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Infrared structure of the leading conformal contribution to the electromagnetic vertex function

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We treat the infrared problem directly related to the asymptotic behavior of the form factor of a particle belonging to the fermion-antifermion channel, with anomalous dimension $\bar{d}$. For $\bar{d} < 2$ we relate it to the off-shell vertex function; for $\bar{d} > 2$ we examine the triangle graph which is considered to be the most infrared divergent. We show that the leading γ_5-even conformal contribution to the wave function does not give rise to infrared divergences, provided the dimensions of the various fields satisfy the conformal bounds. Thus the ensuing contribution to the asymptotic form factor is $(-q^2)^{1-\bar{d}}$.

I. INTRODUCTION

Some interest has been devoted recently to the problem of on-shell large-momentum-transfer processes in field theory by trying to extend renormalization-group ideas to on-shell phenomena.[1-3] The main difficulty in dealing with these problems is that they are not, strictly speaking, short-distance phenomena; thus the Weinberg theorem[4] cannot be directly applied to show that the mass insertion terms are negligible in the Callan-Symanzik equation,[5] nor, as in deep-inelastic scattering, can the light-cone nature of the phenomenon be used[6] to express such mass insertions in terms of the amplitude itself, exploiting the Wilson[7] expansion. In a class of renormalizable theories, excluding theories with vector mesons, such as gauge theories and vector-gluon theories, it has been shown by various authors[2] that in the attractive domain of an ultraviolet fixed point the form factor of a particle, described by a Lagrangian field, behaves asymptotically like $(-q^2)^{-\gamma_0}$, where γ_0 is the anomaly in the dimension of the field.

If one deals with non-Lagrangian fields (i.e., composite fields[8]), which should be used to describe particles which, like hadrons, are believed to be nonelementary, the situation is more complex. The reason is that if the dimension of such fields[9] is larger[10] than 2, a discontinuity occurs between the on-shell and off-shell vertex functions which prevents a simple use of composite field Green's functions. One is thus compelled to work directly on the pole of the external particles; the form factor, to which we shall devote the present paper, is given by the convolution of the residues of the vertex function at the particle pole with the five-point two-particle-irreducible electromagnetic vertex function. If the considered particle is a pole in the channel with the quantum numbers of two fundamental fields the structure of the form factor is as shown in Fig. 1. It is usual as shown in the same figure to distinguish in the five-point-irreducible Green's function a connected (a) and a disconnected (b) contribution. The renormalization group[5,11,12] easily furnishes the behavior of the various vertex functions appearing in Fig. 1 when q^2 and the square momenta of the shown internal legs go to infinity. In addition in the case of anomalous dimensions, to which we shall refer here, one has more detailed information about the vertex functions than that furnished by asymptotic scale invariance. In fact it has been shown[13] that in the attractive domain of an ultraviolet fixed point the theory satisfies the more stringent requirements

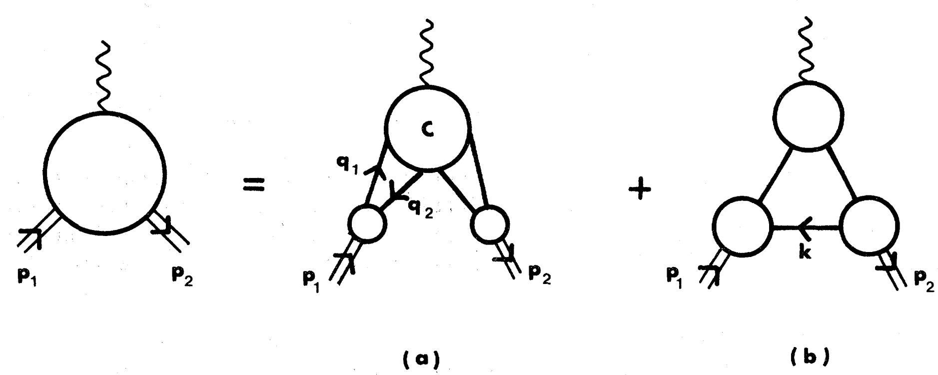

FIG. 1. Structure of the vertex function: (a) connected contribution; (b) disconnected contribution.

of asymptotic conformal invariance.[14] Strictly speaking such information is not sufficient[15,1] to determine the asymptotic behavior of form factors as it is easily realized that one needs to know also the behavior of the vertex function when one squared momentum goes to infinity and the other is kept constant. The information given by the conformal group is nevertheless valuable in the sense that if we restrict ourselves to consider the leading terms in the vertex functions, while conformal invariance gives, as we shall see, a well-defined result for the contribution of these dimensionally leading terms, the simple information supplied by asymptotic scale invariance is insufficient to produce any result. Clearly to complete the treatment one should also consider the influence of conformal-breaking terms. In the present paper we shall limit ourselves to such dimensionally leading terms; the role of the breaking terms treated to first order will be published in another paper.[16,17]

In dealing with this problem, one has to distinguish between $\bar{d}<2$ and $\bar{d}>2$, where $\bar{d}$ is the dimension of the lowest-dimensional field describing the external particle. The problem with $\bar{d}<2$ is much simpler because for $\bar{d}<2$ the light cone of the on-shell wave function of the particle does not differ from the light cone of the off-shell wave function. On the other hand, we know from a general result of Mack and Todorov[18] that the convolution of the conformal-invariant off-shell vertex function over the conformal-invariant five-point function is still conformal invariant, i.e., uniquely determined. Thus for $\bar{d}<2$ one can deal with the off-shell vertex function to get the asymptotic behavior of the form factor $(-q^2)^{1-\bar{d}}$. For $\bar{d}>2$ the situation is more complex as we cannot use the simplifying feature of going off the mass shell. It is well known that one can look at the asymptotic behavior, i.e., $-q^2\to\infty$, as an infrared problem, i.e., $q^2=\text{const}$ and all physical masses going to zero. From this point of view the disconnected graph Fig. 1(b) appears[12,1] to be the most important one as the momentum k during the integration can vanish simultaneously in both wave functions. In contrast with the calculation with an internal boson loop which is rather straightforward the fermion loop is more subtle to treat. In fact the trace over the fermion loop generates terms proportional to $p_1\cdot p_2\simeq -q^2/2$ ($q^2=$ square of the momentum transfer) which for dimensional reasons tend to worsen the infrared behavior of the integral. An infrared divergence would generate a so-called conformal anomaly,[12] i.e., an asymptotic behavior less quickly decreasing at infinity than $(-q^2)^{1-\bar{d}}$. However, as we shall see whenever a $p_1\cdot p_2$ term arises in the trace over the fermion loop it is always accompanied by a number of additional Feynman parameters which reconstitute the infrared convergence. This is due to the conformal structure of the γ_5-even (i.e., containing an even number of γ_μ) wave function which is considered to be the leading contribution,[12,18] and the algebraic structure of the spinor electromagnetic vertex. Thus no conformal anomalies are found at the leading level for the γ_5-even wave function[3] and the behavior of the form factor is still $(-q^2)^{1-\bar{d}}$. Asymptotically free gauge theories,[19] where the content of the conformal group becomes trivial, appear more difficult to treat.

This paper is organized as follows. Section II is concerned with the explicit form of the conformal vertex and wave functions. Section III gives a general discussion of the infrared problem for $\bar{d}<2$ and for $\bar{d}>2$. In Sec. IV the graph of Fig. 1(b) is treated for the sake of clearness in a particular case, i.e., for a fermion field with canonical dimensions. In Sec. V the treatment is extended to the general case. In the appendixes we collect the proof of the equivalence of the wave function we use to other standard forms, the general criterion for infrared convergence we use many times in the text, and a sample calculation with the γ_5-odd wave function.

II. CONFORMAL VERTEX AND WAVE FUNCTIONS

In this section we shall give the vertex functions from which we build up the form-factor graph of Fig. 1(b). As we mentioned in the Introduction we shall consider in this paper the leading behavior of such vertex functions; such a behavior is determined except for a few constants by conformal invariance. The conformal vertex function of two spinor fields and a conserved vector current has been given by Todorov[20] and is written as

$$V_\mu(x;y,z)=\langle 0|T(J_\mu(x)\psi(y)\bar{\psi}(z))|0\rangle$$
$$=c_1S_{3/2}(y-x)\gamma_\mu S_{3/2}(x-z)[-(y-z)^2+i\epsilon]^{3/2-d'}+c_2(\not{y}-\not{z})[-(y-z)^2+i\epsilon]^{1/2-d'}[\Delta_1(y-x)\overleftrightarrow{\partial_\mu}\Delta_1(x-z)]. \tag{2.1}$$

c_1 and c_2 are arbitrary constants while $S_{3/2}$ and Δ_1 are respectively the canonical zero-mass fermion and spin-0 boson propagators. Equation (2.1) satisfies the Ward identity

$$\frac{\partial V_\mu}{\partial x_\mu}(x;y,z)=\text{const}\times S_{d'}(y-z)[\delta(x-z)-\delta(x-y)]. \tag{2.2}$$

By using the standard Feynman parameter technique one can easily calculate the Fourier transform of $V_\mu(x;y,z)$:

$$\int e^{ip_1\cdot y}e^{-ip_2\cdot z}e^{iq\cdot x}V_\mu(x;y,z)d^4x\,d^4y\,d^4z=\delta^4(p_1-p_2+q)V_\mu(p_1,p_2). \tag{2.3}$$

The part proportional to c_1 in (2.1) gives rise to two terms

$$A_{1\mu}=\text{const}\times\gamma_\mu\int \mathcal{G}^{d'-5/2}\delta(1-\Sigma')d\alpha' d\beta'\gamma'^{5/2-d'}d\gamma' \tag{2.4}$$

and

$$A_{2\mu}=\text{const}\times\int[\not p_1(1-\beta')-\alpha'\not p_2]\gamma_\mu[\not p_2(1-\alpha')-\beta'\not p_1]\mathcal{G}^{d'-7/2}\delta(1-\Sigma')d\alpha' d\beta'\gamma'^{5/2-d'}d\gamma', \tag{2.5}$$

where

$$\Sigma'=\alpha'+\beta'+\gamma',\quad \mathcal{G}=-q^2\alpha'\beta'-p_1{}^2\beta'\gamma'-p_2{}^2\alpha'\gamma',\quad\text{and } q=p_2-p_1. \tag{2.6}$$

The part proportional to c_2 in (2.1) gives rise to a term $B_{1\mu}$, identical to $A_{1\mu}$ except for the proportionality constant, and to a term $B_{2\mu}$ given by

$$B_{2\mu}=\text{const}\times\int[p_1(1-2\beta')+p_2(1-2\alpha')]_\mu(\beta'\not p_1+\alpha'\not p_2)\mathcal{G}^{d'-7/2}\delta(1-\Sigma')d\alpha' d\beta'\gamma'^{5/2-d'}d\gamma'. \tag{2.7}$$

We notice that in (2.5) and (2.7) $\mathcal{G}$ and α',β',γ' appear with the same exponents; thus they can be treated together.

In addition one needs the vertex function of the composite system. There are two rigorous ways to derive such a wave function; the first is to use the conformal-covariant operator-product expansion.[21] The other is to write down a wave function of the correct light-cone structure and then impose on it the requirements of conformal invariance.[12] Heuristically one can reach exactly the same results by taking the discontinuity of the off-shell fermion-antifermion boson vertex function in the boson channel, the idea being that not the off-shell vertex function but its discontinuity is akin to the residue at the composite particle pole. The general form of the fermion-antifermion spin-0 boson vertex function is given by[18]

$$\langle 0|T(\Phi(x)\psi(y)\bar\psi(z))|0\rangle=g_1S_{\bar d/2}(y-x)[\gamma_5]S_{\bar d/2}(x-z)\Delta_{d'-\bar d/2}(y-z)+g_2S_{d'-\bar d/2}(y-z)[\gamma_5]\Delta_{\bar d/2}(y-x)\Delta_{\bar d/2}(x-z), \tag{2.8}$$

where

$$S_{d'}(x)=\not x(-x^2+i\epsilon)^{-d'-1/2}\quad\text{and}\quad \Delta_{\bar d}(x)=(-x^2+i\epsilon)^{-\bar d}. \tag{2.9}$$

The γ_5 in the square brackets in (2.8) is present if the field Φ is pseudoscalar, otherwise for scalar Φ it has to be replaced by the identity. In any case the part in (2.8) proportional to g_1 is γ_5-even and the other proportional to g_2 is γ_5-odd ($\gamma_5{}^2=1$). It is generally assumed[18] and also supported by calculations in perturbation theory[12] that the leading light-cone contribution is γ_5-even and thus we shall set $g_2=0$ in (2.8). We give in Appendix C a sample computation with the γ_5-odd leading solution. The Fourier transform of (2.8) with $g_2=0$,

$$\int e^{iq_1\cdot y}e^{-iq_2\cdot z}e^{-ip\cdot x}\langle 0|T(\Phi(x)\psi(y)\bar\psi(z))|0\rangle dx\,dy\,dz=\delta^4(q_1-q_2-p)W(q_1,q_2), \tag{2.10}$$

is calculated by means of the usual Feynman parametrization and one gets for $W(q_1q_2)$, except for an overall constant,

$$W(q_1,q_2)=\not q_1\not q_2\int \mathfrak{F}^{\bar d/2+d'-5}\delta(1-\Sigma)(\alpha\beta)^{3/2-\bar d/2}d\alpha\,d\beta\,\gamma^{2-d'+\bar d/2}d\gamma$$
$$+\frac{\bar d/2+d'-2}{\bar d/2+d'-4}\int \mathfrak{F}^{\bar d/2+d'-4}\delta(1-\Sigma)(\alpha\beta)^{3/2-\bar d/2}d\alpha\,d\beta\,\gamma^{1-d'+\bar d/2}d\gamma, \tag{2.11}$$

where

$$\Sigma=\alpha+\beta+\gamma \quad\text{and}\quad \mathcal{F}=-q_1^2\beta\gamma-q_2^2\alpha\gamma-p^2\alpha\beta. \tag{2.12}$$

Taking the discontinuity in p^2 through a standard procedure[22] and calculating the behavior for large $-q_1^2$ and $-q_2^2$ we get for the discontinuity, apart from an over-all factor,

$$\not{q}_1\not{q}_2\int_0^1[-q_1^2z-q_2^2(1-z)]^{-3+d'-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz+\frac{2-\bar{d}/2-d'}{2+\bar{d}/2-d'}\int_0^1[-q_1^2z-q_2^2(1-z)]^{-2+d'-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz. \tag{2.13}$$

The equivalence of this form of the wave function which is especially useful for computing the form factor to other expressions is proved in Appendix A.

In the computation of the form factor we shall need the vertex functions obtained from (2.13) by removing the external fermion conformal propagators $\not{q}(-q^2)^{d'-5/2}$. As is well known from the shadow formalism[23] this is obtained in the off-shell vertex function through the change $d'\to4-d'$. Thus the composite system vertex function (external propagators removed) is given by

$$\Gamma(q_1q_2)=\not{q}_1\not{q}_2\int_0^1[-q_1^2z-q_2^2(1-z)]^{1-d'-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz$$
$$+\frac{d'-\bar{d}/2-2}{d'+\bar{d}/2-2}\int_0^1[-q_1^2z-q_2^2(1-z)]^{2-d'-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz, \tag{2.14}$$

which can also be directly obtained by multiplying (2.13) on the left by $\not{q}_1(-q_1^2)^{3/2-d'}$ and on the right by $\not{q}_2(-q_2^2)^{3/2-d'}$ (see Appendix A).

The vertex functions (2.3) and (2.14) refer to a conformal-invariant massless ($p^2=0$) theory or to the limit of the physical theory for both squared momenta large.

In calculating the form factor using (2.3) and (2.14) we have thus to set $p^2=0$. If we want to stick more closely to the massive theory and set $p^2=M^2>0$ then we must respect the stability condition of this particle which is reflected in a nonzero threshold for the discontinuities of (2.3) and (2.14) obtained by replacing $\mathcal{G}$ in (2.6) with

$$(-q^2+\mu^2)\alpha'\beta'+(-p_1^2+m^2)\beta'\gamma'+(-p_2^2+m^2)\alpha'\gamma' \tag{2.15}$$

and $-q_1^2z-q_2^2(1-z)$ in (2.14) with

$$(-q_1^2+m^2)z+(-q_2^2+m^2)(1-z), \tag{2.16}$$

with the stability condition

$$m^2-\frac{M^2}{4}>0. \tag{2.17}$$

The two calculations, with $p^2=0$ and with $p^2=M^2$ but respecting the stability condition (2.17) (which if violated gives rise to complex form factors in the spacelike region), give the same results. In some sense keeping finite masses is preferable as one understands in more detail how to handle masses which appear in a non-dilatation-invariant theory.

III. GENERAL DISCUSSION

The form factor is given by the convolution of the two external particle wave functions over the five-point two-particle-irreducible electromagnetic vertex function. As we described in the Introduction we are dealing with a theory which is asymptotically conformal invariant. Thus the first job is to calculate such a convolution with conformal-invariant vertex functions. The spin-0 field Φ describing the external particles must have dimension $\bar{d}$ with $1<\bar{d}<3$ according to the general kinematical restriction of conformal invariance.[18] The situation is quite different for $\bar{d}<2$ and for $\bar{d}>2$. For $\bar{d}<2$ the vertex function of the external particle has the same asymptotic behavior as the off-shell vertex function.[10] This is no longer true for $\bar{d}>2$.

In fact we can easily calculate from (2.11) the asymptotic behavior for q_1^2 and q_2^2 going to ∞. The simplest way to do this, for $\bar{d}<2$, is to remove first the external propagator $(-p^2)^{\bar{d}-2}$ which is a constant for p^2 fixed. It corresponds to changing in (2.11) $\bar{d}$ into $4-\bar{d}$. After that a simple calculation[22] gives for the large q_1^2 and q_2^2 behavior, provided $\bar{d}<2$,

$$\not{q}_1\not{q}_2\int_0^1[-q_1^2z-q_2^2(1-z)]^{-3+d'-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz+\frac{2-\bar{d}/2-d'}{2+\bar{d}/2-d'}\int_0^1[-q_1^2z-q_2^2(1-z)]^{-2+d'-\bar{d}/2}[z(1-z)]^{\bar{d}/2-1/2}dz, \tag{3.1}$$

which coincides with the asymptotic behavior of the wave function (2.13). On the other hand, for $\bar{d}>2$ the asymptotic limit of the off-shell vertex function is no longer given by (3.1) (actually it is less rapidly decreasing at ∞). Following Mack[24] we shall call the field Φ with $\bar{d}<2$ elementary and with $\bar{d}>2$ composite. If we convolute now the two off-shell vertex functions $W(q_1,q_2)$ with the conformal five-point-irreducible electromagnetic vertex function we find according to a general theorem by Mack and Todorov[18] the three-point conformal vertex function

$$\langle 0|T(J_\mu(x)\Phi(y)\Phi^\dagger(z))|0\rangle=[-(y-z)^2]^{1-\bar{d}}[-(x-y)^2]^{-1}[-(x-z)^2]^{-1}\left[\frac{(y-x)_\mu}{(y-x)^2}-\frac{(z-x)_\mu}{(z-x)^2}\right], \tag{3.2}$$

whose Fourier transform is given by $(p_1^2=p_2^2)$

$$(p_1+p_2)_\mu\int[-q^2\alpha\beta-p_1^2\beta\gamma-p_2^2\alpha\gamma]^{\bar{d}-3}d\alpha\,d\beta\,\gamma^{3-\bar{d}}d\gamma \times\delta(1-\alpha-\beta-\gamma). \tag{3.3}$$

Removing the external Φ propagators, i.e., performing in (3.3) $\bar{d}\to 4-\bar{d}$, one easily sees that the asymptotic behavior of (3.3) for $q^2\to-\infty$, $p_1^2=p_2^2=\text{const}$ is $(p_1+p_2)_\mu(-q^2)^{1-\bar{d}}$ as

$$\int(\alpha\beta)^{1-\bar{d}}\gamma^{-1+\bar{d}}d\alpha\,d\beta\,d\gamma\,\delta(1-\alpha-\beta-\gamma)$$

is convergent for $\bar{d}<2$. Owing to this infrared convergence we can also choose $p_1^2=p_2^2=0$. Now as the off-shell wave function with external Φ propagators removed coincides, for $p^2=0$, with the on-shell conformal wave function as long as $\bar{d}<2$ as we have shown previously, we conclude that for $\bar{d}<2$ the conformal contribution to the form factor is $(-q^2)^{1-\bar{d}}$. The nontrivial job is to prove the same result for $2<\bar{d}$, i.e., for composite fields, where we cannot use the general result of Mack and Todorov.

Migdal[25] gave a general heuristic argument which essentially amounts to the following. As the on-shell vertex function is related to the discontinuity in p^2 we can consider the off-shell vertex function (3.3) and calculate first the discontinuity in p_1^2 (with $p_2^2<0$) and then the discontinuity in p_2^2 and argue that such double discontinuity gives the form factor. The discontinuity in p_1^2 of the integral in (3.3) is given by

$$\text{const}\times\int_0^1 du\int_{y_0}^\infty dy\,y\,u^{\bar{d}-2}(1-u)[u(1-u)+y]^{-\bar{d}}(y-y_0)^{\bar{d}-3}, \tag{3.4}$$

where

$$y_0=\frac{1}{p_1^2}[-q^2u-p_2^2(1-u)]. \tag{3.5}$$

The discontinuity of (3.4) in p_2^2 behaves for large $-q^2$ as $(-q^2)^{1-\bar{d}}$. Clearly this argument has only a heuristic value. In fact the conformal off-shell vertex function has the structure shown[26] in Fig. 2(a). The discontinuity in p_1^2 due to the lowest cut is shown in Fig. 2(b).

As we saw above we can replace the cut shown in Fig. 2(b) with the conformal on-shell vertex function, and thus the contribution to the double discontinuity due to the two lowest cuts gives asymptotically the form factor. To deduce, however, that such a contribution to the double discontinuity has the same asymptotic behavior as the total double discontinuity would require a proof that no cancellation occurs among the various possible cuts.

We discuss now in more detail the convolution of the wave functions over the five-point irreducible electromagnetic vertex function. From straightforward dimensional counting it follows that the leading dimensional contribution will be an integral of the form $I(-q^2,M^2,m^2)$ with the property

$$I(-q^2;m^2;M^2)=(-q^2)^{1-\bar{d}}I\left(1;\frac{m^2}{-q^2};\frac{M^2}{-q^2}\right). \tag{3.6}$$

From this point of view, as is well known, the study of the large $-q^2$ behavior of the form factor corresponds through (3.6) to an infrared problem, i.e., setting m^2 and M^2 to zero in the right-hand side of (3.6). The behavior $(q^2)^{1-\bar{d}}$ corresponds to $I(1,0,0)$ being finite. In setting m^2 and M^2 equal to zero in I some care has to be taken. In fact if there are spinors in the internal lines a polynomial in p_1,p_2 can occur in the numerator due to the traces over the fermion loops which in the integration can generate, among the others, terms proportional to p_1^2 and p_2^2. In setting these terms equal to zero one has to check that they are accompanied by a not too divergent denominator in the limit $p_1^2,p_2^2\to 0$. This is the reason why for completeness we shall consider $p_1^2=p_2^2=M^2\neq 0$, which on the other side imposes through the stability

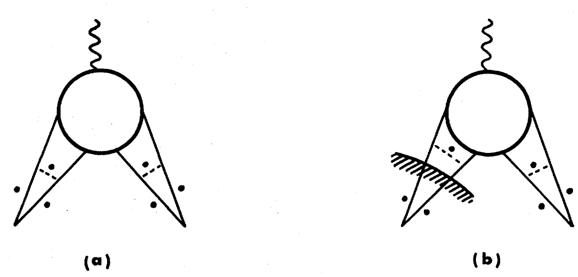

FIG. 2. Off-shell conformal vertex function (a), and its discontinuity in p_1^2 due to the lowest cut (b).

condition the presence of the masses m in the parametric representations (2.4), (2.5), (2.7), and (2.14). We shall see, as a matter of fact, that this rigorous procedure is equivalent to setting $p_1^2=p_2^2=M^2=m^2=0$ straight away in all formulas.

IV. EXAMPLE WITH CANONICAL FERMIONS

In order to illustrate the method we shall deal in this section with the simpler situation in which the fermion has canonical dimension $d'=\frac{3}{2}$ while the composite particle field has dimension $\bar{d}$ only subject to the restriction imposed by conformal invariance $1<\bar{d}<3$. The upper electromagnetic vertex is replaced by γ_μ. The treatment with general anomalous dimensions where the electromagnetic vertex is given by the full conformal vertex function V_μ discussed in Sec. II will be given in Sec. V. In the present section we shall also keep $p^2=M^2>0$ and take into account the nonzero thresholds as in Eqs. (2.15)–(2.17). This will serve to show how one can treat terms arising from having nonzero masses. We can equivalently use the dilatation-invariant function (2.4)–(2.7), (2.14) provided we keep consistently $p^2=0$.

To perform the calculation one combines the three fermion propagators of the form $\not{k}(-k^2+m^2)^{-1}$ with the two vertex functions of the form (2.14) modified through (2.16) by means of the Feynman parameters shown in Fig. 3. One has to perform the trace over the fermion loop and then integrate in d^4k. If we write symbolically $\Gamma(q_1q_2)=\not{q}_1\not{q}_2t+s$ we shall have obviously three kinds of contributions: ss, st (ts), and tt. For the ss part the trace over the fermion loop is

$$\tfrac{1}{4}\,\mathrm{Tr}[(\not{p}_1+\not{k})\gamma_\mu(\not{p}_2+\not{k})\not{k}], \tag{4.1}$$

and we recall that in performing the loop integration one has to perform the momentum translation

$$k=k'-\Delta=k'-ap_1-bp_2, \tag{4.2}$$

where $a=\xi z+\alpha$ and $b=\zeta z'+\beta$. Such a translation symmetrizes the integrand in the form

$$(-k'^2+\mathfrak{D})^{-2-\bar{d}} \tag{4.3}$$

with

$$\mathfrak{D}=-q^2ab+m^2-M^2(a+b)(1-a-b). \tag{4.4}$$

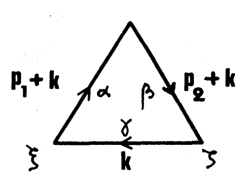

FIG. 3. Feynman parametrization in the simpler case with canonical fermions.

Thus only even terms in k' survive in the trace (4.1) during integration and these are immediately found. The quadratic terms are

$$(p_{1\mu}+p_{2\mu}-\Delta_\mu)k'^2-2k'_\mu\Delta\cdot k' \tag{4.5}$$

while the constant terms are

$$(-\Delta_\mu+p_{1\mu}+p_{2\mu})\Delta^2-p_{1\mu}p_2^{\,2}b-p_{2\mu}p_1^{\,2}a. \tag{4.6}$$

The integration over the quadratic terms (4.5) gives a vector which can be either $p_{1\mu}$ or $p_{2\mu}$ times the integral

$$\int[z(1-z)z'(1-z')]^{\bar{d}/2-1/2}dz\,dz'(\xi\zeta)^{\bar{d}/2-3/2}d\xi\,d\zeta\times d\alpha\,d\beta\,d\gamma\,\delta(1-\Sigma)\mathfrak{D}^{1-\bar{d}} \tag{4.7}$$

with $\Sigma=\alpha+\beta+\gamma+\xi+\zeta$, and where the integration range of all parameters is from 0 to 1.

It is easily checked that the asymptotic behavior of (4.7) is $(-q^2)^{1-\bar{d}}$, i.e., that the integral obtained replacing in (4.7) $\mathfrak{D}$ with ab converges.

In fact one can majorize

$$a^{1-\bar{d}}=(z\xi+\alpha)^{1-\bar{d}}<(z\xi)^{-l_1}\alpha^{-l_2} \tag{4.8}$$

with $-l_1-l_2=1-\bar{d}$ and $l_1>0$, $l_2>0$ and as $1-\bar{d}+\bar{d}/2-\frac{1}{2}+1=\frac{3}{2}-\bar{d}/2>0$ one can choose l_1 and l_2 to achieve convergence. Exactly in the same way one deals with the term

$$(\Delta_\mu-p_{1\mu}-p_{2\mu})(-q^2ab) \tag{4.9}$$

originating from the first term in (4.6) which gives again the asymptotic behavior $(-q^2)^{1-\bar{d}}$. The "mass" terms left over such as $p_{1\mu}p_2^2b=p_{1\mu}M^2b$ are harmless as they are combined in the integration with $\mathfrak{D}^{-\bar{d}}$. Here using the fact that owing to the stability condition

$$m^2-M^2(a+b)(1-a-b)\geq m^2-\frac{M^2}{4}>0 \tag{4.10}$$

we can majorize $\mathfrak{D}^{-\bar{d}}$ with const $\times\,\mathfrak{D}^{1-\bar{d}}$ and we are back to the integral (4.7).

The most important point is the following: In the trace (4.1) terms of the type $k_\mu p_1\cdot k$, which would give a contribution to the asymptotic behavior higher than $(-q^2)^{1-\bar{d}}$, cancel away while the *a priori* dangerous terms $-k_\mu p_1\cdot p_2+p_{2\mu}p_1\cdot k+p_{1\mu}p_2\cdot k$ under the translation (4.2) reduce to harmless mass terms of the type $p_{1\mu}p_2^{\,2}b$. After we have ascertained that the integral in (4.7) with $\mathfrak{D}$ replaced by ab is convergent we can reach the conclusion on the asymptotic behavior of the form factor just by inspection of the trace (4.1) without performing the explicit calculation. In fact as we saw, quadratic terms in k' give rise to (4.7) while the only constant term obtained by replacing in (4.1) k with Δ, which does not contain Δ^2 (i.e., two contiguous $\not{\Delta}$) is $\mathrm{Tr}(\not{p}_1\gamma_\mu p_2\not{\Delta})$ which as $\Delta=ap_1+bp_2$ is just $\mathrm{Tr}(\not{p}_1\gamma_\mu p_2^{\,2}b)+\mathrm{Tr}(ap_1^{\,2}\gamma_\mu\not{p}_2)$, i.e., two harmless mass terms.

Roughly speaking the over-all result is that even if the trace contains potentially dangerous terms, due to the peculiar ordering of the γ matrices in (4.1), only purely over-all dimensional terms such as k'^2 and $-q^2ab$ survive and the remaining becomes harmless through the mass-shell condition $p_1^2=p_2^2=M^2=\text{const}$. On the other hand, the fact that the exponent of the differential in ξ, $\bar{d}/2-\frac{1}{2}$ (and that in z, i.e., $\bar{d}/2+\frac{1}{2}$), plus the exponent of the differential in α which is 1 is greater than $\bar{d}-1$ ensures that when the squared momentum is flowing through the single leg p_1+k in Fig. 3 the contribution to the graph at fixed k' is not higher than $(-q^2)^{1-\bar{d}}$. Integration over k' then brings up the behavior to the dimensional one.

The treatment of the other two contributions st (ts) and tt is much simpler as the trace contains only two γ matrices and no peculiar cancellation is needed.

The integrals, whose convergence is immediately proved, which intervene in these two cases are

$$\int [z(1-z)z'(1-z')]^{\bar{d}/2-1/2}dz\,dz' \times \xi^{\bar{d}/2-3/2}d\xi\,\zeta^{\bar{d}/2-1/2}d\zeta\,d\alpha \times \delta(1-\alpha-\xi-\zeta)[(\alpha+\xi z)z'\zeta]^{1-\bar{d}} \tag{4.11}$$

and

$$\int [z(1-z)z'(1-z')]^{\bar{d}/2-1/2}dz\,dz' \times (\xi\zeta)^{\bar{d}/2-1/2}d\xi\,d\zeta \times \delta(1-\xi-\zeta)(z\xi z'\zeta)^{1-\bar{d}}. \tag{4.12}$$

V. TREATMENT WITH GENERAL ANOMALOUS DIMENSIONS

In the case of general anomalous dimensions one convolutes the upper vertex function (2.1) which includes the external fermion propagators with the two composite particle vertex functions (2.14) which have to be joined by a further (anomalous) fermion propagator. The Feynman parametrization for combining all denominators together is shown in Fig. 4. Here again one has three distinct cases corresponding to the combinations ss, st (ts), and tt of the composite particle vertex function. Each time a trace has to be computed over the internal fermion loop. Both this trace and the power of $\mathcal{G}$ [Eq. (2.6)] depends on whether A_1 ($\equiv B_1$), A_2, or B_2 appear in the upper electromagnetic vertex. The integral in d^4k over the fermion loop is written down directly using a straightforward generalization[22] of Symanzik[27] cutting rules. These loop integrals are of the form

$$\int [-q^2(AB+\eta\alpha'\beta'C)-p_1^2AD-p_2^2BD+m^2C^2+\mu^2\alpha'\beta'\eta C]^E \times C^c dP\,\delta(1-\Sigma)\delta(1-\Sigma')[z(1-z)z'(1-z')]^{\bar{d}/2-1/2}dz\,dz'\,T, \tag{5.1}$$

where

$$\begin{aligned} A&=z\xi+\eta\beta'\gamma',\\ B&=z'\zeta+\eta\alpha'\gamma',\\ C&=\xi+\zeta+\gamma+\eta\gamma'(\alpha'+\beta'),\\ D&=\xi(1-z)+\zeta(1-z')+\gamma, \end{aligned} \tag{5.2}$$

$\Sigma=\xi+\zeta+\eta+\gamma$, $\Sigma'=\alpha'+\beta'+\gamma'$, and the differential dP is given by

$$dP=\prod_p p^{(p)-1}dp = \xi^{(\xi)-1}d\xi\,\zeta^{(\zeta)-1}d\zeta\cdots, \tag{5.3}$$

where the product in (5.3) extends over all parameters $\xi\zeta\eta\gamma,\alpha'\beta'\gamma'$. The exponent (ξ), e.g., is minus the exponent of $[-q_1^2z-q_2^2(1-z)]$ appearing in Eq. (2.14) (η) minus the exponent of $\mathcal{G}$ in (2.4)–(2.7) and $(\alpha')-1$, $(\beta')-1$, $(\gamma')-1$ the exponent of α',β',γ' appearing in the representations (2.4)–(2.7).

c is equal to $-E-n-2$, where n is the power of k'^2 accompanying the differential d^4k' during the integration. Such powers of k'^2 result from the trace over the fermion loop when one keeps in mind the translation in momentum space

$$k=k'-\Delta=k'-\frac{A}{C}p_1-\frac{B}{C}p_2. \tag{5.4}$$

T is what is left over from the trace after transla-

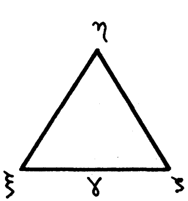

FIG. 4. Feynman parametrization in the general case.

tion (5.4) and is a combination of $p_1^2=p_2^2=M^2$, q^2, and A/C, B/C.

In Eq. (5.1) we have explicitly kept the mass terms inside the bracket; they can be used in the computation of the asymptotic behavior of those terms in which the integral (5.1) is multiplied by mass terms such as p_1^2 through majorizations of the type explained in Sec. IV by using the stability condition. In fact

$$m^2C^2+\mu^2\alpha'\beta'\eta C-M^2(A+B)D>C\left[C\left(m^2-\frac{M^2}{4}\right)+\mu^2\alpha'\beta'\eta\right]\geq \text{const}\times C(C+\alpha'\beta'\eta). \tag{5.5}$$

We have checked that this procedure works in all cases; for simplicity from now on we shall set consistently all masses equal to zero, i.e., $m^2=\mu^2=p_1^2=p_2^2=0$. Clearly (5.1), when on keeps T in mind, has dimension $1-\bar{d}$ in $-q^2$ and thus the behavior of the form factor is $(-q^2)^{1-\bar{d}}$, provided all the integrals of type (5.1) appearing in the form factor converge. Thus we have to examine the convergence properties of integrals of the type

$$\int (AB+\eta\alpha'\beta'C)^E C^c\delta(1-\Sigma)\delta(1-\Sigma')dP, \tag{5.6}$$

$$dP=\prod_p p^{(p)-1}dp = \xi^{(\xi)-1}d\xi\cdots z'^{(z')-1}dz', \tag{5.7}$$

where in (5.7) the product is extended to all parameters $\xi\zeta\eta\gamma,\alpha'\beta'\gamma',zz'$.

This problem is dealt with in Appendix B, where the following sufficient conditions for the convergence of (5.6) are derived:

$$\begin{aligned}
&\text{(i)}\;\; E+m((z),(\xi))+m((\eta),(\beta'))>0,\\
&\text{(ii)}\;\; E+m((z'),(\zeta))+m((\eta),(\alpha'))>0,\\
&\text{(iii)}\;\; E+m((z),(z'))+m((\eta),(\alpha'),(\beta'))>0,\\
&\text{(iv)}\;\; c+2E+(\xi)+(\zeta)+(\gamma)\\
&\qquad +m((\beta')+(\gamma'),(\alpha')+(\gamma'),(\alpha')+(\beta'))>0.
\end{aligned} \tag{5.8}$$

Here $m(,,)$ means the minimum of a set of numbers.

The first two conditions have a very simple interpretation. In fact $(-q_1^2)^{-m((z),(\xi))}$ is the behavior of the integrals in Eq. (2.14) for $-q_1^2\to\infty$, $q_2^2=$ const, while $(-p_1^2)^{-m((\eta),(\beta'))}$, taking into account that $(\eta)<(\alpha')+(\gamma')$, is the behavior of integrals appearing in the representation of the vertex function (2.4), (2.5), and (2.7). Clearly if $-m((z),(\xi))-m((\eta),(\beta'))>E$ the behavior of the integrand as the leg $(p_1+k)^2$ goes to ∞ is higher than $(-q^2)^E$ and thus the integral in (5.6) cannot converge. However, (i) and (ii) are not sufficient to ensure convergence of (5.6) since not only the integrand for fixed k' has to behave properly when $-q^2\to\infty$ but also the over-all integration in d^4k' should not introduce an enhancement in the asymptotic behavior above $(-q^2)^E$. Conditions (iii) and (iv) are sufficient to ensure this property. In particular the fact that C is not identically equal to 1 as in the simple example of Sec. IV is due to the upper vertex function which in the large-k' region of integration introduces a more weakly convergent behavior. Condition (iv) takes care of this fact.

We come now to the actual discussion of the three contributions *ss*, *st* (*ts*), and *tt* to the form factor. As we have pointed out in the Introduction we shall avoid all explicit evaluation of traces. The algebraic properties of the γ-matrices connected with the special ordering in which the matrices occur will be sufficient to extract the relevant information. One has to keep in mind that only terms even in k' survive the symmetric integration; a term of the type $k'_\mu p\cdot k'$ is equivalent, apart from a factor, to $p_\mu k'^2$.

ss contribution

We shall discuss this case in detail and then go over more quickly the other two (*st* and *tt*).

The $A_{1\mu}$ $(=B_{1\mu})$ part gives a trace

$$\mathrm{Tr}(\gamma_\mu \not{k})\to\Delta_\mu=\frac{A}{C}p_{1\mu}+\frac{B}{C}p_{2\mu}\to p_\mu, \tag{5.9}$$

where A/C and B/C have been majorized by 1. The arrow means that, apart from a factor, the replacement is equivalent in the computation of the asymptotic behavior. p_μ means $p_{1\mu}$ and/or $p_{2\mu}$. We are left now with an integral of type (5.6) with $E=1-\bar{d}$, $(\xi)=(\zeta)=\bar{d}/2+d'-2$, $(\eta)=\frac{5}{2}-d'$, $c=\bar{d}-3$, $(\alpha')=(\beta')=1$, $(\gamma')=\frac{7}{2}-d'$. It is immediately noted that, provided the conformal bounds on the dimensions

$$\tfrac{3}{2}<d'<\tfrac{5}{2},\quad 1<\bar{d}<3$$

are respected, all inequalities (5.8) are satisfied which ensures the convergence of our integral and thus the behavior $(-q^2)^{1-\bar{d}}$ of this contribution.

The B_2 and A_2 parts of the vertex function give the traces

$$\mathrm{Tr}[\beta'(\not p_1+\not k)\not k+\alpha'(\not p_2+\not k)\not k][(p_1+k)(1-2\beta')+(p_2+k)(1-2\alpha')]_\mu \to p_\mu(k'^2+\Delta^2+\alpha' p_2\cdot\Delta+\beta' p_1\cdot\Delta) \tag{5.10}$$

and

$$\mathrm{Tr}\{(\not p_1+\not k)(1-\beta')-\alpha'(\not p_2+\not k)]\gamma_\mu[(\not p_2+\not k)(1-\alpha')-\beta'(\not p_1+\not k)]\not k\}\to p_\mu(k'^2+\Delta^2+\alpha' p_2\cdot\Delta+\beta' p_1\cdot\Delta). \tag{5.11}$$

The important facts in the coefficient of p_μ in (5.10) and (5.11) are as follows:

(i) the absence of terms containing both p_1 and p_2 because of $\mathrm{Tr}(\not p_1\gamma_\mu\not p_2\not\Delta)=\mathrm{Tr}(\not p_2\gamma_\mu\not p_1\not\Delta)=0$ in (5.11);
(ii) the term $p_1\cdot\Delta$ $(p_2\cdot\Delta)$ is multiplied by β' (α') as again in (5.11) $\mathrm{Tr}[(1-\beta')\not p_1\gamma_\mu\not p_2(1-\alpha')\not\Delta]=0$ and $\mathrm{Tr}[(1-\beta')p_1\gamma_\mu\not\Delta\not\Delta]\to p_\mu\Delta^2$.

k'^2 generates, writing for simplicity $(AB+\eta\alpha'\beta'C)=\mathfrak{D}$, $\mathfrak{D}^{1-\bar d}C^{\bar d-4}$ while Δ^2 generates $(-q^2)ABC^{-2}\mathfrak{D}^{-\bar d}C^{\bar d-2}\le\mathfrak{D}^{1-\bar d}C^{\bar d-4}$. Now we have

$$E=1-\bar d,\quad (\eta)=\tfrac{7}{2}-d',\quad c=\bar d-4,\quad (\alpha')=(\beta')=1,\quad (\gamma')=\tfrac{7}{2}-d', \tag{5.12}$$

which substituted in (5.8) prove the convergence of our integral. The term $\alpha' p_2\cdot\Delta$ gives

$$\mathfrak{D}^{-\bar d}(-q^2)\alpha'\frac{A}{C}C^{\bar d-2}, \tag{5.13}$$

and recalling that $A=\xi z+\eta\beta'\gamma'$ we again obtain convergence through (5.8).
We now examine the next case.

st contribution

Here $A_{1\mu}$ gives the trace

$$k^2\mathrm{Tr}[(\not p_2+\not k)\gamma_\mu]\to p_\mu(k'^2+\Delta^2), \tag{5.14}$$

which generates $\mathfrak{D}^{1-\bar d}C^{\bar d-4}$ giving rise to a convergent integral. $B_{2\mu}$ gives

$$k^2\mathrm{Tr}\{[\beta'(\not p_1+\not k)+\alpha'(\not p_2+\not k)](\not p_2+\not k)\}[(p_1+k)(1-2\beta)+(p_2+k)(1-2\alpha)]_\mu\to p_\mu\{[4]+[2](\beta' p_1\cdot p_2+\Delta\cdot p_2)\}, \tag{5.15}$$

where we indicate with [2] and [4] the general second- and fourth-order polynomials in k' and Δ, even in k'. The nontrivial fact in the fourth-order polynomial in the right-hand side of Eq. (5.15) is that p_1 in $p_1\cdot p_2$ appears necessarily multiplied by β' as is seen from the structure of the trace and that $\Delta\cdot p_2=p_1\cdot p_2 A/C\to(-q^2)A/C$.
$A_{2\mu}$ gives

$$k^2\mathrm{Tr}\{[(\not p_1+\not k)(1-\beta')-\alpha'(\not p_2+\not k)]\gamma_\mu[(\not p_2+\not k)(1-\alpha')-\beta'(\not p_1+k)](\not p_2+\not k)\}, \tag{5.16}$$

which is equivalent to the right-hand side of Eq. (5.15). In fact as $\mathrm{Tr}(\not p_1\gamma_\mu\not p_2\not p_2)=\mathrm{Tr}(\not p_1\gamma_\mu\not p_2\not\Delta)=0$ the terms containing p_1p_2 either are multiplied by β' or are $\mathrm{Tr}(\not p_1\gamma_\mu\not\Delta\not p_2)=8p_{1\mu}(A/C)p_1\cdot p_2$. Thus (5.15) and (5.16) generate integrands of the types $\mathfrak{D}^{1-\bar d}C^{\bar d-5}$, $-q^2\mathfrak{D}^{-\bar d}C^{\bar d-3}\beta'$, $-q^2\mathfrak{D}^{-\bar d}C^{\bar d-3}A/C$, which are checked by means of (5.8) to give rise to convergent integrals.
At last we have the *tt* contribution.

tt contribution

The $A_{1\mu}$ term gives the trace

$$\mathrm{Tr}[(\not p_1+\not k)\gamma_\mu(\not p_2+\not k)\not k]\to p_\mu(k'^2+\Delta^2),$$

as was already examined in Sec. IV. It generates $\mathfrak{D}^{1-\bar d}C^{\bar d-4}$ giving a convergent integral. $B_{2\mu}$ gives, restricting ourselves for symmetry to the terms proportional to β',

$$k^2\mathrm{Tr}[(\not p_1+\not k)\beta'(\not p_1+\not k)(\not p_2+\not k)\not k][(p_1+k)(1-2\beta)+(p_2+k)(1-2\alpha)]_\mu\to p_\mu\{\beta' p_1\cdot p_2[4]+\beta'[6]\}. \tag{5.17}$$

The important fact in the coefficient of p_μ in **(5.17)** is that we can exclude terms containing two p_1 as $\mathrm{Tr}(\not p_1\not p_1\cdots)=0$ and that $p_1\cdot\Delta p_2\cdot\Delta\to p_1\cdot p_2\Delta^2$. $A_{2\mu}$ gives

$$k^2\mathrm{Tr}\{(\not p_1+\not k)[(\not p_1+\not k)(1-\beta')-(\not p_2+\not k)\alpha']\gamma_\mu[(\not p_2+\not k)(1-\alpha')-(\not p_1+\not k)\beta'](\not p_2+\not k)\not k\}\to p_\mu\{[6]+[4]p_1\cdot p_2\}. \tag{5.18}$$

The important feature in (5.18) is the following: There are no terms $(p_1 \cdot p_2)^2$, $(p_1 \cdot p_2)(p_1 \cdot \Delta)$, $(p_1 \cdot \Delta)(p_1 \cdot \Delta)$, i.e., coefficients of p_μ which are second order in p_1 (and the same by symmetry for p_2). In fact such terms would be of the type $(k'^2 + \Delta^2)T$ where T is a trace of $\not{p}$'s and $\not{\Delta}$. As p_1 has to appear at least twice we have the following possibilities:

(i) $\mathrm{Tr}(\not{p}_1 \not{X} \gamma_\mu \not{p}_1 \not{Y} \not{\Delta})$ which for $Y = p_2$ vanishes, and thus we have the only possibility $Y = \Delta$ giving $\Delta^2 \mathrm{Tr}(\not{p}_1 \not{X} \gamma_\mu \not{p}_1) = 0$.

(ii) $\mathrm{Tr}(\not{\Delta} \not{p}_1 \gamma_\mu \not{p}_1 \not{Y} \not{\Delta}) = \Delta^2 p_{1\mu} Y \cdot p_1 - \Delta^2 p_{1\mu} p_1 \cdot p_2$.

The integrands due to (5.17) and (5.18) are therefore of the types $\mathfrak{D}^{1-\bar{d}} C^{\bar{d}-6}$ and $q^2 \mathfrak{D}^{-\bar{d}} C^{\bar{d}-4}$ which again through (5.8) give rise to convergent integrals.

This concludes the proof that the leading dimensional contribution to the form factor behaves like $(-q^2)^{1-\bar{d}}$.

Speaking from a more qualitative point of view what happens is the following. One has to calculate traces containing $\not{p}_1 \not{p}_2$ and $\not{\Delta}$. Whenever these traces are able to produce a $-q^2$ through a scalar product of these vectors, and the related decrease of the exponent of $\mathfrak{D}$ from $1-\bar{d}$ to $-\bar{d}$ would give rise to an infrared divergence, such $-q^2$ is accompanied by additional Feynman parameters which reconstitute the infrared convergence. These additional parameters are generated by, e.g.,

$$p_2 \cdot \Delta = p_1 \cdot p_2 (\xi z + \eta \beta' \gamma') C^{-1} \tag{5.19}$$

or already multiply like $\alpha' \not{p}_2$ in (2.7) the matrix $\not{p}_2$ responsible for the production of the $-q^2$. The decrease in power of C due to (5.19) never destroys the convergence of the integral.

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APPENDIX A

We shall prove in this appendix the equivalence between various forms of the wave function, in particular between Eq. (2.13) derived in the text through the calculation of the p-channel discontinuity of the off-shell vertex function and the one obtained by directly imposing conformal invariance on the light cone.[12] Callan and Gross[12] start from the general γ_5-even expression of the wave function near the light cone

$$\phi(x,p) = (-x^2)^{-d'+\bar{d}/2}\{g_1(p\cdot x, x^2p^2) + [\not{p}, \not{x}] g_2(p \cdot x, x^2 p^2)\} \tag{A1}$$

and by imposing conformal invariance, i.e.,

$$K_\mu(x)\phi(x,p) + i\bar{K}_\mu(p)\phi(x,p) = 0, \tag{A2}$$

with

$$K_\mu(x) = 2x_\mu(d' + x\cdot\partial) - x^2\partial_\mu + \tfrac{1}{2}[\gamma_\mu, \not{x}] \tag{A3}$$

and

$$\bar{K}_\mu(p) = 2\left(\bar{d} + p\cdot\frac{\partial}{\partial p}\right)\frac{\partial}{\partial p^\mu} - p_\mu \frac{\partial^2}{\partial p^2}, \tag{A4}$$

solve for g_1 and g_2 for $p^2 = 0$. The final result for the Fourier transform of the wave function using the same momentum conventions as in Sec. II is

$$\phi(q_1,q_2) = [\not{q}_1 + \not{q}_2, \not{q}_1 - \not{q}_2] \int_0^1 [-q_1^2 z - q_2^2(1-z)]^{-3-\bar{d}/2+d'}[z(1-z)]^{\bar{d}/2-1/2} dz + \frac{\bar{d}-1}{2-d'+\bar{d}/2}\int_0^1 [-q_1^2 z - q_2^2(1-z)]^{-2-\bar{d}/2+d'}[z(1-z)]^{\bar{d}/2-3/2} dz, \tag{A5}$$

where $p = q_1 - q_2$, $p^2 = 0$. The wave function written in this form is less suitable for computing the form factor than the form (2.13); the reason is that $[\not{q}_1 + \not{q}_2, \not{q}_1 - \not{q}_2]$ is equal to $-4\not{q}_1\not{q}_2 + 4q_1\cdot q_2 = -4\not{q}_1\not{q}_2 + 2(q_1^2 + q_2^2)$ and the part $2(q_1^2+q_2^2)$ eliminates the most singular behavior of the second integral in (A5). In fact $2(q_1^2+q_2^2)$ times the first integral in (A5) can be written after integrating by parts in dz as

$$(q_2^2)^{-2-\bar{d}/2+d'}\frac{\bar{d}-1}{2-d'+\bar{d}/2}\,\frac{1+R}{1-R}\int_0^1 (Rz+1-z)^{-2-\bar{d}/2+d'}(1-2z)[z(1-z)]^{\bar{d}/2-3/2}dz \tag{A6}$$

with $R = q_1^2/q_2^2$. We now add this to the second term in (A5), i.e.,

$$(q_2^2)^{-2-\bar{d}/2+d'}\frac{\bar{d}-1}{2-d'+\bar{d}/2}\int_0^1 (Rz+1-z)^{-2-\bar{d}/2+d'}[z(1-z)]^{\bar{d}/2-3/2}dz. \tag{A7}$$

The integrals in (A6) and (A7) are directly expressible as hypergeometric functions, and using the Gauss relation[28]

$$c(1-y)F(a,b;c;y)-cF(a,b-1;c;y)+(c-a)yF(a,b;c-1;y)=0 \tag{A8}$$

we obtain for the sum of (A6) and (A7)

$$-(-q_2{}^2)^{-2-\bar{d}/2+d'}\,4\,\frac{2-d'-\bar{d}/2}{2-d'+\bar{d}/2}\int_0^1 (Rz+1-z)^{-2-\bar{d}/2+d'}[z(1-z)]^{\bar{d}/2-1/2}dz, \tag{A9}$$

which proves that (A5) is equal to (2.13) apart from an over-all factor.

To remove the external fermion (conformal) propagator we multiply the wave function (2.13) by $\not{q}_1(-q_1{}^2)^{3/2-d'}$ on the left and by $\not{q}_2(-q_2{}^2)^{3/2-d'}$ on the right. In this way the first term in (2.13) goes over to

$$(-q_2{}^2)^{2-d'-\bar{d}/2}R^{5/2-d'}\int_0^1 (Rz+1-z)^{-3-\bar{d}/2+d'}[z(1-z)]^{\bar{d}/2-1/2}dz \tag{A10}$$

which through the Kummer relation[28]

$$F(a,b;c;y)=(1-y)^{c-a-b}F(c-a,c-b;c;y) \tag{A11}$$

goes over to

$$(-q_2{}^2)^{2-d'-\bar{d}/2}\int_0^1 (Rz+1-z)^{2-d'-\bar{d}/2}\times[z(1-z)]^{\bar{d}/2-1/2}dz. \tag{A12}$$

Similarly, one deals with the second term, and we obtain the vertex function (2.14).

APPENDIX B

We derive in this appendix the general criterion of convergence of the parametric integrals we encounter in the text.

As we saw in Sec. V the study of the asymptotic behavior of the form factor is reduced to the proof of the convergence of integrals of the type

$$\int (AB+\alpha'\beta'\eta C)^E C^c\, dP\,\delta(1-\Sigma)\delta(1-\Sigma'), \tag{B1}$$

where $\Sigma=\xi+\zeta+\eta+\gamma$, $\Sigma'=\alpha'+\beta'+\gamma'$, $A=z\xi+\beta'\gamma'\eta$, $B=z'\zeta+\alpha'\gamma'\eta$, $C=\eta\gamma'(\alpha'+\beta')+\xi+\zeta+\gamma$, and the differential dP is given by

$$dP=\prod_p p^{(p)-1}dp=\xi^{(\xi)-1}d\xi\,\eta^{(\eta)-1}d\eta\cdots, \tag{B2}$$

where in (B2) the product is extended to all parameters $\xi\eta\zeta\gamma$, zz', $\alpha'\beta'\gamma'$. To make the notation more clear we have indicated, e.g., with (ξ) the "dimension" of the differential in ξ, with (z) the "dimension" of the differential in z near $z=0$, etc. Moreover, in all cases we meet in the text $E<0$ and $c<0$. The conditions for the convergence of (B1) will be derived[29] starting from the fact that not all Feynman parameters ξ,ζ,η,γ, can vanish at the same time in the sense that at least one of them has to be greater than $\frac{1}{4}$ and also that at least one of the parameters α',β',γ' has to be greater than $\frac{1}{3}$. We examine first the region $\xi>\frac{1}{4}$. We can write

$$\begin{aligned}(AB+\alpha'\beta'\eta C)^E &\le \text{const}\times[z(z'\zeta+\alpha'\gamma'\eta)+\alpha'\beta'\eta]^E\\ &\le \text{const}\times[z(z'\zeta+\alpha'\eta(\beta'+\gamma'))+\alpha'\beta'\eta]^E\\ &\le \text{const}\times[z(z'\zeta+\alpha'\eta(\beta'+\gamma'))]^{E'}(\alpha'\beta'\eta)^{E''}\end{aligned} \tag{B3}$$

with $E'+E''=E$ and $E'<0$, $E''<0$. Then taking E'' as negative as possible, i.e., near $-m'$, $m'=\min((\alpha'),(\beta'),(\eta))$, which does not compromise the convergence of the integral over $d\eta\, d\alpha' d\beta'$, we see that we have convergence if

$$E+m'+(z)=E+(z)+m((\alpha'),(\beta'),(\eta))>0 \tag{B4}$$

and at the same time

$$\begin{aligned}&E+m'+m((z'),(\zeta))\\&\quad+m((\alpha')-m',(\eta)-m',(\beta')+(\gamma')-m')\\&\quad=E+m((z'),(\zeta))+m((\alpha'),(\beta')+(\gamma'),(\eta))>0.\end{aligned} \tag{B5}$$

Here and in the following we indicate with $m(,\,)$ the minimum of a set of numbers. Similarly for $\zeta>\frac{1}{4}$ we obtain

$$E+(z')+m((\alpha'),(\beta'),(\eta))>0 \tag{B6}$$

and

$$E+m((z),(\xi))+m((\alpha')+(\gamma'),(\beta'),(\eta))>0. \tag{B7}$$

For $\gamma>\frac{1}{4}$ we shall show that (B5) and (B7) already ensure convergence. First of all one notes from (2.4), (2.5), and (2.7) that one always has $(\eta)<(\alpha)'+(\gamma')$ and $(\eta)<(\beta')+(\gamma')$, which by the way are necessary conditions for the existence of the integrals representing the upper vertex functions, as is seen looking at the corner, e.g., $\alpha'\to 0$, $\gamma'\to 0$ $(\beta'\to 1)$. If $(\eta)<(\alpha')$, $(\eta)<(\beta')$ one majorizes with $(z\xi z'\zeta)^{E'}(\alpha'\beta'\eta)^{E''}$ with E'' near $-(\eta)$. If $(\alpha')<(\eta)$, $(\alpha')\le(\beta')$ one majorizes with $[(z\xi+\eta\beta'(\gamma'+\alpha'))z'\zeta]^{E'}\times(\alpha'\beta'\eta)^{E''}$ with E'' near $-(\alpha')$ and in both cases (B5) and (B7) ensure convergence.

We examine now $\eta > \frac{1}{4}$ and $\alpha' > \frac{1}{3}$. In this case we majorize with

$$[z\xi(z'\zeta+\gamma')]^{E'}(\beta' C)^{E''} \tag{B8}$$

with E'' near (but larger than) $-(\beta')$. Thus we have two conditions for convergence:

$$E+(\beta')+m((z),(\xi))>0 \tag{B9}$$

and

$$E+(\beta')+m((z'),(\zeta))+(\gamma')>0. \tag{B10}$$

We must now make sure that $C^{c+E''}$ which this time can diverge does not introduce a divergence in the integration. Majorizing $C^{c+E''}<(\xi+\zeta+\gamma)^{c+E''}$ and taking into account (B9) and (B10) we have the sufficient condition of convergence

$$c-(\beta')+(\xi)+E+(\beta')+(\zeta)+E+(\beta')+(\gamma')+(\gamma)$$
$$=c+2E+(\xi)+(\zeta)+(\gamma)+(\beta')+(\gamma')>0. \tag{B11}$$

We notice that (B9) and (B10) are already implied by (B7) and (B5). One similarly works out the conditions for convergence in the regions $\beta' > \frac{1}{3}$ and $\gamma' > \frac{1}{3}$ which simply add two relations obtained substituting in (B11) $(\beta')+(\gamma')$ with $(\alpha')+(\gamma')$ one time and $(\alpha')+(\beta')$ the other.

Summing up, the sufficient conditions for the convergence of the integral (B1) are, taking into account that $(\eta)<(\alpha')+(\gamma')$ and $(\eta)<(\beta')+(\gamma')$,

(i) $E+m((z),(\xi))+m((\eta),(\beta'))>0$,

(ii) $E+m((z'),(\zeta))+m((\eta),(\alpha'))>0$,

(iii) $E+m((z),(z'))+m((\eta),(\alpha'),(\beta'))>0$,

(iv) $c+2E+(\xi)+(\zeta)+(\gamma)$
$\quad +m((\beta')+(\gamma'),(\alpha')+(\gamma'),(\alpha')+(\beta'))>0.$

The qualitative interpretation of these conditions is given in Sec. V of the text.

APPENDIX C

As we mentioned in the text, various arguments[12,18] have been put forward supporting a γ_5-even (even number of γ_μ) leading behavior of the wave function on the light cone for the scalar (pseudoscalar) which decays virtually in the fermion-antifermion pair. For completeness, however, we want to report in this appendix the calculation for the leading γ_5-odd (one γ_μ). Starting from the g_2 part of the vertex function (2.8) and proceeding as in Sec. II, i.e., calculating the discontinuity in the boson squared momentum, or alternatively imposing

$$K_\mu(x)\phi(x,p)+i\overline{K}_\mu(p)\phi(x,p)=0$$

with (C1)

$$\phi(x,p)=(-x^2)^{-d'+\overline{d}/2-1/2}\not{x}f(x\cdot p),$$

we obtain for the wave function (i.e., fermion propagators included)

$$V(q_1,q_2)=\int_0^1[\not{q}_1 z+\not{q}_2(1-z)]$$
$$\times[-q_1^2 z-q_2^2(1-z)]^{-5/2+d'-\overline{d}/2}$$
$$\times[z(1-z)]^{\overline{d}/2-1}dz. \tag{C2}$$

We shall restrict ourselves to the simple situation $d'=\frac{3}{2}$. The trace due to the fermion loop is given by

$$\tfrac{1}{4}\mathrm{Tr}\{[(\not{p}_1+\not{k})z+\not{k}(1-z)]\gamma_\mu[(\not{p}_2+\not{k})z'+\not{k}(1-z')]\not{k}\}$$
$$=zz'(p_{1\mu}p_2\cdot k+p_{2\mu}p_1\cdot k-k_\mu p_1\cdot p_2)+zp_{1\mu}k^2+z'p_{2\mu}k^2+k_\mu k^2$$
$$\to zz'(p_{1\mu}p_2^2z'\zeta+p_{2\mu}p_1^2z\xi)+zp_{1\mu}(k'^2+\Delta^2)+z'p_{2\mu}(k'^2+\Delta^2)-\Delta_\mu(k'^2+\Delta^2)-2k'_\mu k'\cdot\Delta. \tag{C3}$$

The zz' term in (C3) which is of the type of a mass term gives rise to the integral

$$\int(p_{1\mu}p_2^2z'\zeta+p_{2\mu}p_1^2z\xi)zz'[zz'(1-z)(1-z')]^{\overline{d}/2-1}dz\,dz'(\xi\zeta)^{\overline{d}/2}d\xi\,d\zeta(-q^2z\xi z'\zeta+\text{mass terms})^{-\overline{d}},$$

which decreases at infinity faster than $(q^2)^{1-\overline{d}}$. The other terms give, for $\overline{d}>2$, a behavior $(-q^2)^{-\overline{d}/2}$ which (for $\overline{d}>2$) is higher than $(-q^2)^{1-\overline{d}}$. For $\overline{d}<2$ we have always the behavior $(-q^2)^{1-\overline{d}}$. If one should consider a loop with a scalar field ϕ of dimension d, using as upper vertex (3.2) (with $\overline{d}\to d$) we would find again for the form factor $(-q^2)^{1-\overline{d}}$, provided $\overline{d}/2<d$ and $\overline{d}/2<4-d$. These last two relations follow by imposing the convergence of the integral representing the conformal vertex function $\langle 0|\,T\Phi\phi\phi\,|0\rangle$ in momentum space, with and without external ϕ propagators.

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