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Calculations of baryon parameters in the extended models of hadrons

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Within the frame of the extended models of hadrons ("bag" theories), in the semiclassical (spherical cavity) approximation, we discuss the computation of baryon parameters taking into account the interaction among quarks. We consider, as trial wave functions, arbitrary sums of antisymmetrized products of one-quark wave functions obeying the Dirac equation for an arbitrary central potential. It is shown that if the electromagnetic interaction is the minimal one $\bar{\psi}\gamma_\mu\psi$, then $\rho \equiv 9\mu_p{}^2/\langle r_p{}^2\rangle \le 1$, where μ_p and $\langle r_p{}^2\rangle^{1/2}$ are the magnetic moment and root mean square radius of the proton (to be compared with the experimental value $\rho \simeq 1.06$). It is also shown that if $\rho = 1$, then the axial nucleon charge $g_A/g_V = 5/9$.

I. INTRODUCTION

Even though free quarks have never been observed, the idea of hadrons built of quarks has been of great importance in order to describe and predict not only the low-lying hadronic states,[1] but also the deep-inelastic phenomena.[2] However, the negative results of the experiments designed to observe free quarks have spurred the theoretician to investigate possible mechanisms to confine the quarks. In particular, during the last year, a family of extended models of hadrons has been considered. In those models, the quarks are confined by a constant pressure B by unit volume in a finite region of space (the so-called MIT bag model[3]) or by coherent states built of canonical boson fields.[4-8] Both types of model are Lorentz-invariant and, at the classical level, Creutz and Soh[9] have shown that one may obtain the MIT boundary conditions from a conventional local field theory in a strong coupling limit. Although the solution of these strong-coupling or unconventional field theories is a very difficult problem, preliminary results have been obtained based on semiclassical approximations.[4-8] Within the frame of these semiclassical approximations, one considers the particular case when the boundary in the bag model or the confining coherent bosonic states in the canonical field-theory models are static and spherically symmetrical.[5,10] It is worth remarking that these semiclassical "static solutions" are incompatible with quantum mechanics; the coupling of the confined fields with the surface or wall implies fluctuations of the "wall region" due to the quantum fluctuations of the confined fields. However, by assuming that in a first approach one may forget the "wall dynamics," the magnetic moment μ, the root mean square radius $\langle r^2\rangle^{1/2}$, and axial charge g_A/g_V of the nucleon have been computed. In these calculations it is assumed that the three quarks of the nucleon occupy the lowest quantum level of the spherical potential. For the MIT cavity the potential is an infinite square well with the radius R related to the pressure B, which may be fixed by fitting the nucleon mass. Considering the quarks inside the cavity to be free and massless, Chodos *et al.*[10] have obtained, for the proton, the values

$$2m_p\mu_p = 2.6,\quad \langle r^2\rangle_p{}^{1/2} = 1.04\ \text{fm},\quad g_A/g_V = 1.09,$$

to be compared with the experimental values[11]

$$(2m_p\mu_p)_{\text{exp}} = 2.79,\quad \langle r_E{}^2\rangle_p{}^{1/2} = 0.88 \pm 0.03,$$

$$\langle r_M{}^2\rangle_p = 0.83 \pm 0.07,\quad (g_A/g_V)_{\text{exp}} = 1.25 \pm 0.09.$$

In particular, for the quantity $\rho \equiv 9\mu^2/\langle r^2\rangle$, which in the model is independent of the radius R, they obtained a value of 0.69 to be compared with the experimental value $\rho \approx 1.06$. De Grand *et al.*[12] have included, in lowest perturbation order, color-gluon interactions between the quarks. They obtain $\langle r_p{}^2\rangle^{1/2} = 0.73$ fm, but $2m_p\mu_p = 1.9$ (too low), and as a matter of fact the values for ρ and g_A/g_V remain unchanged.

In the model considered by the SLAC group[5] the quark wave function is concentrated in the region $r \approx R$ (where R is related to the parameters of the theory). They find $\langle r_p{}^2\rangle^{1/2} = R$, $\mu_p = \frac{1}{3}R$ (therefore $\rho = 1$), and $g_A/g_V = \frac{5}{9}$. Now ρ is close to the experimental value but g_A/g_V is too small by more than a factor of 2.

One may wonder if by considering different interactions between the quarks one might fit, within the frame of the "spherical approximation," both ρ and g_A/g_V. We shall show in the next section that if the wave function may be represented by an arbitrary sum of antisymmetrized products of one-quark wave functions obeying the Dirac equation for an arbitrary central potential, then $\rho \leq 1$, with $g_A/g_V = \frac{5}{9}$ if $\rho = 1$. Note that these trial wave functions are rather general; a Hartree-Fock type of treatment would include only one antisymmetrized product of three one-quark ground-state wave functions. Although in order to obtain the preceding results one assumes for the electromagnetic interaction the minimal structure $\bar{\psi}\gamma_\mu\psi$, the analysis seems to suggest that the "wall dynamics" is important. A few remarks on this topic are the subject of Sec. III.

II. BOUNDS ON THE NUCLEON PARAMETERS IN THE SPHERICAL CAVITY APPROXIMATION

As mentioned in the Introduction, in the extended models of hadrons a strongly interacting particle consists of fields confined to a finite region of space. The confinement is accomplished in a Lorentz-invariant way by assuming that the bag possesses a constant positive energy B by unit of volume.[3] In other models the confinement is due to a surface tension.[13] In models where coherent states confine the fermion fields,[4-8] both volume and surface energy appear. In all cases, the geometry and configuration of the wall region depend on the fields inside and the wall must follow the change of pressure and momentum of those fields. As we mentioned before, due to the great difficulty of these strong coupling or unconventional quantum field theories, tentative results have been obtained based on semiclassical approximations. These approximations allow solutions with static spherical symmetry and, taking into account the spherical equilibrium shape of a droplet of liquid or a bubble of steam, it is reasonable to think that such spherical shape is relevant for the ground state of hadrons. Due to the interaction between the wall and the confined constituent fields, the wave function for these constituents must also be spherically symmetric; otherwise the pressure on the wall would not be balanced by the momentum of the constituents. Even in the presence of interactions among the constituents, the wave function must obey such spherical symmetry.

In order to impose this spherical symmetry, we shall consider trial wave functions built as an arbitrary sum of antisymmetrized products of one-quark orthogonal wave functions obeying the Dirac equation for an arbitrary central potential. Each term of the sum will be antisymmetrized with respect to the product SU_3(color) $SU_3(\vec{T},Y)$ (where $\vec{T}$ is the isospin and Y the hypercharge), spin and configuration space transformations. [For the case of nonstrange baryons one may consider $SU_2(\vec{T})$ instead of $SU_3(\vec{T},Y)$.] These trial wave functions are much more general than a Hartree-Fock-type wave function. The baryon wave function from a Hartree-Fock treatment would contain only one antisymmetrized product of three one-quark ground-state wave functions.

As we are interested in the magnetic moment, root mean square radius, and axial charge of the proton, we proceed to summarize the relevant information which is needed to compute those parameters. Then we shall discuss the restrictions that the structure of our trial wave functions impose on the values of the proton parameters.

A. Wave functions

The wave function ψ of a Dirac particle in an arbitrary central potential obeys the equation

$$H\psi = \left[\frac{\vec{\alpha}\cdot\vec{\nabla}}{i} + \beta M(r) + V(r) \right]\psi = E\psi, \qquad (1)$$

where $\vec{\alpha}$ and β are the usual Dirac matrices, E is the energy, M is a scalar potential (which includes the mass of the Dirac particle), and V is the fourth component of a four-vector.

In our case only spherically symmetric wave functions are allowed and we may write[14]

$$\psi_n = \begin{pmatrix} iG_n(r)\chi \\ F_n(r)\vec{\sigma}\cdot\hat{r}\chi \end{pmatrix}, \qquad (2)$$

where χ is a Pauli spinor, the lower index n labels different wave functions, and G and F obey two coupled first-order differential equations.

Taking into account the internal symmetries SU_3(color) and $SU_2(\vec{T})$ we write

$$\psi_n(t,k;s,r) = \begin{pmatrix} iG_n(r)\chi_s \\ F_n(r)\vec{\sigma}\cdot\hat{r}\chi_s \end{pmatrix} \varphi_t^{(k)}, \qquad (3)$$

where $k = 1, 2, 3$ is the color index and $t = \pm\frac{1}{2}$ is the isospin index. The wave functions obey the orthogonality condition

$$\int r^2 dr [G_n^*(r)G_{n'}(r) + F_n^*(r)F_{n'}(r)] = \delta_{nn'}. \qquad (4)$$

The trial wave functions for the nonstrange baryon are

$$\Psi(\vec{T};\vec{S};r_1,r_2,r_3)=\sum_{n_1n_2n_3}\sum_{t_1t_2t_3}\sum_{s_1s_2s_3}C_{n_1n_2n_3}S(t_1t_2t_3;s_1s_2s_3)$$
$$\times\begin{pmatrix} iG_{n_1}(r_1)\chi_{s_1}(1) \\ F_{n_1}(r_1)\vec{\sigma}_1\cdot\hat{r}_1\chi_{s_1}(1)\end{pmatrix}\begin{pmatrix} iG_{n_2}(r_2)\chi_{s_2}(2) \\ F_{n_2}(r_2)\vec{\sigma}_2\cdot\hat{r}_2\chi_{s_2}(2)\end{pmatrix}\begin{pmatrix} iG_{n_3}(r_3)\chi_{s_3}(3) \\ F_{n_3}(r_3)\vec{\sigma}_3\cdot\hat{r}_3\chi_{s_3}(3)\end{pmatrix}\varphi^{(1)}_{t_1}(1)\varphi^{(2)}_{t_2}(2)\varphi^{(3)}_{t_3}(3),$$
$$\sum_{n_1n_2n_3}|C_{n_1n_2n_3}|^2=1. \qquad (5)$$

In Eq. (5) we have assumed that Ψ is fully antisymmetric under SU_3(color). For the nucleon and the $\Delta(1236)$ [and all particles in the 56 representation of SU_6] S is symmetric under the simultaneous exchange of both s_i and t_i indices and C is symmetric under the exchange of the n_s.

B. One-quark operators

Within the frame of the model the magnetic moment, root mean square radius, and axial charge are given in terms of one-quark operators.

(i) Magnetic moment. For the minimal electromagnetic interaction $\bar{\psi}\gamma_\mu\psi$ the magnetic moment depends on the one-quark matrix elements

$$\int d^3r\,\tfrac{1}{2}\vec{r}\times(\psi^\dagger\vec{\alpha}Q\psi), \qquad (6)$$

where Q is the matrix in isospin space,

$$Q=\begin{pmatrix}\frac{2}{3} & 0\\ 0 & -\frac{1}{3}\end{pmatrix}.$$

From Eq. (6) and the wave functions (4) we obtain for the magnetic moment of the proton[15]

$$\mu_p=\tfrac{1}{3}\sum C^*_{n_1n_2n_3}C_{n_1n_2n_3'}\times\int r^2dr\,r[G^*_{n_3}(r)F_{n_3'}(r)+F^*_{n_3}(r)G_{n_3'}(r)], \qquad (7)$$

or, introducing the quantities

$$a_{n_1n_2}(r)=\sum_{n_3}C_{n_1n_2n_3}F_{n_3}(r),$$
$$b_{n_1n_2}(r)=\sum_{n_3}C_{n_1n_2n_3}G_{n_3}(r), \qquad (8)$$

which satisfy

$$\sum_{n_1n_2}\int r^2dr[|a_{n_1n_2}(r)|^2+|b_{n_1n_2}(r)|^2]=1,$$

we can rewrite Eq. (7) as

$$\mu_p=\tfrac{1}{3}\sum\int r^2dr\,r(2\,\mathrm{Re}\,a^*b), \qquad (9)$$

where we suppress summation indices.

(ii) Mean square radius. For the mean square radius, we have the expression

$$\langle r^2\rangle=\sum\int r^2dr\,r^2(|b|^2+|a|^2). \qquad (10)$$

(iii) Axial charge of the nucleon. If we define the axial charge g_A/g_V of β decay as the expectation value of the one-quark operator

$$\int d^3r\,\psi^\dagger\tau_3\sigma_z\psi$$

we obtain

$$\frac{g_A}{g_V}=\tfrac{5}{3}\sum\int r^2dr(|b|^2-\tfrac{1}{3}|a|^2). \qquad (11)$$

C. Bounds on $9\mu_p^2/\langle r_p\rangle^2$ and g_A/g_V

From Eqs. (7) and (9) we may write for the proton

$$\rho_p=\frac{9\mu_p^2}{\langle r_p^2\rangle}=\frac{\sum\int\int r^2dr\,r'^2dr'(2\,\mathrm{Re}\,a^*b)2rr'(2\,\mathrm{Re}\,a'^*b')}{\sum\int\int r^2dr\,r'^2dr'(|a|^2+|b|^2)(r^2+r'^2)(|a'|^2+|b'|^2)}, \qquad (12)$$

with

$$a'=a_{n_1'n_2'}(r'),\quad b'=b_{n_1'n_2'}(r'). \qquad (13)$$

Taking into account the inequalities

$$2rr'\leq r^2+r'^2, \qquad (14)$$
$$2\,\mathrm{Re}\,a^*b\leq|a|^2+|b|^2, \qquad (15)$$

we have that

$$\rho_p\leq 1. \qquad (16)$$

Also, from Eq. (11) we may write

$$\left(\frac{1}{2}+\frac{9}{10}\frac{g_A}{g_V}\right)=2\int r^2dr|b|^2, \qquad (17)$$

and this equation implies

$$0 \leqslant \frac{1}{2} + \frac{9}{10}\frac{g_A}{g_V} \leqslant 2$$

and

$$-\frac{5}{9} \leqslant \frac{g_A}{g_V} \leqslant \frac{5}{3}.$$

The experimental values[11] are

$$\rho_p \simeq 1.06, \quad g_A/g_V \simeq 1.25 \pm 0.09.$$

Note that in order to reach in Eq. (12) the upper bound $\rho_p = 1$ we need $M_{n_1n_2} = a_{n_1n_2}/b_{n_1n_2} = 1$ for all n_1n_2 and all values of r. However, from Eq. (17) we find that in this case $g_A/g_V = \frac{5}{9}$. That was the case for the computation of the SLAC group.[5]

In general, the M's will not be near unity and the sign may be different for different n's or for different values of r in the integration range. In particular, when in the wave function the contribution of "excited radial states" becomes increasingly important, the M's will change sign in the integration range and $\rho \to \ll 1$. Note that in order to fit the experimental value of g_A/g_V we need for the M's an average value of 0.48. This value implies $\rho_p < 0.61$. (In models with a positive mass for the nonstrange quark one may fit g_A/g_V but ρ_p will be too small by a factor of 2.)

III. FINAL REMARKS

We have shown in Sec. II that within the frame of the "spherical approximation," even in the presence of interactions among the quarks, it is not possible to get a good fit for the nucleon parameters. However, in addition to the "spherical approximation," the results depend on the validity of the one-quark operators. In particular, if the electromagnetic interaction, in addition to the minimal current $\bar{\psi}\gamma_\mu\psi$, includes anomalous terms one might fit ρ_p.

However, this possibility, in addition to perhaps being at odds with renormalizability, does not seem to be suggested by the rather simple picture that emerges in deep-inelastic experiments. More than anything, the analysis seems to suggest the importance of the "wall dynamics." [Remember that PCAC (partially conserved axial-vector current) and VMD (vector-meson dominance) within the frame of bag theories are related to fusion and fission of extended objects and therefore are "wall phenomena."] The above result will not necessarily be valid for the more complicated wave functions that would arise from the interaction with gluons. One would expect anyway that these wave functions would give corrections of the same order of magnitude as those due to "wall dynamics." Although when one leaves the spherical approximation the analysis becomes rather complicated,[16] it is important to try to understand this wall dynamics. The deep-inelastic regime suggests a simple picture for the inside of the baryon, and a better knowledge of the outer region would allow us to achieve a rather complete understanding of the baryon structure.

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