Shielded Pomeron model for high-energy cross sections

Sudhir Paranjape*
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 28 August 1975)

Starting with a shielding mechanism which makes hard Regge surfaces compatible with two-particle unitarity in the t channel, we present a model for the Pomeron and its associated shielding cut. The model provides a unified description of pp, πp, and Kp scattering while conforming to the usual tenets of factorization and exchange degeneracy.

We assume that cross sections rise asymptotically so that the Pomeron singularity is harder than an ordinary first-order Regge pole. Under these circumstances shielding cuts are required in order to make the amplitudes compatible with two-particle unitarity in the t channel. Threshold branch points of the Pomeron trajectory $\alpha(t)$ in the t plane, which are sufficient and natural for ordinary Regge poles, are generally no longer adequate for the harder Regge surface, in particular if they are branch-point surfaces.[1]

The Pomeron may well be a hard branch-point surface with a complicated structure. For simplicity, we choose here a double pole which gives an asymptotic increase as $\ln s$, and is the hardest singularity which does not require a branch point in $\alpha(t)$ at $t=0$.

Hard singular surfaces which do not possess threshold singularities can be made compatible with elastic t-channel unitarity in $\pi\pi$ scattering using an ansatz proposed by Oehme.[2] Under the above assumptions we write the continued partial-wave amplitude F in the form

$$(t-t_\pi)^j F^{-1}(t,j)=(j-\alpha)^2\left\{\Phi(t,j)-\frac{1}{\pi}(j-\alpha)\int_{-\infty}^{\alpha_s(t)}\frac{d\lambda}{\lambda-\alpha-i\epsilon}\,\frac{[(\alpha_s-\lambda)/c]^{j+1/2}K_{\pi\pi}(t,\lambda)}{[(\alpha_s-\lambda+ct_\pi)/c]^{1/2}(j-\lambda)^3}\right\}. \quad (1)$$

Here $\alpha_s(t)=\alpha(t)+c(t-t_\pi)$, $c>0$ and $t_\pi=4m_\pi^2$, $\alpha(t)=1+\alpha' t$. The functions $\Phi(t,j)$ and $K_{\pi\pi}(t,\lambda)$ are regular at $t=t_\pi$, and the latter is defined so that $K_{\pi\pi}(t,\alpha(t))=1$. Expression (1) satisfies the elastic t-channel unitarity relation.

For phenomenological purposes we must consider amplitudes of the form (1) in the context of coupled-channel unitarity. This has been done in Ref. 3, where it was shown that a simple factorizable solution to the unitarity equations can be found in the neighborhood of $j\sim\alpha(t)$ provided amplitudes in all channels possess a shielding cut. In our model we make a more general ansatz and write the relevant spin-averaged positive-signatured amplitudes in the form

$$(t-t_\pi)^{-j}F_a(t,j)=\frac{f_a(t,j)}{(j-\alpha)^2}\left\{\Phi(t,j)-\frac{1}{\pi}(j-\alpha)\int_{-\infty}^{\alpha_s(t)}d\lambda\,\frac{[(\alpha_s-\lambda)/c]^{j+1/2}}{[(\alpha_s-\lambda+ct_\pi)/c]^{1/2}}\,\frac{K_a(t,\lambda)}{\lambda-\alpha-i\epsilon}\,\frac{1}{(j-\lambda)^3}\right\}^{-1}, \quad (2)$$

where, for example, $a=NN, \pi N, KN$ for nucleon-nucleon, pion-nucleon, and kaon-nucleon scattering, respectively.

Unitarity equations then require that $f_{\pi\pi}(t,j)\equiv 1$ and $K_{\pi\pi}(t,\lambda)=K_{\pi N}(t,\lambda)=K_{\pi\pi}(t,\lambda)\equiv K(t,\lambda)$ while the remaining functions K_a are left arbitrary up to the requirement that $K_a(t,\alpha(t))=1$. It also follows from the multichannel unitarity equations that

$$\lim_{j\to\alpha} f_{\pi N}^2=\lim_{j\to\alpha} f_{NN}\,,\qquad \lim_{j\to\alpha} f_{\pi N}f_{\pi K}=\lim_{j\to\alpha} f_{KN}\,, \quad (3a)$$

and

$$\lim_{j\to\alpha_s} f_{\pi N}^2=\lim_{j\to\alpha_s}\left[f_{NN}\left(\frac{1+\gamma_{\pi\pi}ct_\pi}{1+\gamma_{NN}ct_\pi}\right)^2\right],\qquad \lim_{j\to\alpha_s} f_{\pi N}f_{\pi K}=\lim_{j\to\alpha_s}\left[f_{KN}\left(\frac{1+\gamma_{\pi\pi}ct_\pi}{1+\gamma_{KN}ct_\pi}\right)^2\right], \quad (3b)$$

where for definiteness we have assumed that

$$K_a(t,\lambda)=1+\gamma_a(t)[\alpha(t)-\lambda].$$

Thus the generalized residue functions obey the factorization condition (3a) at the pole, and the corresponding condition (3b) at the tip of the cut, $j\to\alpha_s(t)$.

It should be noted that the kinematic factors coming from $P_l(\cos\theta_t)$ in the Sommerfeld-Watson representation are absorbed in the definition of $f_a(t, j)$, so that the same scale of energy appears in all channels.

By construction, the amplitudes in (2) have singularities at $j=\alpha(t)$ and $j=\alpha_s(t)$ whose contributions at high energies are obtained by taking the Sommerfeld-Watson transform. At $t=0$, we find

$$\frac{1}{(cs)}[\mathrm{Im}F_t(s,0)]_a = A_a + B_a(\ln cs) + C_a(cs)^{-ct_\pi}[ct_\pi \ln cs]^{-3/2-ct_\pi}, \tag{4}$$

where

$$A_a = f_a[3\pi^2\varphi_0^{-1}(\tfrac{5}{3}-\varphi_1/\varphi_0)],$$

$$B_a = f_a(3\pi^2\varphi_0^{-1}),$$

$$C_a = \frac{f_a}{(1+\gamma_a ct_\pi)}(\cos^2\tfrac{1}{2}\pi ct_\pi - \sin\tfrac{1}{2}\pi ct_\pi)4^{1-ct_\pi} \times B(3,\tfrac{1}{2}-ct_\pi)\frac{\Gamma(\frac{5}{2}-ct_\pi)}{\Gamma(2-ct_\pi)}.$$

Here we have defined

$$\Phi(0,j) = \frac{1}{\pi c^j}[\varphi_0 + \varphi_0(j-1)$$

and assumed that only the tip of the cut contributes at high energies. In order to compare our model also with the low-energy ($15 \lesssim P_L \lesssim 40$ GeV/c) data, our final results include secondary terms which are <10% of the primary cut contribution; the inclusion of these terms has an insignificant effect on the parameters A_a, B_a, and C_a. It is interesting to note that the secondary terms become successively less important in going from pp to πp to Kp, which is also the trend observed in the primary term.

From (4) we see that c sets the common scale of energy in all channels, and hence we set $c=1$ GeV^{-2}. This also means that the intercept of the cut is close to unity ($ct_\pi \approx 0.08$). Taking account of Regge poles with $\alpha_R(0)\approx 0.5$, the total cross sections are parameterized by

$$\sigma_a = A_a + B_a \ln s + C_a s^{-ct_\pi}(ct_\pi \ln s)^{-3/2-ct_\pi} + A_{R_a} s^{\alpha_{R_a}-1} \quad (a = pp, \pi p, Kp), \tag{5}$$

and if we assume an exponential t dependence for the generalized residue functions, the amplitudes near $t=0$ are given by

$$\frac{1}{s}\mathrm{Im}F_a(s,t) = [(A_a + B_a \ln s)s^{\alpha(t)-1}e^{-b_a|t|} + C_a s^{\alpha_s(t)-1}(ct_\pi \ln s)^{\alpha_s(t)-5/2}e^{-b_a'|t|} + A_{R_a}s^{\alpha_{R_a}(t)-1}e^{-b_{R_a}|t|}]. \tag{6}$$

The slope parameters are then obtained (assuming $|\mathrm{Re}F/\mathrm{Im}F| \ll 1$) from

$$\tfrac{1}{2}b(s,t) = \left|\frac{d}{dt}\ln(\mathrm{Im}F)\right|.$$

In Eqs. (5) and (6) the parameters of the leading singularity, A_a and B_a, are constrained by factorization: $(A,B)_a = \epsilon_a(A,B)_{pp}$, with $\epsilon_a \equiv f_a/f_{pp}$, deter-

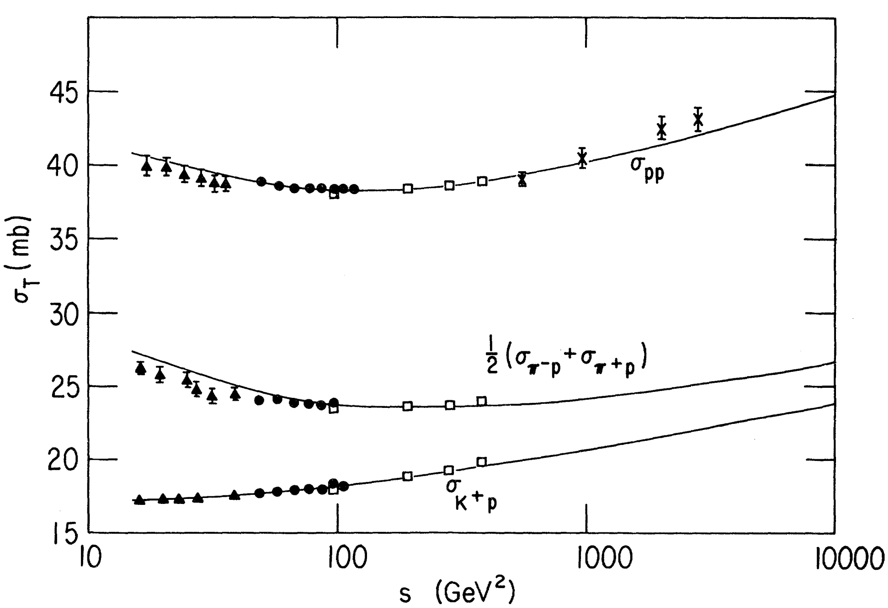

FIG. 1. Fits to total cross sections (see Ref. 4). (i) σ_{pp} : ▲ Foley (67), ● Denisov, □ Baker, × Amaldi. (ii) $\frac{1}{2}(\sigma_{\pi^-p}+\sigma_{\pi^+p})$: ▲ Foley (67), ● Denisov, □ Baker. (iii) σ_{K^+p} : ▲ Galbraith, ● Denisov, □ Baker.

mined phenomenologically. [The f's are defined in Eq. (2).]

The results of fits to the data[4] on $\sigma_{pp}^{\text{total}}$, $\sigma_{K^+p}^{\text{total}}$ and $\frac{1}{2}(\sigma_{\pi^-p}^{\text{total}}+\sigma_{\pi^+p}^{\text{total}})$ above $P_L \approx 20$ GeV/c are shown in Fig. 1, and in Table I we give the parameters. In Fig. 2 we compare with experiments,[5] our prediction for $\alpha_{pp} \equiv \text{Re}F_{pp}(s,0)/\text{Im}F_{pp}(s,0)$. In all our fits it was tacitly assumed that differences of total cross sections are adequately described by ω and ρ exchanges.[6]

The present analysis in terms of the Pomeron and its associated shielding cut points to the following systematic features of the data.[4,5,7]

(i) Exchange degeneracy, which was invoked at lower energies to account for the absence of a falling contribution in the exotic channels, continues to be a stable feature of the data: $A_{R_X} \approx 0$ for $X = p, K^+$. Moreover, our determination of $A_{\pi p}^f$ is consistent with f-ρ exchange degeneracy in πp scattering.

(ii) From Table I we see that the requirement of factorization for the leading singularity is satisfied quite well by the data, and $(A,B)_{Kp}/(A,B)_{\pi p} = \epsilon_{Kp}/\epsilon_{\pi p} \approx 1$ indicates that this singularity may well be an SU(3) singlet.

On the other hand, the contribution of the shielding cut is considerably suppressed in Kp scattering relative to πp scattering: $C_{Kp}/C_{\pi p} \approx \frac{1}{3}$. The shielding cut thus possesses a nonsinglet SU(3) structure, which is plausible in view of its intimate association with the 2π threshold in the present model. Indeed it is possible to obtain the magnitude of the suppression by making an octet-dominance model at an effective particle–particle–shielding-cut vertex if the strength of the cut is determined by coupling via 2π intermediate states. Observe that the nonsinglet character of the shielding cut reflects an important feature of the data, namely, the precocious dominance of the leading singularity in K^+p scattering, as manifested in the early rise of the total cross section and the early onset of logarithmic shrinkage of the slope parameter.

TABLE I. Parameters determined from fits to total cross sections $P_L \gtrsim 20$ GeV/c. See Eq. (5).

a	A_a	B_a	ϵ_a	C_a	A_a^R	A_a^R from exchange degeneracy
pp	18.56	2.64	1.0	2.16	0	0
πp	10.95	1.56	0.59	1.27	14.42	16.2
Kp	10.21	1.45	0.55	0.39	−0.05	0

Unlike many other models[8] which ascribe a nonsinglet structure to the singularity at $(t, j) = (0, 1)$, the present model retains the SU(3) singlet character of this singularity while exploiting the nonsinglet properties of a nearby singularity $[\alpha_s(0) \approx 0.92]$. A similar approach has been advocated by Lipkin.[9]

(iii) The large f contribution in π-p scattering is relevant to the nonshrinking diffraction peak in that channel. Similar effects of secondaries

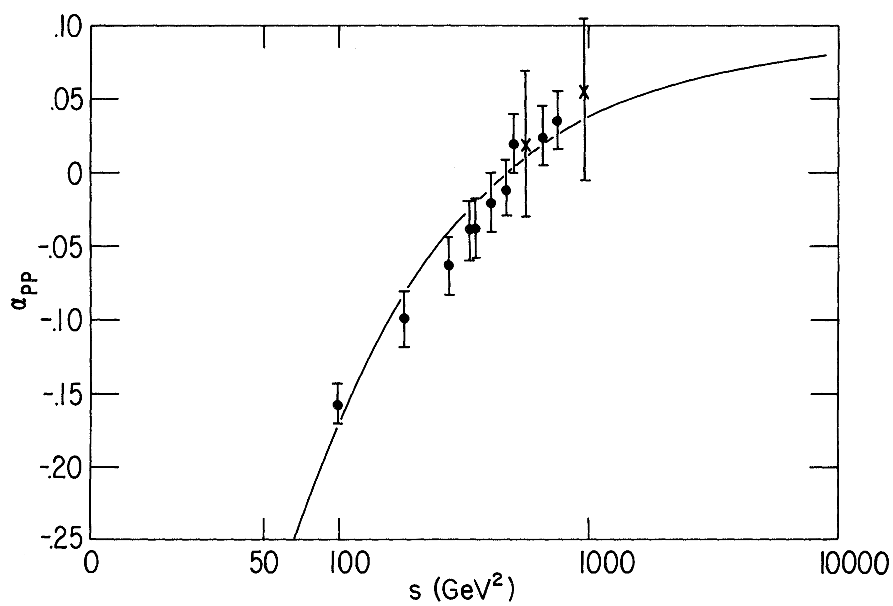

FIG. 2. Prediction for $\alpha_{pp} = \text{Re}F_{pp}/\text{Im}F_{pp}$. Data from Baretnev *et al.* (Ref. 5) (●) and Amaldi *et al.* (Ref. 5) (×).

are observed in K^-p and $\bar{p}p$ scattering, a quantitative analysis of which will be presented elsewhere. It is interesting to note that there are two unambiguous methods of determining the f contribution in π-p scattering—exchange degeneracy, and SU(3)—and both predict a large value.

(iv) In pp scattering the model predicts a change of slope in the t distribution around $t=-0.15$ GeV^2, the magnitude of which decreases slowly with energy. Insofar as this phenomenon depends on the size of the (positive) cut contribution, it appears that, at comparable energies, the K^+p channel will not exhibit a similar change. We believe that definitive statements on this phenomenon at accessible energies can only be made for the two exotic processes, since large secondary contributions would mask this tiny effect in other channels.

It is a pleasure to thank Professor R. Oehme for his continuing guidance and support.

---

*Work supported in part by the National Science Foundation under Contract No. MPS74-08833.

[1]R. Oehme, Phys. Rev. D 11, 1191 (1975).

[2]R. Oehme, Phys. Rev. D 9, 2695 (1974).

[3]R. Oehme and S. Paranjape, Phys. Rev. D 10, 2229 (1974).

[4]Particle Data Group, University of California Report No. UCRL-20000NN, 1970 (unpublished); CERN Report No. CERN/HERA 72-2, 1972 (unpublished); LBL Report No. LBL-63, 1973 (unpublished); S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); W. F. Baker *et al.* (Brookhaven-Fermilab-Rockefeller Collaboration), report presented at the XVII International Conference on High Energy Physics, report, 1975 (unpublished); U. Amaldi *et al.*, Phys. Lett. 44B, 112 (1973); S. R. Amendolia *et al.*, *ibid.* 44B, 119 (1973).

[5]O. V. Dumbrais, Yad. Fiz. 13, 1096 (1971) [Sov. J. Nucl. Phys. 13, 626 (1971)]; V. Bartenev *et al.*, Phys. Rev. Lett. 31, 1367 (1973); U. Amaldi *et al.*, Phys. Lett. 43B, 231 (1973).

[6]See, for example, R. E. Hendrick *et al.*, Phys. Rev. D 11, 536 (1975).

[7]V. D. Bartenev *et al.*, Phys. Rev. Lett. 31, 1088 (1973); G. Barbiellini *et al.*, Phys. Lett. B39, 663 (1972); B35, 355 (1972).

[8]R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. D 4, 3439 (1971); J. Pumplin and G. L. Kane, *ibid.* 11, 1183 (1975).

[9]H. J. Lipkin, Phys. Lett. 56B, 76 (1975).