PHYSICAL REVIEW D VOLUME 13, NUMBER 5 1 MARCH 1976

Direct lepton production and the Drell-Yan mechanism*

J. D. Bjorken
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

H. Weisberg
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19174
(Received 21 August 1975)

We suggest that the Drell-Yan mechanism, when applied to low-mass dilepton production, underestimates the yield by a factor ~ 25 because it neglects the contribution coming from annihilation of produced parton-antiparton pairs. If this is correct, the observed direct lepton production may be accounted for by electromagnetic production of low-mass dileptons possessing high total transverse momentum.

Considerable evidence exists[1-6] that, despite some indications to the contrary,[7-11] the direct lepton production observed in hadron collisions satisfies the following properties[12]:

(i) At least at $\theta_{\text{C.M.}} \sim 90°$, $e/\pi \approx \mu/\pi \sim 10^{-4}$, independent of $p_\perp$ ($1-1.5 < p_\perp < 6$ GeV), of atomic number of the target, and of s ($5 \lesssim \sqrt{s} \lesssim 60$ GeV).

(ii) In the forward direction, the mass spectrum of directly produced dimuons peaks at low values, less than 1 GeV.

(iii) Decays of vector mesons ρ, ω, ϕ into lepton pairs cannot account for even half the observed yield.

This strongly suggests that the origin of the dileptons is electromagnetic: A virtual photon is produced in the collision which then converts (internally) into a lepton pair. This may either be via bremsstrahlung of a virtual photon from some charged constituent present during the collision, or via annihilation of such a constituent with its antiparticle. However, both mechanisms have their difficulties. Bremsstrahlung mechanisms have in general no way of suppressing real photon emission, as well as virtual photons. And the relationship between real and virtual emission is governed by the usual Kroll-Wada type of factor for internal conversion. Experimentally the required amount of internal conversion is not present, as determined in the experiments of CERN-Columbia-Rockefeller-Saclay[3] and Pennsylvania-Stony Brook,[5] which reject leptons that are members of low-mass pairs. Furthermore, the bremsstrahlung mechanism implies a dilepton mass distribution $dN/dm \sim 1/m$ for small m. This leads to the number of leptons produced being proportional to $\log(m_{\max}/m_i)$. The e/μ ratio would then be $\sim [\log(m_{\max}/m_e)]/\log(m_{\max}/m_\mu)]$, and for $m_{\max} \lesssim 1$ GeV one should have $e/\mu \gtrsim \log 10^3/\log 10 = 3$, quite inconsistent with the observations. Therefore, *at least for* $\theta_{\text{C.M.}} \sim 90°$, bremsstrahlung mechanisms do not easily explain the data.

This leaves the annihilation of constituents as the leading candidate for an electromagnetic mechanism. Provided the effective mass or mean momenta of the constituents exceed the muon mass (an eminently reasonable assumption) the minimum virtual photon mass will exceed $2m_\mu$, and there is no problem with the e/μ ratio. The problem is only of rate. Annihilation calculations have generally used the Drell-Yan parton-antiparton annihilation mechanism,[13] and the results have been a lepton inclusive spectrum with the wrong $p_\perp$ dependence, as well as a total dilepton yield much too small. However, the emphasis in the theoretical considerations has been production of massive lepton pairs. It has been recognized for a long time, and especially by Landshoff and Polkinghorne,[14] that the Drell-Yan formula needs to be corrected when the dilepton mass is small. The problem is how to estimate the modification and to ascertain that the observed dilepton yield is "reasonable."[15] This is the purpose here. Our strategy is to first estimate from the inclusive lepton spectra the total dilepton production cross section (per unit rapidity) $d\sigma/dy$. Having determined the experimental yield, we then estimate $d\sigma/dy$ from theory, first from an unmodified Drell-Yan formula, and then with a modification which takes into account the presence of the many *wee* partons and antipartons *produced* in the collision process and which may reannihilate into leptons before the products of the collision emerge. While the unmodified Drell-Yan formula fails by nearly two orders of magnitude, we recover a factor 20-30 by using the annihilation into leptons of *produced* partons and antipartons. This brings theory and experiment within reach of each other for $d\sigma/dy$.

If this mechanism is in fact correct, it implies that the bulk of the high-$p_\perp$ inclusive lepton yield is make up of low-mass dileptons produced at high total transverse momentum.

ESTIMATE OF TOTAL DILEPTON PRODUCTION

At large $p_\perp$, the e^-/π^- ratio is, within 20–30%, 10^{-4}. At large s, and $\theta_{\rm C.M.}=90°$, the integrated yield of π^- is, for pp collisions and per unit of rapidity,

$$\left(\frac{d\sigma}{dy}\right)_{\pi^-} \approx 2\times 10^{-26}\ \text{cm}^2 . \tag{1}$$

We assume $e^-/\pi^- \leqslant 10^{-4}$ *for all* $p_\perp$, and thus

$$\left(\frac{d\sigma}{dy}\right)_{\text{direct } e^-} \leqslant 2\times 10^{-30}\ \text{cm}^2 . \tag{2}$$

However, for $p_\perp < 1$ GeV, the e^-/π^- ratio may well decrease; if the minimum dilepton mass is of order M perhaps

$$\left(\frac{d\sigma}{dy}\right)_{\text{direct } e^-} \approx \exp[-b(p_\perp{}^2+M^2)^{1/2}] . \tag{3}$$

With such a parametrization, one obtains upon integration over $p_\perp$

$$\left\langle \frac{e^-}{\pi^-}\right\rangle \approx 10^{-4}(1+bM)e^{-bM} . \tag{4}$$

Experimentally there seems to be little effect of such a cutoff at $p_\perp \sim 1$ GeV. To bound M, we compute the e^-/π^- ratio at smaller $p_\perp$. With $b\sim 6$,

$$\frac{e^-}{\pi^-} \approx 10^{-4}\ \frac{\exp[-6(p_\perp{}^2+M^2)^{1/2}]}{e^{-6p_\perp}} \approx 10^{-4}e^{-3M^2/p_\perp} . \tag{5}$$

Thus we should have $M \leqslant 0.5$ GeV. With $M=0.5$ GeV, we get $\langle e^-/\pi^-\rangle \sim 2\times 10^{-5}$; hence

$$4\times 10^{-31}\ \text{cm}^2 \lesssim \left(\frac{d\sigma}{dy}\right)_{e^+e^-} < 2\times 10^{-30}\ \text{cm}^2 . \tag{6}$$

DRELL-YAN CALCULATION

The standard Drell-Yan formula for production of a dilepton of (large) mass Q and longitudinal momentum P is, at very high c.m.s. energy $\sqrt{s}$,

$$\frac{d\sigma}{dQ^2dP} = (\tfrac{1}{3})^a \int \frac{dx_1}{x_1}\int\frac{dx_2}{x_2} \sum_{i=\mathcal{P},\mathfrak{N},\lambda,\bar{\mathcal{P}},\bar{\mathfrak{N}},\bar{\lambda}} e_i{}^2 f_i(x_1)\bar{f}_i(x_2)\, \frac{4\pi\alpha^2}{3Q^2}\,\delta(Q^2-x_1x_2s)\,\delta\left(P-\frac{\sqrt{s}}{2}(x_1-x_2)\right) . \tag{7}$$

Here x_1 and x_2 are logitudinal fractions of the incident partons i, e_i are their charges, and f_i is the density of parton of type i in rapidity space. The sum is over both partons *and* antipartons. The factor $(\frac{1}{3})^a$ accounts for the possibility of color:

$$a = \begin{cases} 0 & \text{no colored quarks} \\ 1 & \text{colored quarks .}\end{cases}$$

The normalization of f_i is such that for electroproduction

$$\nu W_2 = \sum_{i=p,n,\lambda} e_i{}^2[f_i(x)+\bar{f}_i(x)] . \tag{8}$$

For $\nu W_2 \sim \frac{1}{3}$ and $f_i=\bar{f}_i$ for small x, we get

$$\nu W_2 = (\tfrac{4}{9}+\tfrac{1}{9}+\tfrac{1}{9})+2\langle f\rangle \approx \tfrac{1}{3}$$

or (9)

$$\langle f\rangle \sim \tfrac{1}{4} .$$

Reduction of the Drell-Yan formula gives for large s and for the dilepton produced in the central region ($x_1, x_2 \ll 1$)

$$\frac{d\sigma}{dQ^2dy} = (\tfrac{1}{3})^a\, \tfrac{4}{3}\,\frac{\pi\alpha^2}{Q^4}\,(\tfrac{4}{9}+\tfrac{1}{9}+\tfrac{1}{9})2\langle f\rangle^2 . \tag{10}$$

Putting in the numbers including $\langle f\rangle \sim \frac{1}{4}$ gives

$$\frac{d\sigma}{dy} \sim (\tfrac{1}{3})^a\left(\frac{1\ \text{GeV}^2}{Q^2{}_{\min}}\right)(7\times 10^{-33}\ \text{cm}^2) . \tag{11}$$

Probably $0.1\ \text{GeV}^2 \lesssim (Q^2)_{\min} \lesssim 0.5\ \text{GeV}^2$. Hence

$$(\tfrac{1}{3})^a\,(1.4\times 10^{-32}\ \text{cm}^2) < \left(\frac{d\sigma}{dy}\right)_{\text{Drell-Yan}} < (\tfrac{1}{3})^a\,(7\times 10^{-32}\ \text{cm}^2) . \tag{12}$$

Even ignoring the factor 3 from color, this estimate is a factor ~ 30 too small to account for the observation.

However, the unmodified Drell-Yan calculations is most likely an underestimate. It assumes a quark density $\langle dN_i/dy\rangle \simeq \langle f\rangle \sim \frac{1}{4}$ (per parton type) during the collision. This may be acceptable for leading partons, or for the density as measured by a lepton probe. However, during the hadron-hadron collision, partons and antipartons are *produced:* This is a certainty because we observe the produced hadrons which also must contain partons. We can in fact estimate the number of produced partons from the number of emerging pions. With $dN_\pi/dy \sim 2.5$, we have $dN_q/dy \simeq dN_{\bar{q}}/dy \geqslant 2.5$ and $dN_p/dy \approx dN_n/dy = \langle f\rangle \geqslant \frac{5}{4}$. Therefore, $\langle f\rangle_{\text{enchanced}} = \frac{5}{4} = 5\,\langle f\rangle_0$. With the cross section proportional to $\langle f\rangle^2$, we obtain a very crude estimate of an enhancement by a factor ~ 25 relative to the straight Drell-Yan formula:

$$(\tfrac{1}{3})^a\,(3.5\times10^{-31}\,\mathrm{cm}^2)\lesssim\left(\frac{d\sigma}{dy}\right)_{\substack{\text{Drell-Yan}\\ \text{enhanced}}}\lesssim(\tfrac{1}{3})^a\,(2\times10^{-30}\,\mathrm{cm}^2). \tag{13}$$

This is at worst a factor 3 away from the observations.[16] Therefore, the parton-annihilation mechanism becomes a quite credible interpretation. Additional enhancement factors can be envisaged, including final-state interactions (which increases the time in which the annihilation partons are in proximity) or perhaps clustering of the partons into protohadrons which have integral charge and thus a large annihilation probability. But it is hard to estimate any numbers from such general statements.

It is likewise hard to deduce theoretically the $p_\perp$ dependence of the inclusive lepton yeild from these crude ideas. The only argument in support of constancy with $p_\perp$ of the l/π ratio is that the formation of a low-mass dilepton of given $p_\perp$ and the formation of a meson with the same $p_\perp$ both depend on very similar joint momentum distributions of the partons, and therefore may vary with $p_\perp$ in a similar way. This is in contrast with the usual Drell-Yan calculations where the single-lepton $p_\perp$ dependence comes out to be too flat,[17] yielding an l/π ratio that grows with $p_\perp$ in disagreement with experiment. In these usual calculations the initial partons' transverse momenta are neglected and the single-lepton $p_\perp$ dependence is simply a power law reflecting the power-law distribution of dilepton mass, which in turn depends only on the longitudinal momentum distribution of the incident partons.

CONSEQUENCES

Of first importance is to determine whether the inclusive direct lepton production is of electromagnetic origin. The tests are obvious:

(i) The leptons must come in pairs.

(ii) For a given total momentum of a pair, the angular distribution in the pair rest frame must be consistent with the decay of a spin-one virtual photon (i.e., nearly isotropic).

We also expect a dilepton mass distribution $d\sigma/dQ$ roughly independent of dilepton total momentum, and peaked at small mass (probably below the ρ).[18] For a while $d\sigma/dQ$ should fall faster than Q^{-3} (the Drell-Yan estimate) but eventually should take that form (at sufficiently high s, and for dileptons in the central plateau in rapidity).

At some level, bremsstrahlung of slightly virtual photons should also exist, with a mass spectrum $d\sigma/dQ\sim Q^{-1}$, extending all the way to threshold (~1 MeV for e^-, ~200 MeV for μ^-). However, at least at large c.m.s. angles, this does not seem to be a dominant mechanism, as evidenced by the e/μ ratio of approximatley unity.

ACKNOWLEDGMENT

One of us (J.D.B.) thanks F. Gilman and Minh Duong-van for helpful discussions.

---

*Work supported by U. S. Energy Research and Development Administration.

[1]J. P. Boymond *et al*,. Phys. Rev. Lett. 33, 112 (1974).

[2]J. A. Appel *et al*., Phys. Rev. Lett. 33, 722 (1974).

[3]F. W. Büsser *et al*., Phys. Lett. 53B, 212 (1974).

[4]L. B. Leipuner *et al*., Phys. Rev. Lett. 34, 103 (1975).

[5]E. W. Beier *et al*., paper B-05, submitted to the International Conference on High Energy Physics, Palermo, 1975 (unpublished).

[6]B. Knapp *et al*., Phys. Rev. Lett. 34, 1044 (1975).

[7]R. C. Lamb *et al*., Phys. Rev. Lett. 15, 800 (1965).

[8]R. Burns *et al*., Phys. Rev. Lett. 15, 830 (1965).

[9]S. Nurashev, in *Proceedings of the XVII International Conference on High Energy Physics*, London, 1974, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. V-53.

[10]D. Bintinger et al., Phys. Rev. Lett. 35, 72 (1975).

[11]K. Winter, Phys. Lett. 57B, 479 (1975).

[12]Recent reviews are given by B. Pope, in Proceedings of the International Conference on High Energy Physics, Palermo, Italy, 1975 (unpublished); J. Cronin, lectures given at the 1975 Erice Summer School on High Energy Physics, proceedings, edited by A. Zichichi (unpublished); and L. Lederman, in Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies (unpublished). These reviews make it clear that the experimental evidence for all these properties is far from decisive.

[13]S. D. Drell and T. M. Yan, Phys. Rev. Lett. 25, 316 (1970).

[14]P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B33, 221 (1971).

[15]A new study, with emphasis on the low-mass dilepton production, has also been made by Minh Duong-van (private communication). He obtains a reasonable $p_\perp$ distribution.

[16]Note that if $(Q^2)_{\min}$ is small (so that the upper limit of the estimate is attained) we expect that the characteristic mass M cutting off the lepton inclusive spectrum should likewise be small. This gives the upper limit of the experimental bound on $d\sigma/dy$ as the most reasonable estimate. Therefore, there is considerably less than a factor 5 uncertainty in the comparisons of

theory and experiment. That is, it does not seem reasonable to compare the largest theoretical estimate with the smallest estimate of the experimental yield.

[17]G. R. Farrar, Nucl. Phys. B77, 429 (1974).

[18]A crude estimate of the mean dilepton mass at low $p_\perp$ is attainable from the assumption that the four-momentum of a parton is half that of a produced wee meson. Then

$$\begin{aligned}\langle m^2_{l\bar{l}}\rangle &\gtrsim \tfrac{1}{4}\langle (p_{\pi^+}+p_{\pi^-})^2\rangle \sim \tfrac{1}{2}\langle p_{\pi^+}\rangle\cdot\langle p_{\pi^-}\rangle\\ &\sim \tfrac{1}{2}\langle E_\pi\rangle^2\\ &\sim \tfrac{1}{2}\times\tfrac{3}{2}\langle p_{\perp\pi}\rangle^2\\ &\sim 0.08\ \text{GeV}^2,\end{aligned}$$

which gives $\langle m_{l\bar{l}}\rangle \gtrsim 280$ MeV.