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Black holes and thermodynamics*

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A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole. If one makes the hypothesis that the entropy is finite, one can deduce that the black holes must emit thermal radiation at some nonzero temperature. Conversely, the recently derived quantum-mechanical result that black holes do emit thermal radiation at temperature $\kappa\hbar/2\pi kc$, where κ is the surface gravity, enables one to prove that the entropy is finite and is equal to $c^3A/4G\hbar$, where A is the surface area of the event horizon or boundary of the black hole. Because black holes have negative specific heat, they cannot be in stable thermal equilibrium except when the additional energy available is less than 1/4 the mass of the black hole. This means that the standard statistical-mechanical canonical ensemble cannot be applied when gravitational interactions are important. Black holes behave in a completely random and time-symmetric way and are indistinguishable, for an external observer, from white holes. The irreversibility that appears in the classical limit is merely a statistical effect.

I. INTRODUCTION

The aim of this paper is to discuss some consequences of the recently discovered quantum effects on black holes. According to the classical theory of general relativity, a gravitationally collapsing star of mass M will shrink, in a short time as measured by an observer on the surface, to a radius of order $2GM/c^2$, at which the gravitational field becomes so strong that no further radiation or anything else can escape to infinity. The region of space-time from which it is not possible to escape to infinity is said to be a *black hole*, and its boundary is an outgoing null hypersurface, called the *event horizon*, which just fails to reach infinity. To an observer at infinity the star will appear to take an infinite time to reach the event horizon. However, as the observer can only ever receive the finite number of photons emitted by the star before it crossed the event horizon, the luminosity of the star appears to him to decrease exponentially with a time scale of order

$$\frac{2GM}{c^3} \sim 10^{-5}\left(\frac{M}{M_\odot}\right) \text{sec.}$$

Thus, after a few milliseconds the star has effectively gone out and what remains is an object which still exerts gravitational influence and which is aptly named a black hole.

This is the classical picture as described by, for example, Misner, Thorne, and Wheeler.[1] However, when quantum effects are taken into account it turns out that a "black hole" is not completely black: Radiation tunnels out through the event horizon and escapes to infinity at a steady rate.[2,3] (These results have been confirmed by several other authors.[4-6]) Even more remarkable than this steady rate of emission is that it turns out to have an exactly thermal spectrum: The expectation value $\langle N\rangle$ of the number of particles of a given species emitted in a mode with frequency ω, angular momentum $m\hbar$ about the axis of rotation of the hole, and charge e is

$$\langle N\rangle = \Gamma\{\exp[k^{-1}T^{-1}(\omega - m\Omega - e\Phi)] \mp 1\}^{-1}. \quad (1)$$

In this expression the − sign is for bosons and the + sign is for fermions. The quantity Γ is the fraction of the mode that would be absorbed were it incident on the black hole. The temperature is $T=\kappa\hbar/2\pi kc$, where κ is the *surface gravity* of the black hole. Ω is the angular frequency of rotation of the black hole and Φ is the potential of the event horizon. For a black hole with mass M, angular momentum J, and charge Q these quantities are

$$\kappa = \frac{4\pi(r_+c^2 - GM)}{A}, \quad (2)$$

$$\Omega = \frac{4\pi J}{MA}, \quad (3)$$

$$\Phi = \frac{4\pi Q r_+}{A}, \quad (4)$$

where

$$r_+ = c^{-2}[GM + (G^2M^2 - J^2M^{-2}c^2 - GQ^2)^{1/2}] \quad (5)$$

and

$$A = 4\pi G c^{-4}[2GM^2 - Q^2 + 2(G^2M^4 - J^2c^2 - GM^2Q^2)^{1/2}] \quad (6)$$

is the area of the event horizon. The radiation is described not by a pure quantum state but by a density matrix. It is completely thermal in that the probabilities of the emission of particles in different modes and the probabilities of emitting different numbers of particles in the same mode are completely uncorrelated.[7] The probabilities for different numbers of particles agree exactly with thermal radiation.[7-9]

Bekenstein[10,11] was the first to suggest that some multiple of κ should be regarded as representing in some sense the temperature of black holes. He also pointed out that one has the relation[10,12]

$$d(Mc^2) = \frac{\kappa c^2}{8\pi G}\, dA + \Omega\, dJ + \Phi\, dQ \quad (7)$$

which connects the difference in energy of two nearby black-hole equilibrium states to the differences in the area A of the event horizons, in the angular momentum J, and in the charge Q. This is very similar to the first law of thermodynamics,

$$dU = TdS - PdV, \quad (8)$$

suggesting that one should regard some multiple of A as the entropy of a black hole. Bekenstein therefore postulated a "generalized second law": [entropy of matter outside black holes] + [some constant times the sum of areas of black-hole event horizons] never decreases.

Bekenstein[10,11] was able to establish the validity of this law in certain *gedanken* experiments, but, because it was thought that black holes could not emit anything, he was not able to obtain a general proof, and indeed there appeared to be situations in which the law could be violated. With the discovery of the quantum thermal emission these violations no longer occur; a general proof of the second law will be given in Sec. II.

The fact that the temperature of a black hole decreases as the mass increases means that black holes cannot be in stable thermal equilibrium in the situations in which there is an indefinitely large amount of energy available. As described in Sec. III, this implies that the normal statistical-mechanical canonical ensemble cannot be applied to gravitating systems. Instead one has to use microcanonical ensembles in which one considers all the possible configurations of a system with a given energy. In Sec. IV this idea is applied to a *gedanken* experiment in which a certain amount of energy is placed in a box. One obtains the surprising conclusion that because a black hole can form by statistical fluctuations in black-body radiation and can then decay quantum mechanically with the reemission of radiation, the time-reverse process must also be possible in which a number of photons or gravitons annihilate to form a *white hole* which then explodes, emitting radiation. Thus the "cosmic censorship hypothesis" is violated by quantum effects. To an external observer a white hole is indistinguishable from a black hole. The process of hole formation and evaporation is completely time-symmetric. The irreversibility which arises in the classical limit is just a statistical effect.

In the rest of this paper dimensionless units will be used in which $G = c = \hbar = k = 1$. The unit of mass is then the Planck mass $\hbar^{1/2} G^{-1/2} c^{-1/2} \sim 10^{-5}$ g. The unit of length is the Planck length $\hbar^{1/2} G^{1/2} c^{-3/2} \sim 10^{-33}$ cm. The unit of temperature is the Planck temperature $k^{-1}\hbar^{1/2} G^{-1/2} c^{3/2} \sim 10^{32}$ °K.

II. THE GENERALIZED SECOND LAW

When a black hole is formed by gravitational collapse it settles down very rapidly to a quasi-stationary state characterized by only three parameters: the mass M, the angular momentum J, and the charge Q. (It is a quasistationary state because the mass, angular momentum, and charge are decreasing slowly because of the quantum emission.) This is known as the "no hair" theorem.[13-19] A black hole of given mass, angular momentum, and charge can have a very large number of unobservable internal configurations which reflect the different possible configurations for the body that collapsed. If quantum effects were neglected, the number of different internal configurations would be infinite because one could form the black hole out of an indefinitely large number of indefinitely small mass particles. However, Bekenstein[10] pointed out that the Compton wavelengths of these particles might have to be restricted to be less than the radius of the black hole and that therefore the number of possible internal configurations might be finite though very large.

Let $\sigma\, dM\, dQ\, d^3J$ be the number of internal configurations or quantum states of a black hole in the range M to $M + dM$, Q to $Q + dQ$ and angular momentum in the element d^3J about a given angular momentum $\vec{J}$. By the "no hair" theorems one has no information about the internal state of the hole, and therefore all these configurations are equally probable. Thus the entropy S_h of the black hole is

$$S_h = -\sum p_i \ln p_i = \ln\sigma. \quad (9)$$

One can also express the entropy in terms of the number of initial states that give rise to a black hole in the above range. There is a slight complication here in that a given initial state will give rise to a black hole only with some quantum-mechanical probability.

Let $\{|\alpha_i\rangle\}$ be a complete orthonormal basis of initial states and let $f_i\, V\, d^3P\, dM\, dQ\, d^3J$ be the probability that the initial state $|\alpha_i\rangle$ gives rise only to a black hole with parameters in the above range and in a normalization volume V with linear momentum in the element d^3P about zero. Then $q_i = f_i(\sum f)^{-1}$ is the probability that the given black hole arose from the initial state $|\alpha_i\rangle$. The entropy $S_h = -\sum q_i \ln q_i$.

The entropy S_h would have to be a function only of M, J, and Q with the following properties:

(1) It always increased when matter or radiation fell into the hole.

(2) When two black holes collided and merged together, the entropy of the final black hole was bigger than the sum of the entropies of the initial holes.

The only such quantity is a monotonic function $f(A)$ with $d^2f/dA^2 \geq 0$, where A is the area of the event horizon. The simplest such function is γA, where γ is a constant. Bekenstein suggested the value of $(\ln 2)/8\pi$ in dimensionless units for γ. (It can now be seen that the correct value is $\gamma = \frac{1}{4}$.)

In order to show that the hypothesis of a finite number of internal configurations is consistent one has to show that if the parameters change by amounts ΔM, ΔJ, and ΔQ by the accretion of more matter and radiation, the new value of σ is at least the old value times the number of possible configurations of the accreting matter or radiation. This is equivalent to showing that the increase in S_h is greater than or equal to the amount of entropy of the accreting matter. In other words, one has to prove the generalized second law: $S_h + S_m$ never decreases with time (where S_m is the entropy of matter and radiation outside black holes).

Given the entropy of a system as a function of the energy E of the system and various other macroscopic parameters, one can define the temperature as $T^{-1} = \partial S/\partial E$. Thus one can define the temperature of a black hole to be

$$T_h^{-1} = \left(\frac{\partial S_h}{\partial M}\right)_{J,Q} . \tag{10}$$

The generalized second law is then equivalent to the requirement that heat should not run uphill from a cooler system to a warmer one.

Consider a situation in which a black hole is surrounded by blackbody radiation at some temperature T_m. The blackbody radiation is here taken to mean all the possible species of particles (both zero and nonzero rest mass) in thermal equilibrium with zero chemical potentials. For any nonzero T_m there will be some rate of accretion of this radiation into the black hole. If $T_m > T_h$, it follows from the definition of temperature that the decrease of S_m caused by the accreting radiation is less than the increase in S_h. Thus the generalized second law holds. However, if $T_m < T_h$, the accretion violates the law. There are only two ways in which consistency can be maintained: Either T_h is identically zero, in which case S_h is infinite and the concept of black-hole entropy is meaningless, or black holes have to emit thermal radiation with some finite nonzero temperature. The first case is what holds in purely classical theory, in which black holes can absorb but do not emit anything. Bekenstein ran into inconsistencies because he tried to combine the hypothesis of finite entropy with classical theory, but the hypothesis is viable only if one accepts the quantum-mechanical result that black holes emit thermal radiation. Conversely, the fact that the black holes emit quantum radiation with a temperature $T_h = \kappa/2\pi$ enables one to prove the generalized second law and hence establish that the entropy of a black hole is finite. If $T_m > T_h$ the accretion is greater than the emission and hence the increase in S_h is greater than the decrease in S_m, while if $T_m < T_h$ the emission is greater than the accretion and the increase in S_m is greater than the decrease in S_h. If the accreting matter or radiation is not in thermal equilibrium with zero chemical potentials at some temperature, its ratio of entropy density to energy density is not as high as it would be if it were blackbody radiation. Hence the accretion is even further from violating the generalized second law than in the case considered above.

The quantum result that the temperature is $\kappa/2\pi$ allows one to integrate the first law of black holes [Eq. (7)] and deduce that the entropy $S_h = \frac{1}{4}A + \text{constant}$. If one makes the reasonable assumption that the entropy tends to zero as the mass tends to zero, the constant must be zero. One might wonder why the value of entropy does not depend upon the details of how many elementary particles there are. For example, if there were 10^9 different kinds of neutrinos, then one might think that the number of different ways one could make a black hole would be multiplied by about $(10^9)^{S_h}$ and so the entropy would be increased by about $S_h \ln 10^9$. The answer is that if there were 10^9 different kinds of neutrinos, a black hole would emit them *all* thermally and so its rate of energy loss would be greatly increased. This means that, if one wished to increase the mass of a black hole, one would have to throw neutrinos in at a much higher rate in order to beat the emission. The fact that the neutrinos have to be accreted in a shorter time means that there are fewer possible configurations for them. Thus the number of ways in which the mass of black holes can be increased is independent of the spectrum of elementary particles. However, the more species there are the harder it is to form black holes, because they radiate faster. If the spectrum of elementary particles increases exponentially, as is suggested by the statistical bootstrap[20,21] and dual resonance models of strong interactions, it may be impossible[3] to form black holes of less than about 5×10^{13} g.

A particularly interesting case of accretion and emission is that in which the particles themselves are small black holes. The number of internal configurations for a black hole in the mass range m to $m+dm$ is

$$\int_{-m}^{m}\int_{0}^{m^2} e^S\,dQ\,J^2 dJ\,dm \sim m^3 \exp(4\pi m^2)dm. \quad (11)$$

The rate of emission of small black holes in this mass range will therefore be greater by this factor than the emission of a single species of particle of mass m, which will be governed by the thermal factor $[\exp(8\pi mM)-1]^{-1}$ for a nonrotating hole of mass M. Thus, since $M\gg 1$ and $m>1$ the rate of emission will be proportional to

$$m^3 \exp[4\pi m(m-2M)]\,dm. \quad (12)$$

This shows that a black hole emits small black holes at a rate which is exponentially small since m will be less than M. In other words, the probability of a black hole bifurcating quantum-mechanically is very small. Classically, bifurcation is completely forbidden.[13]

III. THERMAL EQUILIBRIUM

Consider a black hole surrounded by blackbody radiation in a large container at the same temperature as the black hole. In order to be in thermal equilibrium, the black hole must be nonrotating and electrically neutral since otherwise it will preferentially emit particles with its sign of angular momentum or charge. Suppose now that, as a result of a statistical fluctuation, the black hole accretes a bit more energy than it emits. Because black holes have negative specific heat, the temperature of the black hole will go down. This will decrease the rate of emission and slightly increase the rate of absorption of a black hole. If the blackbody radiation is maintained at a constant temperature by some reservoir of energy, the black hole will grow indefinitely. Similarly, if a statistical fluctuation caused the black hole to emit slightly more than it absorbs, the emission rate would continue to rise until the black hole disappeared completely. In other words, black holes cannot be in stable thermal equilibrium with an indefinitely large reservoir of energy. The consequence of this is that one cannot use the normal statistical-mechanical canonical ensemble when gravitational interactions are important. In the canonical ensemble one considers a very large number n of similar systems loosely coupled together. Each system is supposed to have a number of energy levels E_i, and the total energy of the whole collection of systems has some given very large value E. By considering all the ways in which this total energy can be distributed among the various systems, one finds that the expected number n_i of systems in a given energy state E_i is proportional to $\exp(-E_i T^{-1})$, where T is a Lagrange multiplier which is interpreted as the temperature of the ensemble. Now suppose that the number of energy levels of one of the systems between E and $E+dE$ is $\rho(E)dE$. Then the probability of the system having energy in the range E to $E+dE$ is $\rho(E)\exp(-ET^{-1})dE$. In the systems that are commonly considered the density of energy levels $\rho(E)$ increases with energy but not exponentially, so the probability converges. However, for black holes, $\rho(E)dE=$ number of internal configurations of black holes with masses between $M=E$ and $M=E+dE$ is

$$\int\int e^S J^2 dJ\,dQ\,dE \sim E^3 \exp(4\pi E^2)dE. \quad (13)$$

This grows faster than the thermal factor $\exp(-ET^{-1})$ goes down, so that the probability of a black hole being in a given interval of mass increases with the mass, and the total probability diverges, indicating a breakdown of the canonical ensemble. In any system which includes gravitational interactions there will always be the possibility of forming black holes by a statistical fluctuation causing many particles to get together in a small volume of space. Thus one cannot, strictly speaking, apply the concept of a canonical ensemble to such systems.

Although the canonical ensemble does not work for black holes, one can still employ a microcanonical ensemble of a large number of similar insulated systems each with a given fixed energy E. Each of these systems will have a number of different configurations compatible with the given energy. These configurations will form a surface in the configuration space of the system. As time passes the system will move from one point to another on this surface. By the assumption of ergodicity, the probability of finding a given system in a given region of configuration space is proportional to the number of configurations in that region compatible with the given energy.

In most cases there will be one macroscopic state that has many more possible microscopic configurations than any other macroscopic state. Consider, for example, a certain amount of energy E placed in an insulated box of volume V. Assume, for simplicity, that this energy can be distributed only among gravitons and black holes: Either all the energy could be in gravitons or the energy could be divided among gravitons and one or more black holes. For a given energy E_1 in gravitons, the number of microscopic configurations is a sharp maximum when the gravitons are distributed as blackbody radiation at a temperature

$$T_1=\left(\frac{15E_1}{\pi^2 V}\right)^{1/4} . \tag{14}$$

Thus a good estimate of the number of configurations of gravitons with energy E_1 in a volume V is $\exp S_1$, where

$$S_1=\frac{4\pi^2 V T_1^{\,3}}{45} \tag{15}$$

is the entropy of blackbody radiation at temperature T_1. The number of internal configurations of black holes with total energy E_2 will be $\exp S_2$, where $S_2=\frac{1}{4}\sum A$ is the entropy of the black holes. One can see immediately that the probability of having more than one black hole is very small, since for a given E_2 the entropy S_2 is greatest when there is only one hole. The entropy is also greatest when this one black hole is nonrotating and uncharged. Thus, one can take $\exp(4\pi E_2^{\,2})$ as a reasonable estimate for the number of configurations of the black hole. (One can ignore the factor of order VP^3 which arises from the motion of the black hole.) The total number of configurations ror the system with energy E_1 in gravitons and energy E_2 in black holes is $\exp(S_1+S_2)$. The most probable values of E_1 and E_2 will be those which maximize S_1+S_2 subject to the constraint $E_1+E_2=E$; i.e.,

$$(1)\ \frac{\partial S_1}{\partial E_1}-\frac{\partial S_2}{\partial E_2}=0 ,$$

$$(2)\ \frac{\partial^2 S_1}{\partial E_1^{\,2}}+\frac{\partial^2 S_2}{\partial E_2^{\,2}}<0 .$$

Condition (1) implies $T_1=T_2$; i.e., the black hole is at the same temperature as the blackbody gravitons. Condition (2) implies

$$T_1^{-1}\,\frac{\partial T_1}{\partial E_1}>\frac{1}{E_2} \quad \text{or } E_1<\tfrac{1}{4}E_2 . \tag{16}$$

In other words, in order for the configuration of a black hole and gravitons to maximize the probability, the volume V of the box must be sufficiently small that the energy E_1 of the blackbody gravitons is less than $\frac{1}{4}$ the mass of the black hole. (Note that this result depends only on the T^4 dependence of the energy density of zero-rest-mass blackbody radiation and therefore remains true if one considers in addition to gravitons other zero-mass particles such as photons and neutrinos.) If this condition on V is satisfied, the equilibrium between the black hole and the blackbody radiation at the same temperature will be stable because, if a statistical fluctuation causes a slight excess of radiation to be absorbed by the hole, the temperature of the radiation will fall more than that of the hole, and so the rate of absorption will decrease more than the rate of emission.

To see what the condition on V implies consider the limiting case in which $E_1=\frac{1}{4}E_2=\frac{1}{5}E$. Then $T_1=T_2=5/32\pi E$; therefore

$$E=(n_b+\tfrac{7}{8}n_f)\,\frac{\pi^2}{15}\left(\frac{5}{32\pi E}\right)^4 V$$

or

$$V=V_h=\frac{3\times 2^{20}\pi^2 E^5}{125(n_b+\frac{7}{8}n_f)} \tag{17}$$

when n_b is the number of zero-mass boson fields and n_f is the number of fermion fields. If $V>V_h$, the state of maximum probability will be blackbody radiation without any black hole. From time to time statistical fluctuations in the blackbody radiation will cause black holes to form, but they will tend to evaporate again, so that most of the time there will not be any black hole. If $V_h>V>V_c\sim 64\pi E^3/3$, the most probable state will be a single black hole surrounded by blackbody radiation at the same temperature. Statistical fluctuations will cause the black hole mass to vary and will, on occasion, lead to the complete disappearance of the black hole. If $V<V_c$ (Schwarzschild volume), the whole box will undergo gravitational collapse, so the above analysis cannot be applied.

IV. WHITE HOLES

Consider a *gedanken* experiment as in the previous section in which a certain amount of energy E is put in an insulated box of volume $V>V_c$. By the ergodic assumption the system will pass through every possible configuration and will eventually lose all memory of its initial state.

This means that, if one examines the behavior of the system over a long period of time, one cannot distinguish an arrow of time; any given pattern of behavior should occur equally often in the time-reversed form. It was shown above that at some times the system would contain only zero-mass particles and at other times statistical fluctuations in these particles would cause the formation of black holes which would later evaporate. The time reverse of this behavior must therefore also occur; it must sometimes happen that a number of zero-mass particles annihilate to produce the time reverse of a black hole, a white hole, which at a later time explodes, emitting zero-mass particles.

In the classical theory of general relativity it is thought that there is a "cosmic censorship principle" according to which an initially nonsingular state can never evolve to give a "naked singularity," that is, a singularity which is not hidden by an event horizon from an observer at infinity. This principle would exclude the possibility of white holes forming in the classical theory. However, if one accepts that quantum effects can cause a black hole to disappear, then naked singularities must be present if one tries to describe the situation by a classical metric. Moreover, the time reversibility of the coupled Einstein-Maxwell neutrino equations implies that the time-reversed situation must also be possible in which a white hole forms and then evaporates.

Consider a box with $V_h > V > V_c$. In the forward direction of time one will have most of the time a black hole which is emitting and absorbing approximately blackbody radiation at about the same rates. Considered in the reverse direction of time, this will be a white hole absorbing and emitting approximately blackbody radiation at about the same rates. Thus, if one makes the reasonable assumption that the emission of a white hole is independent of its surroundings, it follows that it is the same as the rate of emission of a black hole of the same mass, angular momentum, and charge. By the thermodynamic interpretation of black holes the processes of formation and evaporation of a black hole are statistically the time reverse of each other. That is to say, a black hole can be formed from gravitons, photons, and neutrinos in any of a large number of different ways, the largest number of possibilities occurring when the particles have an approximately thermal distribution. Similarly, the evaporation will result in any of a large number of different final configurations, the number being greatest when the emission is approximately thermal. This and the previous result mean that black and white holes are identical to an external observer. It is a general principle that a physical theory ought not to contain elements that are not physically measurable and ought not to describe differently situations which cannot be distinguished by observation. In the classical theory of general relativity there is a unique space-time metric which is different for black holes and white holes. However, if space-time is quantized, one has to abandon the idea of a unique space-time metric which is independent of the observer just as in special relativity one had to abandon the idea of a unique time which was independent of the observer. The reason is that in order to determine where one is in space-time one has to measure the metric and this act of measurement places one in one of the various different branches of the wave function in the Wheeler-Everett[22] interpretation of quantum mechanics; if one describes space-time by some sort of Feynman integral over metrics, then the metrics that one will include in the integral will depend on the situation one wishes to describe, which will in turn depend on the measurements that have been made. An observer who measures himself to be outside the hole can use both black- and white-hole metrics. In the black-hole metrics particles accreting will fall into the singularity while particles being emitted will appear to be created somewhere outside the event horizon. In the white-hole metrics the accreting particles will appear to be annihilated by the gravitational field while the emitted particles will appear to come from a singularity in the past. An observer who measures himself as having fallen inside the hole will not see the emitted particles[3,7] and will use only the black-hole metrics. Similarly, in the very unlikely event of the black hole creating and emitting an observer, he would regard himself as having come out of a white hole and would use the white-hole metrics.

It is shown in another paper[7] that the process of formation and evaporation of a hole cannot be described by an S matrix. The reason is that one does not have a Cauchy surface in the future. What gets out to infinity does not completely determine the state of the system because this depends also on what went down the hole. The result of this is that by measurements at future infinity one cannot determine the pure quantum state of the system but only a density matrix which describes the probabilities of different combinations of particles being emitted. Instead of an S matrix one has a new entity called a super scattering operator which maps density matrices describing the initial situation to density matrices describing the final situation.[7] This operator will be invariant under the CPT operator Θ, i.e., the probability of going from an initial density matrix ρ_1 to a final density matrix ρ_2 will be the same as the probability of going from $\Theta\rho_2\Theta^{-1}$ to $\Theta\rho_1\Theta^{-1}$. The super scattering operator provides a description for observers at infinity which satisfies the principle mentioned above that it does not make the observationally undeterminable distinction between black and white holes.

The picture of white holes that has been commonly used hitherto is the time reverse of the collapse of a cloud of dust. The white hole emits nothing before a certain time and then suddenly sends out a cloud of matter. This picture has been criticized by Zel'dovich,[23] who pointed out that one would not expect a nonemitting phase because there would be pair creation in the strong gravitational fields near the singularity. The conclusion of this paper is very similar: White holes emit thermal radiation continuously. The emission is completely random and all possible configurations for the emitted particles are equally probable. (All configurations do not get out to infinity with equal probability because there is a potential barrier around a hole which depends on the angular momentum, etc. of the particles and which can reflect particles back into the hole.) It is indeed possible that a white hole could emit nothing until a certain time and then shoot out a cloud of dust, but the number of possible configurations that this would represent is small. It is much more probable that a white hole would emit thermal radiation, because there are many more configurations.

V. SUMMARY

The conclusions of this paper are that there is an intimate connection between holes (black or white) and thermodynamics which arises because information is lost down the hole. If one makes the hypothesis that the maximum amount of information which can be lost down a hole of a given mass, angular momentum, and charge is finite, it follows that one can associate an entropy with the hole and can deduce that it must emit thermal radiation at some finite nonzero temperature. Conversely, the quantum-mechanical results that black holes emit thermal radiation with a temperature $T_h = \kappa/2\pi$ enables one to prove the above hypothesis and to evaluate the entropy as $S_h = \frac{1}{4}A$. The fact that black holes have negative specific heat implies that they can be in stable thermal equilibrium only when the amount of energy available is restricted in a certain way. Consideration of thermal equilibrium or the time reversibility of the equations implies that black holes are indistinguishable from white holes to an external observer and behave in a time-symmetric manner. The irreversibility associated with classical black holes is merely a statistical effect. For example, in the classical theory two black holes can coalesce but a black hole can never bifurcate. The corresponding result in the quantum theory is that there is a high probability for two black holes to coalesce into one because this involves going from a state with a lower number of configurations to one with a higher number, but there is a low probability for the reverse process.

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