PHYSICAL REVIEW D VOLUME 13, NUMBER 7 1 APRIL 1976

Large-$P_\perp$ distribution and characteristics of the fireball*

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(Received 17 November 1975)

Large-$P_\perp$ distributions for $\pi^\pm$ and $K^\pm$ from p-nucleus collisions of the Chicago-Princeton experiment are analyzed using a Bose-type distribution. A discussion is presented on the behavior of two parameters, temperature and transverse velocity of the fireball. The A dependence of cross sections is found to follow a power law A^α, with $\alpha = 0.86$ for $\pi^\pm$ as well as for $K^\pm$. A comparison with data from a CERN ISR experiment shows a striking similarity in meson production by p-p and p-nucleus collisions. Remarks from the viewpoint of the Boltzmann factor are made about the $P_\perp$ distribution of the new particle $J(3.1)$ from nucleon-nucleus collisions.

I. INTRODUCTION

In recent years, there is a growing interest in multiple-meson production by hadron-nucleus collisions. The remarkable feature of such reactions is that recent experiments at the Fermi National Accelerator Laboratory (Fermilab) confirm earlier observations with cosmic rays, namely, in spite of the complexity due to nuclear target, the characteristics of meson production are found to be very similar to those observed in p-p collisions.[1]

In this paper, we propose to investigate the properties of mesons emitted at large $P_\perp$ from nuclei in terms of a Bose-type distribution, which proves to be adequate to describe the simple particle distribution of mesons from p-p collisions.[2] For this purpose, we shall make use of the large-transverse-momentum data of a recent experiment by the Chicago-Princeton groups,[3] then extend the analysis to discuss the production of the new particle $J(3.105)$ from nuclei. We shall consider the original experiment by the MIT-BNL groups[4] as well as a recent experiment by the Columbia-Hawaii-Cornell-Illinois-Fermilab groups.[5]

Consider first the experiment by the Chicago-Princeton groups[3] using Be, Ti, and W targets at 200, 300, and 400 GeV/c. We recall that their data are taken for mesons emitted near 90° c.m. angle and with a c.m. momentum $P \geq 0.76$ GeV/c, and that the average c.m. momentum of π's and K's produced by p-p collisions is about 1.2 GeV/c. Consequently, as a working hypothesis, we assume that most of the mesons measured in their experiment are produced in the primary act of collisions between the incident proton and a bound nucleon of the target nucleus. Tests of this assumption will be discussed in Secs. III and V.

As regards the Fermi motion of the bound nucleon, its effect is negligible as discussed by the authors of that experiment.[3] Indeed, according to their estimate, the smear of the c.m. angle amounts to less than 6°. This corresponds to an uncertainty of <0.12 GeV/c for the longitudinal momentum $P_\parallel$ in the c.m. system, and is negligible in comparison with measured transverse momentum $P_\perp \geq 0.76$ GeV/c. Therefore, in the case of the Chicago-Princeton experiment, we may write

$$P = P_\perp, \quad E = (P_\perp^2 + m^2)^{1/2}, \tag{1}$$

m being the mass of the meson under consideration. With this remark, we now proceed to analyze their data.

II. PARAMETRIZATION OF LARGE-$P_\perp$ DISTRIBUTION

We assume the following empirical expression to describe the invariant cross sections measured with high accuracy and over a wide range by the Chicago-Princeton groups:

$$E\frac{d\sigma}{d^3P} = \frac{C}{P_\perp} e^{-(E-aP_\perp)/T}, \tag{2}$$

where C is the normalization constant and a and T are two parameters. The physical measuring of a and T will become apparent, if we consider the total cross section σ. Denoting by $\Delta\omega$ the solid angle, we obtain from (1)

$$\sigma = C \int \frac{1}{P_\perp} e^{-(E-aP_\perp)/T} \frac{\Delta\omega P^2 dP}{E}$$
$$= C\Delta\omega \int e^{-(E-aP_\perp)/T} dE, \tag{3}$$

which represents the partition function of a Boltzmann distribution evaluated in a specific system, referred to as the fireball, moving with a transverse velocity a (in units of $c=1$) with respect to the c.m. system of the colliding proton and a target nucleon, T being the temperature (the Boltzmann constant is set equal to 1). For a further discussion on the fireball motion in relation to the Bose distribution, we refer to a previous paper, Ref. 2(c).

TABLE I. Estimates of parameters, Chicago-Princeton experiment, Ref. 2.

$P_{\rm lab}$ (GeV/c)	Target	Particle	T (GeV)	a	$\left(\frac{4\pi}{\Delta\omega}\right)\sigma$ (cm^2)
200	W	π^+	0.154 ±0.004	0.44 ±0.03	$(3.35\pm0.16)\times10^{-25}$
		π^-	0.159 ±0.003	0.43 ±0.09	$(2.85\pm0.49)\times10^{-25}$
		K^+	0.144 ±0.016	0.48 ±0.05	$(8.82\pm1.34)\times10^{-26}$
		K^-	0.129 ±0.019	0.50 ±0.13	$(3.99\pm1.26)\times10^{-26}$
300	W	π^+	0.164 ±0.005	0.45 ±0.02	$(3.96\pm0.18)\times10^{-25}$
		π^-	0.158 ±0.004	0.46 ±0.02	$(3.84\pm0.17)\times10^{-25}$
		K^+	0.167 ±0.015	0.45 ±0.08	$(1.87\pm0.47)\times10^{-25}$
		K^-	0.150 ±0.007	0.48 ±0.14	$(5.16\pm1.51)\times10^{-26}$
400	W	π^+	0.192 ±0.019	0.38 ±0.06	$(4.79\pm0.70)\times10^{-25}$
		π^-	0.164 ±0.016	0.46 ±0.06	$(4.34\pm0.55)\times10^{-25}$
		K^+	0.133 ±0.020	0.56 ±0.09	$(1.03\pm0.27)\times10^{-25}$
		K^-	0.192 ±0.016	0.35 ±0.16	$(0.59\pm0.17)\times10^{-25}$
300	Be	π^+	0.120 ±0.004	0.57 ±0.02	$(2.77\pm0.09)\times10^{-26}$
		π^-	0.118 ±0.005	0.59 ±0.02	$(2.85\pm0.19)\times10^{-26}$
		K^+	0.100 ±0.010	0.64 ±0.10	$(4.17\pm1.27)\times10^{-27}$
		K^-	0.120 ±0.002	0.56 ±0.16	$(3.39\pm1.30)\times10^{-27}$
	Ti	π^+	0.133 ±0.003	0.52 ±0.10	$(1.25\pm0.28)\times10^{-25}$
		π^-	0.143 ±0.003	0.49 ±0.11	$(1.23\pm0.28)\times10^{-25}$
		K^+	0.111 ±0.012	0.60 ±0.13	$(2.30\pm0.79)\times10^{-26}$
		K^-	0.100 ±0.010	0.63 ±0.15	$(1.50\pm0.65)\times10^{-26}$

We have analyzed the data of the Chicago-Princeton experiment[3] for $\pi^\pm$ and $K^\pm$ from three targets, and at three incident proton momenta. The parameters a and T (in GeV) as well as the normalization constant C of (2) are estimated by least-squares fits. The values of a and T are listed in Table I, whereas the constants C have been converted into cross sections (cf. Sec. V) which are listed in the last column of Table I.

Generally speaking, the fits are satisfactory. For illustration, we present in Fig. 1 the fits to π^- from W target at $P_{\rm lab}=200$ and 300 GeV/c, the χ^2/point being 0.50 and 0.54, respectively.

As regards the reliability of the fits, we have checked this by refitting some data with $P_\perp \geq 1.53$ GeV/c and found that the new estimates of a and T thus obtained are, within fitting errors, consistent with those listed in Table I. Furthermore, the extrapolation of the fitted curve passes very close to the two points left aside by the cutoff on $P_\perp$.

This indicates that our method of analysis is satisfactory and supports our working hypothesis that the mesons with $P_\perp \geq 0.76$ GeV/c of the Chicago-Princeton experiment are mostly produced in the primary collision so that they can be adequately described by (2). We shall discuss this point further in the following section.

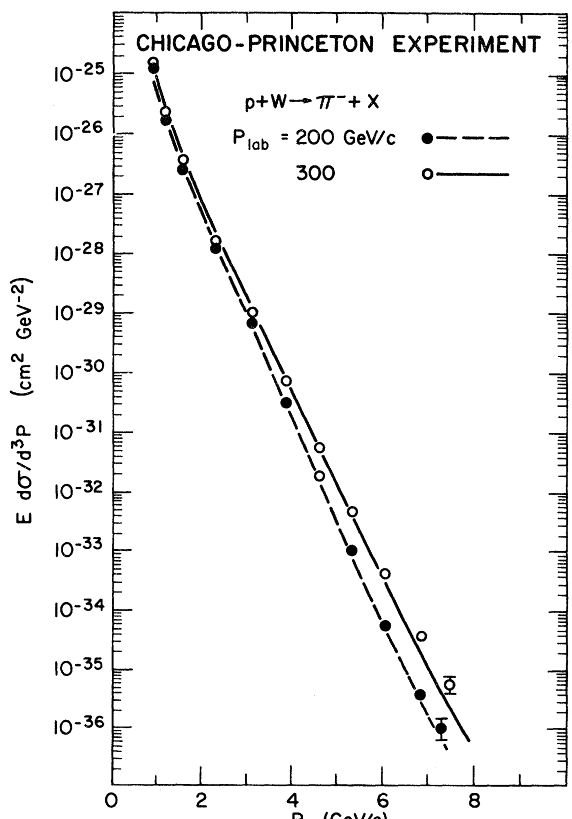

FIG. 1. $P_\perp$ distribution fit with a Bose-type distribution. The parameters are the temperature T and the transverse velocity a of the fireball. Estimates of T and a are listed in Table I.

III. VALIDITY TEST

We now proceed to investigate if the empirical expression (2) is actually adequate to describe the large-$P_\perp$ data of the Chicago-Princeton experiment.[3] For this purpose, we use the measured invariant cross sections and the fitted parameter T to compute the quantity

$$X = \ln\left[P_\perp\left(E\frac{d\sigma}{d^3P}\right)\right] + \frac{E}{T} \tag{4}$$

and plot X against $P_\perp$ for each set of data. In Fig. 2, the open circles represent the case for π^- from W target and at 200 GeV/c; the invariant cross sections being expressed in units of 100 mb/GeV^2 whereas E, P, and T are in GeV.

We note that the points lie on a straight line as expected from (2). This indicates that the distribution here considered is adequate to represent the experimental data under consideration.

It is interesting to compare this plot with that for π^- emitted at 90° in the c.m. system from the CERN colliding proton beams of the British-Scandinavian experiment.[6] The plot for $\sqrt{s} = 52$ GeV is shown by crosses in Fig. 2, the invariant cross sections being in mb/GeV^2 in this case. The parameters are listed in Table II.

The striking similarity of these two plots gives strong indication that the mesons with $P_\perp \geq 0.76$ GeV/c from W target may be regarded as produced by a single p-nucleon collision as in the case of colliding proton beams. This similarity, which has been noted by the Chicago-Princeton groups, see Ref. 3, gives further support to our working hypothesis mentioned earlier in Sec. I. We shall discuss this point again after investigating the behavior of production cross sections in Sec. V.

IV. BEHAVIOR OF THE TRANSVERSE VELOCITY AND THE TEMPERATURE OF THE FIREBALL

Referring to the parameters a and T summarized in Table I, we note that in each case of $\pi^\pm$ and $K^\pm$, we have at our disposal the estimates of a and T for targets of atomic numbers $A = 9$, 48, and 184, and for $\sqrt{s} = 19.4$, 23.8, and 27.4 GeV. This extensive information enables us to investigate the properties of the fireballs resulting from the collision between the incident proton and a nucleon of the target. In this regard we note that the condition of the fireball we are dealing with here differs from the case of p-p collision; in the present case the two fireballs are surrounded by other nucleons of the target nucleus instead of being in an empty space as in the case of p-p collisions.

Considering now the values of a and T at fixed

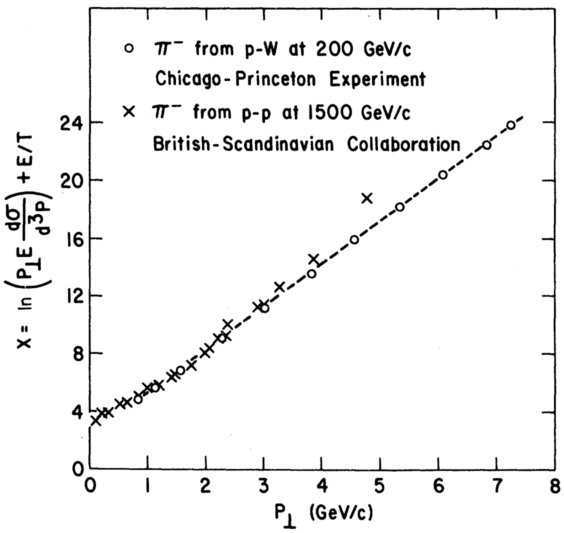

FIG. 2. Plots of the test function X vs $P_\perp$. Open circles represent π^- cross sections (100 mb/GeV^2) for π^- from W target at 200 GeV/c of the Chicago-Princeton experiment, Ref. 3, and crosses are those (in mb/GeV^2) from p-p at 1500 GeV of the British-Scandinavian experiment, Ref. 5. Validity test requires the plot to be linear, see text in Sec. III.

A and $\sqrt{s}$, we note that they are equal within fitting errors, although a small difference between T_π and T_K, as reported previously in Ref. 2(a), cannot be excluded. Note that the errors on the parameters for $K^\pm$ are in general larger than those for $\pi^\pm$. We therefore use only a_π and T_π for the investigation of A and $\sqrt{s}$ dependence.

In Fig. 3 we have plotted the averages of a and T of $\pi^\pm$ against A and $\sqrt{s}$. Consider first the behavior of T; we note that it increases with A. As regards its dependence on $\sqrt{s}$, we recall that a previous analysis of p-p data indicates an increase following a power law as is expected from Stefan's law [see Ref. 2(a)]. Therefore we are led to fit the trend of T with an empirical relation as follows:

$$T = \alpha(1 + \beta A^\gamma)s^\delta. \tag{5}$$

As we have only 5 points at our disposal, therefore we content ourselves only with a rough esti-

TABLE II. Estimates of parameters, British-Scandinavian experiment, Ref. 6. $P_{lab} = 1500$ GeV/c.

Particle	T (GeV)	a	$\left(\frac{4\pi}{\Delta\omega}\right)\sigma$ (cm^2)
π^+	0.110 ± 0.010	0.61 ± 0.03	$(4.78 \pm 0.58) \times 10^{-27}$
π^-	0.165 ± 0.005	0.46 ± 0.05	$(4.34 \pm 0.45) \times 10^{-27}$
K^+	0.120 ± 0.005	0.54 ± 0.05	$(1.13 \pm 0.16) \times 10^{-27}$
K^-	0.148 ± 0.005	0.62 ± 0.05	$(9.29 \pm 0.11) \times 10^{-28}$

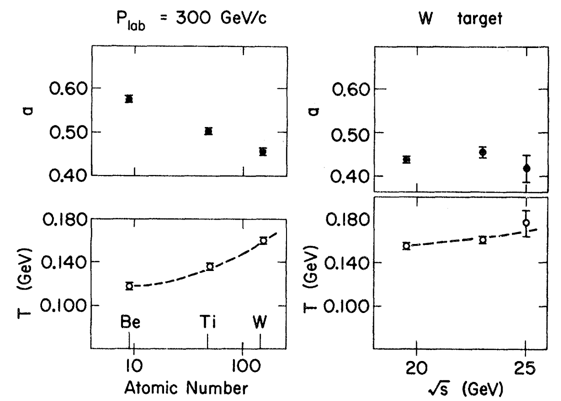

FIG. 3. Plots of the behavior of parameters a and T against the atomic number of the target and the energy. Dotted lines represent an empirical relation (5) for A and s dependence.

mate of the parameters and find

$$\alpha \simeq 0.071, \quad \beta \simeq 0.0069, \quad \gamma \simeq 0.78, \quad \delta \simeq 0.15.$$

The fits are shown by dotted lines in Fig. 3.

Finally, we note that the temperatures listed in Table I are comparable to those of large-$P_\perp$ data of CERN ISR experiments, but significantly higher than those for $P_\perp < 1.5$ GeV/c [see Refs. 2(b) and 2(c)].

Consider next the behavior of a. First, we note that a is not sensitive to the increase of $\sqrt{s}$. This indicates that there is no scaling for a, in contrast with the longitudinal velocity. This result, already mentioned by the Chicago-Princeton groups, has been noticed in a previous study, Ref. 2(c), for the case of p-p collisions.

On the other hand, we note that the transverse velocity a of the fireball decreases with A. This may be regarded as an effect due to a drag of nuclear matter by the moving fireball. Note that the dependence on A may be expressed by means of an angular distribution of the velocity $\vec{v}$ of the fireball with respect to the direction of the incident proton. If we assume an empirical distribution as follows,

$$p(\theta) \propto \cos^n\theta, \tag{6}$$

then we obtain

$$\frac{a}{v} = \frac{\int_0^{\pi/2} \sin\theta \cos^n\theta \, d\cos\theta}{\int_0^{\pi/2} \cos^n\theta \, d\cos\theta} = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\frac{1}{2}n + \frac{3}{2})}{\Gamma(\frac{1}{2}n + 2)}. \tag{7}$$

Figure 4 shows a/v as a function of the power n in (6). The experimental values of a/v for Be, Ti, and W targets at $\sqrt{s} = 23.8$ GeV have been marked on the curve; the value v has been computed using

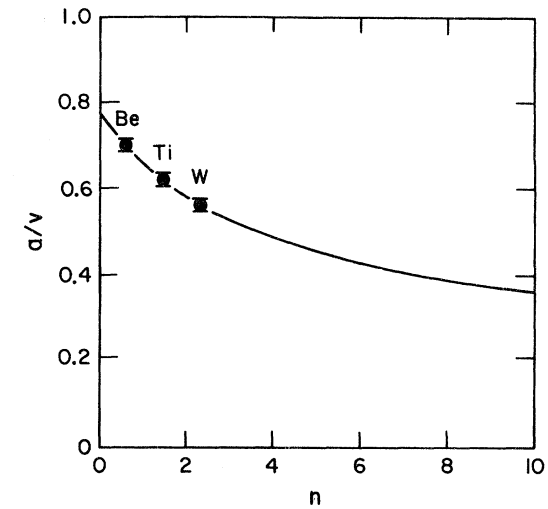

FIG. 4. Plot of a/v against n. For definitions of n and a/v, see (6) and (7).

$$v = 1 - 4m_p/\sqrt{s}, \tag{8}$$

where m_p is the proton mass [see Ref. 2(c)].

We find that n increases from ~0.5 for the Be target to ~2.5 for the W target. This corresponds to an increase in the anisotropy of the orientation of $\vec{v}$ with respect to the line of collision. Finally, we note that the mass of the fireball for $\sqrt{s} = 23.8$ GeV is estimated to be $M^* \simeq m_p(\frac{1}{2}\gamma_{c.m.})^{1/2} = 6.4$ GeV, $\gamma_{c.m.} = \sqrt{s}/2m_p$ being the Lorentz factor for the c.m. system [see Ref. 2(c)], and that its transverse momentum is ~3.5 GeV/c in the case of W target.

V. PRODUCTION CROSS SECTIONS

We now use the fitted parameters a and T to estimate the measured total cross sections. For this purpose we integrate (3) and obtain

$$\sigma = C \frac{\Delta\omega}{4\pi} \frac{T}{1-a} e^{-(E - aP_\perp)\gamma_0/T}, \tag{9}$$

where the subscript 0 indicates that the quantity within the parentheses is to be evaluated at $P_0 = 0.76$ GeV/c. The cross sections thus estimated are listed in the last column of Table I, the solid angle $\Delta\omega/4\pi$ being excluded ($\Delta\omega = 1.7 \times 10^{-6}$ sr according to the experiment, Ref. 3). It should be mentioned that the values of σ thus estimated are independent of our working hypothesis concerning the mechanism of meson production from nuclei.

If we plot on log scale the cross sections listed in Table I corresponding to $P_{lab} = 300$ GeV/c against the atomic number A of the target (see Fig. 5), we find that the points for π's and K's are practically on parallel lines. This indicates a unique

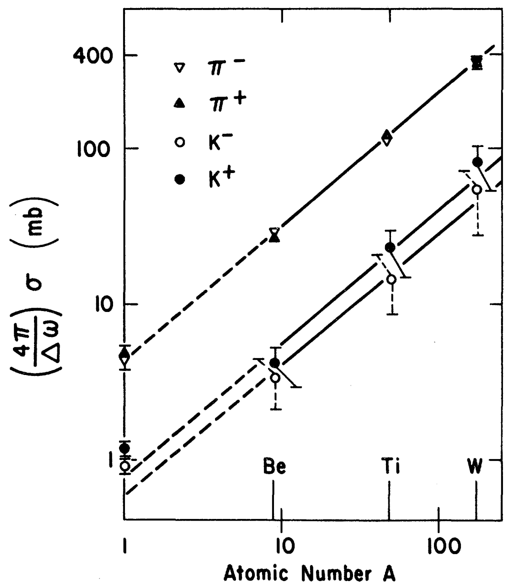

FIG. 5. Cross sections for $\pi^\pm$ and $K^\pm$ at 300 GeV/c of the Chicago-Princeton experiment, Ref. 3, with Be, Ti, and W targets. Solid straight lines represent one fit for A dependence assuming the same power law A^α for $\pi^\pm$ and $K^\pm$; $\alpha = 0.87 \pm 0.01$. Dotted line represents the extrapolation to $A = 1$ for comparison with p-p data.

power-law dependence for all the data here considered, $\pi^\pm$ as well as $K^\pm$. Therefore, we may describe the dependence as

$$\frac{4\pi}{\Delta\omega}\sigma_j(i) = C_j[A(i)]^\alpha , \tag{10}$$

where $i = 1$–3 refers to the targets Be, Ti, and W and $j = 1$–4 refers to π^-, π^+, K^+, K^-, respectively. We have 5 parameters to be estimated by fitting simultaneously 12 data points shown in Fig. 5. We find by least-squares[7]

$$\alpha = 0.87 \pm 0.01,$$

$$C_1 = 4.19 \pm 0.13, \quad \chi^2 = 0.12 \quad \text{for } \pi^-,$$

$$C_2 = 4.14 \pm 0.04, \quad \chi^2 = 0.06 \quad \text{for } \pi^+,$$

$$C_3 = 0.73 \pm 0.06, \quad \chi^2 = 0.79 \quad \text{for } K^+,$$

$$C_4 = 0.52 \pm 0.11, \quad \chi^2 = 0.02 \quad \text{for } K^-.$$

The fit is shown by solid lines in Fig. 5. Note that $C_1 = C_2$ as predicted by Watson[8] and that, in the case of like nuclei, $\alpha = \frac{3}{4}$ as predicted by Landau.[9]

If we extrapolate the fit to $A = 1$ as shown by the dotted lines in Fig. 5, we may compare the intercepts with the cross sections obtained from the p-p data of the British-Scandinavian experiment[6] for $P_\perp \geq 0.76$ GeV/c as in the Chicago-Princeton experiment.[3] The results are shown in Fig. 5. We note that in spite of the difference in energy, namely $\sqrt{s} = 52$ instead of 23.8 GeV, the points for $\pi^\pm$ cross sections are close to the extrapolated straight lines, whereas those for $K^\pm$ are ~14% higher than the dotted lines. This observation indicates that the cross sections for mesons produced at large $P_\perp$ and near 90° c.m. angle are not sensitive to the incident energy for $\sqrt{s}$ varying from 23.8 to 52 GeV.

On the other hand, the A dependence of cross sections are quite important. Indeed, from Fig. 5, we note that for $\pi^\pm$, as for $K^\pm$, the production increases almost by a factor of ~100 from p target at ISR energy to W target at Fermilab energy.

Finally, it should be mentioned that in their investigation of cross sections, the authors of the Chicago-Princeton experiment[3] have used a more detailed analysis by assuming a $P_\perp$-dependent power $\alpha(P_\perp)$ to describe the A dependence for each meson, $\pi^\pm$ and $K^\pm$, considered separately. If we take the average of their values of $\alpha(P_\perp)$ for π's and K's at $P_\perp = 0.76$ GeV, we find 0.86 ± 0.05, in agreement with the estimate according to (10).

VI. $P_\perp$ DISTRIBUTION FOR THE PARTICLE $J(3.1)$

We now turn to the $P_\perp$ distribution of the $J(3.1)$ particle produced near 90° c.m. angle by p-Be collisions of the MIT-BNL experiment,[4] although in this case the range of $P_\perp$ hardly exceeds 1 GeV/c, in contrast with large $P_\perp$ of π's and K's discussed in the previous sections. Nonetheless, because the mass of $J(3.1)$ is much heavier, its total energy E turns out to be comparable to those observed with large-$P_\perp$ data. Consequently, as regards the Boltzmann factor $e^{-E/T}$, there is much in common with these reactions.

We therefore propose to investigate certain properties of $J(3.1)$ production in the context of the Bose-Einstein distribution [see Ref. 2(c)]:

$$\frac{d\sigma}{dP_\perp^2 dP_\parallel} \propto \frac{1}{e^{(E-\vec{v}\cdot\vec{P})/T} - 1}, \tag{11a}$$

where E and $\vec{P}$ are the c.m. energy and momentum of the particle under consideration, and $\vec{v}$ is the velocity (in units of $c = 1$) of the fireball with respect to the c.m. system of the colliding nucleons. Since the temperature T is of the order of m_π, the exponential term in the denominator of (11a) is much greater than 1; thus we may drop the term 1. Consequently, the distribution reduces to that of Boltzmann.

In the MIT-BNL experiment,[4] the particle J is detected near 90° c.m. angle and its momentum $P = (P_\perp^2 + P_\parallel^2)^{1/2} \ll m$. Therefore, $E = m + P^2/2m \simeq m + P_\perp^2/2m$, if we neglect $P_\parallel^2/2m$ in comparison to m. As a further simplification, we recall that for $P_\perp < 1.5$ GeV/c, the transverse motion of the fireball is not perceptible [see Ref. 2(c)], thus we may drop the $\vec{v}\cdot\vec{P}$ term compared to E. This leads to

$$\frac{d\sigma}{dP_\perp^2} \propto e^{-m/T - P_\perp^2/2mT}, \tag{12}$$

i.e., a Gaussian distribution for $P_\perp$, which has been used by the MIT-BNL groups.[10] Their preliminary data yield a slope ~1.6 corresponding to a temperature $T \simeq 0.102$ GeV, which is comparable to ~0.080 GeV computed from the Chicago-Princeton experiment[3] using the empirical formula (5).

Next, consider the Columbia-Hawaii-Cornell-Illinois-Fermilab experiment.[5] Here we are dealing with the J production in the forward direction of n-Be collisions at 205 GeV; contrary to the case of large-$P_\perp$ experiment, we now have $P_\parallel \gg P_\perp$.[11] From a previous study of the fireball, Ref. 2(c), we know that, in general, $v_\perp \ll v_\parallel$. Therefore in this case we may write

$$\vec{v}\cdot\vec{P} \simeq (1-\lambda)P, \tag{13}$$

$(1-\lambda)$ being the longitudinal component of $\vec{v}$, and (11a) becomes

$$\frac{d\sigma}{dP_\perp^2 dP_\parallel} = C\exp[-(P_\perp^2 + \lambda^2 P_\parallel^2 + m^2)^{1/2}/T], \tag{11b}$$

which is more suitable for the computation.

The $P_\perp$ distribution is obtained from (11b) by integrating over $P_\parallel$ between limits corresponding to the acceptance of the set-up. For simplicity, we assume the limits to be 0 and $+\infty$; this enables us to obtain a closed form as follows:

$$\frac{d\sigma}{dP_\perp^2} = C\int_0^\infty \exp[-(P_\perp^2 + \lambda^2 P_\parallel^2 + m^2)^{1/2}/T]\,dP_\parallel$$
$$= C\frac{m_\perp}{\lambda}K_1\left(\frac{m_\perp}{T}\right), \tag{14}$$

where $m_\perp$ denotes the transverse mass

$$m_\perp = (P_\perp^2 + m^2)^{1/2} \tag{15}$$

and K_n denotes the modified Bessel function of the second kind of order n. Noting that $m \gg T$, we may approximate $K_n(x) \simeq (\pi/2x)^{1/2}e^{-x}$. This leads to

$$\frac{d\sigma}{dP_\perp^2} = C'\sqrt{m_\perp}e^{-m_\perp/T}, \tag{16}$$

$C' = (C/\lambda)(\frac{1}{2}\pi)^{1/2}$ being a new constant. An integration of (14) gives

$$\sigma = \frac{2CT^2}{\lambda}K_2\left(\frac{m}{T}\right). \tag{17}$$

A validity test of (16) may be performed by plotting $\log[(d\sigma/dP_\perp^2)/\sqrt{m_\perp}]$ versus $m_\perp$; we should expect the points to lie on a straight line. In Fig. 6 we have presented the plot with the ρ data of

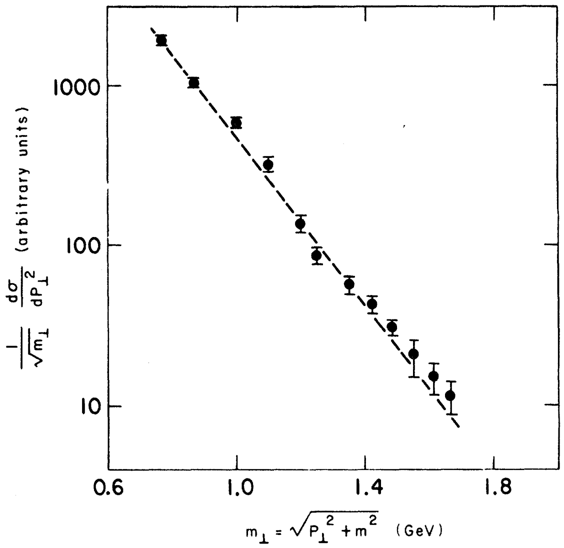

FIG. 6. Comparison of a one-parameter Bose-type distribution (15) fit with the data of ρ produced by n + Be at 205 GeV/c of the Columbia-Hawaii-Cornell-Illinois-Fermilab experiment, Ref. 5. The fitted temperature is $T = 0.163 \pm 0.015$ GeV.

the same experiment. The dotted line represents the least-squares fit with (16), and the corresponding $P_\perp^2$ distribution is shown in Fig. 7. The parameters are

$$T = 0.163 \pm 0.015, \quad \ln C' = 12.39 \pm 0.14,$$

with χ^2/point = 18.7/12. Note that within fitting

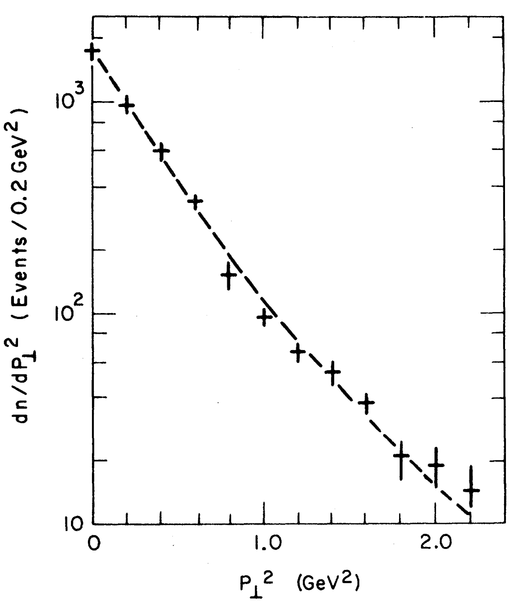

FIG. 7. Fitted $P_\perp^2$ distribution for ρ; same data as in Fig. 6.

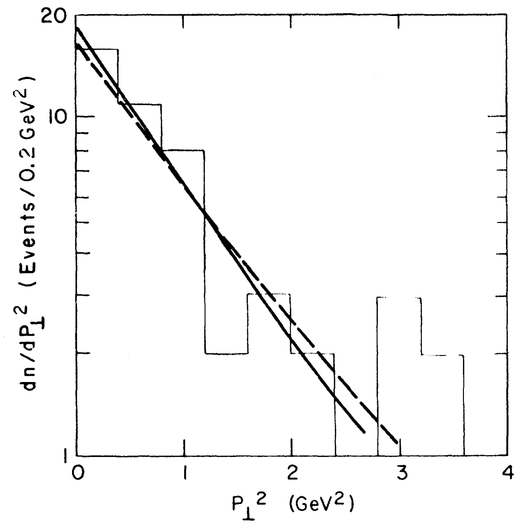

FIG. 8. Fits with a Bose-type distribution (15) to the $J(3.1)$ data of the Columbia-Hawaii-Cornell-Illinois-Fermilab experiment, Ref. 5. The solid line represents the fit similar to that for ρ of Fig. 6, $T=0.142\pm 0.025$ GeV. The dotted line is a fit with $T=0.163$ taken from ρ (see Fig. 7).

errors, the temperature thus estimated agrees with that expected from the Chicago-Princeton experiment (see Table I).

We now proceed to fit the $P_\perp^2$ distribution of $J(3.1)$. By the same procedure as in the case of ρ, we find

$$T=0.142\pm 0.025,\quad \ln C'=24.22\pm 0.25.$$

The fit is shown by the solid curve in Fig. 8, with χ^2/point = 5.7/8. Note that within fitting errors, the temperature thus estimated is consistent with that for ρ of the previous fit.

As a check of this fit, we have made another trial assuming T to be the same for both ρ and $J(3.1)$. In this case we find

$$\ln C'=21.26\pm 0.22.$$

The fitted curve is shown by the dotted line in Fig. 8, χ^2/point = 6.1/8. Note that in spite of the fact that the χ^2 for both fits seems to be reasonable, the uncertainties of these fits are quite large, as is shown by the change in the normalization constant, which amounts to a factor ~20.

Finally, we note that the ratio of $J(3.1)$ to ρ may be estimated by (17) without knowing the parameter λ, which will be discussed in the next section. For this purpose, we rather assume T as a free parameter and obtain for the ratio of J to ρ observed in the $\mu^+\mu^-$ decay mode

$$\frac{\sigma(J)}{\sigma(\rho)}=\frac{C'_J}{C'_\rho}\frac{K_2(m_J/T_J)}{K_2(m_\rho/T_\rho)} =(2.4\pm 0.3)\times 10^{-4}, \qquad (18)$$

the error being only statistical. In this regard we recall that the uncertainties inherited in the fit to J data may alter the above estimate by a factor ~20.

VII. REMARKS

We have presented the analysis of large-$P_\perp$ distributions of mesons using a Bose-type distribution, account being taken of the transverse motion of the fireball described by the parameter a. In this regard, we mention that this parameter is needed not only to obtain a good fit covering the whole range of data, but especially to obtain a fitted temperature T less than the limit $T_0=0.160$ as required by certain models.[12] Indeed without a the temperature, which remains the only free parameter, is found to be higher, about twice the value we have listed in Table I and II, thus unacceptable from the viewpoint of the thermodynamical model.[12]

As regards the longitudinal motion of the fireball, clearly its investigation required additional information on the $P_\parallel$ distribution of the particle under consideration, see (11a). We note that the parameter λ, which we have introduced in (13) to describe the longitudinal motion, is related to the scaling [see Ref. 2(c)] and that λ can be easily estimated by observing the characteristic feature of the angular distribution which can be derived from (11b) as follows.

Let $\mu=\cos\theta$, θ being the c.m. angle of the par-

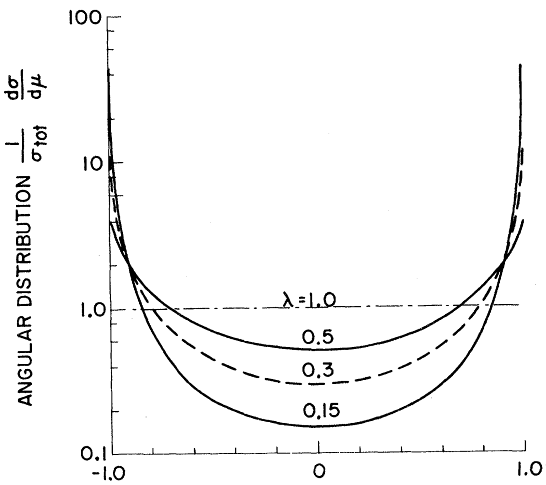

FIG. 9. Plots of c.m. angular distribution according to a modified Bose-Einstein distribution, see (17) and (11b). The anisotropy is characterized by the parameter λ related to the longitudinal velocity of the fireball (see text).

ticle with respect to the line of collision. An integration of (11b) over $P=(P_\perp^2+P_\parallel^2)^{1/2}$ leads to

$$\frac{d\sigma}{d\mu}\propto\frac{1}{[1-(1-\lambda^2)\mu^2]^{3/2}}\,. \quad (19)$$

Figure 9 shows the characteristics of this distribution for some values of λ. Because of the symmetry, we may restrict θ to one hemisphere and obtain

$$\langle\mu\rangle=1/(1+\lambda),\quad \langle\mu^3\rangle=1/(1+\lambda)^2, \quad (20)$$

which may be used to estimate λ and to test the validity of the Bose-type distribution (11b) by checking if $\langle\mu^3\rangle=\langle\mu\rangle^2$.

Finally, we note that the cross sections derived from the Bose-type distribution contains a Boltzmann factor $e^{-m/T}$ characteristic of the production [see (9), (12), and (17)]. In this regard we note that this factor is in part counterbalanced by another one from the density of states $\rho(m)$ which is contained in the normalization constant, although not explicitly specified. The form of $\rho(m)$ depends on the model[12] and is beyond the scope of the present work.[13]

ACKNOWLEDGMENTS

The author wishes to thank Professor J. Cronin and Dr. H. Frisch for invaluable help and inspiring interest in this work. Acknowledgment is also due to Dr. G. Thomas for a discussion.

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*Work done under the auspices of the U. S. Energy Research and Development Administration.

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$\alpha=0.86\pm0.01$, $C_1=4.21\pm0.11$,

$C_2=4.16\pm0.04$, $\chi^2=0.08+0.07$,

consistent with the previous over-all fit. Next, we make a fit to K^+ and K^- and obtain

$\alpha=0.96\pm0.06$, $C_3=0.54\pm0.04$,

$C_4=0.36\pm0.06$, $\chi^2=0.06+0.10$;

these parameters are slightly different from those of the over-all fit.

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[11]The momentum cutoff is ~ 70 GeV/c in the lab system [see Ref. 6, Fig. 3(c)]. This corresponds to $P_\parallel > 3.6$ GeV/c in the c.m. system.

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