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Quark models and radiative pion decay

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In the framework of relativistic quark models, we consider the structure-dependent part of the decay $\pi^+ \to e^+\nu_e\gamma$ and evaluate the ratio, γ, of the axial-vector form factor to the vector form factor. For masses of the quarks much larger than the pion mass we find $\gamma \lesssim 1$. This is different from previous predictions, made in the static quark model, which yield $\gamma = 0$. If the pion is treated as a nucleon-antinucleon pair, $\gamma \simeq 0.4$ in better agreement with experiments.

In a recent letter A. Stetz *et al.*[1] obtain a new experimental determination of γ, the ratio of the axial-vector form factor to the vector form factor which contains all the structure-dependent information in the radiative pion decay $\pi \to e\nu\gamma$,

$$\gamma_{\rm exp} = -0.15 \pm 0.11 \text{ or } 2.07 \pm 0.11 . \tag{1}$$

One must recall that the determination of $\gamma_{\rm exp}$ in Eq. (1) depends on the conserved vector current (CVC) hypothesis and is sensible to the width of $\pi^0 \to \gamma\gamma$ which is not yet well established. This new determination can be compared with the theoretical predictions[2] made within the framework of current algebra, dispersion relations, pole dominance, and hard-pion techniques.[3] In this framework it is possible, with one parameter, to relate together the rates of $\rho \to \pi\pi$ and $A_2 \to \rho\pi$ decays, the pion electromagnetic radius, and γ. But the new determination of γ is in strong disagreement with these predictions[2] (see for example Table I in Ref. 1). On the contrary, the static quark model presented in Ref. 4 seems to give a result, $\gamma_{\rm SQ} = 0$, in agreement with $\gamma_{\rm exp}$. In this version of the static quark model, the vector and axial-vector currents are treated on different footings: An anomalous magnetic moment shows up in the electromagnetic and weak vector currents of the quarks while in the axial-vector current only the $\gamma_5\gamma_\mu$ term appears.

The model we consider is graphically shown in Fig. 1; it can be regarded as a refinement of the static quark models in several ways. First, the quarks are taken to be elementary objects and do not have anomalous magnetic moment. Second, it is relativistic and makes use of field theory in the evaluation of the loops. The only restriction is $M \gg m_\pi/2$, where M and m_π are the quark and pion masses, respectively. It is necessary because the model does not have a mechanism to forbid hadron decays into light quarks. Furthermore, the answer it gives for the ratio γ is dependent on M, and in the infinite-quark-mass limit our result does not agree with the static quark model.[4] Instead of $\gamma_{\rm SQ} = 0$, we get

$$\gamma(M = \infty) = 1 , \tag{2}$$

for the Gell-Mann–Zweig, the colored, and the Han-Nambu versions of the quark model. The discrepancy arises from the fact that in the infinite quark mass limit both the axial-vector form factor and the vector form factor vanish, but their ratio remains different from zero. We remark that the form factors may not vanish if the pion-quark coupling constant, g_0, is proportional to the quark mass. However, this situation will leave the ratio γ unaffected because g_0 enters in both form factors multiplicatively.

Let us specify the model. We have taken for the pion-quark interaction[5]

$$\mathcal{L}_{\pi q\bar{q}} = ig_0 \sum_j \bar{q}_j \vec{\pi} \cdot \vec{\tau} \gamma_5 q_j , \tag{3}$$

where g_0 is the neutral pion-quark coupling constant, the summation is over all nonstrange quarks in a given model, $\vec{\tau}$ are the Pauli matrices, and q_j and $\vec{\pi}$ are the quark and pion fields, respectively. The axial and vector parts of the hadronic current are

$$V_k{}^\mu = g_V \left[\sum_j \bar{q}_j \gamma^\mu \tau_k q_j + 2\epsilon_{klm} \pi_l \partial^\mu \pi_m \right] \tag{4}$$

and

$$A_k^\mu = g_A \sum_j \bar{q}_j \gamma^\mu \gamma_5 \tau_k q_j , \tag{5}$$

respectively, with μ, the Lorentz indices, k, the isovector ones, ϵ_{klm}, the Levi-Civita tensor, g_V and g_A, the vector and axial-vector coupling constants (in $V-A$ theory, $g_A = g_V$).

If one applies the model to the nonradiative decay $\pi^+ \to e^+\nu_e$, Fig. 1(a), divergent integrals are found and a regularization procedure is needed to give a physical meaning to the on-shell pion decay coupling constant, f_π, defined in terms of the $\pi^+ \to e^+\nu_e$ decay as

$$\Gamma_{\pi^+ \to e^+\nu_e} = \frac{(G\cos\theta)^2}{8\pi} \frac{m_e^2}{m_\pi^3} (m_\pi^2 - m_e^2)^2 f_\pi^2 , \tag{6}$$

where G is the Fermi coupling constant, θ the Cabibbo angle, and m_e the electron mass. Computing Fig. 1(a), we find

$$f_\pi = i\frac{\sqrt{2}\, g_0 g_A}{(2\pi)^4}\, 4M \int \frac{d^4r}{(r^2 - M^2 + i\epsilon)[(r-p)^2 - M^2 + i\epsilon]}$$

$$+ (\text{regularization terms}), \qquad (7)$$

where p is the pion four-momentum. Therefore, f_π is not predicted in the model.

The radiative decay, $\pi^+ \to e^+\nu_e\gamma$, to lowest order in all interactions is given by Figs. 1(b)–1(e). Here also divergent integrals appear, but all of them can be cast into a form proportional to the one for f_π. Furthermore the structure part[6] which is the relevant part of the amplitude for the determination of γ is at this order free of divergences.

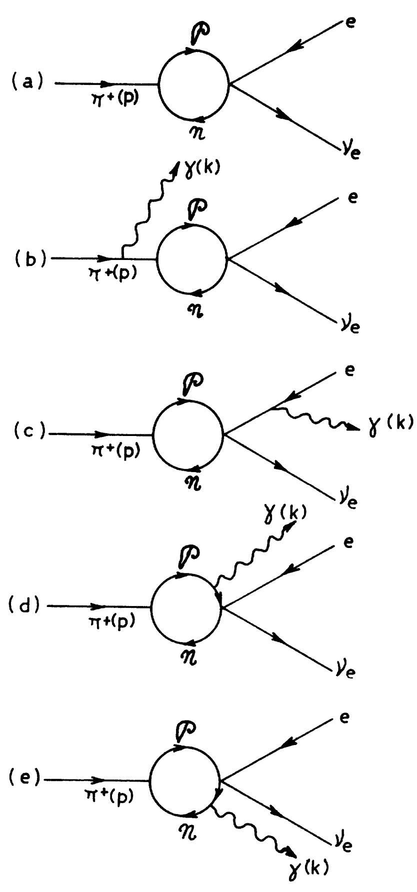

FIG. 1. (a) The nonradiative decay $\pi^+ \to e^+ \nu_e$ in our model. The pion annihilates itself into a virtual pair of quark-antiquark. (b), (c), (d), and (e): These are the different contributions to the radiative decay $\pi^+ \to e^+ \nu_e \gamma$ in the same model. (b) and (c) contain the inner bremsstrahlung contributions and (d) and (e) the structure-dependent terms of the model. The sum of all possible intermediate fermion (quark) pairs is to be understood.

Indeed the only diagrams from which the structure terms stem are those on Figs. 1(d) and 1(e), the so-called triangle diagrams. The vector parts of them are precisely the ones related to the triangle anomaly.[7]

For definiteness we recall here that in process $\pi \to e\nu\gamma$ the amplitude's structure part is the one which is left undetermined after using Low's theorem.[6] This theorem gives the bremsstrahlung part, pion bremsstrahlung included, and the zeroth-order term in the photon energy of the amplitude.[8] Thus the structure-dependent terms are of order unity and higher with respect to the photon energy. Furthermore, gauge invariance leaves undetermined in the structure-dependent amplitude, $\mathfrak{M}_{\rm SD}$, only two form factors: the vector form factor $F_V(t)$ and the axial-vector one $F_A(t)$, where $t = (p-k)^2$, p and k being the pion and photon four-momenta. $\mathfrak{M}_{\rm SD}$ is written in terms of the form factors as[6]

$$\begin{aligned}\mathfrak{M}_{\rm SD} &= e_\pi \epsilon_\nu l_\mu \mathfrak{M}_{\rm SD}^{\mu\nu}\\ &= e_\pi \epsilon_\nu l_\mu [\epsilon^{\mu\nu\rho\sigma} k_\rho p_\sigma F_V(t)\\ &\qquad - i(k^\mu p^\nu - g^{\mu\nu} k\cdot p) F_A(t)],\end{aligned} \qquad (8)$$

where e_π is the pion charge, ϵ_ν the photon polarization vector, and l_μ the leptonic current,

$$l_\mu = \frac{G\cos\theta}{\sqrt{2}}\bar{u}_{\nu_e}\gamma_\mu(1-\gamma_5)\, v_e. \qquad (9)$$

One gets in the model

$$\mathfrak{M}_{\rm SD}^{\mu\nu} = \frac{\sqrt{2}\, g_0 M}{8\pi^2}\frac{1}{p\cdot k}\Big[g_V \sum_{i=1}^{N}(2e_{\mathcal{P}_i} - 1)\epsilon^{\mu\nu\rho\sigma} k_\rho p_\sigma I_{-1}$$

$$- i g_A N(p^\nu k^\mu - g^{\nu\mu} p\cdot k)(I_{-1} - 2I_0)\Big], \qquad (10)$$

where $e_{\mathcal{P}_i}$ is the charge of the proton-like quark (in units of pion charge), N the number of quark pairs which can run through the loops in Fig. 1, and I_{-1} and I_0 are defined from the class of integrals

$$I_n\left(\frac{t}{4M^2}, \frac{m_\pi^2}{4M^2}\right) = \int_0^1 dx\, x^n \ln\left[\frac{1 - x(1-x)t/M^2}{1 - x(1-x)m_\pi^2/M^2}\right]. \qquad (11)$$

Equation (10) has been evaluated with the restriction

$$M > \frac{m_\pi}{2} \qquad (12)$$

because otherwise the pion would decay into two quarks. Evaluating the integrals in (11) and taking into account the definitions of the structure form factors in (8) leads to

$$F_V(t)=g_V\frac{\sqrt{2}g_0M}{2\pi^2}\left[\sum_{i=1}^{N}(2e_{\mathcal{P}_i}-1)\right]\frac{\theta^2-\alpha^2}{m_\pi^{\,2}-t} \tag{13}$$

and

$$F_A(t)=g_A\frac{\sqrt{2}g_0M}{2\pi^2}N\frac{[\theta^2-\alpha^2+2(\theta\cot\theta-\alpha\cot\alpha)]}{m_\pi^{\,2}-t}, \tag{14}$$

where

$$\theta=\sin^{-1}\frac{m_\pi}{2M},\quad \alpha=\sin^{-1}\frac{\sqrt{t}}{2M}. \tag{15}$$

From this one gets immediately

$$\gamma(t)\equiv\frac{F_A(t)}{F_V(t)}=Q\left[1+2\frac{\theta\cot\theta-\alpha\cot\alpha}{\theta^2-\alpha^2}\right], \tag{16}$$

with

$$Q\equiv\frac{g_A}{g_V}\frac{N}{\sum_{i=1}^{N}(2e_{\mathcal{P}_i}-1)}. \tag{17}$$

Our statement about previous computations is now clear: In the limit $M\to\infty$ both F_A and F_V are zero (except if g_0 is proportional to M) but their ratio γ is not zero. Instead γ is in this limit a constant different from zero for any value of t. Although Q in the last equation depends on the quark model, it has the *same* value $Q=3g_A/g_V$ for the most interesting cases: the Gell-Mann–Zweig model with $N=1$, $e_{\mathcal{P}_i}=\frac{2}{3}$ the colored quark model $N=3$, $e_{\mathcal{P}_i}=\frac{2}{3}$ and the original Han-Nambu model $N=3$, $e_{\mathcal{P}_1}=e_{\mathcal{P}_2}=1$, $e_{\mathcal{P}_3}=0$. In the frame of $V-A$ theory, $g_A=g_V$, then $Q=3$. But if one takes a nucleon-antinucleon loop in Fig. 1, one has $N=1$, $e_{\mathcal{P}}=1$ and $g_A/g_V=1.2$. Then $Q=1.2$.

For an arbitrary quark mass in the range of Eq. (12) and in the physically important region near $t=0$ one gets from Eqs. (13) to (16) the following:

(i) The form factor F_A and F_V practically do not change for variations $\Delta t\ll m_\pi^{\,2}$.

(ii) The value of $\gamma(t=0)$ is in the quark models mentioned before:

$$\gamma\equiv\gamma(t=0)=3\left(1+2\frac{\theta\cot\theta-1}{\theta^2}\right), \tag{18}$$

where θ was defined in Eq. (15). In Fig. 2 we have plotted γ as a function of the parameter $m_\pi^{\,2}/4M^2$.

By comparing with the recent experiment of Ref. 1 or the previous value[9] $\gamma'_{\rm exp}=-0.4$ or 2.1, one observes a poor agreement of the quark model with the data because, in this case, $\gamma\lesssim 1$. On the other hand, the nucleon-antinucleon model with $\gamma\simeq 0.4$ is in better agreement with both experiments.[1,9] However, in view of the unreliability of the vector form factor determination it would be better to be cautious in drawing conclusions from this fact until a more precise determination of the $\pi^0\to\gamma\gamma$ lifetime is available. Even more, the model we have used has several disadvantages. From a theoretical point of view the most important flaw is that a direct comparison with current-algebra predictions is not possible because this model does not fulfill current algebra and does not contain PCAC as an operator identity.[10] This can be cured using the σ model of Lévy and Gell-Mann.[10,11]

The first conclusion that should be drawn from this paper is that nonrelativistic models such as the static quark model[4] do not give consistent answers when applied to processes, as $\pi\to e\nu\gamma$, in which annihilation of quarks occurs because they treat the vector and the axial-vector form factors on different footings. Indeed, we observe that the Adler theorem[7] about the triangle anomaly tells us that in the soft-pion limit ($p_\mu\to 0$) the vector form factor, $F_V(t)$, is entirely given in terms of the lowest-order triangle diagrams, namely those in Figs. 1(d) and 1(e). In particular no anomalous magnetic moment term is needed, contrary to the case of the static quark model.[4]

Although there is no analogous theorem for the axial-vector form factor $F_A(t)$, direct computation shows that an induced pseudoscalar term in the axial-vector current-quark vertex never contributes owing to kinematical reasons.

Finally, we are pleased to acknowledge interesting remarks of Professor W. Kummer and several discussions with Dr. H. R. Rubinstein.

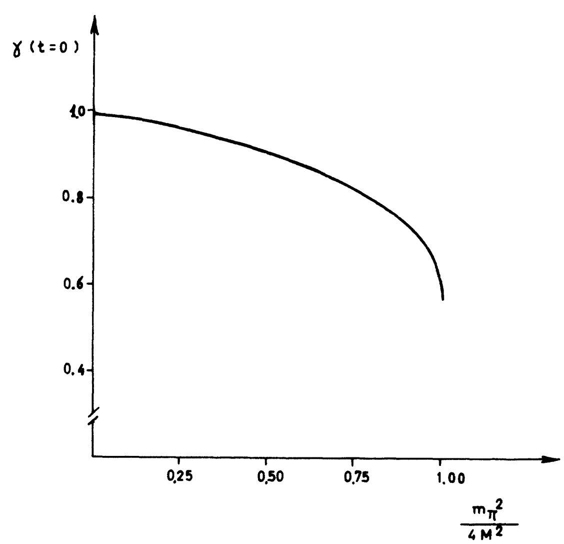

FIG. 2. Prediction for γ, the ratio of the axial-vector form factor to the vector form factor in radiative pion decay, in the framework of Gell-Mann–Zweig, colored, and Han–Nambu quark models. M is the quark mass.

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[1]A. Stetz, J. Carroll, D. Ortendahl, V. Perez-Mendez, G. Igo, N. Chirapatpimol, and M. A Nasser, Phys. Rev. Lett. 33, 1455 (1974).

[2]See T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. 19, 859 (1967); J. S. Vaishya, Phys. Rev. 173, 1757 (1968); V. S. Berezinskii, Yad. Fiz. 8, 1208 (1968) [Sov. J. Nucl. Phys. 8, 700 (1969)]; S. G. Brown and G. B. West, Phys. Rev. 168, 1605 (1968); R. E. Marshak, Riuzuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969); Riazuddin and Fayyazuddin, Phys. Rev. 171, 1428 (1968); F. Scheck and A. Wullschleger, Nucl. Phys. B67, 504 (1973).

[3]Note that hard-pion techniques are compatible with algebra of fields but not with quark or σ models. See, for example, S. G. Brown and G. B. West, Phys. Rev. 180, 1613 (1969); P. Horwitz and P. Roy, *ibid.* 180, 1430 (1969).

[4]W. Kummer and J. Kuti, Nuovo Cimento 45, 242 (1969); R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50A, 617 (1967). See also F. Scheck and A. Wullschleger, Nucl. Phys. B67, 504 (1973).

[5]Our notation is that of J. Bjorken and S. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[6]For definitions of the form factors, see S. Bludman and J. Young, Phys. Rev. 118, 602 (1960); D. Neville, *ibid.* 124, 2037 (1961); S. Brown and S. Bludman, *ibid.* 136, B1160 (1964); M. G. Smoes, Nucl. Phys. B20, 237 (1970); D. Bardin and S. Bilen'kii, Yad. Fiz. 16, 557 (1972) [Sov. J. Nucl. Phys. 16, 311 (1972)]; P. De Baenst and J. Pestieau, Nuovo Cimento 53A, 407 (1968).

[7]S. L. Adler, Phys. Rev. 177, 2426 (1969); S. L. Adler, in *Lectures on Elementary Particles and Quantum Field Theory*, 1970 Brandeis Lecture Notes, edited by S. Deser *et al.* (M.I.T. Press, Cambridge, Mass., 1970).

[8]After this work was submitted for publication, we learned that other authors have studied the same model [N. Cabibbo, Nuovo Cimento 11, 837 (1959) and L. Resnick and J. H. Kim, Carleton University report, 1975 (unpublished)]. While the latter authors agree with us, the result of the former is not correct because he has forgotten the term proportional to I_0 in Eq. (10); see below. This term is required if the inner bremsstrahlung contributions are correctly separated from the structure part of the amplitude (see Ref. 6). We notice that the γ dependence on the quark mass is precisely due to this term. In the infinite-quark-mass limit his γ is three times bigger than ours, as can be seen from our Eq. (18).

[9]P. Depommier, J. Heintze, C. Rubbia, and V. Soergel, Phys. Lett. 7, 285 (1963).

[10]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 53 (1960).

[11]After this work was submitted for publication, we analyzed the process $\pi \to e\nu\gamma$ in the σ model (see Ref. 10). We find that at the zero- and one-loop levels $\gamma = 0$ in the soft-pion limit, $p \to 0$. In other words, the σ-π loop contributions exactly cancel the quark-antiquark loop contributions in that limit. This result is therefore in good agreement with the latest determination of γ (see Ref. 1).