Mass mixing and the Cabibbo angle*

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Spontaneous symmetry breaking does not always induce the Cabibbo form of weak currents. The latter with the renormalizability argument in general constrains the Cabibbo angle. An example is given which gives $\tan\theta = (\sqrt{3}-1)/(\sqrt{3}+1)$, in agreement with experiment.

It is by now well known that weak interactions can be effectively described by current-current coupling with the (charged) hadronic currents satisfying the Cabibbo structure. This fact has been interpreted as the rotation of the weak-symmetry-breaking axis from the strong-symmetry-breaking axis known as the Cabibbo rotation. However, group theoretically no *a priori* reason has been found which enables one to determine the Cabibbo angle. In gauge theories, the arguments of renormalizability are often very restrictive. If the Cabibbo structure of weak currents is obtained by Cabibbo rotation in addition to spontaneous symmetry breaking, the Cabibbo angle remains as an unpredicted parameter of the theory. On the other hand, if the weak-current structure is induced by spontaneous symmetry breaking, the theory is not always renormalizable and at the same time maintaining the Cabibbo form except for some specific values of the Cabibbo angle consistent with the minimum of the potential. The theory might shed some light on probing the dynamics behind the weak interactions. The purpose of this note is to give an example in a unified gauge model.

When the symmetry is spontaneously broken, one often finds natural zeroth-order relations[1] consistent with the minimum of the potential (such as isospin symmetry). If the Cabibbo structure of weak currents should be induced by spontaneous symmetry breaking, it would in general require a relation between the vacuum expectation values (VEV's). Unless this relation is also obtained naturally, it would be violated by renormalization. The example in this note makes use of additional global symmetry in order to satisfy the Cabibbo structure and not violate renormalizability. The above condition already constrains the Cabibbo angle.

We shall adopt the M model of Bars, Halpern, and Yoshimura.[2] The quarks and the leptons are gauged separately with separate commuting groups. The semileptonic decays occur when the strong gauge gluons interacting with the quarks get contaminated with the weak gauge gluons interacting with the leptons. In other words, the physical (diagonal) gluons are mixtures of strong gluons and weak gluons. The strangeness-changing neutral strong gluons are *not* mixed with anything else in order to forbid $\Delta S \neq 0$, $\Delta Q = 0$ weak currents. Several aspects are different from the model of Bars *et al.* First of all, we believe in hadron-lepton symmetry. We shall assume a three-triplets model for both hadrons and leptons as follows:

$$\text{quarks:}\quad \begin{matrix} q^0 & q^+ & q^+ \\ q^{-1} & q^0 & q^0 \\ q^{-1} & q^0 & q^0 , \end{matrix}$$

$$\text{antileptons:}\quad \begin{matrix} \bar{l}^0 & e^+ & \mu^+ \\ l^- & \bar{\nu}_e & \bar{\nu}_\mu \\ L^- & \bar{E}^0 & \bar{M}^0 . \end{matrix}$$

We also assume that l^0 has a light mass ($\lesssim 1$ GeV), l^- has mass of a few GeV, and L^- is heavier. l^- (and l^+) are probably already produced in e^+e^- annihilation as observed in the $e^\pm\mu^\mp$ events at SLAC.[3] Note that l^- (L^-) decays dominantly semileptonically by $l^- \to l^0 +$ hadrons *with* $V+A$ interaction (which should be the signature for testing experimentally).[4] Thus, in e^+e^- annihilation it is contributing to the hadronic final states. This model gives $R = (e^+e^- \to \text{hadrons})/(e^+e^- \to \mu^+\mu^-) = 5$ and 6 at energies below and above L^+L^- production. The crucial test of the lepton spectrum lies in the search for E^0 and M^0 in neutrino scattering[5] and L^- in e^+e^- annihilation. We conjecture that M^0 might already be produced besides the heavy meson in the dimuon events.[6] Secondly, we note that when the Higgs scalars responsible for contaminating the strong and weak gluons carry a representation higher than 3 with respect to strong SU(3) symmetry, the Cabibbo weak currents cannot in general be obtained by Cabibbo rotation and rather have to be induced by spontaneous symmetry breaking. We encounter no difficulty with strangeness-changing neutral currents even for triplets of leptons. Thirdly, we shall maintain left-right symmetry of strong interaction as a natural symmetry such that parity violation is calculable and small.

Denoting the quark and lepton columns respectively as Ψ^i and ψ^i triplets, $i=1,2,3$, we write the following gauge-invariant Lagrangian[7]:

$$-\sum_i \overline{\Psi}^i_L\gamma_\mu[\partial_\mu - if(\tfrac{1}{2}\vec{\lambda}\cdot V_{L\mu}) - ig'y_iB_\mu]\Psi^i_L - \sum_i \overline{\Psi}^i_R\gamma_\mu[\partial_\mu - if(\tfrac{1}{2}\vec{\lambda}\cdot V_{R\mu}) - ig'y_iB_\mu]\Psi^i_R - \sum_i \overline{\psi}^i_L\gamma_\mu[\partial_\mu - ig(-\tfrac{1}{2}\vec{\lambda}^*\cdot W_\mu) - ig'y_iB_\mu]\psi^i_L - \sum_i \overline{\psi}^i_R\gamma_\mu[\partial_\mu - ig'y_iB_\mu]\psi^i_R\,, \tag{1}$$

where we have gauged fully the strong $SU_L(3)\otimes SU_R(3)$ symmetry and only locally the $SU^{(l)}_L(3)$ and U(1) symmetries. The latter can be imbedded in larger groups such as $SU^{(l)}_L(3)\otimes SU^{(l)}_R(3)$ and $SU^{\text{color}}(3)$. y_i are the corresponding weak hypercharges determined by charge assignment. Since we are here interested only in processes involving observed leptons, we shall denote the gluons coupling to observed leptons by $W^\pm, W^3, \ldots$, whereas the strong gluons will be labeled by their SU(3) indices as V_ρ, $V_{K^*}, \ldots$. The coupling constants of the effective current-current interaction for leptonic, $\Delta S=0$, and $\Delta S=1$ semileptonic decays are then given as follows:

$$\begin{aligned}
&\text{leptonic:}\quad \frac{g^2}{8m_W{}^2}=\frac{G_W}{\sqrt{2}},\\
&\Delta S=0\ \text{semileptonic:}\quad \frac{gf}{8m_W{}^2}\frac{1}{m_\rho{}^2}\Delta m_{\rho W}{}^2\frac{1}{m_W{}^2}=\frac{G_W}{\sqrt{2}}\left(\frac{\Delta m_{\rho W}{}^2 f}{m_\rho{}^2 g}\right),\\
&\Delta S=1\ \text{semileptonic:}\quad \frac{gf}{8m_W{}^2}\frac{1}{m_{K^*}{}^2}\Delta m_{K^*W}{}^2\frac{1}{m_W{}^2}=\frac{G_W}{\sqrt{2}}\left(\frac{\Delta m_{K^*W}{}^2 f}{m_{K^*}{}^2 g}\right),
\end{aligned}\tag{2}$$

where m_ρ, m_{K^*} denote the masses of $V_{L\rho^\pm}$, $V_{LK^{*\pm}}$ and $\Delta m_{\rho W}{}^2$, $\Delta m_{K^*W}{}^2$ denote the mass-mixing terms between $V_{L\rho^\pm}$, $V_{LK^{*\pm}}$, and $W^\pm$. The Cabibbo structure of weak currents requires

$$\frac{\Delta m_{\rho W}{}^2 f}{m_\rho{}^2 g}=\cos\theta,\quad \frac{\Delta m_{K^*W}{}^2 f}{m_{K^*}{}^2 g}=\sin\theta,\quad \text{and thus}\quad \left[\left(\frac{\Delta m_{\rho W}{}^2}{m_\rho{}^2}\right)^2+\left(\frac{\Delta m_{K^*W}{}^2}{m_{K^*}{}^2}\right)^2\right]\frac{f^2}{g^2}=1\,. \tag{3}$$

The Cabibbo angle is given by

$$\tan\theta=\frac{\Delta m_{K^*W}{}^2}{\Delta m_{\rho W}{}^2}\left(\frac{m_\rho{}^2}{m_{K^*}{}^2}\right). \tag{4}$$

We introduce Higgs scalars which transform only nontrivially under $SU^{(l)}_L(3)$ to give the weak gluons heavy masses, e.g. triplet representations. The Higgs scalars which give the strong gluons masses and simultaneously contaminate the strong and weak gluons need to be discussed in detail. Let these be called ϕ's and have representation $(8,I)$ under $SU_L(3)$ and $SU^{(l)}_L(2)$. [We shall *localize* our attention to $SU^{(l)}_L(2)$ for mixing of $W^\pm$ with the V_L's.] The gauge-invariant Lagrangian for ϕ is given by

$$-\tfrac{1}{2}\sum_i \mathrm{Tr}\,|\partial_\mu\phi^i - ig(\vec{T}\cdot\vec{W}_\mu)\phi^i - if\phi^i(\vec{\Lambda}\cdot\vec{V}_{L\mu}) - ig'yB_\mu\phi^i|^2 + \text{potential}\,, \tag{5}$$

where $(\Lambda_i)_{jk}=if_{ijk}$ and the T's are the well-known isospin representations.[8] We shall label the SU(3) states by definite charge (and U spin) states:

$$\beta=(\rho^+,\rho^-,u_v,K^{*+},K^{*-},K^{*0},\overline{K}^{*0},u_s)\,,$$

where

$$F_{\rho^\pm}=\frac{1}{\sqrt{2}}(F_1\pm iF_2)\,,$$

$$F_{K^{*\pm}}=\frac{1}{\sqrt{2}}(F_4\pm iF_5)\,,$$

$$F_{K^{*0}(\overline{K}^{*0})}=\frac{1}{\sqrt{2}}(F_6\pm iF_7)\,,$$

$$F_{u_v}=\tfrac{1}{2}(F_3-\sqrt{3}F_8)\,,$$

and

$$F_{u_s}=\tfrac{1}{2}(\sqrt{3}F_3+F_8)\,.$$

ϕ^i is a $(2I+1, 8)$-dimensional matrix with indices labeled by $\phi_{s\beta^*}$, where $s=I, I-1,\ldots,-I$, and the row index is always complex conjugated. In the new basis, the SU(3) matrix representation can be obtained by rotation from the adjoint representation or be expressed analytically as $\Lambda_{\beta^*}V_\beta$, where $(\Lambda_{\beta^*})_{\alpha\gamma^*}=if_{\beta^*,\alpha\gamma^*}$, $f_{\beta^*,\alpha\gamma}$ satisfies $[F_\alpha, F_\gamma]=if_{\beta^*,\alpha\gamma}F_\beta$. From $\phi_j(if_{ijk}V_i)\phi_k = \phi_{\alpha^*}(\Lambda_{\beta^*}V_\beta)_{\alpha\gamma^*}\phi_\gamma$ one finds

$$\Lambda_{\beta^*}V_\beta=\begin{bmatrix} V_3 & 0 & -\frac{1}{2}V_{\rho^+} & -\frac{1}{\sqrt{2}}V_{\bar{K}^{*0}} & 0 & 0 & \frac{1}{\sqrt{2}}V_{K^{*+}} & -\frac{\sqrt{3}}{2}V_{\rho^+} \\ 0 & -V_3 & \frac{1}{2}V_{\rho^-} & 0 & \frac{1}{\sqrt{2}}V_{K^{*0}} & \frac{1}{\sqrt{2}}V_{K^{*-}} & 0 & \frac{\sqrt{3}}{2}V_{\rho^-} \\ -\frac{1}{2}V_{\rho^-} & \frac{1}{2}V_{\rho^+} & 0 & \frac{1}{2}V_{K^{*-}} & -\frac{1}{2}V_{K^{*+}} & V_{\bar{K}^{*0}} & -V_{K^{*0}} & 0 \\ -\frac{1}{\sqrt{2}}V_{K^{*0}} & 0 & \frac{1}{2}V_{K^{*+}} & \frac{1}{2}(V_3+\sqrt{3}V_8) & 0 & \frac{1}{\sqrt{2}}V_\rho & 0 & -\frac{\sqrt{3}}{2}V_{K^{*+}} \\ 0 & \frac{1}{\sqrt{2}}V_{\bar{K}^{*0}} & -\frac{1}{2}V_{K^{*-}} & 0 & -\frac{1}{2}(V_3+\sqrt{3}V_8) & 0 & -\frac{1}{\sqrt{2}}V_{\rho^-} & \frac{3}{2}V_{K^{*-}} \\ 0 & -\frac{1}{\sqrt{2}}V_{K^{*+}} & V_{K^{*0}} & \frac{1}{\sqrt{2}}V_{\rho^-} & 0 & -\frac{1}{2}(V_3+\sqrt{3}V_8) & 0 & 0 \\ \frac{1}{\sqrt{2}}V_{\rho^-} & 0 & -V_{\bar{K}^{*0}} & 0 & -\frac{1}{\sqrt{2}}V_{\rho^+} & 0 & \frac{1}{2}(V_3-\sqrt{3}V_8) & 0 \\ -\frac{\sqrt{3}}{2}V_{\rho^-} & \frac{\sqrt{3}}{2}V_{\rho^-} & -\frac{\sqrt{3}}{2}V_{K^{*-}} & \frac{\sqrt{3}}{2}V_{K^{*+}} & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{6}$$

The above explicit representation is presented for the convenience of the reader.

After spontaneous symmetry breaking, (5) gives the following masses for the strong gluons:

$$m_\beta{}^2=\frac{f^2}{2}\sum_i\sum_s\sum_{\alpha,\gamma}|if_{\beta^*,\alpha\gamma}\langle\phi^i_{s,\alpha^*}\rangle|^2\,, \tag{7}$$

where $\beta=\rho^\pm$, $K^{*\pm}$, $K^{*0}(\bar{K}^{*0})$, etc. Note that only the following members have the same charges: ϕ_{s,ρ^+}, $\phi_{s,K^{*+}}$; $\phi_{s-1,K^{0*}}$, $\phi_{s-1,\bar{K}^{0*}}$, ϕ_{s-1,u_s}, ϕ_{s-1,u_v}; ϕ_{s-2,ρ^-}, $\phi_{s-2,K^{*-}}$. In order not to break the electromagnetic gauge invariance, only those with charge zero can have nonvanishing VEV (by the charge assignment, we know in advance the photon field). Assuming nonvanishing real VEV for $\langle\phi_{s,\rho^+}\rangle$, $\langle\phi_{s,K^{*+}}\rangle$; $\langle\phi_{s-1,u_s}\rangle\langle\phi_{s-1,u_v}\rangle$, (5) gives the following mass-mixing terms:

$$\begin{aligned}\Delta m_{\rho-W^+}{}^2&=\frac{fg}{2\sqrt{2}}\sum_i\{[(T_1-iT_2)_{s,s-1}\langle\phi^i_{s-1,u_s}\rangle][\langle\phi^i_{s,\rho^+}\rangle if_{\rho^+,\rho^-u_s}]^*\\&\qquad+[(T_1-iT_2)_{s,s-1}\langle\phi^i_{s-1,u_v}\rangle][\langle\phi^i_{s,\rho^+}\rangle if_{\rho^+,\rho^-u_v}]^*+\text{H.c.}\}\\&=\frac{fg}{\sqrt{2}}b_s\sum_i[if_{\rho^+,\rho^-u_s}\langle\phi^i_{s-1,u_s}\rangle+if_{\rho^+,\rho^-u_v}\langle\phi^i_{s-1,u_v}\rangle]\langle\phi^i_{s,\rho^+}\rangle\end{aligned} \tag{8}$$

and

$$\Delta m_{K^*-W^+}{}^2=\frac{fg}{\sqrt{2}}b_s\sum_i[if_{K^{*+},K^{*-}u_s}\langle\phi^i_{s-1,u_s}\rangle+if_{K^{*+},K^{*-}u_v}\langle\phi^i_{s-1,u_v}\rangle]\langle\phi^i_{s,K^{*+}}\rangle\,, \tag{9}$$

where b_s is the matrix element of $T_1\pm iT_2$ (note: $[T_1\mp iT_2]_{i,j}=0$ except for $j=i\mp1$).[8] In order for the strong gluons $V_{LK^{0*}}$, $V_{L\bar{K}^{0*}}$ to remain unmixed, we cannot assign all $\langle\phi\rangle$'s to one (irreducible) representation; a choice is given by the following nonvanishing VEV's:

$$\begin{aligned}&\langle\phi^1_{s,\rho^+}\rangle,\langle\phi^1_{s-1,u_s}\rangle\,;\quad\langle\phi^2_{s,K^{*+}}\rangle,\langle\phi^2_{s-1,u_s}\rangle\,;\\&\langle\phi^3_{s,\rho^+}\rangle,\langle\phi^3_{s-1,u_v}\rangle\,;\ \langle\phi^4_{s,K^{*+}}\rangle,\langle\phi^4_{s-1,u_v}\rangle\,;\\&\langle\phi^5_{s-1,\bar{K}^{*0}}\rangle\,;\quad\langle\phi^6_{s-1,\bar{K}^{*0}}\rangle\,.\end{aligned} \tag{10}$$

So far, the theory has a lot of freedom. One thing should be obvious: With arbitrary VEV's allowed by the potential, (3) is not satisfied and will be violated by renormalization. However, there is an interesting case: When we impose a global symmetry on the Lagrangian as discussed below, we find that the potential has a minimum consistent with all the VEV's (10) being equal (denoted as σ). One finds[9]

$$m_{\rho^\pm}{}^2=m_{K^{*\pm}}{}^2=m_{K^{*0}}{}^2=2\sigma^2f^2 \tag{11}$$

and

$$\tan\theta=\frac{f_{K^{*+},K^{*-},u_s}+f_{K^{*+},K^{*-},u_v}}{f_{\rho^+,\rho^-,u_s}+f_{\rho^+,\rho^-,u_v}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\,; \tag{12}$$

$\tan\theta$ is independent of the $SU^{(I)}(2)$ representation of ϕ. Now one notices that (3) becomes $b_s=[(I+s)(I-s+1)]^{1/2}=2$, which is satisfied for $I=2$, with $s=2$ or -1. In other words, the Cabibbo structure is obtained naturally for the angle given by (12) and obviously is not if we shift the VEV's around this solution.

Let us write the Higgs scalar in a supermatrix form:

$$\phi^T=(\phi^{1T},\phi^{2T},\phi^{3T},\phi^{4T},\phi^{5T},\phi^{6T}).$$

Then (5) can also be written as

$$-\tfrac{1}{2}\mathrm{Tr}\,|\partial_\mu\phi-ig(\vec{\mathrm{T}}\otimes 1)\cdot\vec{\mathrm{W}}_\mu\phi-if\phi(\vec{\Lambda}\cdot\vec{\mathrm{V}}_\mu)-igyB_\mu\phi\,|^2+P(\phi)\,,$$

where $\vec{\mathrm{T}}$ are matrices in the $2I+1$ Hilbert space and 1 is a unit matrix in the U(6) space for $i=1, 2,\ldots,6$. The gauge derivative term is also invariant under the global transformation $\phi\to(1\otimes R)\phi$, where R is an SU(6) transformation. Imposing SU(6) global invariance, the potential has the following form:

$$P(\phi)=a\,\mathrm{Tr}(\phi^\dagger\phi\phi^\dagger\phi)+b[\mathrm{Tr}(\phi^\dagger\phi)]^2+c\,\mathrm{Tr}(\phi^\dagger\phi).$$

With the nonvanishing VEV's in (10) all equal (to σ), one has

$$\langle\phi\rangle^\dagger\langle\phi\rangle\langle\phi\rangle^\dagger=2\sigma^2\langle\phi\rangle^\dagger\,.$$

It is then straightforward to check that the condition of zero slope determines σ^2 and $P(\delta\phi+\langle\phi\rangle)-P(\langle\phi\rangle)\geq 0$. Several comments are in order:

(1) $\mathrm{SU}_L(3)$ [as well as $\mathrm{SU}_R(3)$ discussed below] is a natural symmetry of the strong interaction in this model. In general there seems to be no *a priori* reason to expect SU(3) or $\mathrm{SU}_L(3)\otimes\mathrm{SU}_R(3)$ to be a good symmetry unless it is also a natural symmetry of the strong interaction.

(2) There are presumably other minima of the potential which also give the Cabibbo structure of weak currents naturally. Through our search, we seem to find no other nontrivial solution besides (12) and $\theta=45°$, which is also consistent with a natural SU(3) symmetry.

(3) It is generally known that SU(n) ($n>2$) cannot be fully broken with one irreducible low representation of Higgs scalar and that SU(n) is a natural symmetry only when the VEV's of the different representations of the Higgs scalars are constrained (by imposing additional global symmetry on the Lagrangian). But since the global symmetry is also broken spontaneously, the theory has Goldstone bosons which are not always absorbed away. In the present case, there will be Goldstone bosons associated with the broken SU(6) symmetry. These Goldstone bosons are neutral and belong to the Hilbert space projected out by the (neutral) SU(6) generators $1\otimes\theta^k$, where θ^k is a matrix in SU(6) space. The gluons W^i by gauge invariance only couple to Higgs scalars in $T^i\otimes 1$ space and have no direct coupling with the SU(6) Goldstone bosons. The fermions, being SU(6) singlets, also do not couple to the Goldstone bosons directly; the quarks do not couple to ϕ by $\mathrm{SU}_L(3)$ representation. Thus the effect of the Goldstone bosons is to renormalize Higgs scalars masses and coupling constants without modifying ordinary weak interactions (to order G_W^2). The Goldstone bosons can only be produced by self-interaction with the massive Higgs scalars which could couple to gluons and then couple to quarks—the effective coupling is of higher order in G_W^2 and can be arbitrarily small. Being neutral, the Goldstone bosons are very difficult to detect anyway. The presence of such Goldstone bosons is not ruled out by experiments, and we note that they only play a very minimal role in this model.

To break the $\mathrm{SU}_R(3)$ symmetry spontaneously, we introduce the $\mathrm{SU}_R(3)$ partner of ϕ which transforms under $\mathrm{SU}_R(3)$ and $\mathrm{SU}_L^{(I)}(2)$ as $(8,I)$ also (denoted as ϕ'). We shall impose left-right reflection symmetry for the strong $\mathrm{SU}_L(3)$ and $\mathrm{SU}_R(3)$ symmetry in addition to the global symmetry mentioned above. The only difference is that the VEV's of ϕ' do not contaminate the $\rho^\pm$, $K^{*\pm}$, and $K^{*0}(\overline{K}^{*0})$ as do $\mathrm{SU}_R(3)$ gluons with the weak gluons. This can be accomplished by the following nonvanishing VEV's:

$$\langle\phi'^1_{s,\rho^+}\rangle=\langle\phi'^2_{s,K^{*+}}\rangle=\langle\phi'^3_{s-1,u_s}\rangle=\langle\phi'^4_{s-1,u_v}\rangle=\langle\phi'^5_{s-1,\overline{K}^{*0}}\rangle=\sqrt{2}\sigma\,.$$

Note that $\sqrt{2}$ is fixed by left-right symmetry.

The model has finite and small parity violation essentially because parity is a natural symmetry of the strong interactions (because of the left-right symmetry) and all the parity violations come from the mixing of the strong gluons with the weak gluons induced by the Higgs scalars.[10] The physical gluons are mixture of the strong gluons and the weak gluons, but they are sufficiently pure with the mixing of the order[11] $\Delta m_{vW}^2/m_W^2\sim m_v^2/m_W^2\sim 10^{-5}$. This aspect is similar to the Pati-Salam model,[12] and as pointed out in Ref. 13 the parity violation due to such mixing is finite and small (of the order 10^{-5}). We have gauged chiral symmetry for the strong interaction, but we note that the additional parity violation due to chiralty is also finite and of the order[11,13] $(m_{v_L}^2-m_{v_R}^2)/m_v^2\sim\Delta m_{vW}^2/m_W^2\sim 10^{-5}$ (because of the left-right symmetry). For example, the radiative correction to the difference of the gauge coupling constants f_L-f_R will be of the order $\alpha(m_{\nu_L}{}^2 - m_{\nu_R}{}^2)/m_\nu{}^2 \sim \alpha 10^{-5}$.

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[7]In present forms one needs to double the fermions to cancel the anomaly.

[8]The matrix representations of SU(2) generators are [see for example L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), p. 202]

$$T_3 = \begin{bmatrix} l & & & & & & \\ & l-1 & & & & & \\ & & l-2 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & -l \end{bmatrix},$$

$$T_1 = \tfrac{1}{2}\begin{bmatrix} 0 & b_l & 0 & 0 & & & & \\ b_l & 0 & b_{l-1} & 0 & & & & \\ 0 & b_{l-1} & & b_{l-2} & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & 0 & b_{-l+1} \\ & & & & & & b_{-l+2} & 0 \end{bmatrix},$$

$$T_2 = \tfrac{1}{2}\begin{bmatrix} 0 & -ib_l & 0 & 0 & & \\ ib_l & 0 & -ib_{l-1} & 0 & & \\ 0 & ib_{l-1} & 0 & -ib_{l-2} & & \\ & & & & 0 & -ib_{-l+1} \\ & & & & ib_{-l+1} & 0 \end{bmatrix},$$

where $b_s = [(l+s)(l-s+1)]^{1/2}$.

[9]The result can be easily checked with the matrix representations given in footnote 8 and Eq. (6).

[10]Parity violation in the M model has previously been discussed by I. Bars, Nucl. Phys. B64, 163 (1973); M. B. Halpern, *ibid.* B66, 78 (1973).

[11]Note that $m_{V_i}{}^2 \simeq m_V{}^2(1-\Delta m_{V_iW}{}^2/m_W{}^2)$ for $m_{V_i}{}^2$, $\Delta m_{V_iW}{}^2 \ll m_W{}^2$.

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