Spectra of new hadrons*

R. L. Jaffe† and J. Kiskis

Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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The cavity approximation to the bag model is used to calculate the masses of the new hadrons in models which explain the ψ resonances in terms of new quark flavors. The charmed-quark scheme and Harari's six-quark scheme are considered in detail. Difficulties associated with gluon interactions and with the interpretation of the ψ' are also discussed.

I. INTRODUCTION

Last year's discovery of narrow, massive resonances in pp [1] and e^+e^- [2,3] collisions has been taken as evidence for additional quark degrees of freedom in hadron dynamics.[4,5] Other interpretations of these resonances and of associated experiments abound[6] and the case for a new type (or types) of quark is not yet persuasive. Nevertheless, the "new-quark" interpretation is economical, consistent with traditional ideas about hadron structure, and even attractive from the standpoint of unified field theories of weak and electromagnetic interactions.[7] In these schemes, the new resonances are thought to be composed of quark, antiquark pairs of hitherto unknown type. The new quarks carry new quantum numbers (e.g., charm or heaviness) which will not be observed directly until new hadrons composed of new quarks in conjunction with one or more ordinary quarks are discovered.

An important feature of most "new-quark" schemes is that the interaction between two new quarks and the interaction between a new quark and an ordinary quark is qualitatively the same as the interaction between two ordinary quarks. We will not be using a weaker quark-gluon coupling, α_c, for the new quarks. In our picture α_c refers to the exchange of gluons with wavelengths determined by the hadronic size. The new-quark bound states are not significantly smaller than typical hadronic sizes. If this is true, and if sufficient information concerning the new quarks (e.g., their masses) can be extracted from the observed states, then it should be possible to predict the masses and other properties of hadrons carrying the new quantum numbers. In this paper, we will carry out this program for the conventional charm scheme,[4] for Harari's six-quark scheme,[5] and for a third, illustrative, model of our own.

The "traditional" quark dynamics will be provided by the bag model[8,9] as developed in Ref. 10. It was shown there that the model does a good job of reproducing the spectrum of the light hadrons. The number of arbitrary parameters is small. These parameters have been fixed in that work and will require no further adjustment here. The masses of the new quarks will be fixed by fitting to the masses of the ψ resonances. [We use the symbol ψ to refer to the set of new resonances and denote individual states as $J=\psi$ (3.1), $\psi'=\psi$ (3.7), $\psi''=\psi$ (4.1).]

The spectrum of new hadrons that results from these calculations is given in Sec. III for the charm model and in Sec. IV for Harari's model. However, in both cases serious difficulty is encountered in grafting the new-quark scheme onto the bag model. In the case of Harari's model, it is difficult to understand the narrow width and the proposed quark composition of the ψ resonances. In the case of the charm model, the bag dynamics (as presently understood) does not accommodate the observed $J-\psi'$ splitting. This is discussed in Sec. VI.

It is important to keep in mind that the new resonances have been observed only in a neutral, nonstrange meson channel. The SU(3)-singlet interactions which can appear in this channel are not very well understood. Consider, for example, the η,η' system. These states appear to deviate considerably from a simple quark-model assignment. Their masses seem to be affected by a strong singlet interaction. The predicted pseudoscalar partners of the ψ's also appear in such a channel. At present we have no way of estimating the effect of the singlet interaction on the masses and composition of these states. Consequently, a determination of a new-quark mass which neglects this effect may not be too reliable. A prediction of the relative position of the vector and pseudoscalar states is even less reliable. Thus, although we expect the masses of the new hadrons to be accurately predicted in terms of the new-quark masses, the determination of the new-quark masses themselves from the available data may not be very accurate. This is discussed in some detail in Sec. V.

Although there are difficulties associated with the new-quark interpretations of the ψ's we feel that it is useful to "suspend our disbelief" for a moment and to determine as accurately as possible the spectrum of new hadrons implied by these models. This should be a useful device for focusing both experimental and theoretical effort.

II. QUARK-GLUON BAG MODEL

In this section, we will briefly outline the quark dynamics which are used in the remainder of the paper. A complete account can be found in Ref. 10. A hadron is taken to be a finite region of space containing quark and/or gluon fields. The field pressure is balanced by a universal, constant pressure B whose origin is not explained by the theory. The dynamics are specified by equations of motion and boundary conditions for each field, and one further, nonlinear boundary condition which balances the pressures.[8] The equations of motion are simply those of colored quarks of arbitrary mass, coupled in the manner of Yang-Mills to eight massless colored vector gluons.

These equations are very difficult to solve in general. Instead, we have solved a similar and simpler model[9] which we expect to be an approximation to the actual bag dynamics. In this model, we take the bag's surface to be a fixed sphere of radius R. Inside this cavity there are quarks and colored gluons which are treated perturbatively in the quark-gluon coupling constant $g(\alpha_c \equiv g^2/4\pi)$. The quark and gluon fields are constrained to obey the appropriate linear boundary conditions at the cavity surface. This cavity approximation differs from the true bag theory in the treatment of the nonlinear boundary condition: We demand only that the expectation value of the field pressure equal B. This is possible locally on the surface only for Dirac modes with total angular momentum $\frac{1}{2}$. Higher angular momentum modes produce a nonspherically symmetric pressure.

The procedure[10] for finding the mass of a hadron is first to choose the appropriate quarks and/or antiquarks to obtain the desired quantum numbers. Then choose a value of R. Next solve Dirac's equation subject to the bag boundary condition and select the lowest frequency $j=\frac{1}{2}$ mode. (We are not interested in radial or orbital excitations at the moment.) This fixes the quark's frequency in terms of its mechanical mass and the radius R. Now calculate the gluon electrostatic and magnetostatic energy of the quark configuration to lowest order in α_c. Then add the energy due to the bag pressure, $\frac{4}{3}\pi BR^3$, and the finite energy associated with zero-point fluctuations of the fields $-Z_0/R$. Add these all together and minimize the total energy with respect to the radius R. This is equivalent[9] to balancing the field pressure against B. The resulting expression depends parametrically upon B, α_c, Z_0, and the quark masses. In Ref. 10, we found that a very reasonable fit to the light hadrons was obtained with the choice:

$$B^{1/4}=0.145 \text{ GeV},$$

$$Z_0=1.84,$$

$$\alpha_c=0.55,$$

$$m_u=m_d=0,$$

$$m_s=0.279 \text{ GeV}.$$

A nonzero u and d quark mass effects the spectrum only slightly.[11]

Once the masses on new quarks are known the recipé outlined above can be repeated to calculate the masses of the new hadrons. The large mass(es) of the new quark(s) (1500–2000 MeV) has several effects which should be mentioned apart from any specific model. First, the new quarks move nonrelativistically. For example, in the charm scheme a quark in the J has a momentum of 815 MeV and a mass of 1551 MeV. The relativistic correction to the kinetic energy, $\frac{1}{8}p^4/m^4$, is about 1%. This can be compared with the strange quark in the ϕ meson which has a momentum of 535 MeV and a rest mass of 279 MeV. Second, the gluon magnetostatic interaction energy between very massive quarks is weak in comparison with that between ordinary quarks. For very large quark mass the magnetostatic energy decreases like the product of nonrelativistic magnetic moments, $E_{\text{mag}} \sim (1/2m)^2$. The gluon magnetostatic energy of the $c\bar{c}$ pair in the J should be compared with that of the $u\bar{u}(d\bar{d})$ in the ρ^0 and that of the $c\bar{u}$ pair in the $(J=1)D^{0*}$, which contains one heavy and one light quark:

$$E_{\text{mag}}(c\bar{c})=28 \text{ MeV},$$

$$E_{\text{mag}}(u\bar{u})=109 \text{ MeV},$$

$$E_{\text{mag}}(c\bar{u})=47 \text{ MeV}.$$

As a consequence the hyperfine splittings (between 0^- and 1^- mesons or $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons) among the new hadrons will be substantially smaller than among ordinary hadrons. Third, the electrostatic gluon interaction energy which was negligible among ordinary hadrons is larger, but not important among heavy hadrons. To lowest order in α_c the electrostatic energy is nonzero only if the expectation value of the color charge density is locally nonzero. This occurs since quarks with different mass have different wave functions. For the Σ^+ (uus) $E_{\text{elec}}=4.6$ MeV, for C^{++} (uuc) $E_{\text{elec}}=33.1$ MeV. Finally, the new hadrons tend to be smaller than equivalent light hadrons. The size is determined by pressure balance. For a given radius a massive quark exerts less pressure than a massless one. For comparison,

$R_\rho = 0.94$ fm,

$R_\phi = 0.92$ fm,

$R_J = 0.71$ fm.

With these generalities in mind, the spectra of the subsequent sections will be easier to understand.

III. CHARM

According to the tenets of the charm model, the J (3095) is the lowest $J^{PC}1^{--}$ $c\bar{c}$ state of a charmed quark with charge $+\frac{2}{3}e$. Our spectroscopy does not depend on any details of the model and, therefore, applies to any scheme where the J (3095) is an unmixed state of a massive quark and antiquark. In models with only one new quark, the ψ' (3684) must be an internal excitation. The mass of the lowest state is sufficient to determine m_c, the charmed-quark mass. Discussion of ψ' (3684) is postponed until Sec. VI.

The charmed-quark mass is determined to be 1.551 GeV in order to place the J at 3.095 GeV. We have computed the masses of all new hadrons composed of charmed and ordinary quarks in the lowest cavity mode. In the notation of SU(4) this includes the $\underline{20}$'s of $\frac{1}{2}^+$ baryons and $\frac{3}{2}^+$ baryons (including the octet and decuplet of conventional baryons), and the $\underline{15}$'s of 0^- and 1^- mesons. The spectrum is listed in Table I. There we list the particle's name,[12] quark composition, its mass, and the contributions of the five terms in the bag Hamiltonian (quark kinetic energy, E_Q; bag energy, E_V; zero-point energy, E_0; gluon magnetostatic interaction energy, E_M; and gluon electrostatic energy, E_E) to its mass.

These results indicate a low threshold for charmed meson production at SPEAR. We find that two D mesons can be produced at energies above 3.45 GeV. This is below the ψ' mass. It is hard to gauge the reliability of this result. An increase of only 125 MeV in the D mass is sufficient to move the $D\bar{D}$ threshold above the ψ' mass. 125 MeV corresponds to a change of 7% in the D mass. For comparison, among the ordinary baryons, if we were to fit the strange quark mass to the ϕ meson (rather than the Ω^- as we have done), the predicted K mass would differ from its actual value of 5%. We expect heavier states to be more accurately described by our semiclassical calculations; and, therefore, we must consider a 7% discrepancy to be somewhat disturbing. The experimental identification of charm threshold and its relationship to the ψ' mass should clarify this situation.

Also we should point out that there is a relatively large spacing between the J and η_c in our model. The gluon magnetostatic interaction which provides the splitting is smaller than in the ρ-π system ($m_\rho - m_\pi = 503$ MeV in the model), but not as small

TABLE I. Mass spectrum of new hadrons composed of charmed and ordinary quarks in lowest cavity mode. We list for each particle its name, quark composition, its mass, and the contributions of the five terms in the bag Hamiltonian (quark kinetic energy, E_Q; bag energy, E_V; zero-point energy, E_0; gluon magnetostatic interaction energy, E_M; and gluon electrostatic energy, E_E) to its mass.

Multiplet	Particle	Quark content	M (MeV)	E_Q (MeV)	E_V (MeV)	E_0 (MeV)	E_M (MeV)	E_E (MeV)	R (GeV^{-1})
$\frac{1}{2}^+$	C_1^{++}	cuu	2357	2522	206	−384	−20	33	4.79
Baryons	C_0	$c(ud)_{\text{anti}}$	2214	2559	186	−397	167	34	4.63
	S^+	$c(su)_{\text{sym}}$	2507	2691	201	−387	−24	26	4.75
	A^+	$c(su)_{\text{anti}}$	2396	2730	180	−401	−139	26	4.58
	T^0	css	2653	2860	196	−390	−26	13	4.71
	X_u^{++}	ccu	3538	3869	146	−430	−81	35	4.27
	X_s	ccs	3690	4031	144	−432	−68	15	4.25
$\frac{3}{2}^+$		cuu	2461	2453	252	−359	83	32	5.12
Baryons		$c(su)_{\text{sym}}$	2603	2625	244	−362	71	25	5.07
		css	2742	2797	237	−366	61	12	5.02
		ccu	3661	3780	194	−391	45	33	4.69
		ccs	3795	3950	188	−395	40	14	4.64
		ccc	4827	5095	140	−436	27	0	4.21
0^-	D^+	$c\bar{d}$	1726	2579	41	−656	−277	38	2.80
Mesons	F^+	$c\bar{s}$	1885	2714	43	−647	−244	20	2.84
	η_c	$c\bar{c}$	2931	3920	23	−794	−217	0	2.31
1^-	D^{+*}	$c\bar{d}$	1969	2189	137	−439	47	35	4.18
Mesons	F^{+*}	$c\bar{s}$	2099	2358	131	−446	41	15	4.12
	ψ	$c\bar{c}$	3095	3505	82	−520	28	0	3.53

as in other models.[12] The use of nonrelativistic kinematics for ordinary quarks in Ref. 12 makes the resulting estimates of the magnetostatic interaction unreliable. As a consequence we predict a rather large width for the decay $J \to \eta_c \gamma$, $\Gamma(J \to \eta_c \gamma) \simeq 5\,\text{keV}$. This width is proportional to the cube of the mass difference $m_J - m_{\eta_c}$.

Neither this nor any other detailed result concerning the J, η_c system should be taken too seriously. As will be discussed in Sec. V, both states can be expected to mix with nearby gluon states. This will shift their masses and alter results like $\Gamma(J \to \eta_c \gamma)$ which are very sensitive to masses.

IV. HEAVY QUARKS

Since the discovery of the J, ψ', and ψ'' (the 4.1-GeV bump), other models involving new quark flavors have appeared.[5,13] The model which was introduced[5] and developed[14] by Harari has a number of attractive features. In this section, we will calculate the bag-model predictions for the masses of the new hadrons which are implied by Harari's model.

We will begin with a brief introduction to the model. However, the reader is urged to study Ref. 14 where the justification for and the details of the model are presented. We will limit ourselves to a discussion of the U-scheme[14] symmetry-breaking version of the model. In this scheme, the hadronic world has three SU(3)'s. The first is $\text{SU}(3)_{\text{Light}}$, the usual (broken) symmetry of the u, d, and s quarks. The second is $\text{SU}(3)_H$ which is an analogous (broken) symmetry of the three new quarks. The three new quarks have large masses and are referred to as heavy quarks. They carry one unit of a new quantum number called heaviness (H). The familiar light quarks have $H=0$. The heavy quarks have $B=\frac{1}{3}$ as usual. They arrange themselves into a fundamental *antitriplet* basis for a 3×3 matrix representation of the $\text{SU}(3)_H$ symmetry. The r quark is isosinglet with charge $+\frac{2}{3}$. The t and b quarks form an isodoublet with charges $\frac{2}{3}$ and $-\frac{1}{3}$, respectively. The $\text{SU}(3)_H$ symmetry is broken by giving the t and b quarks a larger mass than the r quark. The third SU(3) is the unbroken color symmetry. Each quark flavor comes in three colors.

In the heavy-quark model, the J, ψ', and ψ'' are related to the three neutral $J^{PC}=1^{--}$ states that can be formed with three new quarks. Excited states are shunned. The ψ'' is taken to be the neutral member of an isotriplet:

$$\psi'' = \frac{1}{\sqrt{2}}(t\bar{t} - b\bar{b}). \tag{4.1}$$

It would be natural to identify the ψ' and the J with the states

$$\psi_M \equiv \frac{1}{\sqrt{2}}(t\bar{t} + b\bar{b})$$

and

$$\psi_R = r\bar{r},$$

respectively, in analogy with the ρ, ω, ϕ system. However, this would lead to a degenerate ψ'' and ψ' and to other difficulties discussed by Harari.[14] He argues that a strong singlet interaction which would drive the mixing in the octet-singlet direction is desirable. That is, he would like to identify the ψ' and J with the states

$$\psi_8 \equiv \frac{1}{\sqrt{6}}(t\bar{t} + b\bar{b} - 2r\bar{r})$$

and

$$\psi_1 \equiv \frac{1}{\sqrt{3}}(t\bar{t} + b\bar{b} + r\bar{r}),$$

respectively, in analogy with the π, η, η' system.

In general, we have a two-state mixing problem to solve. A very simple version which does not involve the bag model can be treated analytically. In the basis in which

$$\psi_M = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and (4.2)

$$\psi_R = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

the Hamiltonian which follows from the assumption of a SU(3)-singlet interaction characterized by strength S is

$$H = \begin{pmatrix} E_M + \frac{2}{3}S & \frac{\sqrt{2}}{3}S \\ \frac{\sqrt{2}}{3}S & E_R + \frac{S}{3} \end{pmatrix}. \tag{4.3}$$

E_M is the energy of the state ψ_M. It is the mass of the ψ'' since ψ'' of Eq. (4.1) and ψ_M are degenerate in the absence of mixing. The energy of the ψ_R state, E_R, and S are adjustable. The object of the exercise is to adjust E_R and S so that the eigenvalues $E_\pm$ of H are the J and ψ' masses. In fact, this cannot be done. With

$$E_M = 4.1\ \text{GeV}$$

and

$$\frac{E_+ + E_-}{2} = 3.4\ \text{GeV}.$$

The minimum value possible for $E_+ - E_-$ is 808 MeV. 600 MeV is required. This discrepancy is not huge, and we are not inclined to take it too seriously *at this point*.

When the mixing problem is repeated in the bag model, we find the same difficulty. In this case, the free parameters are the interaction strength S which is characterized by the dimensionless parameter s through

$$S=\frac{s}{2}(m_M+m_R), \tag{4.4}$$

and the mass of the r quark m_R. The mass of the t and b quarks is m_M. It is adjusted to obtain the correct value for the ψ'' mass. The mixing calculation is then carried out as described in Appendix B of Ref. 10. [The singlet interaction is now the S of Eq. (4.4) rather than a/R.] As we have said it is impossible to adjust s and m_R to obtain

$$E_+=M(\psi')$$

and

$$E_-=M(J).$$

Our procedure is to adjust s and m_R to minimize

$$[E_+-M(\psi')]^2+[E_--M(J)]^2.$$

The result of all this was

$$\begin{aligned} &M_M=2.105\text{ GeV},\\ &M_R=1.740\text{ GeV},\\ &S=-0.654\text{ GeV},\\ &E_+=3.822\text{ GeV (138 MeV too high)},\\ &E_-=3.059\text{ GeV (36 MeV too low)}. \end{aligned} \tag{4.5}$$

These numbers are reasonable. Not so reasonable is the implied mixing angle. It comes out to be

$$\tan\theta=-0.51$$

versus the value $-\sqrt{2}$ required by Harari. The difficulty here is that it leads to

$$\frac{\Gamma(J\to e^+e^-)}{\Gamma(\psi'\to e^+e^-)}\cong 57.$$

The experimental ratio of about 2 would be obtained with

$$\tan\theta\cong-\sqrt{2}.$$

Furthermore, the value of S is very large. If the singlet mixing mechanism is blind to heaviness as it is blind to $SU(3)_{\text{Light}}$, then the J, ψ', and ψ'' must mix strongly with conventional quark states and cannot be narrow. Colored-gluon interactions would provide just such a singlet-mixing interaction. They are discussed further in the next section.

Although the details of these numerical results are not to be taken too seriously, the semiquantitative features are certainly correct. The heavy-quark model as implemented in the bag model contains three new quarks. The lighter one has a mass which must be heavier than the corresponding charmed-quark mass by a few hundred MeV. (We will use 1740 MeV.) The other two have the same mass and are heavier still by a few hundred MeV. (We will use 2105 MeV.)

This concludes the prelude. It is now straightforward to determine the masses of the new hadrons implied by these new heavy quarks. In the baryon sector, we will consider only those new baryons which contain only one heavy quark. It is easy to think in the following way: Consider the $C=1$ baryons. By replacing the charmed quark with a heavy quark each of these will give rise to three $H=1$ baryons. There will be one state with a mass around 180 MeV heavier than the charmed state and an isodoublet with a mass about 350 MeV higher than that. There is a similar relationship between the $H=1$ and $C=1$ mesons.

Finally, there are two ($J^P=0^-$ and 1^-) new nonets of $H=0$ mesons formed from the heavy quarks and their antiparticles. As usual, there is a mixing problem in the center of these multiplets. The 1^- case has been discussed. In the 0^- nonet, we have quoted the masses for the simple ψ_M and ψ_R states. A singlet interaction could shift these by an unknown amount. Because of the confusion surrounding the singlet interaction, we do not consider the relative positions of the neutral 0^- and 1^- states to be at all reliable. All these results are given in Table II.

The general conclusion of this section is that the heavy-quark model for the new particles implies a spectrum of new hadrons which is simply related to the charm spectrum. The lightest new hadrons would be a few hundred MeV more massive than the corresponding ones in the charm model. The thresholds in e^+e^- annihilation are correspondingly higher.

V. GLUONS

The new resonances, the J and ψ' and the ψ'', if it is a resonance, have been discovered in a channel to which three gluons can couple. (For definiteness we will work within the standard non-Abelian, colored, vector-gluon model.) The analogous (but as yet unobserved) 0^- particles would be in a channel to which two gluons can couple. The coupling of quarks to these gluons could be the source of the singlet interaction which is responsible for the structure of the π,η,η' system, the ρ,ω,ϕ system, and, in the heavy quark model, the ψ'',ψ',J system. Our understanding of these sys-

TABLE II. Masses of new hadrons implied by new heavy quarks.

Multiplet	Quark content	M (MeV) $h=r$	$h=t,b$
$\frac{1}{2}^+$	huu, etc.	2545	2907
$H=1$	$h(ud)_{\text{anti}}$	2394	2746
Baryons	$h(su)_{\text{sym}}$, etc.	2693	3054
	$h(su)_{\text{anti}}$, etc.	2576	2926
	hss	2839	3198
$\frac{3}{2}^+$	huu	2639	2988
$H=1$	$h(su)_{\text{sym}}$, etc.	2781	3129
Baryons	hss	2920	3268
0^-	$h\bar{d}$, etc.	1916	2284
$H=1$	$h\bar{s}$	2072	2435
Mesons			
1^-	$h\bar{d}$, etc.	2143	2485
$H=1$	$h\bar{s}$	2273	2615
Mesons			
0^-	$r\bar{r}$	3278	
$H=0$	$t\bar{t}$, etc.	3952	
New mesons	$r\bar{t}$, etc.	3617	
1^-	J	3059	
$H=0$	ψ'	3822	
New mesons	ψ''	4100	
	$r\bar{t}$, etc.	3768	

tems is very limited. We have little to guide us in trying to estimate how much singlet interaction may be involved in the new, neutral 1^- and 0^- states. This complicates the problem of inferring a new-quark mass from the observed J mass and the problem of predicting the mass of the 0^- partner of the J.

Many of the difficult problems in hadronic physics have been assigned to the gluons. Various models have them permanently binding the quarks and providing an interaction which is weak at short distances. The spin-dependent part of the quark-gluon interaction can give rise to hadronic mass splittings.[10,12] It may also be the mechanism behind the singlet interaction which is large in the π, η, η' system, small in the ρ, ω, ϕ system, and which "explains" Zweig's rule. Fritzsch and Minkowski[15] have discussed some of these questions. In this section, we will discuss a few topics which relate to the gluon interaction.

The first is the question of gluon resonances. Brower and Primack[16] and Freund and Nambu[17] have made use of such states. By "gluon resonance" *we* shall mean a resonance which is a color singlet bound state of gluons. For $J^{PC}=1^{--}$ this requires three gluons. $J^{PC}=0^{-+}$ requires two gluons.[15]

A very simple argument indicates that it is unlikely that the ψ resonances can be successfully identified as gluon resonances. We can visualize the decay of a gluon resonance into ordinary hadrons by saying that the gluons couple to ordinary quarks, and they then couple to hadrons. Since the gluons are electrically neutral they would decay into e^+e^- by coupling to ordinary quarks which would then annihilate into a virtual photon which decays into the e^+e^- pair. The gluon-quark coupling would cancel in the ratio and we would expect

$$\frac{\Gamma(J\to e^+e^-)}{\Gamma(J\to \text{hadrons})}\sim \alpha^2. \tag{5.1}$$

The observed ratio is 10^{-1}.

The quark, colored gluon version of the bag model predicts the existence of gluon resonances. These states are somewhat more difficult to deal with than quark states. The pressure which arises from a single gluon mode in the cavity is not spherically symmetric. Even though the multigluon color-singlet cavity state generally does not lead to a spherically symmetric pressure we would still expect it to correspond to some nonspherical fluctuating real state. We will *estimate* the energy of this state as follows: The energy will receive contributions from the usual bag volume and vacuum terms and from the gluon kinetic energies. These mode energies are determined by linear boundary condition[8] on the gluons. For this estimate, we will not include any effects that arise from interactions between the gluons. The nonlinear boundary condition is then treated very approximately by minimizing the resulting expression for the energy with respect to the cavity radius. All parameters have been fixed in previous work.[10] We have done this for $J^{PC}=0^{++}$, 0^{-+}, and 1^{--} states. The first two are two gluon states the last has three.

The results of this are the following estimates for the lowest state in each of these channels:

$$M(0^{++})\cong 0.96\text{ GeV},$$
$$M(0^{-+})\cong 1.29\text{ GeV},$$
$$M(1^{--})\cong 1.66\text{ GeV}.$$

Above these energies we would expect a rich spectrum of excited states.

In order to illustrate the effect that the mixing of a quark state with a gluon state may have, we will study the following simple example. Suppose that in the absence of quark-gluon interaction, there is a gluon state at $M+\Delta$ and a quark state at $M-\Delta$. With a quark-gluon mixing of strength ϵ, the Hamiltonian becomes

$$H=\begin{pmatrix} M+\Delta & \epsilon \\ \epsilon & M-\Delta \end{pmatrix}. \tag{5.2}$$

The eigenvalues of this matrix are

$$E_{\pm}=M\pm(\Delta^2+\epsilon^2)^{1/2}. \tag{5.3}$$

If we identify these two levels with the ψ (3100) and the ψ' (3700), then we must take

$$M=3.4\text{ GeV},$$
$$(\Delta^2+\epsilon^2)^{1/2}=0.3\text{ GeV}. \tag{5.4}$$

The condition

$$\frac{\Gamma(\psi'\to e^+e^-)}{\Gamma(J\to e^+e^-)}\cong\frac{1}{2}$$

leads to

$$\Delta=0.1\text{ GeV}$$

and

$$\epsilon=\pm\sqrt{2}\,(0.2)\text{ GeV}.$$

Thus, the *unmixed* new-quark-antiquark bound state had a mass of 3300 MeV. It was shifted down to 3100 MeV by the interaction.

Now, if we use 3300 MeV rather than 3095 MeV to determine the mass of the charmed quark, we find 1665 MeV rather than 1551 MeV. And, finally, if we use this higher charmed-quark mass to calculate the mass of the 0^-D particle we get 1840.3 MeV. Twice this mass (3681 MeV) is sufficiently close to the *observed* ψ' mass (3684 MeV) to provide a huge phase-space suppression in the decay $\psi'\to D^+D^-$.

The point of this exercise has been to illustrate the fact that quark-gluon mixing in the neutral 0^- and 1^- channels could result in significant shifts in the relative positions of states in these channels.

There is an important objection to this and to the J,ψ',ψ'' mixing that the heavy-quark model requires. In both cases, we have found rather strong mixing interactions (characterized by energies in the hundreds of MeV's). If the coupling of the new quarks to the gluons is this strong, one would expect the coupling of the ordinary quarks to be similarly strong. The result would be a width for the ψ resonances which is much larger than the observations.

VI. ψ' IN THE CHARM MODEL

In the charm scheme the ψ' (3684) is understood as an internal excitation of the J. In this section, we will study this idea within the context of the bag model.

The bag model predicts that the ψ' should be found at around 3400 MeV, in rather poor agreement with experiment. If the excitation interpretation of the ψ' is correct, then the cavity approximation to the bag model is inadequate to describe it. The narrow J-ψ' spacing predicted by the bag is a consequence of the large mass of the charmed quark. This effect has been noted already by Vinciarelli.[18] There exist potential models[19] which can accommodate the observed J-ψ' spacing. We will comment on them below.

Excitations are difficult to treat in the bag because of the nonlinear boundary condition. Cavity modes which do not generate a spherically symmetric, classical pressure presumeably correspond to nonspherical, fluctuating bags. The calculation we have carried out is for an excited state which satisfies the bag equations in exactly the same manner as the ground state. However, there are other configurations which do not satisfy the quadratic boundary condition on a static sphere. The energies of these states cannot be calculated with the methods we have developed. However, the linear boundary condition determines the frequencies of these states in a spherical cavity, and they are found to be comparable with the state which we calculate in detail. Thus, we have not one, but several candidates for the ψ', all in the energy region 3400 MeV.

Nonrelativistic Dirac modes in a cavity are labeled by quantum numbers n, l, and j. Relativistically we retain the label l referring to the upper two components of the four-component Dirac spinor. Thus, the ground state of quark and antiquark is the configuration $(1s_{1/2})^2$. The linear boundary condition determines the frequency of the $1s_{1/2}$ mode to be π/R in the nonrelativistic limit. Other modes and their frequencies in the nonrelativistic limit are

Mode	Frequency
$1s_{1/2}$	π
$1p_{1/2}$	4.49
$1p_{3/2}$	4.49
$2s_{1/2}$	2π

Candidates for the ψ' (configurations which may be coupled to $J^{PC}=1^{--}$) are $(1p_{1/2})^2$, $(1p_{3/2})^2$, $(1s_{1/2}2s_{1/2}),\ldots$. Of these, only the first satisfies the quadratic boundary condition and is amenable to calculation. Following the recipe of Sec. II we find

$$M[(1p_{1/2})^2]=3.394\text{ GeV}.$$

The gluon interaction energy is essentially negligible in this state. The splitting $M[(P_{1/2})^2]-M_J=299$ MeV is due almost entirely to the frequency difference $\pi\to4.49$. The small splitting between the J and the ψ' is a consequence of the particular dynamics of the bag model.

The small J-ψ' splitting can be understood in the following way. The kinetic energy of a quark mode of mass m in a cavity of radius R is $x^2/2mR^2$, where x is the dimensionless frequency quoted above (π, 4.49 etc.). Clearly, as m becomes large the spacing of excitations decreases. In the bag, R is not a free parameter, but its variation does not significantly effect the conclusion. The natural scale of spacings of a quark of mass ~1.5 GeV in a cavity of size 1 fm is 300 MeV, not 600 as required. If the quarks were moving in a potential, then excitations could be more widely spaced. For example, a linear potential obeys a theorem $\langle V\rangle = 2\langle T\rangle$ and, therefore, provides a much larger spacing between excitations. Field theories which are known to generate such static, linear potentials are known to be diseased.[20] Theories such as the popular massless Yang-Mills color-gluon scheme are thought by some to generate linear confining forces at large distances. These theories are beset by infrared divergences and no convincing demonstration of this speculation has been presented. The phenomenological treatment of linear potentials[19] is not yet sufficiently developed to be compared critically with the data.

There is some evidence that the bag may be too soft with respect to excitation. There are indications that the same problem arises in the spectrum of ordinary, but excited baryons. Preliminary studies[21] indicate that the low-lying negative-parity baryon resonances appear—on the average—at slightly too low a mass when calculated in the cavity approximation to the bag theory. In conclusion, we are unable to determine whether the predicted low mass of the ψ' in the bag model is an indication of a defect in the cavity approximation, in the model, or is an objection to the interpretation of the ψ' as an excited state of the J in the charm scheme.[22]

Note added. Experimental discoveries proceed at a rapid pace. During the preparation of this manuscript the evidence for several $C=+1$ states between the J and ψ' was announced at the SLAC conference. Harari's model (which suppresses excitations) would have to be reformulated in light of these discoveries (cf. our Ref. 22).

Regardless of the viability of Harari's specific model our analysis of Sec. IV remains applicable generically to models which contemplate mixing of several new types of quarks.

The likely discovery of a 0^{-+} state in the mass region 2.75–2.8 GeV serves to emphasize the warning of Sec. V. Although the bag model predicts a large (163 MeV) J-η_c splitting in comparison with other spectroscopies,[(12)] an experimental splitting of 300–350 MeV can only be explained by mixing in the 0^- or 1^- channels or both.

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[22]Although the heavy quark model discussed in Sec. IV does not make use of excited states of the heavy quark-antiquark system, such excitations would presumably exist. Unless some bizarre mechanism is at work, the excitation energies would be comparable to those discussed in this section. The model would then have the problem of predicting too many states in e^+e^- annihilation.