# Dynamical realization of breaking of exchange degeneracy and violation of the Okubo-Zweig-Iizuka rule

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(Received 11 August 1975)

A multiperipheral model that implements part of the Veneziano topological expansion and in which the cylinder correction breaks exchange degeneracy and produces violation of the Okubo-Zweig-Iizuka (OZI) rule as well as aPomeronabove one is presented. In the negative charge conjugation channel the model is identical to the $O$ meson model of Freund and Nambu if continued to a spin-one pole on the daughter of the Pomeron.

## I. INTRODUCTION

Veneziano[1] has proposed a topological perturbation expansion for unitarization of the dual resonance model. The zeroth-order contribution to the expansion is the sum of all planar unitarity dual diagrams. This term is supposed to produce a set of exchange-degenerate trajectories, and no cuts or exotic trajectories. The first-order unitarity contribution has the topology of a cylinder and produces singularities in both charge conjugation plus and minus channels ($C=\pm$) and breaks exchange degeneracy. The cylinder is said to be associated with the Pomeron in the $C=+$ channel and expected to be above all other singularities in the problem. The singularity in the $C=-$ channel has been associated with the daughter of the Pomeron (PD) by Freund and Nambu[2] (FN), and they have shown that it can account for violations of the Okubo-Zweig-Iizuka[3] (OZI) rule in that channel.

Several groups have attempted to replace dual dynamics with the multiperipheral model, maintaining the internal symmetries, and approximately carry out the Veneziano program for the elastic amplitude. In a detailed treatment, Schmid and Sorensen[4] showed that such models must have only one output pole in each channel and hence cannot have a distinct $P$ and $f$ trajectory. Rosenzweig and Chew[5] (RC) take this single pole in their multiperipheral model to be a Pomeron with intercept 0.85 as suggested in Ref. 6, and are thus able to relate the breaking of the OZI rule to the breaking of exchange degeneracy.

We feel, however, that the high-energy data require a distinct Pomeron trajectory with $\alpha_P(0)$ being above 1.0. (This would be the bare Pomeron in a Gribov Reggon calculus.) Furthermore, the existence of a distinct Pomeron probably implies a distinct PD, thus allowing us to interpret our results in terms of the calculation of the OZI rule violation of FN and Ref. 7.

To motivate our point of view we will outline using multiperipheral ideas how successive twisted links might be summed, although we believe that the dynamics must actually be different from the multiperipheral Fredholm equation. Consider the multiperipheral model for the production amplitude shown in Fig. 1(a) where each exchange is either twisted or planar. After squaring the amplitude, the basic ingredients from which any contribution to the amplitude is constructed are [see Fig. 1(b)] $P$, a pair of planar exchanges, $T$, a pair of twisted exchanges, and $K$, the produced particle. Arguments concerning the absence of interferences between the planar and twisted contribution are given in Ref. 4. The identity of quark lines is handled by matrix techniques. In standard multiperipheral models, $K$ would be expected to be $J$-independent, and $P$ and $T$ would be expected to have similar $J$ dependence. If we strip off the external particles, then the imaginary part of the two-to-two amplitude is given by an integral equation of the form

$$A=[x][K+K(P+T)K+K(P+T)K(P+T)K+\cdots][x] +[x'], \tag{1}$$

where $[x]$ and $[x']$ indicate the way the internal propagator couples to the external particle. There are several possible choices, each of which corresponds to fundamentally different models and has different phenomenological implications. To parallel Ref. 2 with its connection to the OZI rule we take

$$[x]=[x']=P. \tag{2}$$

Following Veneziano[1] we sum just the planar terms to obtain an amplitude $A_p$ given by

$$A_p=P+PKP+PKPKP+\cdots . \tag{3}$$

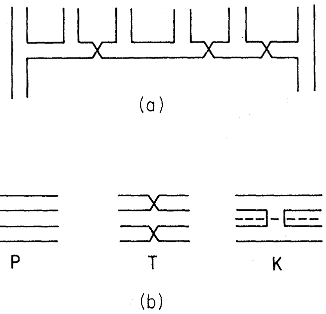


FIG. 1. (a) A dual production amplitude. (b) Ingredients of the multiperipheral model.

In the Veneziano program $A_p$ is expected to have exchange-degenerate poles, $\rho$, $\omega$, $A_2$, and $f$ (we will put the $f'$ and $\phi$ exchange-degenerate, but at lower intercept because of the expected heavier mass for the strange quarks and no cuts or exotic particle trajectories.

Next we must consider the implication of the cylinder corrections, which we will designate by $A_T$, and which results from terms which only contain twisted contributions

$$A_T = T + TKT + TKTKT + \cdots \ . \tag{4}$$

The full amplitude $A$ can now be written in terms of the planar contribution $A_P$ and the cylinder corrections $A_T$

$$A = A_p + A_p K A_T K A_p + A_p K A_T K A_p K A_T K A_p + \cdots \ . \tag{5}$$

As we stated earlier, the detailed construction of $A_p$ and $A_T$ goes beyond the multiperipheral model; however, at this stage of development the application of the multiperipheral model may be justifiable, and if so $A$ will be the solution of the multiperipheral Eq. (5), and thus we find

$$A = (1 - A_p K A_T K)^{-1} A_p \ . \tag{6}$$

Notice that this would be identical to the results of RC if $KA_TK$ were replaced by their $J$-independent cylinder correction $C$. In our approach, however, we will assume that $A_T$ has $J$-dependent singularities.[8] We feel that it is very reasonable to assume that in the $C=+$ channel, $A_T$ contains a high-lying pole since the entire kernel $TK$ gets concentrated into the vacuum channel. In the $C=-$ channel, the singularity will be lower as in Ref. 4 and will be associated with the PD as was suggested by FN.

## II. MODEL

We will work here with an SU(3) model with our channels corresponding to the quark-antiquark states $\mathcal{P}\bar{\mathcal{P}}$, $\mathfrak{N}\bar{\mathfrak{N}}$, and $\lambda\bar{\lambda}$, respectively. [The extension to SU(4) is straightforward and will be presented elsewhere.] The amplitude $A_p$ will be taken to be

$$A_p = \begin{pmatrix} (J-\alpha_0)^{-1} & 0 & 0 \\ 0 & (J-\alpha_0)^{-1} & 0 \\ 0 & 0 & (J-\alpha_3)^{-1} \end{pmatrix} , \tag{7}$$

where $\alpha_0(0) > \alpha_3(0)$ as a reflection of the heavier mass of the $\lambda$ quarks. The kernel of our problem $KA_TK$ which represents the cylinder correction in the $C=+$ or $C=-$ channel will be taken to be completely SU(3)-symmetric, and given by

$$(KA_TK)_\pm = \frac{k_\pm^2}{J-\alpha_4^\pm} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{8}$$

We take $\alpha_4^+ - \alpha_4^- = 1$ in accordance with our expectation of the daughter structure of the cylinder. We should note, however, that the $J$-plane structure in the $C=-$ channel may in fact be more complicated than a simple pole. In multiperipheral models, one would expect a variety of cuts in the region of $\alpha_4^-(0)$ which could be important. For our purposes it is sufficient to say that the PD is the important structure in $(KA_TK)_-$, which we approximate by a pole.

We can partially diagonalize the solution of our model by rotating to a basis corresponding to particles of an ideally mixed multiplet

$$\mu A^\pm \mu^{-1} = A^{-\pm} = \{\mu[1 - A_p (KA_TK)_\pm]\mu^{-1}\}^{-1} A_p \ , \tag{9}$$

where

$$\mu = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

This separates the problem into the isospin-one and -zero components,

$$\bar{A}^\pm = \begin{pmatrix} A_1^\pm & 0 \\ 0 & A_0^\pm \end{pmatrix} , \tag{10}$$

where

$$A_1^\pm = \frac{1}{J-\alpha_0} \tag{11}$$

and

$$A_0^{\pm}=\frac{\begin{pmatrix}(J-\alpha_4^{\pm})(J-\alpha_3)-k_{\pm}^2 & \sqrt{2}k_{\pm}^2\\ \sqrt{2}k_{\pm}^2 & (J-\alpha_4^{\pm})(J-\alpha_0)-2k_{\pm}^2\end{pmatrix}}{(J-\alpha_4^{\pm})(J-\alpha_3)(J-\alpha_0)-k_{\pm}^2(J-\alpha_0)-2k_{\pm}^2(J-\alpha_3)}. \quad (12)$$

The isospin-one exchanges ($\rho$ and $A_2$) will thus remain uncharged at $\alpha_0$, as found by RC, and $\omega$ and $f$ trajectories will split away from them by an amount and in a direction that depends on the parameters $k_{\pm}$. Similarly the $\phi$, $f'$, Pomeron, and PD will move to new intercepts which depend on $k_{\pm}$. The new trajectories will be called $\alpha_\omega$, $\alpha_f$, $\alpha_\phi$, $\alpha_f'$, $\alpha_P$, and $\alpha_{\rm PD}$ respectively and are the solution of the cubic denominator of $A_0^{\pm}$. It is straightforward to write out $A^{+(-)}$ in terms of the new poles:

$$[A^{+(-)}]_{ij}=\frac{g_i^{P(\rm PD)}g_j^{P(\rm PD)}}{J-\alpha_{P(\rm PD)}}+\frac{g_i^{f(\omega)}g_j^{f(\omega)}}{J-\alpha_{f(\omega)}}+\frac{g_i^{f'(\phi)}g_j^{f'(\phi)}}{J-\alpha_{f'(\phi)}}. \quad (13)$$

The couplings $g_i^x$ represent the coupling between the original pure states (i) (that are ideally mixed) and the new mixed states ($x$) that we have generated via our twist operator $KA_TK$. Thus

$$\begin{bmatrix}g_{f(\omega)}^{P(\rm PD)}\\ g_{f'(\phi)}^{P(\rm PD)}\end{bmatrix}=\begin{bmatrix}\left(\dfrac{(\alpha_{P(\rm PD)}-\alpha_4^{+(-)})(\alpha_{P(\rm PD)}-\alpha_3)-k_{+(-)}^2}{(\alpha_{P(\rm PD)}-\alpha_{f(\omega)})(\alpha_{P(\rm PD)}-\alpha_{f'(\phi)})}\right)^{1/2}\\ \left(\dfrac{(\alpha_{P(\rm PD)}-\alpha_4^{+(-)})(\alpha_{P(\rm PD)}-\alpha_0)-2k_{+(-)}^2}{(\alpha_{P(\rm PD)}-\alpha_{f(\omega)})(\alpha_{P(\rm PD)}-\alpha_{f'(\phi)})}\right)^{1/2}\end{bmatrix}, \quad (14)$$

$$\begin{bmatrix}g_{f(\omega)}^{f(\omega)}\\ g_{f'(\phi)}^{f(\omega)}\end{bmatrix}=\begin{bmatrix}\left(\dfrac{(\alpha_{f(\omega)}-\alpha_4^{+(-)})(\alpha_{f(\omega)}-\alpha_3)-k_{+(-)}^2}{(\alpha_{f(\omega)}-\alpha_{P(\rm PD)})(\alpha_{f(\omega)}-\alpha_{f'(\phi)})}\right)^{1/2}\\ \left(\dfrac{(\alpha_{f(\omega)}-\alpha_4^{+(-)})(\alpha_{f(\omega)}-\alpha_0)-2k_{+(-)}^2}{(\alpha_{f(\omega)}-\alpha_{P(\rm PD)})(\alpha_{f(\omega)}-\alpha_{f'(\phi)})}\right)^{1/2}\end{bmatrix}, \quad (15)$$

$$\begin{bmatrix}g_{f(\omega)}^{f'(\phi)}\\ g_{f'(\phi)}^{f'(\phi)}\end{bmatrix}=\begin{bmatrix}\left(\dfrac{(\alpha_{f'(\phi)}-\alpha_4^{+(-)})(\alpha_{f'(\phi)}-\alpha_3)-k_{+(-)}^2}{(\alpha_{f'(\phi)}-\alpha_{P(\rm PD)})(\alpha_{f'(\phi)}-\alpha_{f(\omega)})}\right)^{1/2}\\ \left(\dfrac{(\alpha_{f'(\phi)}-\alpha_4^{+(-)})(\alpha_{f'(\phi)}-\alpha_0)-2k_{+(-)}^2}{(\alpha_{f'(\phi)}-\alpha_{P(\rm PD)})(\alpha_{f'(\phi)}-\alpha_{f(\omega)})}\right)^{1/2}\end{bmatrix}. \quad (16)$$

We see that the pure (ideally mixed) states can communicate via these mixed states. For example, an external $\phi$ state (which we take as ideally mixed) can become an external $\omega$ state via $g_\phi^\phi g_\omega^\phi$. This of course leads to a violation of the OZI rule. To compare our model of the OZI-rule violation with that of FN we continue our $C=-$ amplitude to the pole at $J=1$:

$$A_0^-=-\frac{1}{\alpha'}\,\frac{\begin{pmatrix}(t-m_4^2)(t-m_3^2)-k_-^2/\alpha'\alpha_4' & \sqrt{2}k_-^2/\alpha'\alpha_4'\\ \sqrt{2}k_-^2/\alpha'\alpha_4' & (t-m_4^2)(t-m_0^2)-k_-^2/\alpha'\alpha_4'\end{pmatrix}}{(t-m_4^2)(t-m_3^2)(t-m_0^2)-(k_-^2/\alpha'\alpha_4')(t-m_0^2)-(2k_-^2/\alpha'\alpha_4')(t-m_3^2)}, \quad (17)$$

where

$$\alpha_4^-=1-\alpha_4' m_4^2+\alpha_4' t,\quad \alpha_{0(3)}=1-\alpha' m_{0(3)}^2+\alpha' t.$$

The resulting amplitude is identical to that obtained from the mass matrix of FN. The parameter $f^2$ of FN is related to $k_-^2$ by

$$f^2=k_-^2/\alpha'\alpha_4'. \quad (18)$$

A direct comparison of these parameters of course requires a continuation from $t>0$ to $t=0$.

The parameters of our model are chosen to agree with existing data. We have not attempted to make a systematic $\chi^2$ fit to data, but we have made adjustments to give reasonable fits. Briefly, the way we have chosen the parameters is as follows. $\alpha_0(0)$ was chosen to agree with fits to the $\rho$ trajectory of Ref. 9. $\alpha_P=1.06$ was chosen to agree with the rise of the proton-proton total cross section. $\alpha_4^+(0)$ and $k_+$ are determined from $\sigma_T^{pp}+\sigma_T^{p\bar p}$. $\alpha_4^-$ is taken to be $\alpha_4^+-1$, and $\alpha_\omega(0)$ and $k_-$ are determined from $\sigma_T^{p\bar p}-\sigma_T^{pp}$. The model is relatively insensitive to $\alpha_3$. The parameters are thus determined to be

$$\begin{aligned}&\alpha_4^+=0.85, && \alpha_P=1.06, && k_+^2=0.04252,\\ &\alpha_4^-=-0.15, && \alpha_{\rm PD}=0.004, && k_-^2=-0.01729,\\ &\alpha_0=0.53=\alpha_\rho=\alpha_{A_2}, && \alpha_\omega=0.48, && \alpha_f=0.40115,\\ &\alpha_3=0.2, && \alpha_\phi=0.096, && \alpha_{f'}=0.11885,\end{aligned} \quad (19)$$

The expressions for $\sigma_T^{pp}(J)+\sigma_T^{p\bar p}(J)$ and $\sigma_T^{p\bar p}(J)-\sigma_T^{pp}(J)$ are

$$\sigma_T^{pp}(J)+\sigma_T^{p\bar p}(J)=2\gamma_f^2\left[\frac{(g_f^p)^2}{J-\alpha_p}+\frac{(g_f^f)^2}{J-\alpha_f}+\frac{\frac{1}{9}}{J-\alpha_0}\right] \quad (20)$$

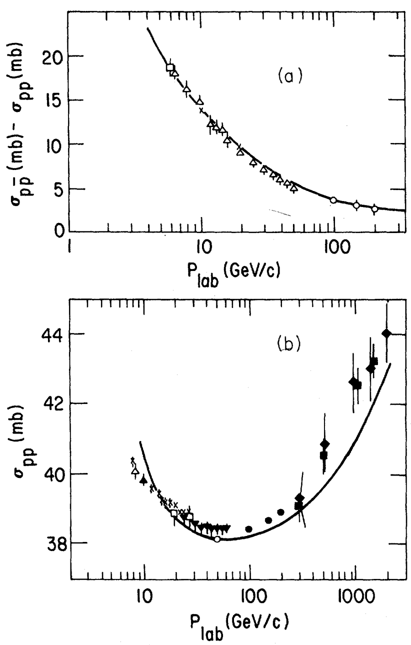


FIG. 2. (a) Fit to $\sigma_T^{p\bar p} - \sigma_T^{pp}$ data. References: □: D. V. Bugg *et al.*, Phys. Rev. 146, 980 (1968); R. T. Abrams *et al.*, Phys. Rev. D 1, 1917 (1970); X: W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965); △: S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); 36B, 528 (1971); Nucl. Phys. B65, 1 (1973); ○: A. S. Carroll *et al.*, Phys. Rev. Lett. 33, 928 (1974). (b) Fit to $\sigma_T^{pp}$ data. References. ◆: S. R. Amendolia *et al.*, Phys. Lett. 44B, 119 (1973); ■: U. Amaldi *et al.*, *ibid.* 44B, 112 (1973); ▼: S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); ○: A. S. Carroll *et al.*, Phys. Rev. Lett. 33, 928 (1974); X: K. J. Foley *et al.*, *ibid.* 19, 330 (1971); □: G. Bellettini *et al.*, Phys. Rev. 146, 980 (1968); △: J. Ginestet *et al.*, Nucl. Phys. B13, 283 (1969).

and

$$\sigma_T^{p\bar p}(J) - \sigma_T^{pp}(J) = 2\gamma_\omega{}^2\left[\frac{(g_\omega^\omega)^2}{J-\alpha_\omega} + \frac{\frac{1}{9}}{J-\alpha_0} + \frac{(g_\omega^\phi)^2}{J-\alpha_\phi}\right], \qquad (21)$$

where $\gamma_{f(\omega)}$ is the coupling of the proton to the ideally mixed $f(\omega)$ channel. We have used the standard relation that $\gamma_{A_2(\rho)} = \frac{1}{3}\gamma_{f(\omega)}$.

The fits to $\sigma_T^{\bar pp} - \sigma_T^{pp}$ and $\sigma_T^{pp}$ are shown in Figs. 2(a) and 2(b). The splitting of the $\alpha_\rho$ and $\alpha_\omega$ trajectories that we obtain appears consistent with other determinations.[9] The breaking of the OZI rule in the $C=-$ channel is determined by $k_-{}^2$. Good fits to direct-channel data have been obtained in Ref. 2 and Ref. 7 with $|f^2| = 0.0256$ to 0.0501 which implies via Eq. (18) that $|k_-{}^2| = 0.0128$ to 0.0167, which is in reasonable agreement with the magnitude of $k_-{}^2$ that we obtain for $t \leq 0$. It is important to note, however, that we find $k_-{}^2 < 0$. While this will not affect any of the predictions for the OZI rule violation, it will make the particle on the PD trajectory a ghost. This lends strength to the argument that the PD is in reality a cut which has been approximated by a pole. In the continuation from $t > 0$ to $t \leq 0$, the parameters could, however, change sign and be consistent with the pole interpretation of the PD.

We have seen that an intermediate step in the Veneziano program can be carried out consistently in the multiperipheral approximation. The calculation gives rise to a relation between the breaking of exchange degeneracy and the violation of the OZI rule that is consistent with data. However, the initial steps of constructing $A_p$ and $A_T$, we believe, require a more sophisticated model, which incorporates the underlying dual structure. (See Ref. 10 and references therein for further discussion of this point.) We have also not attempted to discuss more complicated topologies than the cylinder. It may be that these are best discussed within the context of a Reggeon field theory.


## ACKNOWLEDGMENTS

The authors would like to acknowledge the hospitality of the High Energy Physics Division at Argonne National Laboratory, where the bulk of this work was carried out.



*Work supported in part by the U. S. Energy Research and Development Administration.

†Work supported in part by the National Science Foundation.



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