Phenomenology of the current-constituent quark transformation for the vector-gluon model*

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(Received 22 September 1975)

The transformation relating the expansion of hadronic states in the current- and constituent-quark bases is studied in the context of the vector-gluon model. A phenomenological approach is used to estimate the effects of interaction on the algebraic structure of matrix elements describing pionic, electromagnetic, and weak transitions of hadrons. Contributions of terms with exotic transformation properties are found to be negligible in most instances, but are important for higher moments of deep-inelastic structure functions. The magnitude of such terms is related to that of the mass splitting of SU(6) multiplets.

I. INTRODUCTION

In recent discussions of the relation between constituent quarks and current quarks there is general agreement on at least one fundamental hypothesis. It is assumed that the Hamiltonian is simply expressed in terms of current quarks (or quark partons), at least term by term, but that the eigenstates are simple only in terms of constituent quarks. On the one hand, the interaction of current quarks (at least with currents) is supposedly known, and the hadrons' composition in terms of these current quarks is not known. On the other hand, the hadrons' composition in terms of constituent quarks is known by hypothesis, and the interaction of these quarks is not known. Until recently, the distinction between constituent and current quarks has not been drawn carefully. One may argue that early quark-model calculations were done in the constituent-quark basis after having made some hypothesis for the interaction. More recently, calculations have been made[1] in the constituent-quark basis after choosing an appropriate transition operator, which after all does imply some assumptions about interaction. It is also possible to attack the problem from the opposite direction by employing the transformation[2] V connecting the current- and constituent-quark bases. There may well be some advantages to this approach that are not present in the other.[3]

The transformation V is not trivially unity even in the free-quark model, which is the case investigated by Melosh. As pointed out by Eichten *et al.*,[4] V_{free} is a rotation in spin space, so spin-dependent matrix elements are sensitive to this transformation while spin-averaged quantities are not. Moreover, V_{free} is a one-quark operator and does not create gluons or quark-antiquark pairs. In the present work we undertake the study of V in order to gain some understanding of hadron dynamics. Our approach will be to choose the vector-gluon model as an especially promising interaction and then develop some phenomenological rules for estimating the effect of interaction on the transformation V. To do this, we will take various clues we are given from the experimental data on deep-inelastic leptoproduction as well as on certain static properties of the nucleon and incorporate these clues into the formalism for interacting current and constituent quarks that we have developed earlier.[5]

II. EXPERIMENTAL FRAMEWORK

The cross section for lepton-hadron scattering is proportional to the Fourier transform of the one-particle expectation value of a current commutator. In the Bjorken scaling limit, the integral is presumably dominated by a region near the light cone; thus, the deep-inelastic structure functions can be related[6] to forward matrix elements of the vector bilocal operator $\mathfrak{F}_i^+(x,y)$. Feynman's parton model[7] then follows if the field theory is quantized on a null plane.[8] The structure functions themselves, however, remain unspecified as they are interaction dependent. In this sense, the parton model is a framework in which various interactions (various structure functions) are possible.

Melosh has shown that the relevant operator,

$$\int d^4x\,\delta(x^+)\mathfrak{F}_i^+(\vec{x}_\perp, x^-; \vec{x}_\perp, 0)e^{-iP^+x^-/\omega}, \qquad (2.1)$$

when transformed by V_{free}, has an octet piece that is purely F coupled within the $\frac{1}{2}^+$ octet. This implies that the ratio of neutron-to-proton structure functions R, is bounded below by $\frac{2}{3}$, independent of ω, in disagreement with the data. The bilocal operators thus seem to be sensitive to the existence of quark-antiquark pairs or some other property of the nucleon current-quark space configuration which is due to interaction. On the other hand, the zeroth moments of the neutron and proton structure functions are approximately in the ratio $\frac{2}{3}$. A basic question we must answer, then, is which properties of V are due to interaction and which are present in V_{free}; furthermore, we seek estimates of the magnitudes of these effects.

The transformation V must account for several experimental facts. First, the deviations of G_A/G_V and of the nucleon magnetic moments from their SU(6) symmetric values require that there be an $L=1$ component mixed in the nucleon's expansion in the current-quark basis.[9] This is satisfied by V_{free}. Second, deep-inelastic lepton scattering can be interpreted[7] within the parton model as indicating that the nucleon (in the current-quark basis) is a system with (a) three "valence quarks" carrying about half the total four-momentum and interacting with currents in a pointlike manner; (b) a large, possibly infinite, number of quark-antiquark pairs carrying in all a very small fraction of four-momentum; and (c) neutral hadronic matter carrying about half of the total four-momentum. Third, from deep-inelastic scattering again, the ratio R mentioned above indicates that the configuration mixing induced by V must be momentum dependent as well as interaction dependent. Fourth, it is well known that $\text{SU}(6)_W$ is only an approximate symmetry even for vertex functions and that $\text{SU}(6)_W$ multiplets are certainly not mass degenerate. The free-quark-model transformation V_{free} provides an $\text{SU}(6)_{W,\text{strong}}$ breaking for many vertex functions,[1,2] in agreement with experiment, but the mass breaking has so far resisted efforts at deeper understanding.

In sum, there is abundant evidence for the need for a nontrivial transformation V, and in fact for the need for a transformation more complex than V_{free}. Most of this evidence is derived from deep-inelastic scattering; of course, we cannot ignore other operators for which there is some information on $\text{SU}(6)_W$ symmetry and its breaking. We must take account of the fact that the free-quark algebraic structure abstracted from V_{free} works quite well for matrix elements of F_i^3, describing pionic transitions, and the first moments of F_{em}, describing electromagnetic dipole transitions. Clearly, any reasonable model should add terms to the classification of these operators; however, these terms are suppressed in the case of F_i^3 and F_{em}, but not for $\mathfrak{F}_i^+(x;0)$. We want to find what constraints such observations place on a phenomenological model for hadron interaction based on the vector-gluon model.

III. SOME QUALITATIVE PRELIMINARY REMARKS

The algebraic structure of single-particle matrix elements of an arbitrary Heisenberg operator Q with respect to $\text{SU}(6)_{W,\text{strong}}$ is given by the corresponding properties of $V^{-1}QV$ with respect to $\text{SU}(6)_{W,\text{currents}}$ since

$$\langle b|Q|a\rangle = \langle b,\text{currents}\,|V^{-1}QV|a,\text{currents}\rangle,$$
$$|a,\text{currents}\rangle \equiv V^{-1}|a\rangle, \tag{3.1}$$

just as discussed first by Melosh. We have shown[10] that

$$V=\Omega V_{\text{free}}, \tag{3.2}$$

so that it is convenient to write

$$\langle b|Q|a\rangle = \langle b,\text{currents}|\tilde{\Omega}^{-1}\tilde{Q}\tilde{\Omega}\,|a,\text{currents}\rangle, \tag{3.3}$$

where the tilde denotes the V_{free}-modified operator,

$$\tilde{Q}\equiv V_{\text{free}}{}^{-1}QV_{\text{free}}. \tag{3.4}$$

The interaction appears in the operator $\tilde{\Omega}$, since

$$\tilde{\Omega}=\mathcal{T}\exp\left[-i\int_{-\infty}^{0}\tilde{P}^-_{I,D}(\tau)d\tau\right], \tag{3.5}$$

where $\mathcal{T}$ denotes τ ordering (appropriate to null-plane quantization) and $P^-_{I,D}$ is the interaction part of the Hamiltonian P^- in the Dirac picture.

The known particle and resonance states appear to fall into multiplets with different spins in such a way that we see approximately degenerate $\text{SU}(6)_{W,\text{strong}}$ multiplets. We are thus led to expect

$$[W_i^\alpha, P^-]\approx 0, \tag{3.6}$$

where W_i^α are the generators of $\text{SU}(6)_{W,\text{strong}}$ and the approximate nature of the relation is dependent upon the matrix element chosen. A few steps of algebra then lead to the approximate $\text{SU}(6)_{W,\text{currents}}$ invariance of $\tilde{\Omega}$,

$$[F_i^\alpha,\tilde{\Omega}]\approx 0. \tag{3.7}$$

Now, of course, the strict equality cannot hold since we expect $\tilde{P}_I^-$ to flip quark spin and create soft gluons and quark-antiquark pairs. We do not expect $\tilde{P}_I^-$ to change appreciably quark momenta, quark number (neglecting soft pairs), or SU(3) properties of a state. So, if $\tilde{Q}$ is an integral of a local operator bilinear in quark fields, then $\tilde{\Omega}^{-1}\tilde{Q}\tilde{\Omega}$ will still be (approximately) a one-body quark operator with the same SU(3) transformation properties.

In lowest order, therefore, the operator $\tilde{\Omega}$ should have the sole effect of creating soft gluons and modifying the quark momentum distribution so as to maintain constant total P^+ and $\vec{P}_\perp$. Then $\tilde{\Omega}|a,\text{currents}\rangle$ consists of "valence quarks" and gluons; current operators $\tilde{Q}$ are one-body quark operators (they do not affect gluons), so the free-quark model should work for such operators. With the approximation Eq. (3.7) we conclude that $\tilde{\Omega}|a, \text{currents}\rangle$ transforms under the F_i^α [i.e., under $SU(6)_{W,\text{currents}}$] in the same way as does $|a, \text{currents}\rangle$; we may then use the Wigner-Eckart theorem to extract the SU(6) group-theoretic factors just as if Q transformed under the W_i^α as $\tilde{Q}$ does under the F_i^α. This, after all, is the basic input for the Melosh phenomenology.

Actually, $\tilde{\Omega}$ will create pairs in some approximation, say to order ϵ. Consider the matrix element

$$\langle b|Q|a\rangle = \langle b, \text{currents}|\tilde{\Omega}^{-1}\tilde{Q}\tilde{\Omega}|a, \text{currents}\rangle. \quad (3.3)$$

Then, if Q is a one-quark operator, so is $\tilde{Q}$, and the correction due to pairs enters only to order ϵ^2. However, if Q has a pair creation or destruction piece, then the correction will enter in order ϵ. Since bilocal operators have such pieces, we should thus expect the effect of pairs to be greater for matrix elements of bilocals than for integrated local currents. The point is that the matrix element is taken between states that do not themselves contain soft pairs. Therefore, the only way corrections due to soft pairs can arise is through emission and reabsorption of these pairs.

The preceding discussion is qualitative in the extreme. One would like to make more quantitative statements about the nature of SU(6) breaking and about the effect of interaction on the SU(6) transformation properties of transition operators. In the next section the structure of $\tilde{\Omega}$ is discussed without making use of these questionable arguments. In recent paper, Carlitz and Weyers[11] have developed a scheme for doing just this in a phenomenological and interaction-independent way. We will adapt their techniques to our formalism and apply the method to study the vector-gluon interaction.

IV. THE INTERACTION

The operator $\tilde{\Omega}$, given in Eq. (3.5), contains the dependence on interaction. Therefore, the modification of the algebraic structure of an operator which arises from interaction may be found by examination of the algebraic structure of $\tilde{\Omega}$. But $\tilde{\Omega}$ is an exponential function of $\tilde{P}_I^-$; so, insofar as these modifications are small, they are determined by the structure of $\tilde{P}_I^-$ itself. In this section precisely this question of the algebraic structure of $\tilde{P}_I^-$ in the vector-gluon model is studied. The results are then applied in Sec. V to the problem of the structure of certain interesting transition operators.

The transformation V, connecting current- and constituent-quark bases, is nonlocal; there is an intrinsic spatial dimension characterized by $a = 1/m$, where m is the quark mass. In hadronic matrix elements, however, there is another quantity which enters, namely, the dimension R of the hadronic state itself. For example, derivatives of quark fields have expectation values typically of order $1/R$; this may be thought of as a measure of the momentum of a quark inside the hadron.

Carlitz and Weyers expand local operators in terms of nonlocal constituent quarks. They argue that this expansion, involving powers of the ratio of the parameter a to the hadronic "size" R may very well converge fairly rapidly. They then proceed to use the expansion (suitably truncated) to study pionic and electromagnetic transitions of hadrons. The first question that must be answered is how large is the expansion parameter a/R. An estimate is provided by the deviation of G_A from the SU(6) value of $\frac{5}{3}$, since V_{free} induces such a correction in order $(a/R)^2$, as pointed out by Melosh[3] and in a somewhat different but related context, by Bogoliubov.[12] Thus a/R is of order $\frac{1}{2}$.

The Hamiltonian for the vector-gluon model is easily written down. For the purposes of the present study the internal-symmetry indices are not relevant, so they will be suppressed. The interaction piece of the Hamiltonian, P_I^-, has a term linear in the transverse components of the gluon field, $\vec{A}_\perp$, a term quadratic in $\vec{A}_\perp$, and a "Coulomb" term in which $\vec{A}_\perp$ does not appear. We will discuss these terms in the order mentioned.

The first term has the form

$$P_a^- = \int d^2x_\perp dx^- d\xi \psi_+^\dagger(\vec{x}_\perp, x^-)\vec{\gamma}_\perp\psi_+(\vec{x}_\perp, \xi) \times \epsilon(x^- - \xi)\vec{A}_\perp(\vec{x}_\perp, \xi). \quad (4.1)$$

Clearly, P_a^- transforms under $SU(6)_{W,\text{currents}}$ like $\underline{35}$; furthermore, if ψ_+ and $\vec{A}_\perp$ are expanded in terms of creation and annihilation operators, it is easy to see that this term has a part which creates quark-antiquark pairs with spin $S = 1$ and $S_x = \pm 1$. We propose to estimate the magnitude of contributions of this term and other terms involving $\vec{A}_\perp$. To this end we introduce an additional size parameter, ρ, which characterizes the gluon field. We do not feel that ρ is *a priori* equal to either of the previously introduced parameters a, R. (In this we differ with Carlitz and Weyers.) Then P_a^- has matrix elements characterized by (R/ρ); however, since gluons are not present in hadronic states in the constituent-quark basis, the contribution of this term will be in second order, $(R/\rho)^2$. This is probably small compared to $(a/R)^2$, as we now argue. It cannot be larger, since P_a^- contributes in second order also to SU(6) mass splitting, which is roughly of the order of 25%, or $(a/R)^2$. Now, P_a^- induces a mixing in order (R/ρ) of the baryon $\underline{56}$, $L=0$ with states that transform like $\underline{70}$, $L=0$. There is some evidence[13] for a small amount of such mixing; the dominant effect of mixing is with $\underline{70}$, $L=1$ and occurs in order (a/R). Furthermore, if the quarks are coupled to an octet of colored vector gluons[14] then the $\underline{70}$ mixed with the $\underline{56}$ is a color octet as far as the quark variables are concerned. On the other hand, hadron states are not observed experimentally to have color-octet components in the constituent-quark part of their wave functions; the unitary nature of V would lead one to expect such states to exist, just as one expects (and finds) states of a $\underline{70}$, $L=1$ multiplet which are mixed into the ground state $\underline{56}$, $L=0$ states. The conclusion, therefore, is that P_a^- may be ignored to the 10% level insofar as we are concerned with the algebraic structure of transition matrix elements. Presumably, the nucleon's momentum is shared among "valence quarks" and gluons by virtue of higher-order effects of P_a^-, so it is not completely negligible.[15]

The interaction Hamiltonian has other terms involving $\vec{A}_\perp$, which are expected to be less important. There is a term

$$P_b^- = \int d^2x_\perp dx^- d\xi \psi_+^\dagger(x)\psi_+(x)\epsilon(x^- - \xi)\vec{\partial}_\perp \cdot \vec{A}_\perp , \tag{4.2}$$

which, having a transverse derivative, is of order a/R smaller than P_a^-; there is also a term quadratic in $\vec{A}_\perp$.

The "Coulomb" term,

$$P_c^- = \int d^2x_\perp dx^- d\xi \psi_+^\dagger(\vec{x}_\perp, x^-)\psi(\vec{x}_\perp, x^-) \times |x^- - \xi| \psi_+^\dagger(\vec{x}_\perp, \xi)\psi(\vec{x}_\perp, \xi), \tag{4.3}$$

is the remaining term in the interaction Hamiltonian. It does not depend explicitly on the gluon field variables $\vec{A}_\perp$, so it cannot be immediately neglected on the basis of the preceding discussion. In fact, we will argue contrariwise that it does not in general give small corrections to current matrix elements.

By expanding ψ_+ in terms of creation and annihilation operators, it is easy to see that P_c^- has the ability to create and destroy quark-antiquark pairs. When expressed in momentum space, the $|x^- - \xi|$ factor gives a momentum damping factor of $1/q^2$, where q is the "plus" component of the momentum of the pair. This implies that soft pairs are predominantly created and destroyed.

It is obvious that P_c^- transforms as a singlet under $\mathrm{SU}(6)_{W,\mathrm{currents}}$. However, recall that the relevant question to be asked is how the modified operator $\tilde{P}_c^-$ transforms under $\mathrm{SU}(6)_{W,\mathrm{currents}}$. It is easy enough to compute $\tilde{P}_c^-$, since V_{free} is known, and so $V_{\mathrm{free}}^{-1}\psi_+ V_{\mathrm{free}}$ is known explicitly. An interesting thing occurs now owing to the fact that P_c^- is not bilinear in quark fields. That is, although

$$\int d^2x_\perp dx^- \psi_+^\dagger(x)\psi_+(x) \tag{4.4}$$

is invariant under V_{free} (since the transverse derivatives that V_{free} introduces can be integrated by parts and then they cancel exactly), the same is not true of P_c^-. Indeed, $\tilde{P}_c^-$ will contain a term of the form

$$\int d^2x_\perp dx^- \psi_+^\dagger(x)\vec{\gamma}_\perp \cdot \vec{\partial}_\perp \psi_+(x)|x^- - \xi| \psi_+^\dagger(\vec{x}_\perp, \xi)\psi_+(\vec{x}_\perp, \xi), \tag{4.5}$$

as well as

$$\int d^2x_\perp dx^- \psi_+^\dagger(x)\vec{\gamma}_\perp \cdot \vec{\partial}_\perp \psi_+(x) \times |x^- - \xi| \psi_+^\dagger(\vec{x}_\perp, \xi)\vec{\gamma}_\perp \cdot \vec{\partial}_\perp \psi_+(\vec{x}_\perp, \xi), \tag{4.6}$$

so that $\tilde{P}_c^-$ is *not* $\mathrm{SU}(6)_{W,\mathrm{currents}}$ invariant, but in fact transforms as a sum of terms which occur in the product $\underline{35}\otimes\underline{35}$; its quark spin properties are likewise complex. It is SU(3) invariant. In fact, the part of $\tilde{P}_c^-$ displayed in Eq. (4.6) will cause an SU(3)-invariant breaking of SU(6) mass degeneracy; e.g., it will lead to a splitting of the nucleon and $\Delta(1236)$ states. The magnitude of this splitting, 25%, is consistent with the dimensional argument given earlier as applied to this term, since the two transverse derivatives lead to an estimate of $(a/R)^2$. Note that the term linear in transverse derivatives, displayed in Eq. (4.5), cannot contribute to the octet-decuplet splitting since it transforms as $L_z = \pm 1$.

What is the effect of $\tilde{P}_c^-$ with respect to configuration mixing? In zeroth order in a/R the sole effect is to create a "sea" of low-momentum quark-antiquark pairs without modifying the SU(6) properties of the state in question. In first order in a/R, the effect is the same as the well-known effect of V_{free}. Since V_{free} modifies the state in order a/R also, there is no way of separating the contributions; moreover, since this configuration mixing is in good agreement with experiment, there is no difficulty here with the additional mixing induced by $\tilde{P}_c^-$. In order $(a/R)^2$, states with more complex transformation properties, including exotics, will be mixed. This prediction differs from that of Carlitz and Weyers, who expect such mixing to enter only in order $(a/R)^3$. It is easier to see the consequences of this result by using the complementary method of examining the transformation properties of the modified operators, $\tilde{\Omega}^{-1}\tilde{Q}\tilde{\Omega}$, as outlined above. This will be done next for several specific choices of operators.

V. EFFECT OF INTERACTION ON STRUCTURE OF OPERATORS

Consider first the general class of integrated local operators which are bilinear in quark fields, but have no transverse derivatives acting on these fields. It is easy to see that if $\tilde{Q}$ has such a form then the part of $\tilde{P}_c^-$ which has no such derivatives will commute with $\tilde{Q}$. Furthermore, even if $\tilde{Q}$ has such derivatives, that part of $\tilde{P}_c^-$ which does not will modify $\tilde{Q}$ in such a way as to leave its SU(6) transformation properties unchanged. Therefore, operators $\tilde{Q}$ in this class will transform the same as $\tilde{\Omega}^{-1}\tilde{Q}\tilde{\Omega}$, to lowest order in a/R.

Let us be more specific. The axial charge which is relevant for meson emission, $\tilde{F}_i^3$, has one part which transforms as $(\underline{1},\underline{8})\oplus(\underline{8},\underline{1})$ and another (of order a/R) as $(\underline{3},\underline{\bar{3}})\oplus(\underline{\bar{3}},\underline{3})$. The $(\underline{1},\underline{8})\oplus(\underline{8},\underline{1})$ part consists of the axial generator F_i^3 (without derivatives) and correction terms which enter in order $(a/R)^2$. The interaction $\tilde{P}_c^-$ will modify this only in order $(a/R)^2$, since it commutes with F_i^3. The $(\underline{3},\underline{\bar{3}})\oplus(\underline{\bar{3}},\underline{3})$ part of $\tilde{F}_i^3$ does not commute with $\tilde{P}_c^-$, but terms of different transformation properties only enter in higher order in a/R, namely, order (a/R). The exotic terms present in $\tilde{F}_i^3$ induced by $\tilde{P}_c^-$ transform as $(\underline{8},\underline{8})$ with $L=0,2$ and $(\underline{3},\underline{6})\oplus(\underline{6},\underline{3})$ with $L=2$, in order $(a/R)^2$. Do these exotic terms ruin the phenomenological successes of the model which have been obtained using algebraic properties of $\tilde{F}_i^3$ as abstracted from the free-quark model?

Consider first the matrix elements of the axial charge between states of the $\underline{56}$, $L=0$ multiplets. Since this is a $\Delta L=0$ transition, there is a contribution from $(\underline{8},\underline{8})$ with $L=0$. However, this is $(a/R)^2$ smaller than the dominant term. It would be quite difficult to disentangle this term from the nonexotic correction which is of the same order. For the pionic transitions of the $L=1$ meson $\underline{35}$ and baryon $\underline{70}$ resonances to the ground state $\underline{35}$ and $\underline{56}$ the dominant contribution is of order (a/R) and arises from the $(\underline{3},\underline{\bar{3}})\oplus(\underline{\bar{3}},\underline{3})$ term. The exotic terms of order $(a/R)^2$ cannot contribute to these transitions, so the corrections to this algebraic structure will be of order $(a/R)^2$ relative to the dominant contribution. Experimental data are in agreement with this picture, with room enough for exotic terms of order 25%. For the decays of $L=2$ meson and baryon states to the ground state, the model predicts that the exotic terms should be of the same order as the nonexotic, namely, $(a/R)^2$; there is no evidence for a need for such exotic terms.[16] This may be an indication that the model is in trouble.

The dipole operator $\tilde{D}$, where

$$D=\int d^2x_\perp dx^-(x_1+ix_2)\mathcal{F}^+_{\rm em}(x), \tag{5.1}$$

has been studied in the free-quark model and the attendant algebraic structure has been used in phenomenological analyses of data with good results.[17] What is the effect of $\tilde{P}_c^-$? Since P_c^- commutes with D, the terms of zeroth order in a/R in $\tilde{\Omega}^{-1}\tilde{D}\tilde{\Omega}$ are unchanged. The first-order correction terms will include $\bar{q}q\bar{q}q$ terms, but these will have the same transformation properties under SU(6) as $\tilde{D}$. Therefore, exotic terms will enter only in order $(a/R)^2$. For matrix elements of the dipole operator between states of $L=0$ multiplets of course only $L=0$ terms can contribute. The leading term is of order a/R and transforms as $(\underline{3},\underline{\bar{3}})$. Exotic terms with $L=0$ are induced by the interaction, but it can be shown that these are order $(a/R)^3$, so they are small compared to the dominant nonexotic term, and the predictions of the Melosh approach to these processes are unchanged to order 25%. It is thus still not understood why the famous SU(6) result for the neutron-proton magnetic moment ratio $-\frac{2}{3}$ holds to such fantastic accuracy. A related problem is the vanishing of the $E2$ transition moment for $\Delta^+\to p\gamma$, which SU(6) predicts to vanish; correction terms are much smaller than expected. For transitions from $L=1$ states to $L=0$ ground states the dominant terms are zeroth order in a/R; exotic and nonexotic terms contribute in order $(a/R)^2$. Phenomenological analysis[17] support this scheme, although no need for exotic terms has been found. Present data allow for a 25% violation due to exotic contributions. Photoproduction of $L=2$ baryon states should provide a more sensitive probe of exotic terms since we expect them to be roughly half the size of the leading terms, which are a/R. Analysis of existing photoproduction data[18] does not require exotics.

In a similar way, the second moments of the electromagnetic current can be analyzed. The dominant term is just that obtained by Melosh; this implies that the baryon $\underline{56}$ charge radii are proportional to charge. In particular, the neutron charge radius should vanish. Corrections to this result arise from exotic terms induced by $\tilde{P}_c^-$, and a term proportional to

$$\int d^2x_\perp dx^- d\xi\, \psi_+^\dagger(x)\vec{\gamma}_\perp\lambda_Q\psi_+(x)\,|x^- - \xi|\,\psi_+^\dagger(\vec{x}_\perp,\xi)\vec{\gamma}_\perp\psi_+(\vec{x}_\perp,\xi) \tag{5.2}$$

contributes in order $(a/R)^2$. The experimental value of the neutron charge radius is 15–20% of that of the proton, which we find in satisfactory agreement with expectations.

Finally, we turn our attention to a matrix element which is probably the most promising for our learning about the properties of the current-constituent quark transformation. The particular transition operator we are interested in here is the vector bilocal, $\mathfrak{F}_i^+(\vec{x}_\perp, x^-; \vec{x}_\perp, 0)$, and the expectation value of its Fourier transform for a state of four-momentum P

$$F(\omega)=\int d(P\cdot x)d^2x_\perp e^{-i(P^+x^-)/\omega}\langle P,\text{currents}|\tilde{\Omega}^{-1}\tilde{\mathfrak{F}}_i^+(\vec{x}_\perp, x^-; \vec{x}_\perp, 0)\tilde{\Omega}|P,\text{currents}\rangle. \quad (5.3)$$

The first striking effect of interaction on the form of the bilocal is that P_c^- does not commute with $\mathfrak{F}_i^+$. Note that no tildes are present in the preceding statement. The commutator is proportional to

$$\int d\xi d^2x_\perp \psi_+^\dagger(\vec{x}_\perp, x^-)\lambda_i\psi_+(\vec{x}_\perp, 0)$$
$$\times(|x^- - \xi| - |\xi|)\psi_+^\dagger(\vec{x}_\perp, \xi)\psi_+(\vec{x}_\perp, \xi), \quad (5.4)$$

which means that $\tilde{\Omega}^{-1}\tilde{\mathfrak{F}}_i^+\tilde{\Omega}$ has quark-antiquark pair creation and destruction operators to zeroth order in a/R, contrary to the situation for bilinear operators discussed above. We thus expect that exotic terms may manifest themselves in lower order in a/R than previously.

It is convenient to continue the discussion by treating the various integrated moments of $F(\omega)$, since this is how much analysis of experimental data has been presented. The zeroth moment essentially picks out the $x^-=0$ piece of the bilocal. It is easy to see that the quark-antiquark pair contribution vanishes to zeroth order in a/R. The first-order term is not exotic, nor in fact is the second-order term. Therefore, exotic corrections to the zeroth moment of the structure functions will appear only at the 10% level. This is in good agreement with experiment, as noted in the Introduction. Proceeding to higher moments of the structure functions, we find that exotic corrections appear in order a/R. Again, this is in good agreement with experiment. For example, the ratio of first moments of the neutron and proton structure functions is closer to $\frac{1}{3}$ that the SU(6) value of $\frac{2}{3}$.

VI. DISCUSSION OF RESULTS

There have been two disjoint approximations made heretofore in making phenomenological use of the relation between current and constituent quarks. The approach of Melosh[2] and his followers has been to neglect the effect of interaction while including the spin rotation effects arising from V_{free}. The approach of Yan[19] and others has been to ignore the spin effects while including the interaction in terms of the dressing operators. We have studied[5] the current-constituent quark relation in the context of interacting quark models without a specific choice of interaction and constructed a unitary transformation relating the two bases. The results of Ref. 5 were not given in a form convenient for phenomenological investigation of the effects of interaction; however, we have subsequently obtained[10] an expression for the transformation connecting current- and constituent-quark bases which displays the dependence on interaction in a rather transparent manner. Within the context of this formalism, we have been able to do better than the extreme approximations mentioned above have been able to do. Since V_{free} is known explicitly for the spin-$\frac{1}{2}$ quark model, some spin effects are included completely while effects of interaction are taken into account in the present work by making some hopefully realistic approximations. These approximations were modeled after the approach of Carlitz and Weyers,[11] and we concentrated on the currently popular model of quarks interacting via gauge vector gluons.

We have presented an argument to support the position that the "Coulomb"-like part of the interaction should be the most important insofar as we are concerned with abstracting the algebraic structure of various transition operators. We have estimated the contribution of terms arising from interaction, and especially noted when and at what level one should expect exotic terms to be important. We believe that we have shown how to understand why the free-quark algebraic structure abstracted from V_{free} works quite well for matrix elements of F_i^3, which are relevant to pionic transition, and for the first moments of F_{em}, which are relevant to electromagnetic dipole transitions; furthermore, we have shown that the vector bilocal will have large corrections to the free-quark results, at least for the higher moments of the related structure functions. In addition, the mass splitting of SU(6) multiplets is generated to the correct order of magnitude. In a few cases, we have estimated exotic terms to be larger than they apparently are: for pionic decays of the $L=2$ resonances to their corresponding $L=0$ ground states and for the magnetic moment of the nucleon. This may be an indication of trouble for our model.[20]

A serious drawback in our work is that we estimate only crudely the contributions of terms of different algebraic structure. To do better, one would need to know hadronic wave functions for the model.

*Work supported in part by the U. S. Energy Research and Development Administration.

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