Vertical sea-level muon spectrum at solar minimum near geomagnetic equator in the momentum range 0.5–3 GeV/c

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(Received 13 June 1972; revised manuscript received 25 August 1975)

The absolute sea-level vertical differential muon spectrum in the momentum range 0.5–3 GeV/c has been measured at solar minimum near the geomagnetic equator and compared with other recent measurements at high latitudes. The spectral shape derived from the theory of Olbert is in accord with the experimental results. The observed difference from the high-latitude muon spectrum can be explained in terms of the geomagnetic-latitude effects.

I. INTRODUCTION

The estimation of the absolute sea-level muon spectrum near the geomagnetic equator is necessary for the study of the influence of the long-term solar modulation on the muon spectrum and geomagnetic effects on muon, and to get some information about their primaries.

The absolute sea-level muon intensity around 1 GeV/c has been determined recently by many authors and reviewed by Allkofer and Jokisch.[1] Only a few experimental results on the low-latitude muon spectrum are available and have been reviewed in Refs. 2 and 3.

In the present investigation a detailed study has been made on the sea-level muon spectra at solar maxima and minima in order to explore the latitude effects of muons. The theories of Olbert[4] and Jabs[5] have been used to explain the deviation of our experiment from the high-latitude results of Allkofer *et al.*,[6] Ashton *et al.*,[7] and Ng *et al.*[8]

II. THE EXPERIMENT

The absolute cosmic-ray muon flux in the vertical direction at Calcutta (geomagnetic latitude $\lambda = 12\,^\circ$N, altitude 80 ft, and vertical cutoff rigidity of primaries $P_c = 16.5$ GV) was determined, using a differential range spectrograph, in the momentum range 0.5–3 GeV/c. The experimental arrangement is shown in Fig. 1. The total thickness of the absorbers in the layers Σ and T was varied from 271 to 2042 g cm^{-2} of lead-equivalent material. The detail of the flash tubes used in this experiment was reported in Ref. 9. The geometrical factor of the arrangement was 1.468 cm^2 sr. The counter efficiency correction for the telescope $C_1C_2C_3$-AC_1AC_2 was 2%. The experiment was similar to the one reported in Ref. 3.

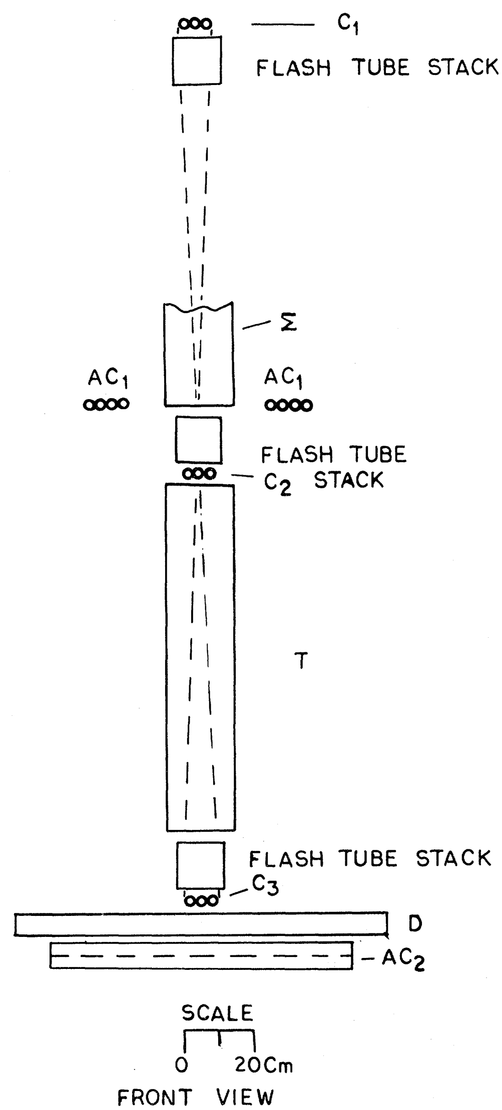

FIG. 1. Schematic diagram of the experimental arrangement.

TABLE I. Muon intensity data at solar minimum. Period: March 1973; Ehmert potential estimated from the Kiel neutron monitor data, $U = 200$ MV.

Range of absorber in $\Sigma + T$ of lead-equivalent material in g cm^{-2}	Run time (hours)	Observed number of events (singles + accompanied)
271–351	147.0	265
655–735	163.9	218
1245–1325	157.3	147
2042–2122	170.0	82

III. RESULTS AND DISCUSSION

The effective run time of the detector system during the period of solar minimum was 638.2 h, and 712 events were observed. These consist of single penetrating particles as well as penetrating particles with accompaniment. Table I gives the number of penetrating particles stopped in the differential layer D (Fig. 1) during the time of run at solar minimum (at Ehmert potential, $U = 200$ MV). The correction factors applied to the experimental data were due to (i) the scattering out of particles, traveling through the differential layer D, whose momenta exceeded that corresponding to the differential layer, and (ii) the scattering loss of particles traveling through the absorber layers $\Sigma + T$. The correction (i) is negative and (ii) is positive with respect to the observed data. The detail of the correction procedure is presented in the Appendix of this paper.

The absolute sea-level muon intensity (singles plus accompanied particles) at solar minimum is presented in Table II. The sea-level absolute muon intensity (singles plus accompanied particles, reported in Ref. 3) determined at solar maximum is presented in Table III.

The theoretical differential spectrum of single muons at $\lambda = 12^\circ$ N has been derived from the production spectrum of Olbert[4]

$$G(R,\lambda) = 7.3 \times 10^4/[a(\lambda) + R]^{3.58}\ \mathrm{g^{-2}\,cm^2\,sec^{-1}\,sr^{-1}}, \quad (1)$$

where $a(\lambda)$ is the latitude parameter, which is found to be 635 g cm^{-2} from

$$I_v(d, R;\lambda) = \int_0^d G(R + s - x, \lambda) \times \exp(-x/L) W(x, R)\,dx \quad (2)$$

at $\lambda = 12^\circ$ N. In this relation $W(x,R)$ is the survival probability of a muon produced at the atmospheric depth x reaching sea-level depth $d = 1033$ g cm^{-2} with residual range R, and $s = 100$ g cm^{-2}. The calculated spectra have been multiplied by 1.16 and 1.27, respectively, and plotted in Fig. 2 along with the experimental results presented in Tables II and III. It is found that the muon-intensity data determined at solar minimum and solar maximum obey the spectral shape, calculated after Olbert,[4] adequately when the calculated muon spectra are normalized to the experimental muon intensity at 1 GeV/c.

Latitude effect on the muon spectrum at solar maximum

The latitude effect is defined as the ratio of the high-latitude intensity to the low-latitude intensity at a particular momentum. The latitude effect on muon intensity at solar maximum has been estimated from the absolute muon spectrum at $\lambda = 12^\circ$ N and the Kiel ($\lambda = 55^\circ$ N) muon spectrum[6] and is shown in Fig. 3. The theoretical results calculated after Olbert[4] and Jabs[5] respectively are displayed in the same figure. The latitude effect decreases with momentum and decreases considerably above 2 GeV/c. It is found that the latitude effect at solar maximum is in accord with

TABLE II. The observed and corrected data at different muon momenta. Telescope efficiency correction is 2%.

Mean muon momentum (GeV/c)	Run time (hours)	Observed number of events	Scattering correction (%) (i) in D	(ii) in $\Sigma + T$	Corrected number of events	Absolute muon intensity per 10^3 cm^2 sec sr GeV/c
0.5	147.0	265	−6.2	1.0	254	3.27 ± 0.21
1.0	163.9	218	−5.0	5.2	220	2.54 ± 0.17
1.8	157.3	147	−5.0	12.0	158	1.90 ± 0.12
3.0	170.0	82	−5.0	23.0	97	1.08 ± 0.11

TABLE III. Absolute sea-level muon intensity data at solar maximum. Period: January 1969; Ehmert potential estimated from the Kiel neutron monitor data, $U = 580$ MV.

Mean muon momentum (GeV/c)	Absolute muon intensity (singles + accompanied) per 10^3 cm^2 sec sr GeV/c
0.5	2.96 ± 0.13
1.0	2.31 ± 0.17
1.8	1.76 ± 0.13
3.0	1.01 ± 0.12

the theoretical curve calculated after Olbert[4] above 1 GeV/c; for low-momenta muons there is an appreciable disagreement. A partial variation of muon intensity is due to atmospheric effects which are caused by changes in the local meteorological conditions. The atmospheric effect, calculated after Olbert,[10] shows a variation of differential muon intensity of 7–2% in the momentum range 0.5–2 GeV/c. In general, the atmospheric effect is about one third of the pure geomagnetic effect on muons.

Latitude effect on the muon spectrum at solar minimum

The vertical sea-level muon spectrum near the geomagnetic equator at solar minimum has been

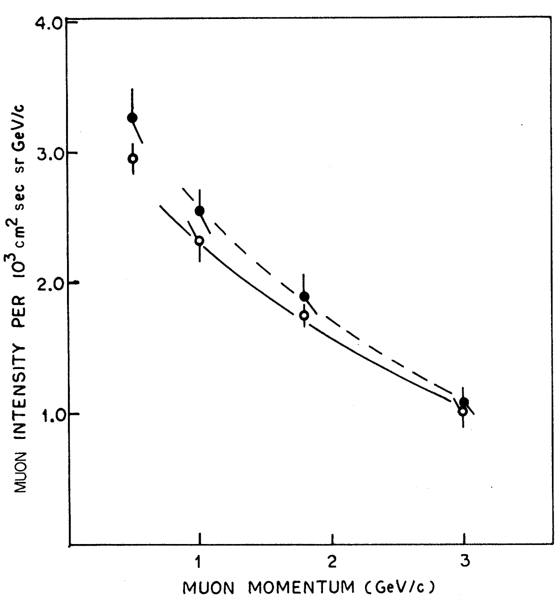

FIG. 2. Differential muon spectra at solar maximum and minimum: Theoretical curves—calculated after Olbert (Ref. 4), solid curve at solar maximum, broken curve at solar minimum. Experimental data solid circles ● at solar minimum (present work), open circles ○ at solar maximum (Ref. 3).

corrected for the geomagnetic-latitude effect on muons, after the theories of Olbert[4] and Jabs,[5] respectively. The theoretical and the experimental results are presented in Fig. 4. It is found that our muon spectrum at $\lambda = 12°$ N when corrected after Olbert[4] agrees with the experimental results of Ashton *et al.*[7] at 1 GeV/c, which were not at solar minimum. Our results when corrected after Jabs[5] agree well with the high-latitude results of Ng *et al.*[8] The Jabs model correlates well the sea-level muon spectra at low and high latitudes during the solar minimum.

IV. CONCLUSION

The spectral shape of the absolute sea-level muon spectrum at solar minimum and solar maximum obeys the spectral shape calculated after the theory of Olbert[4] from 1 GeV/c and above. The disagreement of our results on the recent absolute muon intensity data from those determined by Ashton *et al.*[7] and Ng *et al.*[8] at different latitudes may be due to geomagnetic and atmospheric effect on muons.

ACKNOWLEDGMENTS

The author expresses his thanks to Professor D. Basu for his interest in the work and to Professor R. L. Sen Gupta, Principal, Presidency College, Calcutta for laboratory facilities. Thanks are due to Professor W. B. Fretter, University of California, Berkeley for some helpful correspondence, and to the University Grants Commission, Government of India for partial financial support.

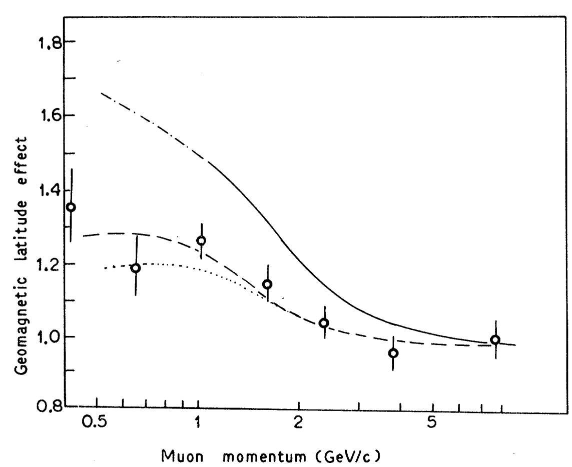

FIG. 3. Geomagnetic effect of muons at different muon momentum. Theoretical curves: solid curve, calculated after Jabs (Ref. 5); dashed curve, calculated after Olbert (Ref. 4). Experimental results: dotted curve, present work; open circles ○ Dau (Ref. 14).

APPENDIX

Scattering correction (i)

Rossi and Greisen[11] have derived a formula for the mean square scattering angle by considering the energy loss of muons in the absorbers. The formula which they give is

$$2\alpha^2 = \overline{\theta}^2 = \frac{E_s^{\,2}}{m_\mu^{\,2}} t\left[\ln\left(\frac{1+y_2}{1+y_1}\frac{p_1}{p_2}\right)\Big/ (y_1+y_1^{-1}-y_2-y_2^{-1})\right], \tag{A1}$$

where α^2 = mean square projected angle of scattering, $E_s = 21$ MeV, $y = [1+(p/m_\mu)^2]^{1/2}$, p = muon momentum, $m_\mu = 106$-MeV muon mass, t = absorber thickness in radiation lengths, and p_1 and p_2 are the momenta of the particles at the time of entering and leaving the absorber. The fraction of the particles received by the apparatus is calculated by using the expression

$$\frac{1}{\sqrt{\pi}}\int_{-\phi}^{\phi}\exp(-x^2)dx, \tag{A2}$$

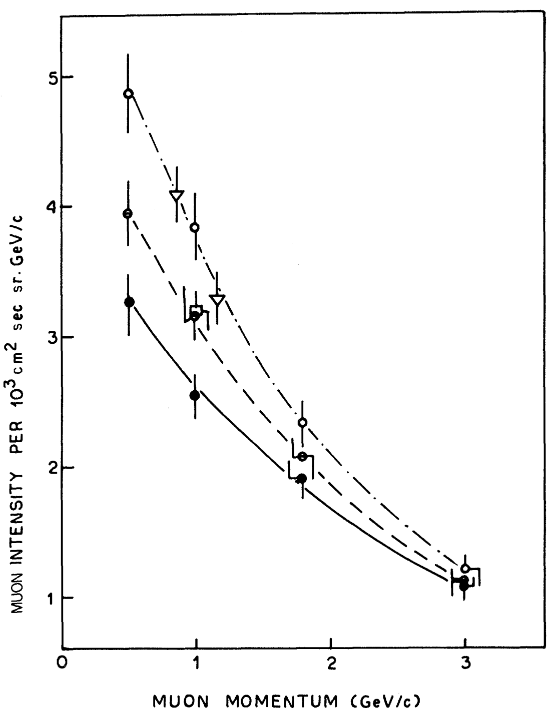

FIG. 4. Muon spectra at different latitudes during solar minimum. Experimental data ●, at $\lambda = 12°$ N (present work); (Ref. 4) ϴ, present work corrected for $\lambda = 55°$ N after Olbert (Ref. 4); ○, present work corrected for $\lambda = 55°$N after Jabs (Ref. 5); □, Ashton *et al.*, (Ref. 7); ▽, Ng *et al.* (Ref. 8).

where $\phi = \theta_1/\sqrt{2}\,\alpha$ and θ_1 = maximum aperture half angle which is given by the geometry of the counter C_3, placed above the absorber D, and the anticoincidence counters AC_2. It is assumed that all particles incident inside the solid angle covered by the threefold coincidence unit reach the counter placed at C_3. Then the fraction of particles $C(p_1)$, for a particular incident muon momentum p_1, which are scattered out from the anticoincidence counters AC_2, causes an increase of the anticoincidence rate in our experimental counts, and this fraction is calculated by the relation

$$C(p_1) = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\phi}\exp(-x^2)dx. \tag{A3}$$

Taking the calculated spectrum of muons as $N(p)dp = Kp^{-\gamma}\,dp$ from Olbert's[4] theory at $\lambda = 12°$ N we have estimated the correction factor $C'(p_1)$ from the relation

$$C'(p_1) = \frac{\int_{p_1+100}^{\infty} N(p)C(p)dp}{\int_{p_1}^{p_1+100} N(p)dp} \tag{A4}$$

in the muon momentum range p_1 MeV/c (incident muon momentum on counter tray C_1) to $p_1 + 100$ MeV/c. The calculated correction factors for different values of p_1 found for the different thicknesses of $\Sigma + T$ are shown in Table IV. The maximum value of ϕ was 1.32 rad.

Scattering correction (ii)

For our counter telescope, which has a finite aperture, a correction can be made for the loss of muons due to scattering in the absorber layer $\Sigma + T$. The particles which are scattered out of the absorber and do not hit the counters C_2 and C_3 contribute to this loss. The problem has been worked out in a two-dimensional case by Germain.[12] We follow Germain's procedure.

The percentage loss of muons can be calculated by using the relation

$$f = \sqrt{2\pi}\,\frac{5A_1^{\,2}}{6H^2}\times 100. \tag{A5}$$

In the present telescope geometry $H = 130$ cm,

TABLE IV. Scattering correction (i) in the layer D.

Absorber thickness in $\Sigma + T$ ($\mathrm{g\,cm^{-2}}$ lead-equivalent)	271	655	1245	2042
Fraction to be subtracted from the observed data (%)	6.2	6.2	5	5

$A_1^2 = 3\overline{r}^2$, and $\overline{r}^2 = t_1^2 \overline{\theta}^2/3$, where t_1 is the thickness in cm of the absorber. The mean squared scattering angle $\overline{\theta}$ of a muon which traverses a material of thickness t_1 is given by the Williams[13] formula

$$\overline{\theta}^2 = 16 \frac{NZ^2}{A} \rho r_0^2 \frac{(m_e c^2)^2}{p^2 \beta^2} \ln(181 Z^{1/3}) t_1 , \qquad \text{(A6)}$$

where Z = atomic number of the target material of density ρ and atomic weight A, N = Avogadro's number, $r_0 = e^2/m_e c^2 = 2.87 \times 10^{-13}$ cm, and p = momentum of the incident particle. $\overline{r}^2$ comes out to be of the form

$$\overline{r}^2 = 254 t^3/p_1^2 . \qquad \text{(A7)}$$

As shown by Wouthuysen (private communication, cited by Germain[12]), in order to account for the energy loss, A_1^2 must be multiplied by a function $f(\lambda)$ where $\lambda = (p_1 - p_2)/p_2$, where p_1 and p_2 are the muon momenta at the time of entering and leaving the absorber of thickness t_1. The function

TABLE V. Scattering correction (ii) in the layer $\Sigma + T$.

Absorber thickness in layer $\Sigma + T + \frac{1}{2}D$ ($\mathrm{g\,cm^{-2}}$ lead-equivalent)	311	700	1285	2082
λ	0.92	0.96	0.97	0.986
$f(\lambda)$	2.25	2.50	2.75	2.90
A_1^2	161	517	717	1765
$f(\%)$ Fraction to be added to the observed data	2	6.4	8.8	21.7

$f(\lambda)$ is given by

$$f(\lambda) = \frac{3}{\lambda^3} \{\lambda + (1-\lambda)[\lambda + 2 \ln(1-\lambda)]\} , \qquad \text{(A8)}$$

and $A_1^2 = 3 f(\lambda) \overline{r}^2$. Table V gives the values of $f(\lambda)$, A_1^2, and the estimated percentage of particles scattered out of the telescope for different absorber thicknesses.

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