{d\Omega}(\phi)+\frac{d\sigma}{d\Omega}(\phi+\pi). \qquad (7)$$

One may also choose[5] data at $\phi=\pi/2$ so that terms proportional to $\sin\theta$ are absent. We propose a different approach, that of calculating the polynomials multiplying the $\sin\theta$ terms explicitly, rather than eliminating them by another extrapolation to $\phi+\pi$ or by the use of the Berends[12] dispersion relation.

For a successful extrapolation, the present method requires cross-section data at a large number of angles for fixed values of $-K^2$, W, and ϵ. Therefore, from among all the data available, we have chosen those of the Cambridge Electron Accelerator (CEA) group[13] with an intention of establishing a model-independent method of extrapolation.

Since these data are given for a small range of θ in the forward direction ($1.8°\le\theta\le21.0°$), we make the mapping to spread them over the entire physical region ($-1\le x\le1$)

$$x=\frac{2\cos\theta-\cos\theta_1-\cos\theta_N}{\cos\theta_1-\cos\theta_N}, \qquad (8)$$

where the subscripts 1 and N refer to the first and the last data points, respectively. Table I gives the positions of the singularities in the $\cos\theta$ and x planes. Evidently, one of the effects of this mapping is to increase the separation between the positions of the pion pole and the 2π cut thereby making the pole more "visible" to the extrapolation procedure.

Consider a quantity

$$f(x,F_\pi)=(x-x_\pi)\left(\frac{d\sigma}{d\Omega}\bigg|_{\text{exp}}-\frac{d\sigma}{d\Omega}\bigg|_{\text{Born}}\right), \qquad (9)$$

where x_π is the position of the pion pole in the x plane. If F_π is chosen incorrectly in the Born cross section, $f(x,F_\pi)$ has a first-order pole at $x=x_\pi$ which, since it is the singularity nearest to the physical region, defines the *figure of conver-*

TABLE I. Position of the singularities in the $\cos\theta$ and x planes.

K^2 (GeV^2)	θ_1 (deg)	θ_N (deg)	$\cos\theta$ plane t_{pole}	t_{cut}	u_{pole}	u_{cut}	x plane t_{pole}	t_{cut}	u_{pole}	u_{cut}
0.354	1.8	16.2	1.012	1.040	−1.334	−1.472	1.649	3.104	−118.0	−125.0
0.396	1.8	15.0	1.016	1.050	−1.377	−1.539	2.026	4.040	−140.5	−150.2
0.426	1.8	19.8	1.022	1.062	−1.416	−1.605	1.792	3.133	−81.41	−87.86
0.451	1.8	21.0	1.030	1.075	−1.452	−1.669	1.948	3.316	−73.38	−79.96

gence[14] of the polynomial sequence that is made to approximate $f(x,F_\pi)$. However, if F_π is correctly chosen, $f(x,F_\pi)$ becomes free from the pion pole, and the figure of convergence gets enlarged to touch the next nearest singularity, which is the start of the 2π cut in the t channel. The enlargement of the figure of convergence is manifested in an increased rate of convergence of the polynomial series. Thus, when χ^2 of the fit is plotted against $F_\pi(K^2)$ for a given order of approximation, it exhibits a dip at the correct value of $F_\pi(K^2)$.

CEA data are given at both $\phi=0°$ and 180° for fixed values of $-K^2$, ϵ, and W. For these data $f(x,F_\pi)$ has the general form

$$f(x,F_\pi)=P(x)+\sin\theta\cos\phi\, Q(x). \tag{10}$$

Following a suggestion by Cutkosky[15] we use Chebyshev polynomials to approximate the functions $P(x)$ and $Q(x)$:

$$f(x,F_\pi)=\sum_n a_n T_n(x)+\sin\theta\cos\phi\sum_n b_n T_n(x). \tag{11}$$

The coefficients a_n and b_n are found with the help of *biorthogonal functions*[16] as described below. We have

$$f(x,F_\pi)=\sum_n c_n h_n(x), \tag{12}$$

where

$$\begin{aligned} h_1(x)&=T_0(x),\\ h_2(x)&=T_0(x)\sin\theta\cos\phi,\\ h_3(x)&=T_1(x),\\ h_4(x)&=T_1(x)\sin\theta\cos\phi,\\ &\cdots. \end{aligned} \tag{13}$$

The h_n's are known but nonorthogonal functions. The c_n's can be evaluated by finding a set of biorthogonal functions $\psi_n(x)$ which satisfy the condition

$$(\psi_n(x),h_m(x))=\delta_{nm}. \tag{14}$$

Then

$$c_n=(\psi_n(x),f(x,F_\pi)). \tag{15}$$

Klink[17] has given the following prescription for evaluating the ψ's: If

$$(h_n(x),h_m(x))=\lambda_{nm}, \tag{16}$$

then

$$\psi_n(x)=\sum_j(\lambda^{-1})_{nj}h_j(x).$$

We determine the ψ's as follows. The complete set of functions $h_1(x),h_2(x),\ldots,h_l(x)$ is orthogonalized by the Schmidt procedure to give orthogonal polynomials $\mathcal{P}_1(x),\mathcal{P}_2(x),\ldots,\mathcal{P}_l(x)$. Then $(\mathcal{P}_l(x),h_j(x))=0$ for $j=1,2,\ldots,l-1$ since $h_j(x)$ can be represented as a linear combination of $\mathcal{P}$'s of order less than l. Thus $\mathcal{P}_l(x)$ is to be identified as the biorthogonal function $\psi_l(x)$. Therefore, to obtain any $\psi_k(x)$, we treat $h_k(x)$ as being of the highest order among the basic set of functions. Then the Schmidt procedure gives $\psi_k(x)$ as the highest-order polynomial constructed out of this basic set.

However, in order to make use of the biorthogonal functions, we must first know the complete set of linearly independent functions that can represent $f(x,F_\pi)$ adequately. If the data were free from experimental errors, we would have to take N such functions. However, for error-affected data the number of linearly independent functions required is much less. To determine this number, we adopt the following procedure.

We choose the set of nonorthogonal functions

$$T_0(x),\,T_0(x)\sin\theta\cos\phi,\,T_1(x),\,T_1(x)\sin\theta\cos\phi,\ldots, \tag{17}$$

and construct out of them orthogonal polynomials by the Schmidt procedure. $f(x,F_\pi)$ are fitted with a series of these polynomials and the number (say, l) of terms for which χ^2 of the fit falls below NDF (number of degrees of freedom) is taken, for practical purposes, to comprise the complete set of functions required for constructing the biorthogonal functions.

$f(x,F_\pi)$ can also be represented by an expansion in a complete set of the ψ's, i.e.,

$$\begin{aligned} f(x,F_\pi)&=\sum_{n=1}^{N}c_n h_n(x)\\ &=\sum_{m=1}^{N}\alpha_m\psi_m(x), \end{aligned} \tag{18}$$

where

$$\alpha_m=(f,h_m)\text{ and }c_m=(f,\psi_m). \tag{19}$$

Then, χ^2 for an l term approximation to $f(x,F_\pi)$ is given by

$$\chi_l{}^2=\sum_i\frac{1}{(\Delta f_i)^2}\left[f(x_i,F_\pi)-\sum_{n=1}^{l}c_n h_n(x_i)\right]\times\left[f(x_i,F_\pi)-\sum_{m=1}^{l}\alpha_m\psi_m(x_i)\right], \tag{20}$$

where i runs over all data points:

$$\chi_l{}^2=\sum_i\frac{1}{(\Delta f_i)^2}\sum_{n=l+1}^{N}c_n h_n(x_i)\sum_{m=l+1}^{N}\alpha_m\psi_m(x_i). \tag{21}$$

Using

$$\sum_i \frac{1}{(\Delta f_i)^2} h_n(x_i)\psi_m(x_i)$$

as the definition of the scalar product $(h_n(x_i),\psi_m(x_i))$ we obtain

$$\chi_l{}^2 = \sum_{n=l+1}^{N} c_n \alpha_n . \qquad (22)$$

To maximize the rate of convergence of the expansions in Chebyshev polynomials we then performed the elliptical mapping of Cutkosky and Deo[7] and repeated the whole extrapolation procedure in the mapped variable.

III. RESULTS AND DISCUSSION

In our analysis we have used the virtual photoproduction cross-section data of the CEA group[13] at $K^2 = 0.396$, 0.354, 0.426, and 0.451 GeV^2 for fixed values of ϵ, W, and $\phi = 0, \pi$. For all these data it was found that five linearly independent functions were enough to give acceptable fits over a wide range of values for F_π.

With 5-term fits to $f(x, F_\pi)$, the following choice of terms was observed to yield the most rapidly converging sequence

$$(x - x_\pi)\left(\frac{d\sigma}{d\Omega}\bigg|_{\text{exp}} - \frac{d\sigma}{d\Omega}\bigg|_{\text{Born}}\right) = C_0T_0(x)\sin\theta\cos\phi + C_1T_0(x) + C_2T_1(x) + C_3T_2(x) + C_4T_3(x) . \qquad (23)$$

Comparing the coefficients of $\cos\phi$ on both sides of Eq. (23), we obtain an expression for the transverse-scalar interference term

$$D_{\text{exp}} = D_{\text{Born}} + \frac{C_0T_0(x)\sin\theta}{(x - x_\pi)[0.5\epsilon(1+\epsilon)]^{1/2}} , \qquad (24)$$

where $D = S\sin\theta$ and S is defined through Eq. (A5) of the Appendix.

In Fig. 1 the Born contribution for D is plotted (curve I) along with the expression on the right-hand side of Eq. (24) (curve II) for typical experimental data. The excellent agreement of curve II with experimental values indicates that the choice of the $\sin\theta$ terms in Eq. (23) has been adequate. Therefore, we carried out the extrapolation procedure with this choice of the terms.

χ^2 values given by the expression

$$\chi^2 = \sum_i \frac{[f_i - \sum_{j=1}^{N} c_j h_j(x_i)]^2}{(\Delta f_i)^2} , \qquad (25)$$

where i runs over all the data points for a given value of K^2, were determined for different values of F_π ranging from 0 to 1. Figures 2(a)–2(e) depict the plots of χ^2 versus $F_\pi(K^2)$ for those values of n for which the χ^2 curve exhibited a dip. The values of $F_\pi(K^2)$ determined from these curves are presented in Fig. 2(f) and Table II. The extrapolated value of F_π corresponds to χ^2_{min} in these curves. The error in the extrapolated value of F_π is obtained by noting the variation in F_π when χ^2 exceeds χ^2_{min} by 1. Unique values for $F_\pi(K^2)$ were obtained at $K^2 = 0.354$, 0.426, and 0.451 GeV^2. Only at $K^2 = 0.396$ GeV^2 are acceptable fits ($\chi^2_{\text{min}}/\text{NDF} < 1$) obtained for both 2 and 3 terms giving $F_\pi = 0.45 \pm 0.03$ and 0.59 ± 0.05, respectively. More accurate data may be able to distinguish between these two fits. Brown *et al.*[13] have obtained $F_\pi = 0.577 \pm 0.016$ at $K^2 = 0.396$ GeV^2, which agrees with our result obtained with a 3-term fit.

When the extrapolation was carried out in the elliptically mapped variable, the results were no better than those obtained with the variable x. This is due to the small number of terms required

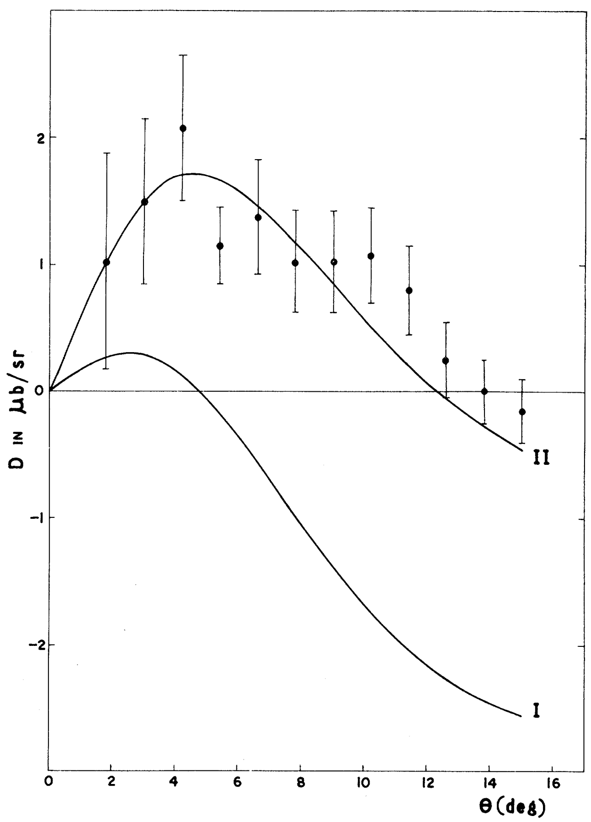

FIG. 1. A plot of the transverse-scalar interference term D for $W = 2.15$ GeV and $K^2 = 0.396$ GeV^2 versus θ. Curve I is the Born approximation and curve II is a plot of the expression on the right-hand side of Eq. (24). In calculating the theoretical curves F_π has been taken to be 0.6.

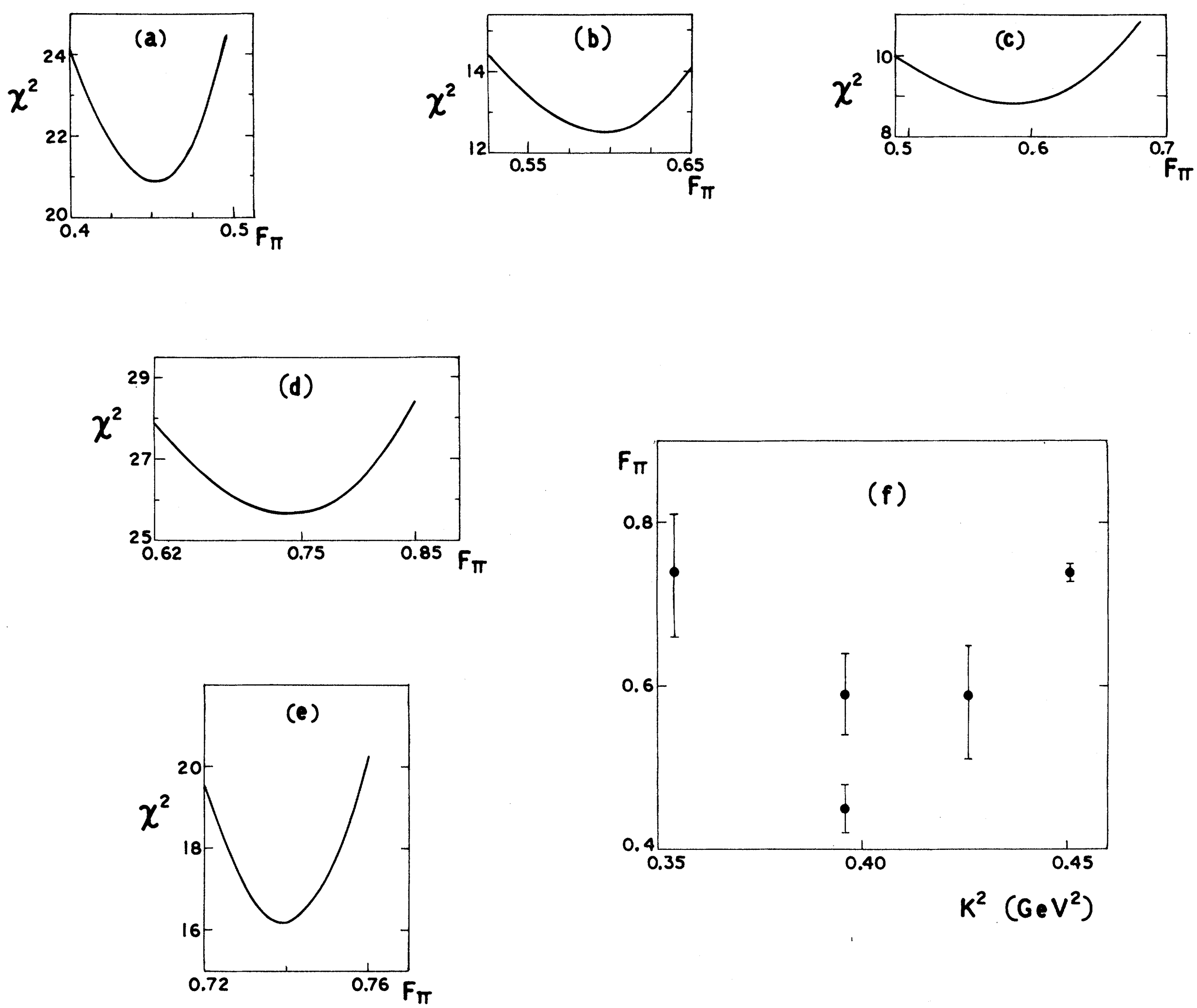

FIG. 2. Plot of χ^2 versus $F_\pi(K^2)$ for (a) $K^2 = 0.396$ GeV^2 with $n=2$, (b) $K^2=0.396$ GeV^2 with $n=3$, (c) $K^2=0.426$ GeV^2 with $n=3$, (d) $K^2=0.354$ GeV^2 with $n=3$, and (e) $K^2=0.451$ GeV^2 with $n=1$. (f) depicts values of $F_\pi(K^2)$ calculated from the curves in (a)–(e).

for a good fit. It is to be noted that the data have already been treated and extrapolated by the CEA group and their extrapolation has been done in the normal $\cos\theta$ plane. Further, they have only quoted values of $\phi=0$ and $\phi=\pi$. If there were a good spread in the values of ϕ, this method of extrapolation using biorthogonal functions could give more unambiguous results. For such data Eq. (10) gets modified to

$$f(x,F_\pi)=P(x)+\sin\theta\cos\phi\, Q(x)+\cos2\phi\, R(x).$$

One can then approximate $P(x)$, $Q(x)$, and $R(x)$ by series in Chebyshev polynomials and carry out extrapolation as in the present analysis.

In conclusion we wish to remark that even though the elliptical mapping of Cutkosky and Deo has not been helpful in the present analysis, it has been shown that the Cutkosky-Deo extrapolation procedure has distinct advantages over the Chew-Low procedure in that unambiguous values for $F_\pi(K^2)$ are obtained at three out of the four values of K^2 considered.

ACKNOWLEDGMENT

The use of computational facilities at the Computer Centre of the Utkal University is gratefully acknowledged.

TABLE II. Result of extrapolation in the x plane.

K^2 (GeV^2)	Number of data points	n, the number of terms in the fitted series	$F_\pi(K^2)$	$\chi^2_{\min}$/NDF
0.354	20	3 [Fig.2(d)]	$0.74^{+0.07}_{-0.08}$	1.5
0.396	24	2 [Fig.2(a)]	0.45 ± 0.03	0.9
		3 [Fig.2(b)]	0.59 ± 0.05	0.6
0.426	23	3 [Fig.2(c)]	$0.59^{+0.06}_{-0.08}$	0.4
0.451	17	1 [Fig.2(e)]	0.74 ± 0.01	1.01

APPENDIX

Invariant amplitudes. The contributions of the Born poles to the invariant amplitudes of Dennery[18] are

$$
\begin{aligned}
A_1^- &= \tfrac{1}{2} eg F_1^v(K^2)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right),\\
A_1^0 &= \tfrac{1}{2} eg F_1^s(K^2)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right),\\
A_2^- &= -eg F_1^v(K^2)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right)\frac{1}{t-m_\pi{}^2},\\
A_2^0 &= -eg F_1^s(K^2)\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)\frac{1}{t-m_\pi{}^2},\\
A_3^- &= -\tfrac{1}{2} eg F_2^v(K^2)\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right),\\
A_3^0 &= -\tfrac{1}{2} eg F_2^s(K^2)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right),\\
A_4^- &= -\tfrac{1}{2} eg F_2^v(K^2)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right),\\
A_4^0 &= -\tfrac{1}{2} eg F_2^s(K^2)\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right),\\
A_5^- &= -\tfrac{1}{2} eg F_1^v(K^2)\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)\frac{1}{t-m_\pi{}^2}\\
&\quad+\frac{2eg}{K^2}\,\frac{[F_\pi(K^2)-F_1^v(K^2)]}{(t-m_\pi{}^2)},\\
A_5^0 &= \tfrac{1}{2} eg F_1^s(K^2)\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right)\frac{1}{(t-m_\pi{}^2)},\\
A_6^- &= A_6^0 = 0.
\end{aligned}
\tag{A1}
$$

For π^+ electroproduction $A_i=\sqrt{2}\,(A_i^- + A_i^0)$, $i=1,2,\ldots,6$.

Form factors. $F_\pi(K^2)$ is the pion electromagnetic form factor, $F_1^{(v,s)}(K^2)$ and $F_2^{(v,s)}(K^2)$ are the Dirac and Pauli form factors for the nucleon with the superscripts v and s denoting isovector and isoscalar states respectively. These form factors are normalized as follows:

$$
\begin{aligned}
&F_\pi(0)=1,\\
&F_1^{(v,s)}(0)=1,\\
&F_2^v(0)=\frac{1}{2M}(\kappa_p-\kappa_n)\\
&F_2^s(0)=\frac{1}{2M}(\kappa_p+\kappa_n),
\end{aligned}
\tag{A2}
$$

where κ_p and κ_n are the anomalous magnetic moments of the proton and neutron, respectively.

In terms of the Sachs form factors $G_{E,M}^{(v,s)}$ the Dirac and Pauli form factors of the nucleon read

$$
\begin{aligned}
&F_1^{(v,s)}=\left(G_E^{(v,s)}+\frac{K^2}{4M^2}G_M^{(v,s)}\right)\bigg/\left(1+\frac{K^2}{4M^2}\right),\\
&2MF_2^{(v,s)}=(G_M^{(v,s)}-G_E^{(v,s)})/(1+K^2/4M^2),
\end{aligned}
\tag{A3}
$$

where

$$G_M^{(v,s)}=G_{Mp}\mp G_{Mn}$$

and

$$G_E^{(v,s)}=G_{Ep}\mp G_{En}.$$

For the Sachs form factors we use the "scaling law" and dipole fit

$$
\begin{aligned}
\frac{G_{Mp}(K^2)}{1+\kappa_p}&=\frac{G_{Mn}(K^2)}{\kappa_n}\\
&=G_{Ep}(K^2)\\
&=\frac{1}{(1+K^2/0.71\ \text{GeV}^2)^2}
\end{aligned}
\tag{A4}
$$

and assume that $G_{En}(K^2)=0$.

Cross-section formulas. Following Brown *et al.*[13] we use the conventions of Berends[12] for defining the cross sections. The virtual-photon differential cross section in the πN c.m. system can be written as

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_T}{d\Omega}+\epsilon\frac{d\sigma_0}{d\Omega}+\cos 2\phi\sin^2\theta\, T+[\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2}\cos\phi\sin\theta\, S\,, \tag{A5}$$

where θ is the π scattering angle in the πN c.m. system and ϕ is the angle between the πN plane and the electron scattering plane. Donnachie[19] has summarized the formulas that relate $d\sigma_T/d\Omega$, $d\sigma_0/d\Omega$, T, and S to the invariant amplitudes.

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