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Speculations regarding a scalar partner of the hypothetical *O* particle*

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Decays of ψ are studied in a resonance-domination model. The Okubo-Zweig-Iizuka rule is realized dynamically by the propagation of quarkless mesons. Free parameters in the model are fixed by using available experimental data. Then a number of specific predictions are made, which test the dynamical scheme. In particular, suggestions for where to look for the quarkless mesons as resonant enhancements in spectra are discussed quantitatively.

I. INTRODUCTION

Most currently popular dynamical models of hadrons employ color-quark and color-gluon degrees of freedom, for a variety of reasons. Three specific models seem to allow the possibility of quarkless, all-gluon matter existing on equal footing with normal quarkful matter[1–3]: the string, the bag, and the lattice. Dual dynamical models enjoy two topologically inequivalent "chemical" structures—open strings, associated with ordinary hadrons, and closed strings, which carry no quark end points. A similar situation obtains on a lattice. In the case of the bag, one simply has an option as to what to put inside, provided it is color singlet. The models cited could support quarkless low-lying vector and scalar states in the 1- to 3-GeV mass region.

One way to produce quarkless states in any of these models is to annihilate quarks with antiquarks. Recently,[1] Freund and Nambu, and independently Clavelli, have explored the possibility that when such annihilations occur, as in OZI (Okubo-Zweig-Iizuka[4])-rule–violating meson decays, the hypothetical quarkless resonances actually dominate the dynamics.

In this paper, the Freund-Nambu-Clavelli (FNC) dynamical mechanism will be expanded and developed further, and applied to the phenomenology of ψ decays. It must be admitted that there are few firm theoretical results on which to base a study of quarkless states (q.s.'s), so many speculative assumptions are required if progress is to be made. The principal ones are:

(1) The ψ is a bound state of some "new" species of $q\bar{q}$, and couples directly to "ordinary" hadrons only by $q\bar{q}$ annihilation, thus violating the OZI rule. Evidence that ψ is an ordinary SU_3 singlet has been given in Ref. 5.

(2) The blank spaces which appear in Harari-Rosner pictures of OZI-rule–violating decays, as in Fig. 1, are to be filled in by propagators for q.s.'s. Two candidates for such states are studied in this paper. The color singlet, flavor[6] singlet, 1^{--} O_V particle of FNC (which may be considered to be a bound state with the quantum numbers of three color gauge vector gluons) will be assigned $M_{O_V} = 1.4$ GeV, $\Gamma_{O_V} = 50$ MeV, based on arguments presented in Ref. 1. In a Regge-trajectory picture such as was used in that reference, an accompanying 0^{++} q.s. O_S, should be approximately degenerate with O_V. A value $M_{O_S} = 2$ GeV will be taken for purposes of illustration. Motivations for this value will be discussed in Sec. II, along with estimates for its width. There is no denying, however, that one is basically guessing.

(3) The O_V and O_S give access to examining several, though by no means all, of the decay modes of ψ, provided that in $\psi \to \phi\pi^+\pi^-$, $\psi \to \omega\pi^+\pi^-$, e.g., the $\pi^+\pi^-$ are in S wave. Guided by observations[5] that $\psi' \to \psi 2\pi$ does occur with the π's in S wave, and with the (2π) system in S wave relative to ψ, it will be assumed that in $\psi \to VPP$ decays, the two pseudoscalars are in a relative S wave. The PP resonate at ordinary quark-model excited 0^{++} states, such as $\epsilon(660)$, $S^*(993)$, and $\kappa(1300)$. Assuming the ϵ, S^* are analogous to ω, ϕ, and so contain flavor singlet components, these couple directly to O_S. This is described in full in Sec. II.

The most drastic part of the scalar dominance assumption is not with ϵ and S^*, however. It is in the $\psi \to K^*(890)K\pi$ decays. The present experimental reports list the $K\pi$ as resonating in the tensor $K^*(1420)$ mass region. It is the author's understanding that this identification is really around (1420± several hundred MeV); and that there is no analysis to show the $K\pi$ are really in

FIG. 1. Conventional Harari-Rosner picture for OZI-rule violation. This could represent $\phi \to \omega$, so "a" are strange quarks, "b" are usual nonstrange quarks. For $\psi \to \omega$, "a" could be charmed quarks.

d wave.[7] Presumably the main reason for calling the enhancement $K^*(1420)$ is that this tensor particle is much better established than the evanescent scalar κ, currently believed to have a mass of ≈ 1300 MeV. Until the true experimental situation is clarified, it will be assumed that part of the $K\pi$ in $\psi \to K^*(890)K\pi$ can actually be scalar $\kappa(1300)$, in addition to tensor $K^*(1420)$. A fuller discussion of the relevance of this assumption is given in Sec. III.

(4) A double OZI-rule–violating decay, such as $\psi \to \phi\pi^+\pi^-$, could occur via the diagram of Fig. 2. This diagram involves form factors $F_{\psi O_S}(\sigma_1^2)$, $F_{\psi O_V}(\sigma_2^2)$, $F_{\phi O_V}(\sigma_2^2)$, and $F_{\epsilon O_S}(\sigma_1^2)$ in addition to the propagators already discussed. An indispensable assumption is that all these form factors are slowly varying, and can be approximated by constants.

It is also possible to draw a direct $(\psi O_S O_V)$ vertex, as in Fig. 3(c). If that coupling is comparable to, say $g_{\psi O_V}$ or $g_{\epsilon O_S}$, the diagram will dominate that of Fig. 2 because Fig. 2 is suppressed by an extra propagator. To hold the proliferation of coupling constants down, it will be assumed that sequential emissions of q.s.'s by the same particle are heavily suppressed, so only Fig. 3(c) needs to be considered. This is not one of the crucial assumptions, but the magnitude of $g_{\psi O_S O_V}$ turns out to be consistent with this assumption.

Section II contains the determination of parameters for the O_S, based on the decay rates of ϵ and S^*. In Sec. III, the determination of parameters for O_V is reviewed quickly, and the coupling parameters involving O_S and O_V are obtained. Several predictions for OZI-rule–violating decays via O_V and O_S are then made. The most interesting predictions involve the possibility of actually observing these particles as resonant peaks in invariant mass distributions: O_V in K^+K^- and O_S in $\pi^+\pi^-$ for the decay $\psi \to K^+K^-\pi^+\pi^-$.

Unfortunately the fitting of so many parameters feels like the most gross form of phenomenology. It has been undertaken with two hopes in mind—first, that the dynamical scheme as whole is workable, yielding verifiable predictions, and

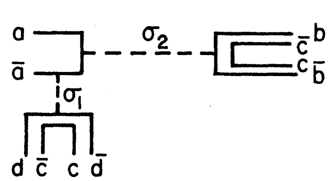

FIG. 2. A double OZI-rule violating decay, as in $\psi \to \phi\pi^+\pi^-$. Here "*a*" would be charmed quarks, "*d*" strange quarks, and "*b*" and "*c*" would be ordinary nonstrange quarks. The σ_i are the invariant masses going to each new pair.

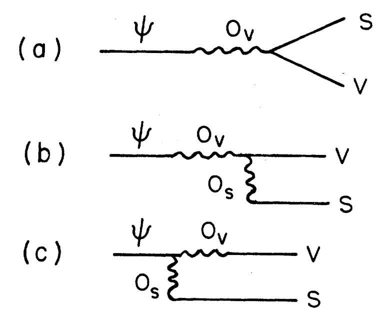

FIG. 3. Couplings of *O* particles to ψ and to ordinary hadrons considered in the paper. Here *S* is a Lorentz scalar channel, *V* a vector channel.

perhaps culminating the actual observation of a totally new form of matter that many theories lead us to expect. And secondly, one hopes that the parameters so obtained may give some insight useful for the construction of a *genuine* theory of bound gluons.

Section IV summarizes the results and contains some speculations. An appendix has been included, in which several useful kinematic formulas are compiled. Also the actual machine-evaluated values for various matrix elements have been tabulated to facilitate modifications in predictions as the experimental values on which these are based become more precisely known.

II. SCALAR BOSONS

A. Quark-model scalars

The standard quark model allows for the existence of ${}^3P_0(C=+,\ P=+)$ scalar states. Experimentally, if such states occur at all, they are observed as broad enhancements detectable through phase-shift analyses in $\pi\pi$, KK, and $K\pi$ channels, and usually not as sharp resonances. This leads to considerable uncertainty as to what "masses" and "widths" they should be assigned in a conventional particle treatment.

Based on the information available in Ref. 8, the $I=0$, $J=0$ $\pi\pi$ enhancement ϵ will be taken to have $M_\epsilon = 660$ MeV, and a width of 640 MeV. With such an incredibly large width, the propagator of the ϵ will not be a simple Breit-Wigner, so the simplest finite-width correction will be employed. See below.

An $I=0$, $J=0$ KK enhancement, $S^*(993)$, is also reported. Its reported width is only 40 MeV. This suggests that $\epsilon(660)$ and $S^*(993)$ are members of the same nonet of scalar mesons, paralleling the nonet structure of 1^{--} vectors. At the ideal mixing level, S^* is the scalar analog of ϕ (i.e., pure $\lambda\bar{\lambda}$), and ϵ is the scalar version of ω. Then S^* should decay *only* into $K\bar{K}$ if the OZI rule is respected. But $2M_K = 988$ MeV. Thus, there is virtually no phase space for the decay. That explains why S^* is so narrow.

The O_S particle allows the OZI rule to be violated. This opens the $S^* \to \pi\pi$ channel. Now, however, the available phase space is respectable. To keep the S^* partial width from being too large, the coupling of S^* to O_S must be small. Thus, the situation parallels the envisioned mechanisms for $\phi \to \rho\pi$ decay. These remarks will be quantified in Sec. II C.

At this point, before coming to $\kappa(1300)$, it is useful to discuss an aborted theoretical argument. The argument was initially motivated by gauge-field-theory models, especially by lattice realizations of these. Since an $L=1$ quark-model state contains momentum operators, gauge invariance demands it must also contain gauge field operators. Lattice models suggest a radical implementation of this requirement[9]: A state of this type must be described as a $q\bar{q}$ pair linked by a unit of concentrated gluon flux. (This has an analog in string models, where $L=1$ states have a unit of transverse oscillator excitation.)

If the tube should be counted as a unit of gluon flux, and gluon flux valence should be treated on equal footing with quark valence number, then ϵ and S^* should have zero projection on $q\bar{q}$ states with zero valence gluon flux. To give an example, the decay $\psi \to \omega\ (\epsilon \to \pi^+\pi^-)$ would only proceed through diagrams 3(b) and 3(c), and not through Fig. 3(a).

An immediate prediction of this scheme is that in *all* direct $\psi \to V(PP)$ decays, V and (PP) must separately have SU_3-singlet components. This is immediately ruled out by the observation of $\psi \to K^*(890)K\pi$ decays. Thus, the coupling of Fig. 3(a) must be present, and, what is more important, consistency with the scalar-dominance assumption for $\pi^+\pi^-$ in $\psi \to (\omega \text{ or } \phi)\ \pi^+\pi^-$ demands that we consider the scalar $K\pi$ resonance.

If one accepts ϵ, S^*, and the quark model, the κ *should* be present. The problem is that, experimentally, the κ is in even worst shape than the ϵ. From Ref. 8 one can arrive at a best estimate for its mass as 1300 MeV, with a width of ≈ 400 MeV. This unfortunate mass value makes the problem of actually identifying it from ψ decays very hard, since there is a better established tensor $K^*(1420)$, as was discussed in Sec. I. The discussion of how $\kappa(1300)$ is used appears in Sec. III.

B. O_S particle

Perhaps the best way to introduce a value for the O_S mass is to briefly review the motivations for O_V presented by FNC. There the basic idea was that in dual models, the Pomeron may be thought of as a closed string, with no q or $\bar{q}$ end points. The Pomeron trajectory has daughters, and among the particles supported by the daughter trajectories are degenerate 1^{--} and 0^{++} states. If the Pomeron trajectory has a slope equal to $\frac{1}{2}$ the normal trajectory slope, these particles should have masses of 1.4 GeV. Thus, while bag and lattice models also give theoretical predictions for these particles' masses, the Regge argument actually tries to incorporate phenomenology into the prediction. For this reason, this value will be used for the O_V mass.

In addition, Freund and Nambu estimated a total width of 50 MeV for the O_V. While this value was based on an overestimate of the $\psi \to \omega \to \rho\pi$ partial width, they did not consider other open channels for O_V decay, such as that of Fig. 3(a). It will simply be assumed that these competing tendencies average out so that 50 MeV remains a reasonable value for the O_V width. This makes it a typical vector-meson width in order of magnitude. The whole point of the OZI rule is that the coupling of q.s.'s to quarks is *small*, so if anything 50 MeV may be an overestimate.

Now, the scalars and vectors that are degenerate in an ideal Regge scheme never turn out to be degenerate in practice. The ϵ and S^* are lighter than their supposed vector partners. On the other hand, if the κ is being interpreted correctly, its mass is larger than its vector partner's. There is, in practice, no rule of thumb. Consequently, any value chosen for the O_S mass (M_O) serves for purposes of illustration only. If a different value is used, various coupling constants which are determined by fitting data acquire different values. The results of Nambu and Freund indicate that this works out to give factor-of-2 changes in subsequent predictions, but no order-of-magnitude effects. Consequently, while ideally a number of values for M_O should be tested, practical considerations constrain us to do our analysis with a single value for M_O—it is not expected that small changes in this value will give order-of-magnitude changes in the over-all results.

Searches by Aubert *et al.*[10] for resonances in two-body channels could by interpreted as placing an upper bound of 2 GeV for the mass of O_S. The experiments that observed ϵ, S^*, $E(1420)$ and κ show no evidence for other scalar enhancements around the 1-GeV region. If M_0 is taken to be 1.5 to 1.8 GeV, the invariant mass spectra of K^+K^- or $\pi^+\pi^-$ in $\psi \to K^+K^-\pi^+\pi^-$, e.g., might be artificially enhanced by nearly simultaneous resonances in the vector and scalar arms—that might be the true situation, but a more conservative attitude will be taken. A value $M_O = 2$ GeV will be used, which is at the edge of being ruled out by Aubert *et al.*, but which gives no double quarkless-resonance enhancement in ψ decays.

C. OZI-rule–violating decay of S^*

Consider the coupling scheme summarized by

$$\mathcal{L}^{\rm eff} = g_{SPP}[\epsilon(\pi_0{}^2 + K^0\bar{K}^0 + 2\pi^+\pi^- + K^+K^-) - \sqrt{2}S^*(K^0\bar{K}^0 + K^+K^-)] + g_{O_SS}(\sqrt{2}\epsilon - S^*)\,. \qquad (2.1)$$

This effective Lagrangian incorporates the ideally mixed nonet scheme for the scalar particles, coupling in a U_3 symmetric manner to the ordinary pseudoscalars. The second term is the direct coupling of quark scalars to the scalar q.s.

An ϵ width of 640 MeV gives $g_{SPP}{}^2 = 15.6$ GeV.[2] The relevant decay kinematics is contained in the Appendix. With this value for $g_{SPP}{}^2$, the phase space for $S^* \to 2K$ is far too small to account for the 40-MeV S^* full width. The parameter g_{O_SS} is obtained by allowing $S^* \to 2\pi$. With $M_{O_S} = 2$ GeV, this gives $g_{O_SS}{}^2 = 0.24$ GeV^4 (for comparison, $M_{O_SS} = 1.5$ GeV gives $g_{O_SS}{}^2 = 0.11$ GeV^4).

This is similar to the method used for obtaining g_{O_VV} from $\phi \to \rho\pi$ decay, as described in Ref. 1. One readily obtains $g_{O_VV}{}^2 = 3.6 \times 10^{-2}$ GeV.[5] This gives $|g_{O_SS}| = 2.6|g_{O_VV}|$, so both couplings are of the same order of magnitude, as desired for a consistent interpretation of OZI-rule violation.

A better measure of how the examples compare in giving small OZI-rule violations is

$$\frac{|g_{O_SO}|}{m_{O_S}{}^2 - m_{S^*}{}^2} = 0.16\,, \qquad (2.2)$$

versus

$$\frac{|g_{O_VO}|}{m_{O_V}{}^2 - m_{\varphi}{}^2} = 0.24\,. \qquad (2.3)$$

An important point to notice is that these numbers are small enough that a "perturbative" approach to the state mixing of (ϵ, S^*, O_S) and (ω, ϕ, O_V) is justified. If one attempts to diagonalize a Lagrangian including mass and kinetic energy terms for (ω, ϕ, O_V), plus the analog of Eq. (2.1) for that system, it turns out that $M_\omega < M_\rho$. As was noted in Ref. 1, this implies that contributions due to continuum states must be included. Simply diagonalizing $\mathcal{L}^{\rm eff}$ will not do. Nevertheless, (2.2) and (2.3) indicate that the use of $\mathcal{L}^{\rm eff}$ for couplings is a good approximation.

Having determined g_{SPP} and g_{O_SS}, the O_S width can also be calculated, using the model of Eq. (2.1). This gives $\Gamma_{O_S} = 20.9$ MeV (compare: $M_{O_S} = 1.5$ GeV gives $\Gamma_{O_S} = 51.9$ MeV. Evidently in these decays the phase-space enhancement with larger mass is overcome by propagator suppression.) Here again a conservative note will be inserted—to guard against possible decays not covered by (2.1), in the subsequent calculations for ψ decay the value $\Gamma_{O_S} = 50$ MeV will be used. Over most of the regions of integration in those calculations, this change is inconsequential. It would be most beautiful if the O particles did present a very sharp spike, but it will turn out that a 50-MeV broad enhancement is quite tolerable.

To summarize this section: We will use $\epsilon(660)$, $\Gamma_\epsilon = 640$ MeV; $S^*(993)$, $\Gamma_{S^*} = 40$ MeV; $\kappa(1300)$, $\Gamma_\kappa = 400$ MeV; $O_S(2000)$, $\Gamma_{O_S} = 50$ MeV. The coupling parameters in Eq. (2.1) were $g_{SPP} = \pm 3.95$ GeV; $g_{O_SS} = \pm 0.49$ GeV^2. Also, $g_{O_VV} = \pm 0.19$ GeV^2, where V refers to ω and ϕ vector mesons.

III. ANALYSIS OF ψ DECAYS

A. $\psi \to \pi^+\pi^-\pi^0$ via ω

Here ψ goes to O_V, which then goes to ω. The ω decay to 3π is assumed to go through $\rho\pi$. The coupling constants that enter are $g_{\rho\pi\pi}{}^2 = 36.3$; $g_{\omega\rho\pi}{}^2 = 4g_{\rho\pi\pi}{}^2/m_\rho{}^2$ (which provides an excellent fit to the ω width under the $\rho\pi$ decay assumption); $g_{O_VV}{}^2$; and $g_{O\psi}{}^2$ which is to be determined. Using $\Gamma(\psi \to \pi^+\pi^-\pi^0) = 0.9$ keV, one obtains $g_{O_V\psi} = \pm 0.05$ GeV^2.

B. $\psi \to (K^*(890)\,\kappa(1300)) \to 2K2\pi$

This decay proceeds entirely via the diagram of Fig. 3(a), and allows us to determine $g_{OVS}{}^2$. The coupling of O_V to other VS combinations is then completely determined by

$$\mathcal{L}^{\rm eff}_{OVS} = g_{OVS}\,O_\mu\,{\rm Tr}V^\mu S = g_{OVS}\,O_\mu[\varphi^\mu S^* + \omega^\mu\epsilon + K^{*\mu+}\kappa^- + K^{*\mu-}\kappa^+ + K^{*0}\bar{\kappa}^0 + \cdots]\,. \qquad (3.1)$$

In a similar manner, $g_{K^*K\pi}{}^2$ is related to $g_{\rho\pi\pi}{}^2$ by ${\rm Tr}(V_\mu P \overleftrightarrow{\partial}^\mu P)$; and $g_{\kappa K\pi}{}^2$ is related to $g_{SPP}{}^2$ by ${\rm Tr}SP^2$.

The $\kappa(1300)$ propagator requires some discussion, since 400 MeV is a sizable width. It turns out that $g_{\kappa K\pi}{}^2$ gives the κ a width of only 150 MeV. Clearly, κ is massive enough to have decay modes other than $K\pi$. But to include all the possible decay kinematics for multiparticle final states as finite-width correction terms in the κ propagator would be rather tedious and uncertain. Consequently, a compromise has been struck:

$$\Delta_\kappa(q^2) = [q^2 - m_\kappa{}^2 + im_\kappa\Gamma_\kappa(q^2)]^{-1}\,, \qquad (3.2a)$$

$$\Gamma_\kappa(q^2) = \frac{g_{\rm eff}{}^2}{q^2}\,k(q^2)\,. \qquad (3.2b)$$

The kinematic factor $\Gamma_\kappa(q^2)$ is described more fully in the Appendix. Then g_{eff}^2, which includes the necessary factors of π, etc., is adjusted to give the κ a 400 MeV width *as if* $K\pi$ were the only decay mode. This device gives the right residue at resonance, and attempts to incorporate some q^2 variation off resonance.

The coupling g_{OVS}^2 will be as large as possible if *all* of $K^*(890)K^*(1420)$ is really $K^*(890)\kappa(1300)$. In that case, $g_{OVS} = \pm 1.85$. More generally, we could insert a factor $(x<1)$ to represent what fraction of the K^*K^{**} is truly $K^*\kappa$. Instead, however, this number will be carried along, and the question of smaller values will be discussed when it is relevant.

C. $\psi \to \phi\pi^+\pi^-$

Consider first the contribution from Fig. 3(a). Equation (3.1) tells us that only the (ϕS^*) combination appears here. The S^* can decay to $\pi^+\pi^-$ only by violating the OZI rule, i.e., passing through O_S and ϵ. Since all the parameters for this graph have now been determined, it can be calculated, as though it were the only contribution to the decay. Numerically, it turns out to be insignificant. The reasons are that, even though g_{OVS}^2 is not too small, it is multiplied by the O_V propagator evaluated at the ψ mass; an extra factor g_{OS}^2 is picked up relative to the other graphs; and an extra scalar propagator is present relative to the other graphs. Thus, it is an excellent approximation to neglect *this kind* of double OZI-rule violation completely.

To compute the remaining graphs [Figs. 3(b), 3(c)], two more coupling constants would have to be known, $g_{O_SO_V\phi}$ and $g_{O_SO_V\psi}$. Are these related? Let $\langle v|1\rangle$ be the amount of the $q\bar{q}$ vectors that is flavor singlet. Freund and Nambu explored the following possibility:

$$g_{O_VV} = \langle v|1\rangle F(m_V^2)\,, \tag{3.3}$$

with $F(m_\phi^2) \approx F(m_\omega^2) \approx F(m_\psi^2)$. F has units of GeV^2. Unfortunately, an SU_4 charm picture for $\langle\psi|1\rangle$ led them to an order of magnitude too large predicted rate for $\psi \to \omega \to 3\pi$.

Assumption (4) of the first section would place the blame for this on the charm value for $\langle\psi|1\rangle$. That interpretation is testable in the following manner. Write

$$g_{O_SO_VV} = \langle v|1\rangle G(m_V^2)\,, \tag{3.4}$$

where G is in units of GeV.

If also $G(m_\phi^2) \approx G(m_\omega^2) \approx G(m_\psi^2)$, then, introducing μ_v with dimensions of GeV to match G with F,

$$\mu_\omega \frac{g_{O_SO_V\omega}}{g_{O_V\omega}} \approx \mu_\phi \frac{g_{O_SO_V\phi}}{g_{O_V\phi}} \approx \mu_\psi \frac{g_{O_SO_V\psi}}{g_{O_V\psi}}. \tag{3.5}$$

Two possible assumptions are $\mu_\psi = \mu_\phi = \mu_\omega$; and $\mu_V = m_V$. [Actually, (3.5) would also be true if G and F form factors *track*, i.e., have the same m^2 dependence.]

Another possibility may be explored in a very crude manner. Suppose that the q.s.'s are actually resonating gauge vector gluons. The $O_V(O_S)$ would be a 3- (2-) gluon bound state in this view. Count a factor γ for each gluon emission, so

$$F(m_V^2) \equiv F_V\gamma_V^3\,,\qquad G(m_V^2) \equiv \mu_V G_V \gamma_V^5\,. \tag{3.6}$$

Then in a charmlike scheme

$$\frac{|g_{O_V\phi}|}{|g_{O_V\psi}|} = \frac{F_\phi}{F_\psi}\left(\frac{\gamma_\phi}{\gamma_\psi}\right)^3 = 3.8\,, \tag{3.7}$$

and

$$\frac{|g_{O_SO_V\phi}|}{|g_{O_SO_V\psi}|} = \frac{\mu_\phi G_\phi}{\mu_\psi G_\psi}\left(\frac{\gamma_\phi}{\gamma_\psi}\right)^5. \tag{3.8}$$

If now F_V (G_V) are equal for all V's, or if they track, then for all μ_V equal

$$\left|\frac{g_{O_SO_V\phi}}{g_{O_SO_V\psi}}\right| \approx \left|\frac{g_{O_V\phi}}{g_{O_V\psi}}\right|^{5/3} = 9.25\,, \tag{3.9}$$

while if $\mu_V = m_V$,

$$\left|\frac{g_{O_SO_V\phi}}{g_{O_SO_V\psi}}\right| \approx \frac{m_\phi}{m_\psi}\left|\frac{g_{O_V\phi}}{g_{O_V\psi}}\right|^{5/3} = 3.05\,. \tag{3.10}$$

A convenient parametrization, in any case, is

$$\frac{g_{O_SO_V\psi}}{g_{O_V\psi}} = \frac{1}{\eta_\phi}\frac{g_{O_SO_V\phi}}{g_{O_V\phi}} = \frac{1}{\eta_\omega}\frac{g_{O_SO_V\omega}}{g_{O_V\omega}}. \tag{3.11}$$

The first possibility discussed above (a "universality assumption") corresponds to $\eta = \pm 1$ (allowing for a possible phase) with all μ_v equal; and to $\eta = \pm\frac{1}{3}$ for $\mu_\phi \approx \mu_\omega = 1$ GeV, $\mu_\psi = 3$. The second possibilities, basically coupling constant counting, correspond to $\eta = \pm 2.4$ or ± 0.79, depending on the μ_V. Unfortunately, nothing close to these possibilities emerges from an analysis of the data. More will be said about this in the final section.

Using the matrix elements in Table I, the coupling constants that have been determined so far, the parametrization of Eq. (3.11), and a reported[5] partial width $\Gamma(\psi \to \phi\pi^+\pi^-) = 0.14$ keV, one obtains the equation (with $g_{O_VO_S\psi} \equiv h$)

$$h^2[0.39\eta_\phi^2 - 6.42\eta_\phi + 26.59] = 1. \tag{3.12}$$

Now, one possibility is to assume that $\eta_\phi = \eta_\omega$. An analysis of the decay $\psi \to \omega\pi^+\pi^-$ gives a second equation for η and h, and the two equations can be used to determine best values for η and h. Unfortunately, this leads to disastrous further predictions, e.g., $\Gamma(\psi \to (O_V \to \pi^+\pi^-\pi^0)\pi^+\pi^-) \approx 25$ keV.

The reason for this problem is clearly that the presence of the O_V keeps the matrix element up

TABLE I. Matrix elements for various processes, calculated as described in the Appendix. These values include all kinematic factors. Only the coupling constants are needed to obtain the partial widths.

Final State	Matrix element No. 1	No. 2	No. 3	No. 4	No. 5	No. 6
$K^*(\kappa\to K\pi)$	1.26×10^{-8}	0	0	0	0	0
$\phi 2\pi$	0	8.74×10^{-9}	5.90×10^{-7}	0	-7.14×10^{-8}	0
$2K2\pi$ via $\phi\epsilon$	0	3.42×10^{-8}	3.98×10^{-6}	0	-2.78×10^{-7}	0
$(O_V\to 2K)2\pi$	0	7.37×10^{-11}	6.86×10^{-7}	0	-1.03×10^{-10}	0
$\omega 2\pi$	2.57×10^{-11}	4.75×10^{-12}	1.51×10^{-10}	-7.91×10^{-12}	-2.68×10^{-11}	4.45×10^{-11}
5π via $\omega\epsilon$	2.25×10^{-10}	2.93×10^{-11}	1.16×10^{-8}	-6.73×10^{-11}	-7.84×10^{-11}	6.41×10^{-11}
$(O_V\to 3\pi)2\pi$	5.18×10^{-12}	4.80×10^{-13}	7.69×10^{-11}	-1.56×10^{-12}	3.22×10^{-13}	5.29×10^{-12}
$\phi 2K$ via ϵ	0	3.58×10^{-9}	2.43×10^{-7}	0	2.294×10^{-8}	0
$4K$ via $\phi\epsilon$	0	1.36×10^{-8}	1.10×10^{-6}	0	-1.12×10^{-7}	0
$(O_V\to 2K)(\epsilon\to 2K)$	0	8.54×10^{-12}	7.94×10^{-8}	0	-2.01×10^{-11}	0
$\omega 2K$ via ϵ	1.88×10^{-12}	2.37×10^{-12}	7.56×10^{-11}	-8.91×10^{-13}	-1.34×10^{-11}	4.94×10^{-12}
$3\pi 2K$ via $\omega\epsilon$	1.17×10^{-11}	9.96×10^{-12}	1.36×10^{-9}	-5.30×10^{-12}	-5.53×10^{-11}	2.07×10^{-11}
$(O_V\to 3\pi)(\epsilon\to 2K)$	3.04×10^{-13}	5.55×10^{-14}	5.17×10^{-10}	-1.27×10^{-13}	-1.60×10^{-14}	-4.86×10^{-13}
$\phi 2K$ via S^*	6.73×10^{-8}	1.26×10^{-8}	8.53×10^{-7}	-2.43×10^{-8}	-1.03×10^{-7}	1.98×10^{-7}
$4K$ via ϕS^*	2.70×10^{-7}	4.94×10^{-8}	5.47×10^{-6}	-9.75×10^{-8}	-4.06×10^{-7}	7.91×10^{-7}
$(O_V\to 2K)(S^*\to 2K)$	9.01×10^{-10}	1.12×10^{-10}	1.05×10^{-6}	-3.16×10^{-10}	-1.65×10^{-10}	-5.19×10^{-10}
$\omega 2K$ via S^*	0	6.67×10^{-12}	2.13×10^{-10}	0	-3.76×10^{-11}	0
$3\pi 2K$ via S^*	0	3.83×10^{-11}	1.46×10^{-8}	0	-1.32×10^{-10}	0
$(O_V\to 3\pi)(S^*\to 2K)$	0	7.32×10^{-13}	6.81×10^{-9}	0	4.37×10^{-13}	0
$\phi\to 5\pi$	3.50×10^{-16}	2.31×10^{-17}	5.85×10^{-16}	-8.98×10^{-17}	1.41×10^{-17}	5.48×10^{-17}

far from the ϕ and ω regions. It turns out that if $\eta_\phi=\eta_\omega$, the magnitudes and signs of η and h are such that it is the interference between Figs. 3(b) and 3(c) that keeps $\psi\to\phi\pi^+\pi^-$ small. This cancellation does not occur at the O_V mass. A similar phenomenon in the $\psi\to 2\pi^+2\pi^-\pi^0$ system gives rise to the quoted large $\psi\to O_V 2\pi$ rate. One must make certain that the total predicted partial widths for $\psi\to(K^+K^-)_p(\pi^+\pi^-)_s$ and $\psi\to(\pi^+\pi^-\pi^0)_p(\pi^+\pi^-)_s$ via the quarkless-resonance mechanism are not enormous. This can only be done if $\eta_\phi\neq\eta_\omega$.

How large could the total $K^+K^-\pi^+\pi^-$ partial width possibly be in the model? Let Γ be that partial width in keV. An analysis identical to that leading to Eq. (3.12), using the appropriate matrix elements from Table I, leads to

$$h^2(0.22\eta_\phi{}^2-3.59\eta_\phi+25.78)=\Gamma\,. \tag{3.13}$$

Equations (3.12) and (3.13) are consistent for real η and h if and only if $66.75>\Gamma>0.57$. The lower limit is 0.83% of $\Gamma(\psi\to\text{all})$, whereas experimentally $\Gamma(\psi\to K^+K^-\pi^+\pi^-)=(0.4\pm0.2)\%\ \Gamma(\psi\to\text{all})$ excluding $K^*(890)K\pi$. At this point the only recourse is to keep in mind the expected "factor of 2" accuracy of the whole scheme, taking comfort from the fact that the order of magnitude is compatible.

To move slightly away from the very edge of compatibility of (3.12) and (3.13), $\Gamma=0.6$ keV $=0.87\%\ \Gamma(\psi\to\text{all})$ will be used. This allows the solutions

$$\eta_\phi=41.75,\quad h=\pm4.78\times10^{-2}, \tag{3.14a}$$

$$\eta_\phi=-19.75,\quad h=\pm5.72\times10^{-2}. \tag{3.14b}$$

D. $\psi\to\omega\pi^+\pi^-$

Proceeding as in the ϕ case, and using $\Gamma(\psi\to\omega\pi^+\pi^-)=0.72$ keV, one obtains

$$1.06\eta_\omega{}^2h^2-12\eta_\omega h^2+34h^2-1.78\eta_1\eta_\omega h+9.93\eta_1 h=100\,. \tag{3.15}$$

In this equation, η_1 is a phase collected from the interference terms. When the coupling constants were determined from rates, one always obtained

g^2. Setting $g=\eta_g|g|$, η_1 is

$$\eta_1 \equiv \eta_{O_V\psi}\eta_{O_V VS}\eta_{O_V\omega}\eta_{O_S\epsilon}\,. \tag{3.16}$$

Fortunately, $(\eta_1 h)$ always appear together, so only $(\eta_1\,\text{sgn}h=\pm1)$ need to be considered.

Each value of h from Eq. (3.14) gives two η_ω solutions to (3.15). Thus, there are eight possible $(\eta_1 h, \eta_\omega)$ combinations in all. The total rate for $\psi\to 2\pi^+2\pi^-\pi^0$ due to this mechanism can then be calculated for each combination. All the results are in the 4.1- to 5-keV range. Experimentally, this rate is only 2.8 keV, so again, the over-all predicted rate is a bit bigger than what has been observed. The smallest prediction is for $\eta_1 h = -5.72\times10^{-2}$ (thus fixing $\eta_\phi=-19.75$) and $\eta_\omega = -179.1$. There are no values of η_ω of the same order of magnitude as η_ϕ (the other $|\eta_\omega|$ solutions range from 226 to 150). This surprising and discouraging turn of events will be discussed in Sec. IV.

At this point, it is natural to wonder if the q.s. dynamics are necessary for this decay at all. Setting $h=0$, one finds that the direct $O_V SV$ graph, Fig. 3(a), accounts for only 0.01 keV. Thus, the other graphs are absolutely necessary.

Another interesting question is how sensitive these results are to the value of g_{OSV}. Instead of assuming it is as large as it can be, assume that it is zero. Then Eq. (3.15) becomes

$$h^2[1.05\eta_\omega^{\,2}-12\eta_\omega+34]=100\,. \tag{3.15$'$}$$

There are now four solutions, two η_ω values for each h^2. The η_ω values are in the same range as before, and the resulting total $\psi\to2(\pi^+\pi^-)\pi^0$ are from 4.4 to 4.7 keV. Thus, the results are fairly insensitive to whether g_{OSV} is there or not, as could have been anticipated from the $h=0$ results. Since g_{OSV} maximal helps keep the $2\pi^+2\pi^-\pi^0$ rate down by a little bit, though (4.1 vs 4.4 keV), it will be used in the predictions that follow, although those are not terribly sensitive to g_{OSV}, either.

Having fixed all the parameters, it is easy to compute the O_V contribution to that $\psi\to2\pi^+2\pi^-\pi^0$ decays. It is

$$\Gamma(\psi\to(O_V\to\pi^+\pi^-\pi^0)\pi^+\pi^-)=0.08\text{ keV}\,, \tag{3.17}$$

which is 11% of the $\omega\pi^+\pi^-$ branching ratio.

E. Predictions for rates and spectra

1. $\phi\to2\pi^+2\pi^-\pi^0$. Nothing prevents this decay from occurring by the same mechanism as ψ decay into that final state. Using the parameters obtained in Secs. IIIC and IIID, one obtains

$$\Gamma(\varphi\to2\pi^+2\pi^-\pi^0)=2.9\text{ eV}\,, \tag{3.18}$$

which is only (7×10^{-5})% of the total ϕ width. Experimentally, only an upper bound of 1% exists.

This decay is highly suppressed because all propagators are far off-shell, unlike the situation for the ψ decay. If future experiments discover a ϕ decay rate at a level much larger than this prediction, that should be considered an indication that direct couplings of O_V to multiparticle states should be taken into account. Possibly even electromagnetic mechanisms could give a larger rate than (3.18).

2. $\psi\to\phi K^+K^-$. There are two reasons why this decay is not directly SU_3 connected to $\phi\pi^+\pi^-$. The first is that the kinematics are different, albeit not too much different. The second reason is that in this case, the $S^*(993)$ also participates in the scalar channel.

From the ϵ, one obtains $\Gamma(\psi\to\phi K^+K^-)=1.5\times10^{-2}$ keV. Via S^*, the partial width is 3.5×10^{-2} keV. *Adding the partial widths*, the result is

$$\Gamma(\psi\to\varphi K^+K^-)=0.05\text{ keV} = 0.07\%\Gamma_{\text{tot}}\,. \tag{3.19}$$

Interference between ϵ and S^* could alter this prediction a bit, but not by an order of magnitude.

In the same way, adding the ϵ and S^* contributions incoherently, the model predicts

$$\Gamma(\psi\to(K^+K^-)_p(K^+K^-)_s)=0.21\text{ keV} = 0.3\%\Gamma_{\text{tot}}\,, \tag{3.20a}$$

$$\Gamma(\psi\to(O_V\to K^+K^-)(K^+K^-)_s)=6.6\text{ eV} = 0.01\%\Gamma_{\text{tot}}\,. \tag{3.20b}$$

Inclusion of the off-shell possibility $\omega\to K^+K^-$ will not alter these predictions by very much.

3. $\psi\to\omega K^+K^-$. Here again both ϵ and S^* participate, and proceeding as in the previous example, the predictions are:

Final state	keV	$\%\Gamma_{\text{tot}}$	
ωK^+K^-	0.4	0.58	(3.21a)
$(\pi^+\pi^-\pi^0)_p(K^+K^-)_s$	2.2	3.	(3.21b)
$(O_V\to\pi^+\pi^-\pi^0)(K^+K^-)_s$	0.04	0.06	(3.21c)

These results are a bit disturbing, since no example of such modes appears in Ref. 5. On the other hand, it seems eminently reasonable that $\pi^+\pi^-\pi^0K^+K^-$ should be of the same order of magnitude as $2\pi^+2\pi^-\pi^0$. If comparable rates for these processes are not established by experiments in the future, it would appear that more than the details of this dynamical model would have to be called to question.

4. Searching for O_V. Since O_V mixes with ω, ϕ, and ψ, it should also appear as a resonance in

e^+e^- annihilation. However, in a perturbative mixing formalism such as the one being employed here to study the OZI-rule violation, it is easy to see that detection of O_V in this manner would be difficult. The ratio of the cross section at O_V resonances (produced through ω's) relative to the cross section at ω resonance is

$$\frac{\sigma_{e^+e^-}(O_V)}{\sigma_{e^+e^-}(\omega)} = \frac{m_\omega{}^4\Gamma_\omega{}^2}{m_{O_V}{}^4\Gamma_{O_V}{}^2} \frac{f_{O\omega}{}^4}{(m_{O_V}{}^2 - m_\omega{}^2)^2} = 1.1\times 10^{-5}\,. \tag{3.22}$$

Similarly small ratios are obtained relative to ϕ and to ψ.

On the other hand, in decays of ψ the situation for O_V becomes more favorable, as (3.20) and (3.21) indicate. As another example, integrating over the resonance widths, one finds

$$\frac{\Gamma(\psi\to(O_V\to K^+K^-)\pi^+\pi^-)}{\Gamma(\psi\to\varphi\pi^+\pi^-)} = 10.7\%\,. \tag{3.23}$$

Figure 4 displays the K^+K^- (mass)^2 spectrum for $\psi\to K^+K^-\pi^+\pi^-$ in the O_V region. There is no reason to expect the K^+K^- spectrum from the $K^*(890)$ $K\pi$ decays to the same final state to be anything but small and smooth in these regions. If S-wave K^+K^- could be separated from P-wave K^+K^-, the possible $\psi\to\rho^0\delta^0$ cascade sequence would not contribute here at all. However, even if this cannot be done, the off-shell $\delta^0\to K^+K^-$ should be a small background.

It cannot be stressed too strongly that this is the best way to rule the whole model out. Either one observes a second resonance in a spectrum such as this one, along with the known quark-containing vector, or one does not. The details of the spectrum in Fig. 4 are determined by fitting the correct $\phi\pi^+\pi^-$ partial width, and by demanding that the integral over the entire spectrum not be orders of magnitude too large. This leaves O_V as a not terribly prominent resonance, but hopefully one that is detectable, given enough resolution and statistics.

5. *Searching for* O_S. The decays of the ψ are also good places to detect O_S. One process which would be relatively uncontaminated by competing processes would be the K^+K^- spectrum in $\psi\to\pi^+\pi^-\pi^0K^+K^-$. As already noted, however, there seems to be a serious scarcity of such events.

The decay $\psi\to K^+K^-\pi^+\pi^-$ will also do, however. Here one scans the $\pi^+\pi^-$ spectrum. The model of Sec. III C predicts the spectrum displayed in Fig. 5.

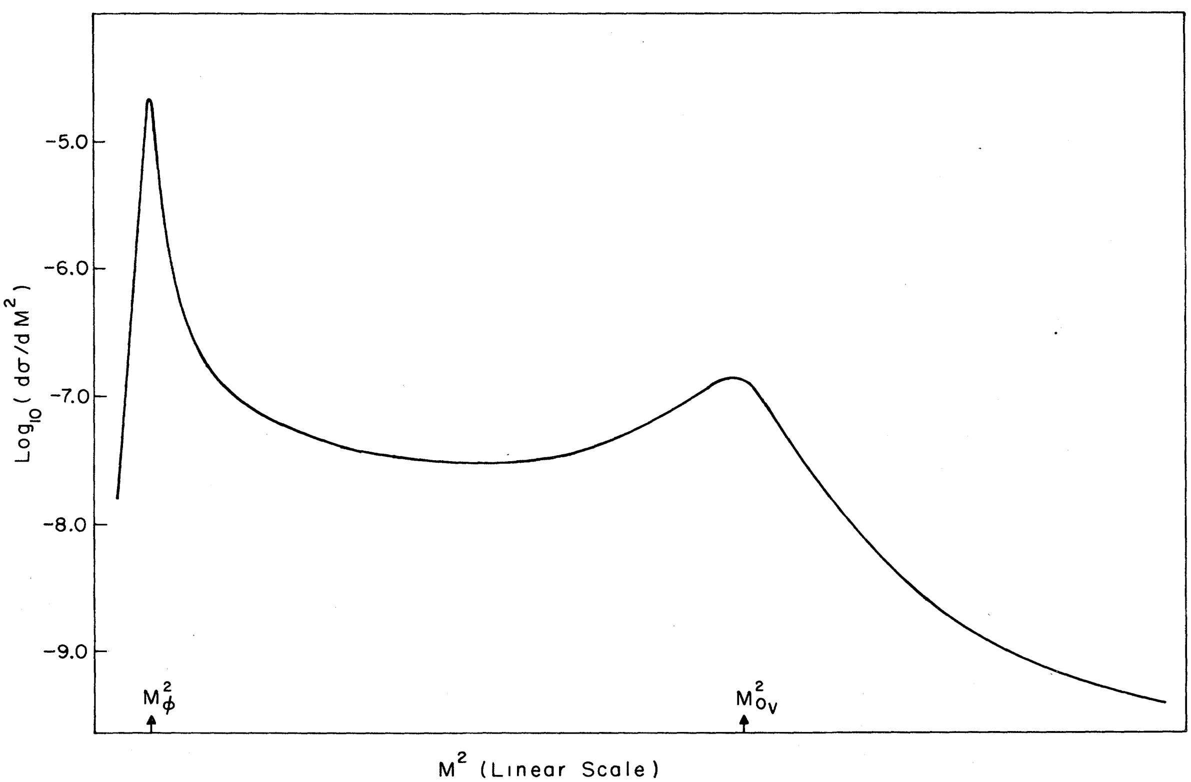

FIG. 4. Predicted spectrum of K^+K^- in $\psi\to K^+K^-\pi^+\pi^-$. Competing backgrounds are not displayed. This spectrum is normalized so that the integral of $(d\sigma/dm^2)$ over all kinematically allowed m^2 (in GeV^2) gives $\Gamma(\psi\to K^+K^-\pi^+\pi^-)$ in GeV.

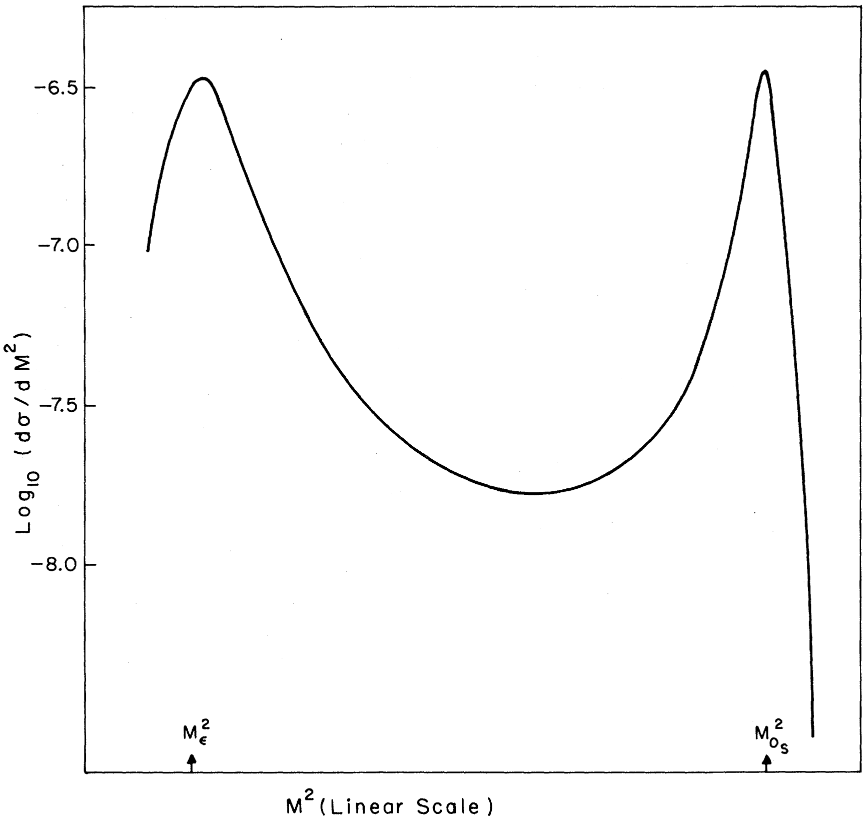

FIG. 5. Predicted spectrum of $\pi^+\pi^-$ in $\psi \to K^+K^-\pi^+\pi^-$. Comments in Fig. 4 caption apply here as well.

Again, there is the possible competing mode for $\psi \to K^+K^-\pi^+\pi^-$ through $\rho^0\delta^0$. Even if $g_{OSV}=0$, there could be a (ρ^0 + tensor) mode to $K^+K^-\pi^+\pi^-$, so Fig. 5 should not lead to an excess of optimism. Nonetheless, the ρ peak should have died well away by M_{O_S} (which is relevant under the assumption that lack of data precludes separation of partial waves in a given channel). Keep in mind this goes *only* through the direct $O_V SV$ coupling.

Theoretically, one is in better shape if only the spectrum of the $\pi^+\pi^-$ associated with a ϕ are plotted. This can be used to search for scalars with masses from $2m_\pi$ up to $(m_\psi - m_\phi) = 2.076$ GeV. The trouble, of course, is that there are very few events, so this is experimentally quite difficult.

IV. CONCLUSIONS AND SPECULATIONS

This paper is illustrative in nature. It explores a possible series of relationships between ψ decay rates in the context of a specific dynamical model for OZI-rule violation. Some of the parameters of the model are almost totally the user's to choose, e.g., the O_V and O_S masses, although it was argued that some values may be more sensible than others. Other parameters are then fixed by comparison with rather uncertain experimentally available numbers. A few predictions then follow, which should be taken as order-of-magnitude estimates at best.

Perhaps the strongest claim that could be made is that the model does provide a reasonably consistent scheme for relating the several decay modes of ψ discussed in detail. In addition, it has been remarked several times in the literature that ψ decays may be a "copious" source for bound gluons (b.g.'s), if these exist at all.[11] The quantitative results of this paper can be viewed as a useful guide as to just how accessible (or inaccessible) to direct detection these new states of matter may actually be.

Of course, even if new resonances in two- or three-body SU_3-singlet channels are observed, this will not prove they are resonant gluons. One will have to eliminate the possibility that they have flavored partners. But the first stage is to either find or rule out the weakly coupling, flavorless vector and scalar bosons. *It should not be hopelessly difficult to do this.*

Pretend, now, that quarkless resonances in the 1- to 2- or 3-GeV region actually exist, and couple more or less as described in this paper. This opens a vast area for theoretical investigation. The assumptions of the first section would have to be studied on solid theoretical grounds. Relationships among several of the coupling constants should be deduced. Let us focus on a few of these points:

(1) If whole Regge families of b.g.'s exist, how would these *not* considered in this paper modify the results? Where could one best look for a pseudoscalar recurrence? What would be the relative importance of scalar and tensor quarkless resonances?

(2) What is the correct way to describe an l-excited state in a quark model with color gauge fields? At the "current quark," operator level, one has ordinary derivatives, to get the l, and one has gauge field terms to preserve gauge invariance. Do these combine an exponentiate, as suggested by lattice models? Or is there a sensible notion of gauge field valence number?

(3) Just how constant can the meson to b.g. form factors actually be? Perhaps the most discouraging results of the entire investigation are that $|g_{\phi O_V O_S}| \approx 75 |g_{\psi O_V O_S}|$, and $|g_{\omega O_V O_S}| \approx 13 |g_{\phi O_V O_S}|$. The qualitative results that $|g_{\phi O_V}| > |g_{\psi O_V}|$, and $|g_{\omega O_V O_S}| > |g_{\phi O_V O_S}| > |g_{\psi O_V O_S}|$, smack of asymptotic freedom. But the actual numbers for the latter case would imply an incredibly fast rate of growth, accelerating rapidly from 1 to 0.8 GeV. Can such behavior be symptomatic of the intermediate region between the free and the enslaved regions? What are the correct mass scales on which to base such an analysis, and to use to relate $g_{O_V V}$ to $g_{O_V O_S V}$?

(4) Why are $g_{O_V \omega}$ and $g_{O_S \epsilon}$ of comparable orders of magnitude, and "small" in the sense that the perturbative mixing approximation is valid? This is the basic reason why quarkless-resonance models can be used to implement the OZI rule dynamically. Presumably asymptotic freedom answers the smallness question for $g_{O_V \omega}$ if the vector b.g., e.g., acts as three gauge fields when probing the quark current at short distances. But is the same true for $g_{O_S \epsilon}$? Related to this is the open question of the strength of b.g.-b.g. interactions, e.g., $O_V \to O_V O_S$ off shell.

There are two basic questions running through the last three points. One is the oft-made observation that with the discovery of ψ, it is really no longer safe to neglect form-factor effects, since extrapolations are not over only 1 GeV or so. So what do we do now? The second is that presumably color as a symmetry is extremely powerful, because only singlet combinations of colorful objects need to be considered. But does not color as a gauge theory tell us any more than color symmetry in describing states and their couplings? Are there possibly algebraic consequences of local non-Abelian gauge invariance that allow one to relate different processes, form-factor dependences aside?

All of this is rather far removed from the phenomenological arena in which the model has been asked to confront data. The truly central question is whether quarkless resonances exist. It would be embarrasing if any of the experimental spectra looked just like Figs. 4 or 5. Yet, the intractability of the problem of strong interactions in conventional theories has led to several inventive and ingenious new approaches: strings, bags, and lattices. All of these predict quarkless states. It might prove even more embarrasing if these particles are not found.

Note added in proof: After completion of this work, several related papers have appeared: J. F. Bolzan, K. A. Geer, W. P. Palmer, and S. S. Pinsky, Fermilab Report No. Fermilab Pub. 75/62 (unpublished); W. F. Palmer and S. S. Pinsky, Ohio State University Report No. COO-1545-171 (unpublished); M. Chaichian and M. Hayaski, CERN Report No. CERN-TH-2082 (unpublished).

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APPENDIX

Referring to Fig. 6, for the decay $A \to BC$ in the rest frame of A, with $k_A{}^2 = m_A{}^2$, etc., one has

$$k \equiv |\vec{k}_B| = |\vec{k}_C| = \frac{1}{2m_A} \left\{ [(m_A - m_B)^2 - m_C{}^2] \times [(m_A + m_B)^2 - m_C{}^2] \right\}^{1/2}, \tag{A1}$$

which is symmetric in m_B and m_C.

Then the phase-space factors for various possibilities are, as is well known:

(1) $V \to VS$. Using $\mathcal{L}^{\text{eff}} = g A_\mu B^\mu C$,

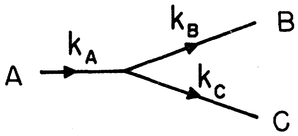

FIG. 6. Momenta in decays $A \to BC$.

$$\Gamma = \frac{g^2}{24\pi} \frac{k}{m_A{}^2}\left(3 + \frac{k^2}{m_B{}^2}\right). \tag{A2}$$

(2) $V \to VP$. Using $\mathcal{L}^{\text{eff}} = ig[\epsilon_{\mu\nu\rho\lambda} C\partial^\mu A^\nu \partial^\rho B^\lambda - \text{H.c.}]$,

$$\Gamma = \frac{g^2}{12\pi} k^3. \tag{A3}$$

(3) $V \to PP$. Here $\mathcal{L}^{\text{eff}} = gA_\mu(B\partial^\mu C - \partial^\mu CB) + \text{H.c.}$ gives

$$\Gamma = \frac{g^2}{6\pi}\frac{k^2}{m_A{}^2}. \tag{A4}$$

(4) $S \to PP$. Using a dimensional coupling constant,

$$\mathcal{L}^{\text{eff}} = gA_S B_P C_P$$

gives

$$\Gamma = \frac{g^2 k}{32\pi m_A{}^2}. \tag{A5}$$

The matrix elements in Table I are calculated using these formulas, with all $g = 1$. Since all the decays are cascaded, e.g., $\psi \to K^*\kappa$, then $K^*, \kappa \to K\pi$, integrals over the invariant masses squared must be performed where appropriate. In this example, to obtain the $\psi \to K^*(K\pi)$ rate, the vector arm is integrated over the K^* resonance width, from $m_{K^*}{}^2 - m_{K^*}\Gamma_{K^*}$ to $m_{K^*}{}^2 + m_{K^*}\Gamma_{K^*}$. At each vector mass, m_v, in this range, the scalar arm is integrated over the total allowed range, $(m_K + m_\pi)^2$ to $(m_\psi - m_v)^2$.

Whenever such integrals are performed, the masses used in the phase-space formulas are the running masses, not the particle masses. Finally, when a particle decays in motion at invariant mass m^2 the kinematic formulas must be modified by a factor (m/π), since (A1)–(A4) were derived in the rest frame.

To explain the meaning of the "matrix element" entries in Table I, let $\Delta_p(q^2)$ be the propagator of particle P with four-momentum q_μ; and label the vector arm's $(\text{mass})^2$ by $\sigma_1{}^2$, the scalar arm's by $\sigma_2{}^2$. The matrix elements of Fig. 3 are of the form

$$T = |a\Delta_{0_V}(m_\psi{}^2) + b\Delta_{0_V}(m_\psi{}^2)\Delta_{0_S}(\sigma_2{}^2) + c\Delta_{0_V}(\sigma_1{}^2)\Delta_{0_S}(\sigma_2{}^2)|^2, \tag{A6}$$

where a, b, and c are real constants.

Evaluating the squared modulus, one obtains six terms. The correspondence between these terms, the graphs they arise from, and the entries in Table I are indicated below:

Matrix element No. 1 = a^2 term
= $(\text{modulus})^2$ of Fig. 3(a),

Matrix element No. 2 = b^2 term
= $(\text{modulus})^2$ of Fig. 3(b),

Matrix element No. 3 = c^2 term
= $(\text{modulus})^2$ of Fig. 3(c).

Matrix element No. 4 = ab term
= Interference between Figs. 3(a) and 3(b),

Matrix element No. 5 = bc term
= Interference between Figs. 3(b) and 3(c),

Matrix element No. 6 = ac term
= Interference between Figs. 3(a) and 3(c).

All dimensional coupling constants should be expressed in units of GeV. The resulting partial widths are then in GeV. There are some couplings implicit in these results through the finite-width corrections to ϵ and κ. The determination of these couplings was, however, independent of any gluon dynamics or quark model.

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[6]"Flavor" refers to the internal degrees of freedom of the quarks other than color, e.g., ordinary SU(3), or SU(4) if a charm scheme is considered.

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