The psion family: Relativistic charmonium*

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(Received 13 November 1975)

The authors have reported earlier that the spectrum of the psion family suggests a treatment based on an O(4) harmonic-oscillator family with O(4) quantum number $M = 0$ (equally spaced, mass-squared linear Regge trajectories). This results in the new J^{PC} quantum numbers: 0^{++} for 2.8 GeV/c^2; 2^{++}, 1^{-+}, 0^{++} for the 3.4-GeV/c^2 mass region. All radiative decays are related through a single O(4) coupling $S_{\mu\nu}F_{\mu\nu}$, where $S_{\mu\nu}$ is the internal-space angular-momentum operator. In this article the details of the calculations of radiative transition widths and of the $\gamma\gamma$ angular correlation functions are given. Experimental signatures are discussed and compared with the nonrelativistic charmonium scheme; among others $J \to O(2.8)+\gamma$ and $\psi' \to O(2.8)+\gamma$ are small, being forbidden in the strict O(4) limit. $O(2.8)$ is expected to be narrow like all other psions below the 4.0-GeV/c^2 region. $\chi_0(3.41)$ and $\chi_1(3.49)$ total widths are expected to be around 630 keV and 185 keV, respectively. The missing decays of ψ' could be due to $\psi' \to O + \omega$ ($\sim 32\%$).

I. INTRODUCTION

In our earlier communication[1] we reported on a new J^{PC} assignment for members of the psion family[2-5] based on an O(4) oscillator family with O(4) quantum number $M=0$ (equally spaced, mass-squared linear Regge trajectories).[6-9] This results in the new J^{PC} quantum numbers: 0^{++} for 2.8 GeV/c^2, 2^{++}, 1^{-+}, 0^{++} for the 3.4-GeV/c^2 mass region, etc. This is to be contrasted with the nonrelativistic charmonium scheme[10-11] where it is 0^{-+} for 2.8 GeV/c^2 and 2^{++}, 1^{++}, 0^{++} for the 3.4-GeV/c^2 region. In the brief note we merely presented the results of an O(4) calculation of the radiative transitions and found a reasonable fit to available data. We call this scheme relativistic charmonium.[12]

As mentioned earlier, the principal motivation for studying the psions in terms of equally spaced linear Regge trajectories is that it gives a good fit ($\Delta M^2/M^2 < 8\%$) to the psion mass that have been observed to date. The mass formula, in (GeV/c^2)^2 units,

$$M^2 = 2n + (2.75)^2 \tag{1.1}$$

works remarkably well if the $J(3.1)$ and $\psi'(3.7)$ states are assigned to $n = 1, 3$ respectively, for the rest of the known psions then all fall into place to within 8% or so accuracy.

Equation (1.1) is suggestive of the levels of an O(4) harmonic oscillator.[13,14] For each nth level, the spin content of the states will then be given by $J = n, n-1, \ldots, 0$, each spin occurring once.

The advantage of this assignment to an infinite family of O(4) oscillator states is that radiative transitions among the psion family are all related through their O(4) transition matrix elements. This comes about through a single radiative coupling $S_{\mu\nu}F_{\mu\nu}$, where $S_{\mu\nu}$ is the internal-space angular-momentum operator. Since the psions are neutral there are no charge couplings and this coupling, at a phenomenological level, is the natural analog of the Pauli-type electromagnetic coupling for the neutron.

We have carried out detailed calculations of the width for psion → psion + γ transitions and, using certain hadronic modes as part of input, have found a good general agreement with data known to date.

The general features of our experimental fit which emerge are the following: $J \to O(2.8)+\gamma$ and $\psi' \to O(2.8)+\gamma$ are small, in fact, in limit of strict O(4), they are forbidden. $O(2.8)$ psion is expected to be narrow like all other psions below 4.0 region, with typical, allowed, channels $\gamma\gamma$, $p\bar{p}$, $\pi\pi$, $K\bar{K}$, $\rho K\bar{K}$, etc. (In contrast, η_c is expected to be a few MeV in width.) The χ_0 and χ_1 psions are expected to be around 630 and 185 keV in total width, respectively, with predominant decay modes $O+2\pi$ (~77%) in χ_0 case and $J+\gamma$ (~80%) in χ_1 case. The missing decays of ψ' could be explained by the new channel $O+\omega$ which could be as large as 32% in branching ratio. These and other details are summarized in Tables I and II as well as Sec. V.

II. REVIEW

Having assigned the psions to an O(4) family we must write down the phenomenological fields associated with the individual O(4) states and invent an interaction responsible for the transitions from one O(4) state to another. The phenomenological fields for these O(4) states have already been written down earlier.[9] We recall here the rationale for that construction and list for later reference the relevant fields that we shall need for our present purposes.

Consider a family of particles, degenerate in mass, with spin content $J=n, n-1, \ldots, 0$. Construct the annihilation operators $a_{\mu_1 \cdots \mu_n}(\vec{p})$, such that[15]

$$[a_{\mu_1\cdots\mu_n}(\vec{p}), a^*_{\nu_1\cdots\nu_n}(\vec{p}\,')] = \frac{p_0}{M}\,\delta(\vec{p}-\vec{p}\,')\Delta_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}, \tag{2.1}$$

where $\Delta_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}$ are momentum-independent matrices satisfying the properties

(i) $\Delta_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}$ totally symmetric in $\mu_1\cdots\mu_n$, and separately in $\nu_1\cdots\nu_n$.

(ii) $\delta_{\mu_1\mu_2}\Delta_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}=0$.

(iii) (Normalization) $\Delta_{\mu_1\cdots\mu_n,\mu_1\cdots\mu_n}=(n+1)^2$.

In other words, Δ are the "spin matrices" of the well-known O(4) propagators. It is helpful to list a few examples for reference purposes:

$$n=1,\quad \Delta_{\mu\nu}=\delta_{\mu\nu}, \tag{2.2}$$

$$n=2,\quad \Delta_{\mu\nu,\sigma\tau}=\tfrac{1}{2}(\delta_{\mu\sigma}\delta_{\nu\tau}+\delta_{\nu\sigma}\delta_{\mu\tau})-\tfrac{1}{4}\delta_{\mu\nu}\delta_{\sigma\tau}, \tag{2.3}$$

$$\begin{aligned} n=3,\quad \Delta_{\mu\nu\lambda,\sigma\tau\rho}=\tfrac{1}{6}(&\delta_{\mu\sigma}\delta_{\nu\tau}\delta_{\lambda\rho}+\delta_{\mu\tau}\delta_{\nu\sigma}\delta_{\lambda\rho}+\delta_{\mu\sigma}\delta_{\nu\rho}\delta_{\lambda\tau}+\delta_{\mu\tau}\delta_{\nu\rho}\delta_{\lambda\sigma}+\delta_{\mu\rho}\delta_{\nu\sigma}\delta_{\lambda\tau}+\delta_{\mu\rho}\delta_{\nu\tau}\delta_{\lambda\sigma}\\ &-\tfrac{1}{18}(\delta_{\mu\nu}\delta_{\sigma\tau}\delta_{\lambda\rho}+\delta_{\mu\nu}\delta_{\sigma\rho}\delta_{\lambda\tau}+\delta_{\mu\nu}\delta_{\tau\rho}\delta_{\lambda\sigma}+\delta_{\mu\lambda}\delta_{\sigma\tau}\delta_{\nu\rho}+\delta_{\mu\lambda}\delta_{\sigma\rho}\delta_{\nu\tau}+\delta_{\mu\lambda}\delta_{\tau\rho}\delta_{\nu\sigma}\\ &+\delta_{\nu\lambda}\delta_{\sigma\tau}\delta_{\mu\rho}+\delta_{\nu\lambda}\delta_{\sigma\rho}\delta_{\mu\tau}+\delta_{\nu\lambda}\delta_{\tau\rho}\delta_{\mu\sigma}). \end{aligned} \tag{2.4}$$

Equation (2.1) thus exhibits the O(4) nature of the family of particles all with mass M appropriate to the nth level. Equation (2.1) has no problem with ghosts. However, the Lorentz transformation properties must be satisfied, and as has been shown the $a_{\mu_1\cdots\mu_n}$ are pseudotensors under Lorentz transformation, viz.

$$U(\Lambda)a_{\mu_1\cdots\mu_n}(\vec{p})U^\dagger(\Lambda)=(R^{-1})_{\mu_1\nu_1}\cdots(R^{-1})_{\mu_n\nu_n}a_{\nu_1\cdots\nu_n}(\vec{p}\,'), \tag{2.5}$$

where

$$R_{\mu\nu}=[L^{-1}(\Lambda p)\,\Lambda L(\vec{p})]_{\mu\nu} \tag{2.6}$$

is the 4×4 matrix representation of the Wigner rotation.

With the auxiliary operators

$$A_{\mu_1\cdots\mu_n}(\vec{p})=[L(\vec{p})]_{\mu_1\nu_1}\cdots[L(\vec{p})]_{\mu_n\nu_n}a_{\nu_1\cdots\nu_n}(\vec{p}), \tag{2.7}$$

where $L(\vec{p})$ are the Lorentz boost matrices, phenomenological fields for these O(4) family states may be easily constructed.

For our purposes it is sufficient to exhibit the covariant spin decomposition for some of the fields so constructed:

$$n=0,\quad T(x)\equiv O(x), \tag{2.8}$$

$$n=1,\quad T_\mu(x)\equiv\psi_\mu(x)+\frac{1}{M}\partial_\mu\psi(x), \tag{2.9}$$

$$n=2,\quad T_{\mu\nu}(x)\equiv\chi_{\mu\nu}(x)+\frac{1}{\sqrt{2}M}[\partial_\mu\chi_\nu(x)+\partial_\nu\chi_\mu(x)]+\frac{2}{\sqrt{3}M^2}(\partial_\mu\partial_\nu-\tfrac{1}{4}\delta_{\mu\nu}\partial^2)\chi(x), \tag{2.10}$$

$$\begin{aligned} n=3,\quad T_{\mu\nu\lambda}(x)\equiv{}&\psi'_{\mu\nu\lambda}(x)+\frac{1}{\sqrt{3}M}(\partial_\mu\psi'_{\nu\lambda}+\partial_\nu\psi'_{\mu\lambda}+\partial_\lambda\psi'_{\mu\nu})\\ &+\left(\frac{2}{5}\right)^{1/2}\frac{1}{M^2}\left[\partial_\mu\partial_\nu\psi'_\lambda+\partial_\mu\partial_\lambda\psi'_\nu+\partial_\nu\partial_\lambda\psi'_\mu-\frac{\partial^2}{6}(\delta_{\mu\nu}\psi'_\lambda+\delta_{\mu\lambda}\psi'_\nu+\delta_{\nu\lambda}\psi'_\mu)\right]\\ &+\frac{\sqrt{2}}{M^3}\left[\partial_\mu\partial_\nu\partial_\lambda-\frac{\partial^2}{6}(\delta_{\mu\nu}\partial_\lambda+\delta_{\mu\lambda}\partial_\nu+\delta_{\nu\lambda}\partial_\mu)\right]\psi'(x). \end{aligned} \tag{2.11}$$

TABLE I. Radiative and total widths (Underlined quantities are based on $S_{\mu\nu}F_{\mu\nu}$ coupling and J^{PC} selection rules).

Mode	Width	Branching ratio (%)	Remarks
$\psi'(3684) \to \chi_2+\gamma$	2.3 keV	1%	Input: $B_{\gamma J}(\chi_1)=3\times10^{a,b}$
$\to \chi_1+\gamma$	8.4 keV	3.7%	$B(\chi_1\to\gamma J)=0.8$
$\to \chi_0+\gamma$	26.6 keV	11.8% [c]	$\Gamma_{tot}(\psi')=225$ keV
$\to O+\gamma$	Forbidden		Hence, $\kappa^2=9.63$
$\chi_2(3530)\to J(3.1)+\gamma$	68.1 keV		
$\to J_0(3.1)+\gamma$	33.6 keV		
$\chi_1(3490)\to J(3.1)+\gamma$	147.3 keV	80%	Branching ratio used as input above
$\to J_0(3.1)+\gamma$	28 keV		Hence, $\Gamma_{tot}(\chi_1)=184$ keV
$\chi_0(3410)\to J(3.1)+\gamma$	79.9 keV	12.7%	Input: $B_{\gamma\psi}(\chi_0)=1.5\times10^{-2}$ [b]
			Hence, $\Gamma_{tot}(\chi_0)\sim 629$ keV
$\to J_0(3.1)+\gamma$	Forbidden		
$\chi_0(3094)\to O+\gamma$	Forbidden		
$\to \pi^0+\gamma$	Forbidden		Parity selection rule

[a] Reference 16.
[b] Reference 5
[c] Reference 17, and see text.

In writing down this decomposition the tensor fields on the right are pure spin fields, i.e., $\chi_{\mu\nu}$ is a $J=2$ field and, thus, satisfies symmetric and traceless ($\delta_{\mu\nu}\chi_{\mu\nu}=0$) conditions as well as $\partial_\mu\chi_{\mu\nu}=0$, similarly for the other fields. Also, for each nth-rank tensor field the mass M appearing in the decomposition is that appropriate to the nth level.

With these fields in hand, we can now go about the task of finding a coupling between two such psion states with, say, the electromagnetic field. Since the psions are neutral, no simple $J_\mu A_\mu$ charge coupling is possible, therefore, some Pauli-type "magnetic moment" coupling should be tried. In terms of the phenomenological fields, we need to specify in

$$T^\star_{\nu_1\cdots\nu_{n'}}C_{\nu_1\cdots\nu_{n'};\mu_1\cdots\mu_n;\sigma\tau}T_{\mu_1\cdots\mu_n}F_{\sigma\tau} \tag{2.12}$$

the general form of the coupling matrix. Such a form can be found by appealing to an analogy with the nonrelativistic physics.

III. O(4)

Suppose we work in an internal (Euclidean) space with coordinate X, X_4 real. In this space let us denote the O(4) wave functions by

$$\phi_{\mu_1\cdots\mu_n}(X)=N_n\{X_{\mu_1}\cdots X_{\mu_n}\}e^{-(1/8)X^2}, \tag{3.1}$$

where

$$\{X_{\mu_1}\cdots X_{\mu_n}\}\equiv X_{\mu_1}\cdots X_{\mu_n}-\frac{X^2}{2n}[\delta_{\mu_1\mu_2}X_{\mu_3}\cdots X_{\mu_n}+\text{permutations}]+\cdots \tag{3.2}$$

is symmetric and traceless under contraction with $\delta_{\mu\nu}$, and N_n is given by

$$N_n{}^{-1}=4\pi(2^n n!)^{1/2}. \tag{3.3}$$

The normalization factor has been chosen such that

$$\int d^4X\phi^*_{\mu_1\cdots\mu_n}(X)\phi_{\nu_1\cdots\nu_{n'}}(X)=\delta_{nn'}\Delta_{\mu_1\cdots\mu_n,\nu_1\cdots\nu_n}. \tag{3.4}$$

In this space, the internal spin operator $S_{\mu\nu}$ is given by ($\overleftrightarrow{\partial}_\mu\equiv\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu$)

$$S_{\mu\nu}=-i\left(X_\mu\frac{\overleftrightarrow{\partial}}{\partial X_\nu}-X_\nu\frac{\overleftrightarrow{\partial}}{\partial X_\mu}\right). \tag{3.5}$$

We are now in a position to make the ansatz, in connection with the psion radiative transition coupling referred to earlier. We assume

$$C_{\mu_1\cdots\mu_n;\nu_1\cdots\nu_{n'};\sigma\tau}=\frac{e\kappa}{4}\int d^4Xe^{iq\cdot X}\phi^*_{\mu_1\cdots\mu_n}S_{\sigma\tau}\phi_{\nu_1\cdots\nu_{n'}}. \tag{3.6}$$

TABLE II. Hadronic decay modes and widths (Underlined quantities are based on J^{PC} selection rules).

Mode	Width	Branching ratio (%)	Remarks
$\psi'(3684) \to J(3.1) + 2\pi$	108 keV [a]	48%	Hence, $f^2/4\pi = 1.95\times10^{-3}$
$\to O + \omega$	72 keV	32%	Assumed $f_0 = f$
$\to J_0(3.1) + 2\pi$	Forbidden		
$\chi_2(3530) \to O + 2\pi$	4.75 eV		See remark under χ_0
$\to 4\pi^{\pm}$	$\gtrsim(20\pm10)$ keV	(20 ± 10)%	$B_{4\pi^\pm} = 0.20\pm0.10$% [b,a]
$\chi_1(3490) \to O + 2\pi$	~2.9 keV		See remark under χ_0
$\to 2\pi, K\bar{K}, \pi K\bar{K}, \rho K\bar{K}$	Forbidden		1^{-+} selection rule
$\to 4\pi^{\pm}$	~5 keV	~2.7%	$B_{4\pi^\pm}(\chi_1) \sim 0.1$% [b,a]
$\chi_0(3410) \to O + 2\pi$	~484 keV	~77%	Since $\Gamma_{tot}(\chi_0) = 629$ keV; Choose $f_\chi{}^2/4\pi \sim 2.5\times10^{-3}$
$\to 4\pi^{\pm}$	~10 keV	~1.2%	$B_{4\pi^\pm}(\chi_0) = (0.14\pm0.07)$% [b,a]
$\to 6\pi^{\pm}$	~5.3 keV	~0.85%	$B_{6\pi^\pm}(\chi_0) \sim 0.1$% [b,a]
$\to \pi^+\pi^-K^+K^-$	~3 7 keV	~0.6%	$B_{\pi\pi K\bar{K}} \sim 0.07$% [b,a]
$\to \pi^+\pi^- + K^+K^-$	~6.9 keV	~1.1%	$B \sim (0.13\pm0.05)$% [b,a]
$J_0(3094) \to 3\pi$	Forbidden		0^{+-} selection rule
$\to 5\pi, 7\pi$	Allowed		
$\to K\bar{K}$	SU_3 forbidden		
$\to K\bar{K}\pi$	Forbidden		0^{+-} selection rule
$\to K\bar{K}2\pi$	Allowed		
$\to K\bar{K}\rho$	Allowed		
$O(2800) \to \gamma\gamma, p\bar{p}, 2\pi$; $\to 4\pi, K\bar{K}, \rho K\bar{K}$	Allowed		
$\to \pi\bar{K}K$	Forbidden		

[a] Reference 16.
[b] Reference 4.

In Eq. (2.12), q_μ is the momentum four-vector for the photon ($q_4 = i\,|\vec{q}\,|$). Because $q^2=0$, the coupling $C_{\mu\cdots;\nu\cdots;\sigma\tau}$ is a local one, in spite of the apparent nonlocal nature of the O(4) integral. Furthermore, the coupling matrix $C_{\mu\cdots;\nu\cdots;\sigma\tau}$ is actually Lorentz covariant, as will be evident by explicit integration.

Before proceeding with an evaluation of the individual radiative transition matrix elements based on this general ansatz, it is worthwhile summarizing the properties of the O(4) representation we are using here.

The O(4) wave functions we use here have been chosen to be eigenfunctions of

$$\left(2\,\frac{\partial^2}{\partial X^2} - \tfrac{1}{8}X^2\right)\phi_{\mu_1\cdots\mu_n}(X) = -(n+2)\phi_{\mu_1\cdots\mu_n}(X)\,. \tag{3.7}$$

In a simple-minded picture of two constituents bound by an oscillator, an equation like

$$\left[\frac{\partial^2}{\partial X_1{}^2} + \frac{\partial^2}{\partial X_2{}^2} - \tfrac{1}{8}(X_1 - X_2)^2 - \frac{(2.75)^2 - 4}{2}\right]\psi(X_1, X_2) = 0 \tag{3.8}$$

would lead to a separable equation, in terms of

$$Y \equiv \frac{X_1 + X_2}{2}\,, \qquad X \equiv X_1 - X_2\,, \tag{3.9}$$

viz.,

$$\left(\frac{\partial^2}{\partial Y^2} - M^2\right)\psi_M(Y) = 0\,, \tag{3.10}$$

$$\left(\frac{M^2}{2} + 2\frac{\partial^2}{\partial X^2} - \tfrac{1}{8}X^2 - \frac{(2.75)^2 - 4}{2}\right)\phi_M(X) = 0\,. \tag{3.11}$$

IV. RADIATIVE-DECAY WIDTHS

With the single $S_{\mu\nu}F_{\mu\nu}$ coupling, all the radiative-decay amplitudes for the psion family follow from Eq. (2.12) and (3.6). In this section we give the explicit forms of the coupling matrix for the psion transitions of interest. Since the coupling matrices are to be contracted with the phenomenological fields $T_{\mu\nu\ldots}$ use of symmetry and tracelessness simplifies considerably their form. It is this simplified, equivalent, form which we list below.

A. Psion radiative transitions into $O(2.8)$

The $S_{\mu\nu}$ operator annihilates the ground state $O(2.8)$. Therefore, in Eq. (3.6), the coupling matrix vanishes in the limit $q=0$. The general result for the transitions $n=1 \to n=0$ and $n=3 \to n=0$ are (in GeV units)

$J \to O\gamma$,

$$C_{;\mu;\sigma\tau} \doteq \tfrac{1}{2}e\kappa\sqrt{2}\,q_\sigma\delta_{\tau\mu}\,, \tag{4.1}$$

$\psi' \to O\gamma$,

$$C_{;\mu\nu\lambda;\sigma\tau} \doteq \tfrac{1}{2}e\kappa\sqrt{3}\,(q_\sigma\delta_{\tau\mu}\delta_{\nu\lambda} - 2q_\sigma q_\nu q_\lambda\delta_{\tau\mu})\,, \tag{4.2}$$

where $\doteq$ denotes equivalence as spelled out above. Since $\partial_\sigma F_{\sigma\tau}(x)=0$, the amplitudes for all single-photon radiative transitions to the $O(2.8)$ vanish.

B. $n \to n'$ Radiative transition

The first two nonvanishing amplitudes involve the $n=2$ level and the couplings for these transitions simplify to

$n=2$ to $n=1$, $\chi \to J+\gamma$,

$$C_{\alpha;\mu\nu;\sigma\tau} \doteq 2e\kappa\,q_\mu\delta_{\alpha\sigma}\delta_{\nu\tau}\,, \tag{4.3}$$

$n=3$ to $n=2$, $\psi' \to \chi+\gamma$,

$$C_{\alpha\beta;\mu\nu\lambda;\sigma\tau} \doteq 2\sqrt{6}\,e\kappa(q_\nu\delta_{\mu\tau}\delta_{\alpha\sigma}\delta_{\beta\lambda} + q_\beta q_\nu q_\lambda\delta_{\alpha\tau}\delta_{\sigma\mu})\,. \tag{4.4}$$

Therefore, by our ansatz, the phenomenological coupling for the radiative decays of the $n=3$ and $n=2$ psions are

$$2e\kappa[T_\sigma(x)\partial_\mu T_{\mu\tau}(x) + \partial_\mu T^\star_\sigma(x)\,T_{\mu\tau}(x)]\,F_{\sigma\tau}(x) + \text{H.c.}\,, \tag{4.5}$$

$$\begin{aligned}2\sqrt{6}\,e\kappa[&T^\star_{\sigma\beta}(x)\partial_\nu T_{\tau\nu\beta}(x) + \partial_\nu T^\star_{\sigma\beta}(x)\,T_{\tau\nu\beta}(x) - T^\star_{\tau\beta}(x)\partial_\beta\partial_\nu\partial_\lambda T_{\sigma\nu\lambda}(x) - \partial_\beta T^\star_{\tau\beta}(x)\partial_\nu\partial_\lambda T_{\sigma\nu\lambda}(x)\\ &- 2\partial_\nu T^\star_{\tau\beta}(x)\partial_\beta\partial_\lambda T_{\sigma\nu\lambda}(x) - \partial_\nu\partial_\lambda T^\star_{\tau\beta}(x)\partial_\beta T_{\sigma\nu\lambda}(x) - 2\partial_\beta\partial_\nu T^\star_{\tau\beta}(x)\partial_\lambda T_{\sigma\nu\lambda}(x) - \partial_\nu\partial_\lambda\partial_\beta T^\star_{\tau\beta}(x)\,T_{\sigma\nu\lambda}(x)]\,F_{\sigma\tau}(x)\\ &+ \text{H.c.}\,.\end{aligned} \tag{4.6}$$

The matrix elements for the transitions of interest can be written down using the spin decomposition as outlined in Sec. II.

For the sake of completeness, we list individual matrix elements so determined. (In the following, P_μ, M denote momentum and mass of initial psion, p_μ, m denote those of final psion.)

$$\mathfrak{M}^*(\psi' \to \chi_2\gamma) = e\kappa\,\frac{2i}{\sqrt{15}}\,\epsilon_{\alpha\beta}(A\epsilon^\gamma_\alpha P_\beta q\cdot\epsilon'^\star + B\epsilon^\gamma\cdot\epsilon'^\star P_\alpha P_\beta + C\epsilon^\gamma_\alpha\epsilon'^\star_\beta)\,, \tag{4.7}$$

$$\mathfrak{M}^*(\psi' \to \chi_1\gamma) = e\kappa\left(\frac{2}{15}\right)^{1/2}\frac{i}{m}(\epsilon^\gamma\cdot\epsilon'^\star P\cdot\epsilon^\chi D + \epsilon^\gamma\cdot\epsilon^\chi q\cdot\epsilon'^\star E)\,, \tag{4.8}$$

$$\mathfrak{M}^*(\psi' \to \chi_0\gamma) = e\kappa\,\frac{4}{3\sqrt{5}}\,\frac{i}{m^2}\epsilon^\gamma\cdot\epsilon'^\star F\,, \tag{4.9}$$

where (all momenta and masses are measured in GeV units)

$$A = -1 - 12\frac{P\cdot q}{M^2} + 18\frac{(P\cdot q)^2}{M^2}\,, \tag{4.10}$$

$$B = 1 + 6\frac{P\cdot q}{M^2} - 6\frac{(P\cdot q)^2}{M^2}\,, \tag{4.11}$$

$$C = -6\frac{(P\cdot q)^2}{M^2}\,, \tag{4.12}$$

$$D = -4P\cdot q - 12\frac{(P\cdot q)^3}{M^2}\,, \tag{4.13}$$

$$E = 11P\cdot q + 18\frac{(P\cdot q)^2}{M^2} + 18\frac{(P\cdot q)^3}{M^2}\,, \tag{4.14}$$

$$F=-5(P\cdot q)^2-6\frac{(P\cdot q)^3}{M^2}-6\frac{(P\cdot q)^4}{M^2}. \tag{4.15}$$

Similarly, we have, using the same convention for notation,

$$\mathfrak{M}^*(\chi_2\to J\gamma)=2e\kappa i(\epsilon\cdot q\,\epsilon^{\star}_{\alpha\beta}q_\alpha\epsilon^\gamma_\beta-\epsilon\cdot\epsilon^\gamma\epsilon^{\star}_{\alpha\beta}q_\alpha q_\beta), \tag{4.16}$$

$$\mathfrak{M}^*(\chi_1\to J\gamma)=\sqrt{2}\,e\kappa i\,\frac{P\cdot q}{M}(2\epsilon\cdot\epsilon^\gamma\epsilon^{\chi\star}\cdot q-\epsilon^\gamma\cdot\epsilon^{\chi\star}\epsilon\cdot q), \tag{4.17}$$

$$\mathfrak{M}^*(\chi_0\to J\gamma)=4e\kappa\frac{i}{\sqrt{3}}\,\frac{P\cdot q}{M^2}(\epsilon\cdot\epsilon^\gamma P\cdot q-\epsilon\cdot qP\cdot\epsilon^\gamma), \tag{4.18}$$

and

$$\mathfrak{M}^*(\chi_2\to J_0\gamma)=2ie\kappa\left(\frac{p\cdot q}{m}\epsilon^{\star}_{\alpha\beta}q_\alpha\epsilon^\gamma_\beta-\frac{p\cdot\epsilon^\gamma}{m}\epsilon^{\star}_{\alpha\beta}q_\alpha q_\beta\right), \tag{4.19}$$

$$\mathfrak{M}^*(\chi_1\to J_0\gamma)=\sqrt{2}\,ie\kappa\frac{P\cdot q}{Mm}(2p\cdot\epsilon^\gamma\epsilon^{\chi\star}\cdot q-\epsilon^\gamma\cdot\epsilon^{\chi\star}p\cdot q), \tag{4.20}$$

$$\mathfrak{M}^*(\chi_0\to J_0\gamma)=0. \tag{4.21}$$

In addition to the rates that have been calculated using these matrix elements, we have also calculated the angular correlation function $W(\theta)$, where θ is the angle between the two photons in the ψ' rest frame. These are listed in Table III.

V. COMPARISON WITH DATA

We identify the P_c resonance 3.507 ± 0.007 GeV which has been studied at DORIS[3] with the $\chi(3.50\pm0.01)$ reported by SPEAR[4,16] and assign it as the χ_1 of our model. Our prediction, therefore, for the J^{PC} of the 3.5 psion is 1^{-+}. We identify the $P_c'(3.407\pm0.008)$ seen in the γJ mode at DORIS[5] with the $\chi(3.41\pm0.01)$ seen at SPEAR[4,16] and since $\chi(3.41)$ has been seen to decay into 2π we assign it to χ_0 of our model, predicting it to be 0^{++}. Finally, we assign $\chi(3.53\pm0.02)$[4,16] to χ_2 of our model, predicting it to be 2^{++}. The rest of

TABLE III. $\gamma\gamma$ correlation functions in ψ' decay.

$\psi'\to\gamma\chi_2\to\gamma\gamma J(3.1)$	$1-0.45\cos^2\theta+0.06\cos^4\theta$
$\to\gamma\chi_1\to\gamma\gamma J(3.1)$	$1-0.42\cos^2\theta$
$\to\gamma\chi_0\to\gamma\gamma J(3.1)$	1
$\to\gamma\chi_2\to\gamma\gamma J_0(3.1)$	$1-0.06\cos^2\theta-0.26\cos^4\theta$
$\to\gamma\chi_1\to\gamma\gamma J_0(3.1)$	$1+0.45\cos^2\theta$
$\to\gamma\chi_0\to\gamma\gamma J_0(3.1)$	Forbidden

the psion assignments can be seen in Fig. 1.

For our comparison with experiment we have chosen as input the values:

$$\kappa^2=9.63,\quad B(\chi_1\to J\gamma)=80\%. \tag{5.1}$$

These have been chosen so that (i) the over-all branching ratio

$$B_{\gamma J}(\chi_1)\equiv\frac{\Gamma(\psi'\to\chi_1\gamma)}{\Gamma(\psi'\to\text{all})}\,\frac{\Gamma(\chi_1\to J+\gamma)}{\Gamma(\chi_1\to\text{all})}=3\times10^{-2} \tag{5.2}$$

agrees with the observed[5] ratio $(4\pm2)\times10^{-2}$ and (ii) the single monoenergetic radiative decays of ψ' are compatible with the Hofstadter limits.[17] For the decay $\Psi'\to\chi_1+\gamma$, our branching ratio of 3.7% is very easily within the Hofstadter limit. For the decay $\psi'\to\chi_0+\gamma$, our branching ratio (11.8%) is outside the quoted limit ($\lesssim 7\%$). However, this limit is contingent upon an assumed mode of decay of χ_0 into either $2\pi^+2\pi^-2\pi^0$ or $3\pi^+3\pi^-2\pi^0$, so that the source of γ-ray background is known. In this model, it will turn out that χ_0 has a relatively large width into $O+\pi^+\pi^-$. The decays of O into $4\pi^0$ could be a prolific source of γ rays which help swamp the signal and thus weaken the limit.

Having thus chosen a value for κ^2 and an input for the branching ratio $(\chi_1\to J\gamma)$, we can confront the rest of the predictions of $S_{\mu\nu}F_{\mu\nu}$ coupling with experiment.[18] These are summarized in Table I, but a brief discussion of the procedure used would be helpful.

A. $\Gamma_{\text{tot}}(\chi)$

From the input, $\Gamma_{\text{tot}}(\chi_1)$ is obviously determined to be 184 keV. The branching ratio for $\chi_0\to J+\gamma$ is obtained from the number[5]

$$B_{\gamma J}(\chi_0)=\frac{\Gamma(\psi'\to\chi_0\gamma)}{\Gamma(\psi'\to\text{all})}\,\frac{\Gamma(\chi_0\to J+\gamma)}{\Gamma(\chi_0\to\text{all})}\sim1.5\times10^{-2}, \tag{5.3}$$

since the $B(\psi'\to\chi_0\gamma)$ has already been fixed in our model. We find $B(\chi_0\to J+\gamma)=12.7\%$. From our knowledge of the width for $\chi_0\to J+\gamma$ we find finally the total width for χ_0 to be large, ~629 keV.

B. Hadronic decay widths

To more completely compare our model with the data, we need some input with regard to hadronic decays. Our approach here is phenomenological. For ψ' decay modes, we learn from the gauge-invariant hadronic coupling $\psi'J\epsilon$ $[M=M(\psi')]$[19]

$$\frac{f}{M}\epsilon(\partial_\mu\psi_\nu'-\partial_\nu\psi_\mu')(\partial_\mu J_\nu-\partial_\nu J_\mu), \tag{5.4}$$

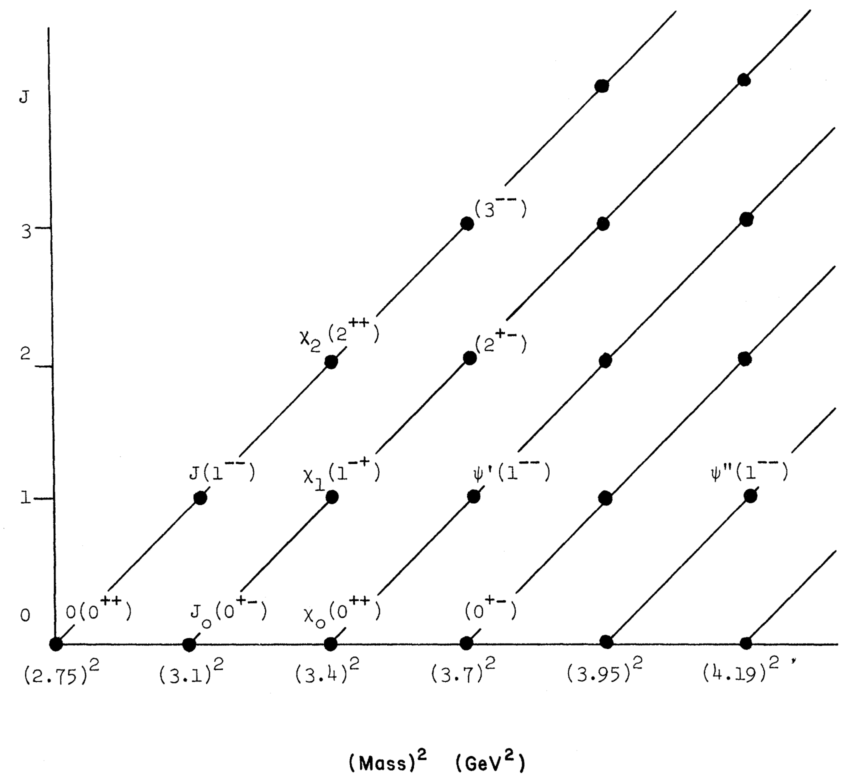

FIG. 1. Psion family described by O(4) Regge trajectories linear in M^2 with slope 0.500 and J mass 3.094 GeV/c^2.

and from the data $\Gamma(\psi' \to J\pi\pi) = 108$ keV. This coupling implies

$$\Gamma(A \to B\pi^+\pi^-) = \frac{1}{\pi}\int_{4\mu^2}^{(M_A-M_B)^2} dq^2 \Gamma_{A\to B\epsilon}(q^2) \times \frac{M_\epsilon \Gamma_{\epsilon\to\pi^+\pi^-}(q^2)}{(q^2-M_\epsilon^2)^2 + M_\epsilon^2\Gamma_\epsilon^2}, \qquad (5.5)$$

where q^2 is the off-shell ϵ mass squared and μ is the pion mass. With $A = \psi'$, $B = J$,

$$\Gamma_{\psi'\to J\epsilon}(q^2) = \frac{f^2}{4\pi}\frac{2}{3}\frac{\mathcal{P}}{M^2}(2\mathcal{P}^2 + 3m_J^2), \qquad (5.6)$$

where $\mathcal{P}$ is the magnitude of the final three-momentum in the ψ' rest frame. For the ϵ decay, we use the coupling

$$g_{\epsilon\pi\pi} M_\epsilon \epsilon \vec{\pi}\cdot\vec{\pi}, \qquad (5.7)$$

so that

$$\Gamma_{\epsilon\to\pi^+\pi^-}(q^2) = \frac{g_{\epsilon\pi\pi}^2}{8\pi}\left(\frac{q^2}{4} - \mu^2\right)^{1/2}, \qquad (5.8)$$

and $\Gamma_\epsilon = \frac{3}{2}\Gamma_{\epsilon\to\pi^+\pi^-}$. For $\Gamma_\epsilon = 0.6$ GeV and $M_\epsilon = 0.7$ GeV, we find $g_{\epsilon\pi\pi}^2/4\pi = 2.5$, and this leads us to find from Eq. (5.5) that

$$f^2/4\pi = 1.95\times 10^{-3}, \qquad (5.9)$$

which is a small number in comparison with ordinary hadronic couplings. Notice that this gauge-invariant $\psi' J\epsilon$ coupling forbids ψ' decay into $J_0 + 2\pi$.

We now use the magnitude of this coupling constant as a basis to estimate the decay $\psi' \to O(2.8) + \omega$ from the gauge-invariant coupling

$$\frac{f_0}{M}\Phi(\partial_\mu\psi'_\nu - \partial_\nu\psi'_\mu)(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu). \qquad (5.10)$$

It gives

$$\Gamma(\psi' \to O + \omega) = \frac{f_0^2}{4\pi}\frac{2\mathcal{P}}{3M^2}(2\mathcal{P}^2 + 3m_\omega^2). \qquad (5.11)$$

If f_0 is taken arbitrarily to be the same as f in the $\psi' J\epsilon$ coupling, then we obtain

$$\Gamma(\psi' \to O\omega) = 72 \text{ keV}$$

or

$$B(\psi' \to O\omega) = 32\%,$$

which could explain the "missing" decays of ψ'.

We return next to the large width we obtained for χ_0. We conjecture that this large width ~629 keV is principally due to the decay $\chi_0 \to O + 2\pi$. To make an estimate about the decay, we use the coupling

$$\frac{f_\chi}{M_\chi}(T_{\mu\nu}\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_\nu\Phi)\epsilon\,, \tag{5.12}$$

which has the (desirable) property of suppressing the χ_1 and χ_2 decays relative to χ_0. Using the spin-decomposition Eq. (2.10), we find the partial widths

$$\Gamma_{\chi_2\to O\epsilon}(q^2) = \frac{f_\chi^{\,2}}{4\pi}\,\frac{\mathcal{P}^5}{15M_\chi^{\,4}}\,, \tag{5.13}$$

$$\Gamma_{\chi_1\to O\epsilon}(q^2) = \frac{f_\chi^{\,2}}{4\pi}\,\frac{\mathcal{P}^3}{3M_\chi^{\,2}}\left(1+\frac{p_0}{M_\chi}\right)^2, \tag{5.14}$$

$$\Gamma_{\chi_0\to O\epsilon}(q^2) = \frac{f_\chi^{\,2}}{4\pi}\,\frac{\mathcal{P}}{24M_\chi^{\,4}}[\mathcal{P}^2+3(M_\chi+p_0)^2]\,, \tag{5.15}$$

where $\mathcal{P}$ is the magnitude of the final three-momentum and p_0 is the energy of $O(2.8)$ in χ rest frame. In order that $\Gamma_{\mathrm{tot}}(\chi_0) \sim 629$ keV, based upon the observed branching ratio $B_{\gamma J}(\chi_0)$ and the $S_{\mu\nu}F_{\mu\nu}$ coupling, we can choose

$$\frac{f_\chi^{\,2}}{4\pi} \sim 2.5\times 10^{-3}\,, \tag{5.16}$$

so that $\Gamma(\chi_0 \to O + 2\pi) \sim 484$ keV. This coupling constant is also the order of magnitude of the $\psi' J\epsilon$ coupling constant.

With these hadronic decays as auxiliary input, the structure of our Tables I and II is clear. To indicate the size of the other hadronic decays of the χ's in our scheme, we have listed in Table II specific channels observed at SPEAR.[10]

VI. COMMENTARY

A. Mass splittings in an O(4) multiplet

In strict O(4) symmetry, all the members of a given nth multiplet are degenerate in mass. They belong to the $M \equiv J_{\min} = 0$ representation of O(4). In group-theoretical language, they belong to the set of states for which $\vec{J}\cdot\vec{R}=0$, $\vec{J}, \vec{R}$ being the two commuting SU(2) generators of O(4).

In order that, under mass breaking, no new ($\vec{J}\cdot\vec{R}\neq 0$) states appear, it is clear that we should require that M^2 operator commute with $\vec{J}\cdot\vec{R}$. An example of a mass-breaking operator which lifts the parent-daughter degeneracy for given n is

$$\Delta M^2 = c_n(\vec{R}^2+2)\vec{J}^2\,, \tag{6.1}$$

where we have identified $\vec{J}$ as the total angular momentum of the state. This operator preserves the tracelessness condition ($\vec{J}\cdot\vec{R}=0$) since

$$[R^2, \vec{J}\cdot\vec{R}] = 0 = [J^2, \vec{J}\cdot\vec{R}]\,. \tag{6.2}$$

For the $n=2$ family this mass-splitting operator leads to the observed spacing rules

$$m^2(\chi_2) - m^2(\chi_1) = \tfrac{1}{2}[m^2(\chi_1) - m^2(\chi_0)]\,. \tag{6.3}$$

In principle, if we had a deeper commitment to the basic $q\bar{q}$ picture for the psions, the mass splittings will result from quark spin-orbit couplings, etc. But at the semiphenomenological level here, we merely take the mass splittings to be fine-structure effects summarized by ΔM^2 operators of the type of Eq. (6.1).

B. Psion $\not\to O(2.8)+\gamma$ selection rule

In strict O(4) symmetry, the coupling $S_{\mu\nu}F_{\mu\nu}$ forbids all psion radiative transitions *into* O(2.8). The question might be raised as to how good a selection rule it is, when O(4) is broken. If mass breaking is due entirely to an operator such as Eq. (6.1), the individual O(4) states remain pure under mass splitting and the selection rule is rigorously valid.

Since the decay of $J(3.1)\to\gamma+O\to\gamma\gamma\gamma$ has been observed with an over-all branching ratio of $\sim 10^{-4}$, it is clear that the selection rule cannot be rigorously valid, some configuration mixing must be present. A crude estimate for the amount of configuration mixing may be obtained as follows: Let δ be the (small) amount of $n=2$, $J=0$ state present in O, the ground state. Then[5]

$$\frac{J\to\gamma O\to\gamma\gamma\gamma}{J\to\mathrm{all}} = \delta^2 B(O\to 2\gamma) \sim 10^{-4}\,.$$

If we assume O branching ratio to be say, of order of tens of percent, then δ, the mixing, is of order of a few percent. In other words, for our scheme, we require O to be a narrow psion, unlike the η_c width of order of MeV.

VII. EXPERIMENTAL SIGNATURES

In this section we summarize the prominent experimental signatures that are unique to our new assignment for the psions.

(1) 1^{-+} Selection rules for χ_1 vs 1^{++} assignment. 1^{-+} assignment *forbids* its decay into $\pi K\bar{K}, \rho K\bar{K}$ while 1^{++} allows for it. Preliminary data from SPEAR on $\pi\pi K\bar{K}$ channel indicates an absence of χ_1 signal in that channel. Of course 1^{-+}, as well as 1^{++}, both forbid decays into $2\pi, K\bar{K}$.

(2) 0^{++} Assignment for $\psi(2.8)$ vs 0^{-+} assignment. The 2.8 psion, O, like other psions below the 4.0 region, is expected to be narrow in width. Lacking a fundamental psion-hadronic coupling scheme, we can make no definitive statements

about the relative branching ratios of O. We assume the 2γ decay mode of O to be one of the prominent decay modes perhaps of the order of tens of percent. The 0^{++} assignment allows for $\pi^+\pi^-, K^+K^-$ decays while 0^{-+} would, of course, forbid it.

Another distinguishing signature is the $\pi K\overline{K}$ decay channel. 0^{++} assignment *forbids* its decay into $\pi K\overline{K}$, while 0^{-+} assignment allows it.

(3) $\chi_0(3.41) \to O+2\pi$. We have conjectured that the large width for χ_0 could be due to the hadronic decay into $O+2\pi$, around 77%. Since O has been seen to decay into $p\overline{p}$ with branching ratio comparable to $O \to \gamma\gamma$, we expect a prominent signature for

$$\chi_0 \to p\overline{p}\,\pi^+\pi^-,$$

with $p\overline{p}$ peaking at 2.8 GeV$/c^2$. The branching ratio is given by

$$B(\chi_0 \to \overline{p}p\pi^+\pi^-) = 0.51 B(O \to p\overline{p}).$$

Similarly, we expect on the basis of simple isospin consideration the relation for the branching ratio

$$B(\chi_0 \to O\pi^+\pi^- \to \pi^+\pi^-\pi^+\pi^-) = 0.34 B(O \to \pi^+\pi^-).$$

The fact that the $4\pi^{\pm}$ decay of χ_0 is seen only at ~1.2% branching ratio, therefore, implies that, in our model, $B(O \to \pi^+\pi^-)$ is at the percent level.

VIII. CONCLUSION

Throughout this paper we have studiously avoided the fundamental question of what the constituents, if any, of the psions really are. This question will, apart from the obvious philosophical implications, bear on the mass-splitting operator for the psions, the configuration mixing for the physical psion states, and the strength of the violation of the selection rules due to $S_{\mu\nu}F_{\mu\nu}$. It is fortunate for us that these mixings and splittings are fine-structure effects and, therefore, at the phenomenological level not so important for this first go-around. But having demonstrated the viability of this scheme of classification of psions, we must face up to this question.

Previous attempts at identification of psions with linear Regge trajectories have involved the so-called "ring" states[20] in the dual-resonance model,[21] quarkless states that have Regge slopes exactly equal to $\frac{1}{2}$. Because of their identification with the Pomeron, the spectrum of ring states extends all the way from zero to the observed psion mass range. Furthermore, only even Regge trajectories were involved. Perhaps the ring states should not be identified with the Pomeron, in which case they could be anchored at 2.8 (GeV$/c^2$).[22] An open question in this approach is the explanation for an apparent threshold effect in R around the 4.0-GeV region.

An alternative interpretation more akin to the spirit of the nonrelativistic charmonium picture is to insist on an O(4) quark dynamics, and let the psions be O(4) $q\overline{q}$ bound states. Obviously, these would have to be fully relativistic bound states. If we take the naive version of a zero-spin "constituent" model, then the physical interpretation for κ is g/m, where g is the Landé g factor. If $g=2$, the fitted value for κ would imply a mass for the "quark" to be around 640 MeV$/c^2$. It is an open question as to what the mass for a spin-$\frac{1}{2}$ quark in an O(4) scheme would have to be for this fit to linear trajectories to work.

Be that as it may, the really crucial issue at hand, it seems to us, is how well do further experimental data support our ideas. To this end, the determination of the parity of O psion at 2.8 GeV$/c^2$, and of the χ_1 psion at 3.49 GeV$/c^2$ would clearly be of the utmost importance.

ACKNOWLEDGMENT

It is a pleasure to thank Professor Drell for his warm hospitality at SLAC where much of this work was done. One of us (N.P.C.) would like to thank the Aspen Center for Physics for the stimulating atmosphere that led to this work. We would also like to thank our colleagues at City College and at SUNY-Binghamton for stimulating discussions.

*Work supported in part by the U. S. Energy and Development Administration.

†Work supported in part by a grant from the CUNY Research Foundation.

‡Work supported in part by a State University of New York JAC/UAC Award.

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