# Radiative decays of strange baryons and the structure of weak interactions*

N. Vasanti
*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540*
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Radiative weak decays such $\Sigma^+ \to p\gamma$, $\Xi^- \to \Sigma^-\gamma$, etc. are considered and various symmetry relations among these amplitudes are derived. They are then analyzed in a quark model and the operator-product expansion is studied. The effects of introducing right-handed currents are discussed. They are found to change drastically the angular distribution of the decay products, depending on the specific form of the right-handed current used.

## I. INTRODUCTION

Although we have a fairly satisfactory understanding of leptonic and semileptonic charged-current weak interactions, the same cannot be said for nonleptonic processes. The structure of the total hadronic weak current is still a matter of conjecture. In this article we look at radiative weak decays of baryons in the context of gauge theories to see if they place any constraint on the underlying structure of the weak Hamiltonian.

There are six possible radiative decays of strange spin-$\frac{1}{2}$ baryons,

$$\Sigma^+ \to p\gamma\ ,$$

$$\Sigma^0 \to n\gamma\ ,$$

$$\Lambda^0 \to n\gamma\ ,$$

$$\Xi^0 \to \Sigma^0\gamma\ ,$$

$$\Xi^0 \to \Lambda^0\gamma\ ,$$

and

$$\Xi^- \to \Sigma^-\gamma\ .$$

Of these the $\Sigma^0$ decay should be swamped by the faster electromagnetic decay $\Sigma^0 \to \Lambda^0\gamma$ and will probably be very difficult to detect experimentally. To date, the $\Sigma^+$ and $\Xi^0 \to \Lambda^0\gamma$ transitions have been observed. If $q$ is the momentum carried by the photon and $\epsilon$ its polarization, the amplitude should have the form

$$\mathfrak{M} = \epsilon_\mu \bar{u}(a + b\gamma_5)\, i\sigma_{\mu\nu} q_\nu u \qquad (1.1)$$

by virtue of gauge and Lorentz invariance. The rate is given by

$$\Gamma = \frac{1}{\pi}\left(\frac{m_i^2 - m_f^2}{2m_i}\right)^3 (|a|^2 + |b|^2)\ ,$$

where $m_i$ and $m_f$ are the masses of the initial and final baryon, respectively. Another interesting decay parameter involves the angular distribution of the emitted baryon with respect to the spin of the decaying baryon in its rest frame. This is given by

$$\frac{d\Gamma}{d\Omega} = (1 + \alpha\, \hat{P}\cdot\vec{S}), \qquad (1.2)$$

where $\hat{P}$ is the unit vector along the momentum direction of the final baryon and $S$ is the polarization of the initial baryon. In terms of the parameters $a$ and $b$ appearing in the amplitude

$$\alpha = \frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2},\quad -1 \leqslant \alpha \leqslant 1 \qquad (1.3)$$

$a$ and $b$ should be relatively real if final-state interactions are ignored and if time-reversal invariance holds.

Experimentally[1] we have measurements of

$$\frac{\Gamma(\Sigma^+ \to p\gamma)}{\Gamma(\Sigma^+ \to p\pi^0)} = (2.76 \pm 0.51)\times 10^{-3}\ ,$$

$$\frac{\Gamma(\Xi^0 \to \Lambda^0\gamma)}{\Gamma(\Xi^0 \to \Lambda^0\pi)} = (2.3 \pm 0.7)\times 10^{-3}\ ,$$

$$\alpha(\Sigma^+ \to p\gamma) = -1.03^{+0.52}_{-0.42}\ .$$

Section II deals with various relations one derives among these amplitudes on the basis of exact symmetries. $U$-spin symmetry relations are derived in detail for theories with only left-handed currents and for theories having both left- and right-handed currents.[2] We also give a brief summary of the low-momentum pole-model calculations.[3] In Sec. III we discuss the quark model based on the usual Weinberg-Salam theory[4] of weak interactions which assigns all quarks to singlets or doublets of the weak $SU(2)_L$. We also discuss the relevant operator-product expansion for the weak Hamiltonian. Section III treats the same topic in the framework of right-handed theories[2] and we find that certain operators that are not important in the usual theory are important in such theories. We assume throughout that $CP$ is not being violated. We also assume that the strong interactions are described by a non-Abelian gauge theory based on SU(3) coupled to color.

The main emphasis of the paper is on the difference in various results predicted by purely left-handed theories and by theories having right-handed currents. We call the latter left-right theories and the former left-left theories. We have found the following:

(a) They change the $U$-spin properties of the Hamiltonian, so symmetry relations based on $U$ spin get altered.

(b) Secondly, they change the relative importance of certain operators in the operator-product expansion for the weak Hamiltonian. Specifically, some operators that we ignore, for very valid reasons, in left-left theories cannot be ignored in left-right theories where they appear with an intrinsic enhancement factor of the heavy-quark (charmed-quark) mass.

(c) Thirdly, the chiral structure of these operators implies that the parity-conserving and parity-violating amplitudes contribute equally. This means they give rise to a large asymmetry parameter. Further, the sign of the asymmetry parameter is found to depend on the exact nature of the right-handed coupling, going from $-1$ to $+1$; its two extreme values in the two left-right theories are considered here.

## II. $U$-SPIN AND SYMMETRY RELATIONS

In deriving symmetry relations one must be careful to note what minimal assumption is sufficient for the derivation. In the conventional Cabibbo theory the $\Delta S=1$ effective Hamiltonian is given by the current product

$$\mathcal{H}_{\text{eff}} = J_\mu^{\dagger\,\Delta S=1} J_\mu^{\Delta S=0} + \text{H.c.}$$

In terms of elementary fermions ($\theta$ is the Cabibbo angle)

$$J_\mu^{\Delta S=0} = \cos\theta\,\overline{\mathcal{P}}\gamma_\mu(1-\gamma_5)\,\mathfrak{N},$$
$$J_\mu^{\Delta S=1} = \sin\theta\,\overline{\mathcal{P}}\gamma_\mu(1-\gamma_5)\,\lambda\ ,$$

so that

$$\mathcal{H}_{\text{eff}} = \sin\theta\cos\theta\,[\overline{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\,\mathcal{P}\overline{\mathcal{P}}\gamma_\mu(1-\gamma_5)\,\lambda + \text{H.c.}]\ . \tag{2.1}$$

This transforms like the first (by first we mean the $x$ component, working in a Cartesian basis) component of a $U$-spin vector. Under a rotation through $\pi$ around the second axis

$$e^{i\pi U_2}\,\mathcal{H}_{\text{eff}}\,e^{-i\pi U_2} = -\mathcal{H}_{\text{eff}}\ . \tag{2.2}$$

If we now introduce a fourth, charmed quark, the $\mathcal{P}'$, via the usual GIM[5] mechanism, $\mathcal{H}_{\text{eff}}$ becomes

$$\mathcal{H}_{\text{eff}} = \sin\theta\cos\theta[\overline{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\,\mathcal{P}\overline{\mathcal{P}}\gamma_\mu(1-\gamma_5)\,\lambda - \overline{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}'\,\overline{\mathcal{P}}'\,\gamma_\mu(1-\gamma_5)\lambda + \text{H.c.}]\ .$$

The $\mathcal{P}'$, like the $\mathcal{P}$, is a $U$-spin singlet, so that the introduction of charm in no way changes the $U$-spin properties of $\mathcal{H}_{\text{eff}}$ and Eq. (2.2) is still true. Using $U$ spin and $P$ and $T$ invariance we get (the electromagnetic current in a $U$-spin singlet)

$$\langle B_f|\mathcal{H}_{\text{eff}}^{\text{pc}}|B_i\rangle = -\langle B_i'|\mathcal{H}_{\text{eff}}^{\text{pc}}|B_f'\rangle\ , \tag{2.3a}$$
$$\langle B_f|\mathcal{H}_{\text{eff}}^{\text{pv}}|B_i\rangle = +\langle B_i'|\mathcal{H}_{\text{eff}}^{\text{pv}}|B_f'\rangle\ , \tag{2.3b}$$

where pc and pv stand for parity-conserving and parity-violating, respectively, $B_i, B_f$ denote the initial and the final baryon, and $B_i', B_f'$ denote the $U$-spin-rotated states

$$|B'\rangle = e^{i\pi U_2}|B\rangle\ .$$

For a $U$-spin doublet such as $p, \Sigma^+$ or $\Sigma^-, \Xi^-$, $B_i' = -B_f,\ B_f' = B_i$, so that Eq. (2.3b) tells us these decays have vanishing parity-violating amplitudes[6]

$$b_{\Sigma^+p} = 0 = b_{\Xi^-\Sigma^-}\ .$$

According to Eq. (1.1), $a$ stands for the parity-conserving part and $b$ for the parity-violating part of the decay amplitude.

Now let us look at the neutral decays involving $n$, $\Lambda^0$, $\Sigma^0$, and $\Xi^0$. $\Lambda^0$ and $\Sigma^0$ are mixtures of $U=0$ and $U=1$ states. Call the corresponding $U$-spin eigenstates $B_0$ and $B_3$ respectively

$$B_0 = \frac{\sqrt{3}\,\Sigma^0 + \Lambda^0}{2}\ ,$$
$$B_3 = \frac{-\Sigma^0 + \sqrt{3}\,\Lambda^0}{2}\ .$$

Then Eq. (2.3) says

$$\begin{aligned}
\langle B_3|\mathcal{H}_{\text{eff}}^{\text{pc}}|\Xi^0\rangle &= -\langle n|\mathcal{H}_{\text{eff}}^{\text{pc}}|B_3\rangle\ ,\\
\langle B_3|\mathcal{H}_{\text{eff}}^{\text{pv}}|\Xi^0\rangle &= +\langle n|\mathcal{H}_{\text{eff}}^{\text{pv}}|B_3\rangle\ ,\\
\langle B_0|\mathcal{H}_{\text{eff}}^{\text{pc}}|\Xi^0\rangle &= +\langle n|\mathcal{H}_{\text{eff}}^{\text{pc}}|B_0\rangle\ ,\\
\langle B_0|\mathcal{H}_{\text{eff}}^{\text{pv}}|\Xi^0\rangle &= -\langle n|\mathcal{H}_{\text{eff}}^{\text{pv}}|B_0\rangle\ ,
\end{aligned} \tag{2.4}$$

where we have used

$$e^{i\pi U_2}\,\Xi^0 = -n\ ,$$
$$e^{i\pi U_2}B_0 = B_0\ ,$$
$$e^{i\pi U_2}B_3 = -B_3\ .$$

Equation (2.4) implies the relations

$$\begin{aligned}
\sqrt{3}\,a_{\Xi^0\Sigma^0} &= 2a_{\Lambda n} + a_{\Xi^0\Lambda^0}\ ,\\
2a_{\Xi^0\Sigma^0} &= \sqrt{3}\,a_{\Lambda n} + a_{\Sigma^0 n}\ ,\\
\sqrt{3}\,b_{\Xi^0\Sigma^0} &= -2b_{\Lambda n} + b_{\Xi^0\Lambda^0}\ ,\\
2b_{\Xi^0\Sigma^0} &= -\sqrt{3}\,b_{\Lambda n} + b_{\Sigma^0 n}\ .
\end{aligned} \tag{2.5}$$

If we now invoke octet dominance which says $\mathcal{H}_{\text{eff}} \sim \lambda_6$ ($\sim$ is to be read as transforms like) we get the additional relation

$$-b_{\Xi^0\Sigma^0} = \sqrt{3}\, b_{\Lambda^0 n} = b_{\Sigma^0 n} = -\sqrt{3}\, b_{\Xi^0\Lambda^0} = \frac{\beta}{\sqrt{3}}\ , \text{ say}\ . \qquad (2.6)$$

SU(4) invariance forbids the decay $\Xi^- \to \Sigma^-\gamma$. This is readily understood in terms of $P$ spin, the SU(2) subgroup of SU(4) that takes $\mathcal{P} \to \mathcal{P}'$. The Hamiltonian is a $P$-spin vector but the $\Xi^-$, $\Sigma^-$, and the electromagnetic current are all $P$-spin singlets. This makes the matrix element $\langle \Sigma^-\gamma | \mathcal{H}_{\text{eff}} | \Xi^- \rangle$ vanish identically. Assumption of specific transformation properties, namely 20 dominance for $\mathcal{H}_{\text{eff}}$ ($\mathcal{H}_{\text{eff}} \sim T^{[ab]}_{[cd]}$) leads to no further relations.

Recently several proposals have been made for changing the form of the charm-changing current.[2] The first proposal was to include a right-handed current $\overline{\mathcal{N}}\gamma_\mu(1+\gamma_5)\mathcal{P}'$ in addition to the usual currents. We call this the DGG model. This would supposedly account for the observed enhancement of nonleptonic $\Delta I = \frac{1}{2}$ decays in a straightforward manner and the occurrence of dilepton events in neutrino charged-current reactions. However, this model runs into trouble with the smallness of the observed $K_L$-$K_S$ mass difference and restrictions imposed by current algebra and partially conserved axial-vector current (PCAC) on the chiral structure of the $\Delta S = 1$ nonleptonic weak Hamiltonian.[7] An alternative version that avoids these difficulties introduces as an extra piece the current $\overline{\lambda}\gamma_\mu(1+\gamma_5)\mathcal{P}'$. We call this the WZKT model.

In the DGG model the effective Hamiltonian has an additional piece $\cos\theta\, \overline{\mathcal{N}}\gamma_\mu(1+\gamma_5)\mathcal{P}'\overline{\mathcal{P}}'\gamma_\mu(1-\gamma_5)\lambda$ and in the WZKT model the additional piece is $\sin\theta\, \overline{\mathcal{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}'\overline{\mathcal{P}}'\gamma_\mu(1+\gamma_5)\lambda$. Let us denote this extra piece by $\mathcal{H}^{\text{LR}}_{\text{eff}}$ for obvious reasons. In either model $(\mathcal{H}^{\text{LR}}_{\text{eff}})^{\text{pc}}$ transforms like the first component of a $U$-spin vector (just like $\mathcal{H}^{\text{LL}}_{\text{eff}}$) but $(\mathcal{H}^{\text{LR}}_{\text{eff}})^{\text{pv}}$ transforms like the second component of a $U$-spin vector.[8] This means Eq. (2.3b) now reads

$$\langle B_f | (\mathcal{H}^{\text{LR}}_{\text{eff}})^{\text{pv}} | B_i \rangle = -\langle B_i' | (\mathcal{H}^{\text{LR}}_{\text{eff}})^{\text{pv}} | B_f' \rangle\ ,$$

so that $b^{\text{LR}}_{\Sigma^+ p}$, $b^{\text{LR}}_{\Xi^-\Sigma^-}$ are not constrained to vanish. Further, we also obtain

$$\sqrt{3}\, b^{\text{LR}}_{\Xi^0\Sigma^0} = 2b^{\text{LR}}_{\Lambda n} + b^{\text{LR}}_{\Xi^0\Lambda^0}$$

and (2.7)

$$2b^{\text{LR}}_{\Xi^0\Sigma^0} = \sqrt{3}\, b^{\text{LR}}_{\Lambda n} + b^{\text{LR}}_{\Sigma^0 n}\ .$$

Combining Eq. (2.6) with Eq. (2.7) we get for the total parity-violating amplitudes

$$\sqrt{3}\, b_{\Xi^0\Sigma^0} = 2b_{\Lambda n} + b_{\Xi^0\Lambda^0} - \tfrac{4}{3}\beta\ ,$$
$$2b_{\Xi^0\Sigma^0} = \sqrt{3}\, b_{\Lambda n} + b_{\Sigma^0 n} - (\sqrt{3}+1)\beta\ . \qquad (2.8)$$

In writing down SU(3)-invariant amplitudes what we have said translates into

$$(\mathcal{H}^{\text{LR}}_{\text{eff}})^{\text{pc}} \sim \lambda_6\ ,$$
$$(\mathcal{H}^{\text{LR}}_{\text{eff}})^{\text{pv}} \sim \lambda_7\ ,$$

but this yields no new relations in addition to Eq. (2.8).

We have seen that $U$ spin forbids parity-violating amplitudes in the decay $\Sigma^+ \to p\gamma$. This would imply the asymmetry parameter $\alpha \simeq 0$. Experimentally $\alpha \simeq -1$. Since $\alpha$ is supposed to arise from $U$-spin-breaking effects we should see how large they could be. Note that $|\alpha| \simeq +1$ indicates that the parity-conserving and parity-violating pieces contribute equally to the amplitude. However, a phenomenological analysis[3] suggests that the parity-violating amplitude is about 10% of the parity-conserving amplitude. We give a brief summary of the analysis below.

One draws pole diagrams as in Fig. 1 for the decay using as an effective electromagnetic vertex

$$e\left(i\gamma_\mu + \frac{\mu}{2m}\sigma_{\mu\nu}q_\nu\right),$$

where $\mu/2m$ is the anomalous magnetic moment. This relates the matrix element $\langle p\gamma | H | \Sigma^+ \rangle$ to $\langle p | H | \Sigma^+ \rangle$. To evaluate $\langle p | H | \Sigma^+ \rangle$ one looks at the dominant decay mode $\langle p\pi | H | \Sigma \rangle$ and relates this to $\langle p | H | \Sigma^+ \rangle$ via PCAC and current algebra. So eventually one gets the matrix element $\langle p\gamma | H | \Sigma^+ \rangle$ in terms of the measured anomalous magnetic moments of the baryons and the measured pionic decay amplitudes.[9] There are various refinements to this approach, but they all yield typically a small value for the asymmetry parameter $\alpha$, in disagreement with experiment. This is primarily an analysis of the nonshort-distance piece of the Lagrangian, i.e., a low-momentum calculation.

## III. QUARK-MODEL CALCULATIONS IN WEINBERG-SALAM MODEL

Let us look at the Weinberg-Salam theory of weak interactions and consider the quarks to be free. We ignore, for the time being, strong interactions. The $\Delta S = 1$ radiative transitions cor-


FIG. 1. Pole-model diagrams relevant for the decay $\Sigma^+ \to p_\gamma$. The shaded box represents the weak Hamiltonian.

respond to the process

$$\lambda \to \mathcal{N}\gamma ,$$

and the contributing diagrams are shown in Fig. 2. If we put the quarks on the mass shell we expect an amplitude of the form

$$\mathfrak{M} = \epsilon_\mu \overline{\mathcal{N}}\, i\sigma_{\mu\nu}(a + b\gamma_5)\, q_\nu \lambda\, \frac{Ge}{\sqrt{2}} \sin\theta\cos\theta \qquad (3.1)$$

as remarked earlier. Dimensional analysis says that $a$ and $b$ have dimensions of mass. We will now show how symmetry arguments are sufficient to fix the form of the matrix element. The Lagrangian of our theory has an invariance under

$$\psi_i \to -\gamma_5\psi_i \text{ so that } \psi_R \to -\psi_R, \quad \psi_L \to \psi_L$$

$$m_i \to -m_i,$$

where $i$ stands for the quark flavor. This is true for each separate flavor. This at once says that $a$ and $b$ cannot be odd in $m_{\mathcal{P}}$ or $m_{\mathcal{P}'}$ and further that if we start with a general form

$$a = \rho m_{\mathcal{N}} + \beta m_\lambda ,$$

$$b = \gamma m_{\mathcal{N}} + \delta m_\lambda ,$$

then $\beta = \delta$ and $\rho = -\gamma$. So then

$$\mathfrak{M} \sim (\rho m_{\mathcal{N}} + \beta m_\lambda)\, \overline{\mathcal{N}} \sigma_{\mu\nu} \lambda + (-\rho m_{\mathcal{N}} + \beta m_\lambda)\, \overline{\mathcal{N}} \sigma_{\mu\nu} \gamma_5 \lambda . \qquad (3.2)$$

Recall now that since $\mathcal{N}$ and $\lambda$ are $U$-spin doublets, when $m_{\mathcal{N}} = m_\lambda$, the parity-violating amplitude is supposed to vanish. This implies $\rho = \beta$. So finally we have

$$\mathfrak{M} = \epsilon_\mu \rho\, (m_{\mathcal{N}} + m_\lambda)\, \overline{\mathcal{N}}\, i\sigma_{\mu\nu} \left(1 - \frac{m_{\mathcal{N}} - m_\lambda}{m_{\mathcal{N}} + m_\lambda}\gamma_5\right) q_\nu \lambda , \qquad (3.3)$$

dropping a factor $(Ge/\sqrt{2})\sin\theta\cos\theta$.

That the amplitude cannot depend on the internal quark mass is evident since $\sigma_{\mu\nu}$ couples left-handed quarks to right-handed quarks and our theory has currents coupled only to left-handed quarks.

Before we do any explicit computation of graphs let us look at the operator structure of the effective Lagrangian.[10] Our aim is to write down an effective Lagrangian that consists of a set of local operators multiplied by coefficients such that we can correctly get weak-interaction amplitudes to order $e/M_W^2$. This means we can have coefficients of order $1/M_W^2$ and evaluate matrix elements to order $e$ or coefficients of order $e/M_W^2$ and matrix elements of order unity. As usual we want an expansion valid when matrix elements are taken between any arbitrary states. We also expect the relevant operators to be gauge-invariant and that the dominant operators have lowest dimension. It

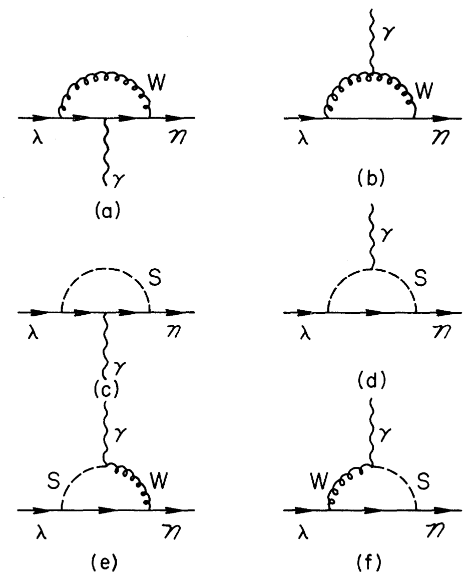


FIG. 2. Diagrams which contribute to the $\lambda \to n\gamma$ amplitude. Script letters refer to quarks. The internal solid lines in this and subsequent figures represent quarks ($\mathcal{P}$ or $\mathcal{P}'$).

is well known that we can discount operators of dimension four as they can be absorbed in the wave-function and coupling constant renormalization counterterms.[11] So we look at operators of dimension five and six.

At the tree level, hadronic weak-interaction processes are represented by Fig. 3, and $\mathcal{L}_{\rm eff}$ which is a product of two currents here is given by a four-quark operator of the form $\overline{\psi}\gamma_\mu(1-\gamma_5)\,\psi\overline{\psi}\gamma_\mu(1-\gamma_5)\,\psi$ to order $1/M_W^2$. In processes with a $W$-boson loop we can get additional operators. Let us look at Fig. 2(a). It surely has contributions from the four-quark operator. It also has contributions from two-quark operators such as $\overline{\mathcal{N}}\sigma_{\mu\nu}(a + b\gamma_5)\,\lambda\, eF_{\mu\nu}$, where $F_{\mu\nu}$ is the photon field tensor. But the GIM mechanism cancels the order $1/M_W^2$ contribution exactly and so we have only an order $1/M_W^4$ contribution in which we are not interested.

We also have, in addition to this diagram, other diagrams where the $W$-boson propagator has interactions in it. We take Fig. 2(b) as a representative of this class. We can think of the $WW\gamma$ vertex as

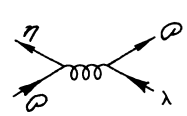


FIG. 3. Four-quark interaction at the tree-approximation level for hadronic weak decays.

an insertion of an operator $\Gamma$ and for this class of processes $\mathcal{L}_{\text{eff}}$ is not given by the product of two current operators but of three operators—$\Gamma$ and two current operators. Note that we also have two $W$-boson propagators. In coordinate space it is easily seen that the large mass of the $W$ boson makes the integral have it main contribution when all three operators converge to the same point. Thus one can still replace $\mathcal{L}_{\text{eff}}$ by a sum of local operators. The operator we are interested in, $\overline{\mathfrak{N}}\sigma_{\mu\nu}(a+b\gamma_5)\lambda eF_{\mu\nu}$, appears here to order $1/M_W^2$ but the GIM mechanism leaves us with only the order $1/M_W^4$ contribution. The same thing happens for Figs. 2(e) and 2(f). For simplicity we are here assuming that the Higgs-scalar mass is the same as the $W$-boson mass. The scalar exchange graphs, Figs. 2(e) and 2(d), are down by an order $1/M_W^2$ even without the GIM mechanism.[12] Thus we have concluded that

$$\mathcal{L}_{\text{eff}} = A\frac{1}{M_W^2}[\overline{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}\overline{\mathcal{P}}\gamma_\mu(1-\gamma_5)\lambda - \overline{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}'\overline{\mathcal{P}}'\gamma_\mu(1-\gamma_5)\lambda], \tag{3.4}$$

where $A$ is some Wilson coefficient and that this gets contributions from Fig. 2(a) alone.

Now let us see what the exact calculations tell us. Note that it is perfectly legitimate to use equations of motion in simplifying operators that appear in the operator-product expansion. But what we will do now is to calculate matrix elements between quark states taken on shell, just to give us a feel for the values. This is not justified certainly, and, as we shall see, we have reasons to mistrust these naive calculations.

Working with the exact Lagrangian that we have, we find that for the on-shell process $\lambda\to\mathfrak{N}\gamma$, Fig. 2(a) gives

$$\rho = \frac{1}{12\pi^2}\frac{\Delta m^2}{M_W^2}\left(2\ln\frac{M_W^2}{m_{\mathcal{P}'}^2}-\tfrac{1}{3}\right), \tag{3.5}$$

where

$$\Delta m^2 = m_{\mathcal{P}'}^2 - m_{\mathcal{P}}^2$$

and where $\rho$ has been defined in Eq. (3.3) and the others give a $\rho$ of the order of

$$\frac{1}{16\pi^2}\frac{\Delta m^2}{M_W^2}$$

without any logarithmic factors. This is expected, but what is unexpected is that Fig. 2(a) also seems to contribute to the amplitude only in order $1/M_W^4$. What happened to the

$$\frac{1}{M_W^2}\langle\mathfrak{N}\gamma|[\overline{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}\overline{\mathcal{P}}\gamma_\mu(1-\gamma_5)\lambda-(\mathcal{P}\to\mathcal{P}')]|\lambda\rangle$$

contribution? Since we cannot think of any symmetry that would say this matrix element vanishes identically (after all, $P$ spin is broken badly) we claim its vanishing is a fortuitous result that occurs when we do not take strong interactions into account. Thus to order $g^2$ ($g$ is the gluon coupling constant) it would receive nonzero contributions from Fig. 4. Its vanishing in order $g^0$ can be understood by the fact that Fig. 5(a) can be Fierz-transformed into Fig. 5(b). But now the loop integral is the same one we encounter in vacuum polarization graphs in QED and is proportional to $(q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma_\nu$, which is zero for a real photon,[13] hence our previous rejoinder not to put too much faith in free-quark-model results.

However, the important points of the free-quark model are already contained in Eq. (3.3), namely that the mass insertion involves the external quarks $\mathfrak{N}$ and $\lambda$ and that the asymmetry parameter

$$\alpha = \frac{2(m_\lambda - m_{\mathfrak{N}})/(m_{\mathfrak{N}}+m_\lambda)}{1+[(m_{\mathfrak{N}}-m_\lambda)/(m_{\mathfrak{N}}+m_\lambda)]^2}. \tag{3.6}$$

This is $\sim+1$ if $m_{\mathfrak{N}} \ll m_\lambda$.

A renormalization-group analysis gives us the Wilson coefficient $A$ appearing in Eq. (3.4) as being really an enhancement factor $(\ln M_W^2/\mu^2)^\phi$ times an undetermined constant reflecting the low-momentum contribution. ($\mu^2$ is the subtraction point.) This has been discussed in great detail in many papers[14] and $\phi$ turns out to be, for the color-symmetric four-quark operator, $12/(33-2n)$, where $n$ = number of quark flavors. However, this changes neither of the two points noted above.

Technically one could improve on this analysis by making a heavy-quark expansion.[15] This says that in taking matrix elements of heavy-quark operators between light-particle states one can replace the heavy-quark operator by a sum of light-quark operators with the same quantum numbers and coefficients which are functions of the heavy-quark mass $m_H$, the coupling constant, and the subtraction point. These coefficients can be computed via a renormalization-group calculation. We feel here that such an expansion is unwarranted because it is true only in the limit $m_H\to\infty$ and even if this were approximately true, we still have no reliable means of estimating matrix elements between bound hadron states.

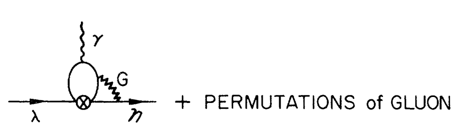


FIG. 4. Typical contribution to lowest-order corrections due to strong interactions for Fig. 5.

## IV. QUARK-MODEL CALCULATION WITH RIGHT-HANDED CURRENTS

We now wish to see what happens in a theory with right-handed currents,[2] specifically the current

$$\bar{\lambda}\gamma_\mu(1+\gamma_5)\mathcal{P}' \ .$$

In addition to the four-quark operator we got in the Weinberg-Salam model we will now get other operators in the expansion for $\mathcal{L}_{\rm eff}$. We focus on these new operators. The relevant diagrams are still those of Fig. 2, where the internal quark is a $\mathcal{P}'$ quark and the $\lambda\mathcal{P}'W$ coupling is $V+A$ rather than $V-A$. Obviously Fig. 2(a) contributes to the four-quark operator $\bar{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}'\bar{\mathcal{P}}'\gamma_\mu(1+\gamma_5)\lambda$ of the LR type. In addition it also gives rise to the operators $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}$ and $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda \times gG_{\mu\nu}$ in order $1/M_W^2$. $G_{\mu\nu}$ is the gluon field tensor. In order to see how this comes about one should replace the photon by a gluon in Fig. 2(a). Strictly speaking, one gets the operator $\bar{\mathfrak{N}}\Box^2(1+\gamma_5)\lambda$, where $\Box^2$ is the covariant operator $D_\mu D^\mu$ but this goes over into $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda gG_{\mu\nu}$ by the equations of motion plus operators that can be absorbed in the counterterms in the Lagrangian. Moreover, since the added coupling has no counterpart with the $\mathcal{P}$ quarks, Figs. 2(b), 2(e), and 2(f) also contribute to $\bar{\mathfrak{N}}q_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}$ in order $1/M_W^2$. However, they cannot contribute to $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda gG_{\mu\nu}$ since the gluon does not couple to scalars or $W$ bosons. Figures 2(c) and 2(d) are still down by $m_{\mathcal{P}'}{}^2/M_W^2$, so these scalar exchange graphs need not be considered.

The significant points to be noted are that these new operators have the chiral structure $(1+\gamma_5)$ (see Ref. 16) and, more important, the mass insertion term multiplying them is the internal quark mass $m_{\mathcal{P}'}$. This, as we have seen, is in accordance with a theory that has right-handed couplings. But this means we have a direct enhancement because of the explicit heavy-quark mass operator and also the parity-conserving and parity-violating amplitudes give equal contributions. This helps to make the asymmetry parameter $\alpha\simeq+1$.

Thus

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\rm LR} &= a_1\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}\\ &\quad + a_2\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda gG_{\mu\nu}\\ &\quad + a_3\bar{\mathfrak{N}}\gamma_\mu(1-\gamma_5)\mathcal{P}'\bar{\mathcal{P}}'\gamma_\mu(1+\gamma_5)\lambda\ , \end{aligned} \qquad (4.1)$$

where the $a_i$'s are Wilson coefficients. Calculations of the graphs of Fig. 2(a), 2(b), 2(e), and

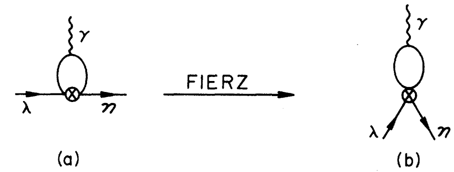


FIG. 5. Diagram for calculating the matrix element of the four-quark operator between a $\lambda$ quark and an $\mathfrak{N}$ quark and photon state, ignoring strong interactions.

2(f) give for the left-right amplitude

$$\begin{aligned}\mathfrak{M}^{\rm LR} &= \epsilon_\mu i\,\frac{Ge}{\sqrt{2}}\,\sin\theta\,\frac{m_{\mathcal{P}'}}{\pi^2}\,\bar{\mathfrak{N}}i\sigma_{\mu\nu}q_\nu(1+\gamma_5)\\ &\quad\times\lambda\left(\tfrac{1}{6}-\tfrac{3}{16}-\tfrac{1}{16}\right)\ ,\end{aligned} \qquad (4.2)$$

where the three contributions correspond, respectively, to Figs. 2(a) and 2(b) and the sum of 2(e) and 2(f),

$$\mathfrak{M}^{\rm LR} = -i\epsilon_\mu\,\frac{Ge}{\sqrt{2}}\,\sin\theta\,\frac{m_{\mathcal{P}'}}{12\pi^2}\,\bar{\mathfrak{N}}i\sigma_{\mu\nu}(1+\gamma_5)q_\nu\lambda\ . \qquad (4.3)$$

Thus to lowest order

$$a_1\sim -m_{\mathcal{P}'}/12\pi^2\ ,$$
$$a_2\sim m_{\mathcal{P}'}/6\pi^2\ ,$$
$$a_3\sim 1\ ,$$

leaving out factors of $1/M_W^2$ and the Cabibbo angle.

A renormalization-group calculation gives,[13] as already mentioned, enhancement factors of $(\ln M_W^2/\mu^2)^\phi$. For both $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}$ and $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda gG_{\mu\nu}$, we have calculated $\phi$ to be $4/(33-2n)$, where $n$ is the number of quark flavors. For the color-symmetric LR four-quark operator $\phi=24/(33-2n)$. Since this operator involves heavy quarks it will also have extra factors when we perform a heavy-quark expansion. But as remarked we will not do this.

In the alternative DGG model which has the extra current

$$\bar{\mathfrak{N}}\gamma_\mu(1+\gamma_5)\mathcal{P}'\ ,$$

our formulas in Sec. III still hold provided one replaces $\sin\theta(1+\gamma_5)$ by $\cos\theta(1-\gamma_5)$ (see Ref. 16) everywhere. We still have the enhancement due to the $m_{\mathcal{P}'}$ factor, but the chiral structure implies an asymmetry parameter $\alpha\simeq-1$ instead of $+1$. Thus the form of the weak current can change the angular distribution dramatically.

What can we say about the matrix elements of the operators appearing in Eq. (4.1)? In the quark model we have already given the matrix elements of $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}$ between a $\lambda$ and an $\mathfrak{N}\gamma$ state. Calculating that of the four-quark operator requires evaluating it to order $g^2$ and so involves two-loop diagrams as in Fig. 4. The matrix element of $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda gG_{\mu\nu}$ between $\lambda$ and $\mathfrak{N}\gamma$ turns out to be divergent because under renormalization it mixes with $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}$. We ignored this mixing because it is small when we calculated their anomalous dimensions.

Strictly speaking, we would like to be able to evaluate these matrix elements between hadron states. For the operator $\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}$ this is possible in a phenomenological way using the bag model. For $B_i \to B_f\gamma$

$$\langle B_f\gamma|\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda eF_{\mu\nu}|B_i\rangle = 2\epsilon_\mu q_\nu\langle B_f|\bar{\mathfrak{N}}\sigma_{\mu\nu}(1+\gamma_5)\lambda|B_i\rangle\,, \quad (4.4)$$

since we are working throughout to lowest order in electromagnetism. Because the photon is real and transverse

$$\langle B_f|\bar{\mathfrak{N}}\sigma_{\mu\nu}\lambda|B_i\rangle = T\bar{u}_f\sigma_{\mu\nu}u_i\,, \quad (4.5)$$

where $T$ is a pure number. Moreover, since

$$\sigma_{\mu\nu}\gamma_5 = -\tfrac{1}{2}i\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho}$$

the pseudotensor current also has the same form factor. Let us denote the tensor current by $T^j$, where $j$ is the SU(3) index. Using SU(3) of states

$$\langle B^j|T^i|B^k\rangle = Dd_{ijk} + iFf_{ijk}\,. \quad (4.6)$$

Our current is really $T^{6+i7}$. Owing to $CP$, only $T^6$ contributes to parity-conserving amplitudes and $T^7$ to parity-violating amplitudes.[17] We have found that the bag model gives[18]

$$F = -0.55, \quad D = 0.83\,.$$

Thus

$$\langle p|T^6|\Sigma^+\rangle = \tfrac{1}{2}(D-F)\,, \text{ etc.}$$

But this is as far as we can go with present-day techniques.

## V. COMMENTS AND CONCLUSIONS

In conclusion, we have found that a large parity-violating amplitude in $\Sigma^+ \to p\gamma$ or $\Xi^- \to \Sigma^-\gamma$ signals the presence of some right-handed currents. The total amplitude would also be larger than what naive estimates would lead us to believe by a factor $(m_H/m_L)^2$, where $H$ and $L$ stand for heavy and light quarks. We would also see a large asymmetry parameter and its sign should distinguish between various right-handed currents that have been proposed. Thus the angular distribution of decay products is very sensitive to the nature of the right-handed coupling introduced. Present data seem to be inadequate for a definite conclusion to be made.

If a charmed baryon were to decay radiatively into an uncharmed one and if the quark process $\mathcal{P}' \to \mathcal{P}\gamma$ were important for this then we could conclude, since the light- and heavy-quark roles are now interchanged, that the amplitude is enhanced by the heavy-quark mass in left-handed theories. If right-handed currents are added, the added effect is not very significant, although the asymmetry parameter might change slightly.[19] However, a charmed baryon by virtue of its high mass has a lot of energy and enough phase space to decay into more than two particles, in which case the structure effects are not so important and it is the bremsstrahlung processes that dominate. We note too that in scalar models of weak interactions such as that proposed by Segrè[20] the mass insertion can occur in the internal quark line, as in left-right theories. This will be discussed elsewhere.

*Note*. After this work was completed I came across a report by Ahmed and Ross[21] dealing with the same topic. I thank Dr. A. Zee for drawing my attention to their work.

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[8]This is best seen by making a Fierz transformation of the Hamiltonian. The DGG Hamiltonian goes into

$$\bar{\mathfrak{N}}(1-\gamma_5)\lambda\bar{\mathcal{P}}'(1+\gamma_5)\mathcal{P}' + \text{H.c.}$$

while the WZKT Hamiltonian goes into

$$\bar{\mathfrak{N}}(1+\gamma_5)\lambda\bar{\mathcal{P}}'(1-\gamma_5)\mathcal{P}' + \text{H.c.}$$

The fact that the parity-violating part of the total Hamiltonian now has a piece transforming like $\lambda_7$ (unlike the usual $\lambda_6$) implies for example that the decay $K_1^0 \to 2\pi$ is not forbidden by SU(3). That this decay is forbidden by SU(3) in the standard theory was noted by N. Cabibbo, Phys. Rev. Lett. 12, 62 (1964) and M. Gell-Mann, *ibid.* 12, 155 (1964).

[9]The pole-model results remain unaltered so long as the Hamiltonian commutes with the sum of the $1 \pm i2$ and 3-vector and axial charges. This is true for the WZKT Hamiltonian. The DGG Hamiltonian commutes with the difference of the vector and axial charges.

[10]Note that for Fig. 2(a) we are making an operator-product expansion of the weak Hamiltonian only. This means we have the product of two currents acting at two points in space time and we let these points coincide because the $W$ boson (which propagates between the two points) has a large mass. We do not pull out the electromagnetic current and try to make an expansion of the product of three currents, letting the three points where they act, all come together. We believe this assumption cannot be justified. However, such a procedure has been followed as, for example, in P. Roy, Phys. Rev. D 5, 1180 (1972).

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[12]To understand this claim one needs to look at the contributions of the various graphs in Fig. 2 when we have only the internal $\mathcal{P}$ quark, i.e., without the GIM mechanism. For the pc amplitudes Fig. 2(b) $\sim Ge[M_W^2/(M_W^2 - m_{\mathcal{P}}^2)](m_{\mathfrak{N}} + m_\lambda)$; Figs. 2(c) and 2(d) $\sim Ge(m_{\mathcal{P}}^2/M_W^2)(m_{\mathfrak{N}} + m_\lambda)$; Figs. 2(e) and 2(f) $\sim Ge[M_W^2/(M_W^2 - m_{\mathcal{P}}^2)](m_{\mathfrak{N}} + m_\lambda)$. It is now clear that without the GIM mechanism Figs. 2(c) and 2(d) are suppressed relative to Figs. 2(b), 2(e), and 2(f) by a factor $m_{\mathcal{P}}^2/M_W^2$. Figures 2(b), 2(e), and 2(f) then contribute at order $Ge$, but the GIM mechanism cancels out this contribution and we are left with an order $Ge\Delta m^2/M_W^2$ contribution for Figs. 2(b) to 2(f) inclusive.

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[16]This can be derived using symmetry arguments similar to those in Sec. III.

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[18]The calculation follows very closely that of S. L. Adler *et al.*, Phys. Rev. D 11, 3309 (1975).

[19]In certain models as in model D in R. L. Kingsley *et al.*, Ref. 3, there exists charm-changing neutral currents, so we would have to take into account $Z$ exchange effects. $Z$ is the neutral intermediate boson.

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