Leptonic decays of mesons in an approximation to the MIT bag model*

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The MIT bag model is used to compute certain electromagnetic and weak leptonic decay rates of the pseudoscalar and vector mesons. Because of the limitations inherent in the cavity approximation we find that generally the results are not in good agreement with experiment.

I. INTRODUCTION

In this work we have used the MIT bag model[1] in the cavity approximation to calculate various weak and electromagnetic decay widths for vector and pseudoscalar mesons. The quark fields are confined by a c-number boundary to the interior of a static sphere. The linear boundary condition determines the frequencies of the quark modes and the nonlinear boundary condition fixes the radius of the spherical bag by requiring that the energy be minimized.[2]

Recently, this simple version of the bag model has been applied to hadron spectroscopy.[3] DeGrand *et al.* also confined gluons to the bag, which, by their interactions with the quarks, provided hyperfine splitting in the SU(3) spectrum. Particular values of the quark masses, bag constant B, gluon coupling constant, and zero-point energy produced good results for many hadron states. In computing the bag decay widths we have used these same values of quark masses and bag radii.

Even with cavity wave functions, approximations are necessary to arrive at numerical results for the bag decay widths. The first approximation, that the boundary dynamics does not affect the decay widths, is unavoidable because the cavity approximation itself neglects the dependence of the boundary points on the fields inside the bag.[2] In the decays with no bag in the final state we need to compute the overlap between the vacuum and the empty bag state. In this work we compute as if this overlap were unity: however, in a more fundamental treatment of the boundary one might be able to compute this overlap directly.

Second, the energy δ function which results from using the cavity wave functions in the S matrix includes only the quark kinetic energy. The additional terms (bag energy BV, zero-point energy, gluon-exchange energy) which were important to the success of the bag spectroscopy are inserted by hand to give a phenomenologically correct energy-conservation δ function.

Finally, because momentum is not conserved in the cavity approximation,[4] the decay width for a particle at rest does not vanish for nonzero total momentum of the final-state particles. This dificulty forces us to make additional approximations. We assume that the momentum nonconservation only appears in the hadronic part of the amplitude and consequently, for a decay at rest, set the total momentum to zero elsewhere. This procedure will be further elaborated in the next section.

The remainder of this paper is organized as follows. In Sec. II we compute the partial widths for $\pi \to l+\nu_l$ and $K\to l+\nu_l$ and thereby determine the phenomenological coupling constants f_π and f_K. Here the approximations are first encountered and discussed. In Secs. III and IV we compute the partial widths for $V\to l^+ + l^-$ and $V\to P+\gamma$, with $V(P)$ representing various low-lying vector (pseudoscalar) meson states.

The work of Secs. II, III, IV may otherwise be organized into two categories: (i) the weak and electromagnetic "annihilation" decays in Secs. II and III in which there is no final-state hadron (bag), and (ii) the electromagnetic "spin-flip" decays in Sec. IV in which there is a final-state hadron (bag). It was the possibility of comparing the success of type-(i) calculations with that of type-(ii) calculations which originally led us to study meson and not baryon decays. Type-(i) reactions are likely to be more sensitive to the neglect of the dependence of the boundary points on the interior fields which is implicit in the cavity approximation.

Unfortunately, none of our numerical results correspond well with experiment in the examples we have been able to check. We attempt, in Sec. V, to sort out the reasons for this failure.

III. ANNIHILATION DECAYS—ELECTROMAGNETIC

In this section we study the reaction $P\to l+\nu_l$, where P is a pseudoscalar meson (either π or K) and l is a lepton (either e or μ). The steps written out are chosen in order to expose the peculiarities and assumptions involved in calculating with the cavity approximation to the bag model.

The interaction Lagrangian is

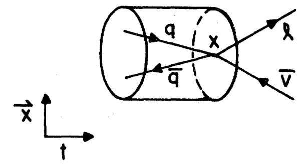

Fig. I

FIG. 1. Lowest-order graph for the weak leptonic decay of a meson.

$$L=\frac{G}{\sqrt{2}}\int d^3x\,\bar{q}_\alpha(x)\gamma_\mu(1-\gamma_5)(\cos\theta_C\tau^+_{\alpha\beta}+\sin\theta_C\sigma^+_{\alpha\beta})q_\beta(x)\bar{l}(x)\gamma^\mu(1-\gamma_5)\nu(x)+\text{H.c.}\,, \tag{2.1}$$

where $G=10^{-5}/M_{\text{proton}}{}^2$, θ_C is the Cabibbo angle, and $\tau^\pm$ ($\sigma^\pm$) are the SU(3) isospin (strangeness) raising and lowering operators. Notice that the unrenormalized vector and axial-vector coupling constants are equal in the above Lagrangian. This is because the ratio G_A/G_V for the *composite* hadrons is calculated within the theory from the fundamental quark fields.[5]

The decays $P\to l+\nu_l$ can be represented by the diagram in Fig. 1, in which the circle means that the vertex X ranges only over the interior of the decaying bag. The amplitude is

$$S_{fi}=\frac{G}{\sqrt{2}}\begin{Bmatrix}\cos\theta_C\\ \sin\theta_C\end{Bmatrix}\int_{V_{\text{bag}}}d^4x\langle l\nu|\bar{\nu}(x)\gamma_\mu(1-\gamma_5)l(x)|0\rangle_B\Big\langle 0\Big|\bar{q}_\alpha(x)\gamma^\mu(1-\gamma_5)\begin{Bmatrix}\tau^\pm_{\alpha\beta}\\ \sigma^\pm_{\alpha\beta}\end{Bmatrix}q_\beta(x)\Big|P\Big\rangle\,. \tag{2.2}$$

The upper (lower) line is appropriate to $\pi^\mp$ ($K^\mp$) decay. The subscript B on the hadronic vacuum state reminds us that when the quark and the antiquark annihilate into leptons, an empty bag is left. However, in our approximation we calculate as if ${}_B\langle 0|=\langle 0|$, the true vacuum.

When the wave functions are substituted, the unconfined time integral gives the energy conservation equation $\omega_q+\omega_{\bar{q}}=E_l+E_{\nu_l}$, where ω_q ($\omega_{\bar{q}}$) is the ground-state frequency of the confined quark (antiquark).[3] Following the discussion in Sec. I, we use the phenomenological energy conservation $M_P=E_l+E_{\nu_l}$ appropriate for a decay at rest.

Therefore,[6]

$$\begin{aligned}\Gamma_{P\to l+\nu_l}&=\sum_{s_l,s_{\nu_l}}\int\frac{Vd^3p_l}{(2\pi)^3}\int\frac{Vd^3p_\nu}{(2\pi)^3}\frac{|S_{fi}|^2}{T}\\ &=\frac{G^2}{2(2\pi)^5}\begin{Bmatrix}\cos^2\theta_C\\ \sin^2\theta_C\end{Bmatrix}\sum_{s_l,s_{\nu_l}}\left|{}_B\Big\langle 0\Big|\sum_{\alpha,m,m'}b_a(m)\begin{Bmatrix}\tau^\pm_{ab}\\ \sigma^\pm_{ab}\end{Bmatrix}d_b(m')\Big|P\Big\rangle\right|^2\\ &\qquad\times\int d^3p_l d^3p_\nu\,\delta(M_P-E_l-E_{\nu_l})\,|J^{\text{leptonic}}_{\mu\,(1-5)}J^{\mu\ \text{hadronic}}_{(1-5)}|^2\,,\end{aligned} \tag{2.3}$$

where

$$J^{\text{leptonic}}_{\mu\,(1-5)}\equiv\left(\frac{m_lm_\nu}{E_lE_\nu}\right)^{1/2}\bar{u}(p_l,s_l)\gamma_\mu(1-\gamma_5)v(p_\nu,s_\nu) \tag{2.4}$$

as usual, and, using the cavity wave functions ψ with their normalizations N,[4]

$$J^{\text{hadronic}}_{\mu\,(1-5)}\equiv N_qN_{\bar{q}}\int_{V_{\text{bag}}}d^3x\,e^{-i(\vec{p}_l+\vec{p}_{\nu_l})\cdot\vec{x}}\,\bar{\psi}_{-1,1,m'}(\vec{x})\gamma_\mu(1-\gamma_5)\psi_{1,-1,m}(\vec{x})\,. \tag{2.5}$$

When the matrix element is simplified by using the semiclassical quark field operators $b_a(m)$, $d_a(m)$ to represent the bag state $|P\rangle$ we are left with

$$\Gamma_{P\to l+\nu_l}\equiv\int d^3P_{\text{tot}}\,\Gamma(\vec{P}_{\text{tot}}) \tag{2.6}$$

after the integration over the relative momentum $\frac{1}{2}(\vec{p}_{\nu_l}-\vec{p}_l)$ of the outgoing leptons has been performed. Of course $|\vec{P}_{\text{tot}}|$ is constrained by energy conservation, but because momentum conservation is violated in the cavity approximation the final-state leptons need not have $|\vec{P}_{\text{tot}}|=|\vec{P}_l+\vec{P}_{\nu_l}|=0$. We must, however, hope that most of $\Gamma_{P\to l+\nu_l}$ comes from $|\vec{P}_{\text{tot}}|$ small compared to M_P if this model calculation is to have any resemblance to the physical decay. We therefore proceed by setting $|\vec{P}_{\text{tot}}| \simeq 0$ in $\delta(E_f - E_i)$ and in $J^{\text{leptonic}}_{\mu\,(1-5)}$ but not in $J^{\text{hadronic}}_{\mu\,(1-5)}$. By this expedient, we keep all quark contributions to the decay and are able to obtain a numerical result for $\Gamma_{P\to l+\nu}$ without fitting parameters. This approximation further recommends itself because only in this way are the conventional kinematic factors obtained.

Now, after standard simplifications, the width can be written as

$$\Gamma_{P\to l+\nu_l} = \frac{G^2\begin{Bmatrix}\cos^2\theta_C\\ \sin^2\theta_C\end{Bmatrix}}{8\pi} M_P\, m_l^2\left(1-\frac{m_l^2}{M_P^2}\right)^2 f_P^2\,, \tag{2.7}$$

with

$$f_\pi^2 = M_\pi^2\,\frac{\dfrac{x^3}{(M_\pi R_\pi)^3\pi}\displaystyle\int_0^x dy\, y^2[j_0^2(y) - j_1^2(y)]^2}{\left[\displaystyle\int dy\, y^2[j_0^2(y)+j_1^2(y)]\right]^2}\,, \tag{2.8}$$

where $x = 2.04$ is the ground-state frequency of the nonstrange quark. The expression for f_K is similar except for the slight complication caused by the massive strange quark.

Numerically, we find

$$f_\pi^2 = 3.2M_\pi^2\,,\quad f_K^2 = 2.25M_\pi^2\,,$$

compared to the experimental values $f_\pi^2 = (0.87\pm0.01)M_\pi^2$ and $f_K^2 = (2.53\pm0.02)M_\pi^2$. Notice from (2.8) that if R_π were increased by about 50%, f_π^2 would then be in substantial agreement with its experimental value. R_π quoted in Ref. 3 is one of the smallest and least reliable radii computed by these authors, and, in general, the radii are only accurate to ±20%. It is unfortunate that f_π^2 and f_K^2 depend so sensitively on the radii.

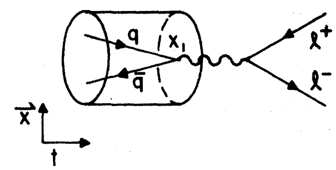

Fig.2

FIG. 2. Lowest-order graph for the electromagnetic decay of a meson.

II. ANNIHILATION DECAYS—WEAK

In this section we study reactions of the type $V\to l^+ + l^-$.[7] These reactions are represented by the diagram of Fig. 2 and the amplitude

$$S_{fi} = ie^2\sum_\alpha\left(\frac{e_\alpha}{e}\right)\int_{V_{\text{bag}}} d^4x_1\int d^4x_2\int\frac{d^4k}{(2\pi)^4}\,\frac{e^{-ik\cdot(x_2-x_1)}}{k^2+i\epsilon}\,\langle l^+l^-|\bar{l}(x_2)\gamma^\mu l(x_2)|0\rangle_B\langle 0|\bar{q}_\alpha(x_1)\gamma_\mu q_\alpha(x_1)|V\rangle\,. \tag{3.1}$$

The four-momentum conservation at the leptonic vertex and energy conservation at the hadronic vertex give

$$\begin{aligned}\Gamma_{V\to l^+l^-} &= \tfrac{1}{3}\sum_{s_V, s_{l^-}, s_{l^+}}\int\frac{Vd^3p_{l^+}}{(2\pi)^3}\int\frac{Vd^3p_{l^-}}{(2\pi)^3}\,\frac{|S_{fi}|^2}{T}\\ &= \frac{\alpha^2}{6\pi^3}\sum_{s_V, s_{l^+}, s_{l^-}}\left|{}_B\!\left\langle 0\left|\sum_{\alpha,m,m'}\left(\frac{e_\alpha}{e}\right)b_\alpha(m)d_\alpha(m')\right|V, S_V\right\rangle\right|^2\\ &\quad\times\int d^3p_{l^+}\int d^3p_{l^-}\,\delta(E_f - E_i)\left|J_\mu^{\text{leptonic}}\,\frac{1}{M_V^2 - (\vec{p}_{l^+}+\vec{p}_{l^-})^2}\,J^{\mu\ \text{hadronic}}\right|\end{aligned} \tag{3.2}$$

with

$$J_\mu^{\text{leptonic}} \equiv \left(\frac{m_l^2}{E_{l^+}E_{l^-}}\right)^{1/2}\bar{u}(p_{l^-}s_{l^-})\gamma_\mu v(p_{l^+}s_{l^+}) \tag{3.3}$$

as usual, and

$$J_\mu^{\text{hadronic}} \equiv N_q^2\int_{V_{\text{bag}}} d^3x\, e^{-i(\vec{p}_{l^+}+\vec{p}_{l^-})\cdot\vec{x}}\,\bar{\psi}_{-1,1,m'}(\vec{x})\,\gamma_\mu\psi_{1,-1,m}(\vec{x}) \tag{3.4}$$

from the bag model.

Following the reasoning in Sec. II we set the total momentum $\vec{P}_{l^+}+\vec{P}_{l^-}$ equal to zero everywhere except in J_μ^{hadronic}, and after summing over the final-state spins we obtain

$$\Gamma_{V\to l^+l^-}=\frac{8\alpha^2 M_V}{3\pi}\left\{\frac{\pi^2}{3M_V{}^3}\left|{}_B\left\langle 0\left|\sum_{\alpha,m,m'}\left(\frac{e_\alpha}{e}\right)b_\alpha(m)d_\alpha(m')\right|V,S_V\right\rangle\right|^2\right.$$
$$\left.\times\sum_{S_V}\int_{V_{\text{bag}}}d^3x[\bar{\psi}_{-1,1,m'}(\vec{x})\vec{\gamma}\psi_{1,-1,m}(\vec{x})][\bar{\psi}_{-1,1,m'}(\vec{x})\vec{\gamma}\psi_{1,-1,m}(\vec{x})]^\dagger\right\}. \tag{3.5}$$

Finally, when the sum over the spin states of the bag has been performed, our explicit result is

$$\Gamma_{V\to l^+l^-}=\left(\frac{8\alpha^2 M_V}{3\pi}\right)C_V{}^2\psi^2\,, \tag{3.6}$$

in which $C_V{}^2$ is the SU(3) coefficient and

$$\psi^2\equiv\frac{\pi}{2}\frac{x^3}{(M_V R_V)^3}\frac{\int_0^x dy\,y^2[j_0{}^4(y)+f^4 j_1{}^4(y)+\frac{2}{3}f^2 j_0{}^2(y)j_1{}^2(y)]}{[\int_0^x dy\,y^2(j_0{}^2(y)+f^2 j_1{}^2(y))]^2} \tag{3.7}$$

expresses our knowledge of hadronic wave functions. f is a normalization for massive-quark wave functions that varies from 1 to 0 as the quark mass varies from 0 to ∞. The results are summarized in Table I.

IV. SPIN-FLIP REACTIONS

In this section we study the reactions $V\to P+\gamma$, where V (P) is any vector (pseudovector) meson for which this reaction is allowed energetically. This single-vertex process is represented by the diagrams in Fig. 3, in which the photon is emitted alternately by the confined quark or antiquark.

In Coulomb gauge, with the usual choice of bag fields, the amplitude is

$$S_{fi}=\frac{ie}{(2\omega_\gamma V)^{1/2}}2\pi\,\delta(E_f-E_i)\sum_{\alpha,m,m'}\left(\frac{e_\alpha}{e}\right)\langle P|\vec{J}_q^{\text{hadronic}}\,b_\alpha^\dagger(m)b_\alpha(m')+\vec{J}_{\bar{q}}^{\text{hadronic}}\,d_\alpha(m)d_\alpha^\dagger(m')|V\rangle\vec{\epsilon}(k,\lambda)\,, \tag{4.1}$$

where

$$\vec{J}_q^{\text{hadronic}}\equiv N_V N_P\int_{V_c}d^3x\,e^{-i\vec{k}\cdot\vec{x}}\bar{\psi}^{(P)}_{1,-1,m}(\vec{x})\vec{\gamma}\psi^{(V)}_{1,-1,m'}(\vec{x}) \tag{4.2}$$

and

$$\vec{J}_{\bar{q}}^{\text{hadronic}}\equiv N_V N_P\int_{V_0}d^3x\,e^{-i\vec{k}\cdot\vec{x}}\bar{\psi}^{(V)}_{-1,1,m}(\vec{x})\vec{\gamma}\psi^{(P)}_{-1,1,m'}(\vec{x})\,. \tag{4.3}$$

It is necessary to distinguish the initial- and final-state bag wave functions and their normalizations by the superscripts "P" and "V" to allow for differing radii of the initial-and final- state bags. Accordingly, the region V_0 of the overlap integral is ambiguous. This point will be resolved shortly. Notice that if we did not patch up the energy δ function in these reactions, then, since V and P are members of the same SU(6) multiplet, no energy would be available for the reaction to proceed.

TABLE I. Summary of the electromagnetic decays of the vector mesons.

Reaction	$C_V{}^2$	ψ	Γ_{pred} (keV)	Γ_{exp} (keV)
$\rho^0\to e^++e^-$	$\frac{1}{2}$	0.085	1.5	8.2–14.8
$\omega\to e^++e^-$	$\frac{1}{18}$	0.085	0.16	0.55–0.96
$\phi\to e^++e^-$	$\frac{1}{9}$	0.053	0.3	1.2–1.5
$\psi\,(J)\to e^++e^-$	$\frac{4}{9}$	0.010	0.6	~4.8

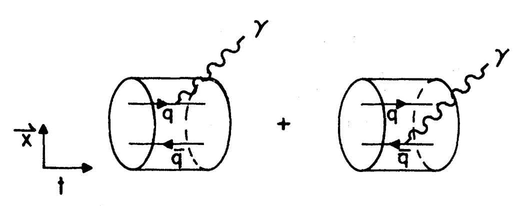

Fig. 3

FIG. 3. Lowest-order graphs contributing to the spin-flip decay.

TABLE II. Summary of the electromagnetic spin-flip decays of the vector mesons.

Reaction	$\Gamma_{\rm pred}$ (keV)	$\Gamma_{\rm exp}$ (keV)	$\Gamma_{\rm VR\text{-}W}$ (keV) (Ref. 7)
$\omega^0 \to \pi^0 + \gamma$	300	790–970	1170
$\omega^0 \to \eta + \gamma$	5.4	<50	6.3
$\rho^0 \to \eta + \gamma$	37.4	...	50
$\phi \to \eta + \gamma$	31.7	76–180	...
$\psi(J) \to \eta_c(2800) + \gamma$	4.9–12.8	...	...

It is easy in this case to use the energy δ function to convert Eq. (4.1) to a partial width:

$$\Gamma_{V \to P+\gamma} = \tfrac{1}{3} \sum_{\lambda, S_V = \pm 1, 0} \int \frac{V d^3k}{(2\pi)^3} \frac{|S_{fi}|^2}{T}$$

$$= \frac{4\alpha}{3} \frac{k_0^3}{M_V^2} \left\{ \frac{1+(M_P/M_V)^2}{[1-(M_P/M_V)^2]^2} \right\} f(V,P,R_0) \sum_{\lambda, S_V} |\langle P|\mathfrak{M}|V\rangle|^2 , \tag{4.4}$$

where $k_0 \equiv (M_V^2 - M_P^2)/2M_V$ is the energy of the outgoing photon, and

$$f(V,P,R_0) \equiv \frac{x_V^3}{x_P^3} \frac{R_0^6}{R_V^3 R_P^3} \left\{ \int_0^{x_P} dy\, y^2 \left[f_V j_1\!\left(\frac{R_0}{R_V}\frac{x_V}{x_P} y\right) j_0\!\left(\frac{R_0}{R_P} y\right) + f_P j_0\!\left(\frac{R_0}{R_V}\frac{x_V}{x_P} y\right) j_1\!\left(\frac{R_0}{R_P} y\right) \right] j_1\!\left(\frac{kR_0}{x_P} y\right) \right\}^2$$

$$\times \left\{ \int_0^{x_P} dy\, y^2 [j_0^2(y) + f_P^2 j_1^2(y)] \right\}^{-1} \left\{ \int_0^{x_V} dy\, y^2 [j_0^2(y) + f_V^2 j_1^2(y)] \right\}^{-1} . \tag{4.5}$$

We have evaluated $f(V,P,R_0)$ by varying R_0 within the range $R_P \leq R_0 \leq R_V$. Some rather *ad hoc* procedure of this sort seems unavoidable in the context of our simple model. The calculated $\Gamma_{V \to P+\gamma}$ for ω, ρ^0, ϕ decay, listed in Table II, occur when $R_0 = R_V$ and are the largest partial widths obtainable by this method. [For $\psi(J) \to \eta_c + \gamma$, we list the range of possible widths obtained by varying R_0.] Because the experimental widths for several of these reactions are not well established, we also list the results based on the nonrelativistic quark model for comparison.[8]

V. DISCUSSION

Our numerical results on both annihilation decays and spin-flip decays were not good, but perhaps for different reasons. The calculations of Secs. II and III were more complicated, and $|\vec{P}_{\rm tot}| \simeq 0$ for the outgoing particles was assumed in order to simplify them. We believe that this approximation was well motivated, and furthermore that it is necessary if the cavity approximation is not to be rejected out of hand. Even so, one might argue that since the quarks are confined to a cavity of definite radius R, total momenta as large as $\sim 1/R$ for the outgoing particles might contribute to the partial widths. It is difficult to see how the widths in both Sec. I and Sec. II can be substantially improved this way. So we are left to conclude that dynamical effects of the boundary must be included in more realistic bag model calculation of these decays.

This conclusion is reinforced when we examine Sec. IV. Although the numerical results are also unimpressive, Sec. IV is not plagued by the same approximations inherent in Secs. II and III. But there is a new problem. We have, for lack of an alternative, used the wave functions of a static sphere to describe the quark fields in the outgoing meson. This is suspect when the velocity of the outgoing meson is large, and indeed might account for the discrepancies in Table II. For $\omega^0 \to \eta + \gamma$, $\rho^0 \to \eta + \gamma$, and $\psi(J) \to \eta_c + \gamma$, the kinetic energy of the pseudoscalar is less than its mass and therefore the results should be in reasonable agreement with experiments. Unfortunately, the experimental rates for these reactions have not yet been determined, and our explanation of Table II remains speculative. On the other hand, in the reaction $\omega^0 \to \pi^0 + \gamma$ the pion is by necessity highly relativistic, and so the static, spherical approximation breaks down. A solution of the MIT bag model equations for a hadron with nonzero total momentum has not yet been found; phenomenological approximations to such a solution would only introduce additional (undetermined) parameters into the calculation of the spin-flip decay.[9]

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[2]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974). Good results were obtained for matrix elements near zero momentum transfer.

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[4]R. L. Jaffe, Phys. Rev. D 11, 1953 (1975). The problem of momentum nonconservation in the cavity approximation for deep-inelastic electroproduction is discussed in Sec. III.

[5]J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), pp. 53–55; see also Ref. 2 above.

[6]In the following formula, the (unbounded) normalization volume V of the leptons is not to be confused with the finite volume V_{bag} to which the quarks are confined.

[7]The assumptions made here have already been pointed out in Sec. II.

[8]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50, 617 (1967).

[9]We have recently learned of another attempt to calculate weak and electromagnetic decay widths of hadrons using the cavity approximation to the bag model: John F. Donoghue, E. Golowich, and Barry R. Holstein, Phys. Rev. D 12, 2875 (1975).