Counting quarks in e^+e^- annihilation*

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A comparison of asymptotically-free-quark-model predictions and e^+e^- annihilation data can be made by using a dispersion relation to continue the data into the spacelike region. We make this comparison for several models, including when appropriate the effect of heavy-quark masses. We conclude that the "old" theory with no charm is excluded, the standard model with charm is acceptable if heavy leptons are produced, and six-quark models are viable if no heavy leptons are produced.

I. INTRODUCTION

Asymptotically free non-Abelian gauge models of the strong interactions[1] provide a partial explanation in the context of quantum field theory for the scaling phenomena observed at spacelike momentum transfers in inclusive electroproduction and neutrino scattering. They also predict logarithmic deviations from scaling.[2,3] Scaling violation has been seen recently in deep-inelastic muon scattering from iron.[4] The data agree qualitatively with the expectations from asymptotic freedom and suggest that deviations from scaling in other reactions will be large enough to be measurable.

In the timelike regime, the high W^2 data on the total cross section for e^+e^- annihilation are often advertized not only as a test of simple scaling ideas but, with charming naïveté, even as a measure of the number of fundamental fermions (quarks or extra leptons) and their charges. Alas, this test of the underlying theory does not lie on such firm ground as its spacelike counterparts. It ignores the difficult problems associated with the breakdown of perturbation theory near thresholds and bound states, and it sweeps under the rug the open question of whether Kinoshita's theorem is valid for non-Abelian theories.[5] Theoretical expectations for e^+e^- annihilation are generally obtained (via a dispersion relation for the vacuum polarization tensor) from problem-free predictions in the unphysical spacelike domain.[6] Fortunately, the logic is invertible: The data can be used to construct, via the inverse dispersion relation, the spacelike quantities that may be directly compared with firm theoretical predictions. This comparison and a quantitative test of asymptotically free gauge models are the purpose of this paper. The present analysis is made possible by the recent accurate data on the total cross section for e^+e^- annihilations in the region of the new resonances[7] ($W\sim 2$ to 5 GeV) and above[8] ($W \sim 5$ to 7.8 GeV).

The paper is organized as follows: In Sec. II we discuss the conversion of e^+e^- data into information on the photon propagator in the spacelike domain; in Sec. III we present the theoretical expectations; in Sec. IV we compare predictions and experiments and we draw our main conclusions on the viability of different models; in Sec. V we study inclusive lepton scattering; and in Sec. VI we discuss the hadronic width of J (ψ).

II. STUDY OF THE PHOTON PROPAGATOR IN THE SPACELIKE DOMAIN

We first convert the e^+e^- data into information about the photon's renormalized hadronic vacuum polarization tensor Π in the spacelike domain. We follow Adler[9] in defining

$$D(Q) = Q^2 \int_{4m_\pi^2}^{\infty} \frac{dW^2 R(W)}{(Q^2+W^2)^2} .$$

$$= W^2 \frac{d\Pi(W^2)}{dW^2}\bigg|_{(W^2=-Q^2)} \times (\text{known factors}), \quad \text{(1a)}$$

$$R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-). \quad \text{(1b)}$$

In Eq. (1a) an unsubtracted dispersion relation has been used for the derivative of Π. We evaluate D as a sum:

$$D = \sum_i D_i + D(\text{background}) + D(\text{asym}). \quad (2)$$

The index i runs over the vector-meson states. For ω, ϕ, J, and ψ' we use the narrow-resonance approximation

$$\alpha^2 D_i = 9\pi Q^2 M_i \Gamma_i/(Q^2+M_i^2)^2, \quad (3)$$

where M_i are masses and Γ_i widths into e^+e^-. For the broader ρ we borrow from Adler[9] a more elaborate analysis based on a Gounaris-Sakurai[10] fit to the pion form factor with an $\omega \to 2\pi$ interference term.

D(background) is estimated by numerical integration. Below $W=2$ GeV we use a fit of Silvestrini[11]; from $W=2$ to 7.8 GeV we use an eyeball fit to the data that excludes the narrow resonances, but includes the structure between 3.8 and 5 GeV.[8]

D(asym) is the contribution to $D(Q)$ from the asymptotic region above $W = 7.8$ GeV, for which no data exist. In an attempt to bound D(asym), we calculate this contribution in two extreme situations. One extreme corresponds to $R(W)$ staying flat forever at the average value measured in the region 5 GeV $< W <$ 7.8 GeV. The other extreme is model-dependent and corresponds to $R(W)$ jumping immediately at $W = 7.8$ GeV to its $W = \infty$ asymptotic value as predicted in the given model. If the actual value of $R(W)$ stays between these two limits, then $D(Q)$ will be between the two extremes we compute. In view of recent data[12] on "μ-e events" we allow in each quark model for the possibility that R is partly mocked up by "semihadronic" decays of one or more species of heavy leptons of mass ~1.8 GeV. We do this by performing the appropriate subtractions from the data, assuming that the semihadronic branching ratio is unity.

Our "experimental" determination of $D(Q)$ is shown in Fig. 1 for the "standard" model with three colors, four flavors ($\mathcal{P}, \mathcal{N}, \lambda, \mathcal{P}'$), and the conventional fractional charges. Upper and lower limits are shown for the cases in which no heavy leptons are being produced at present energies or one heavy-lepton pair is being produced. If two kinds of heavy-lepton pairs are being produced, our upper and lower limits coincide (the asymptotic prediction coincides with the data for $W > 5$ GeV) and are shown as a single curve. Figure 2 shows the "experimental" determination of D in the "old" model with no charm flavor. Figure 3 shows the situation in a six-flavor model to be discussed below. As seen in the figures, the "error" introduced by the theoretical uncertainty is in all cases $\lesssim 10\%$ for $Q < 8$ GeV. This is comparable to the quoted error in the actual data.

III. THEORETICAL EXPECTATIONS

In an asymptotically free color SU(3) gauge theory with n triplets of massless quarks, it is well known[6] that

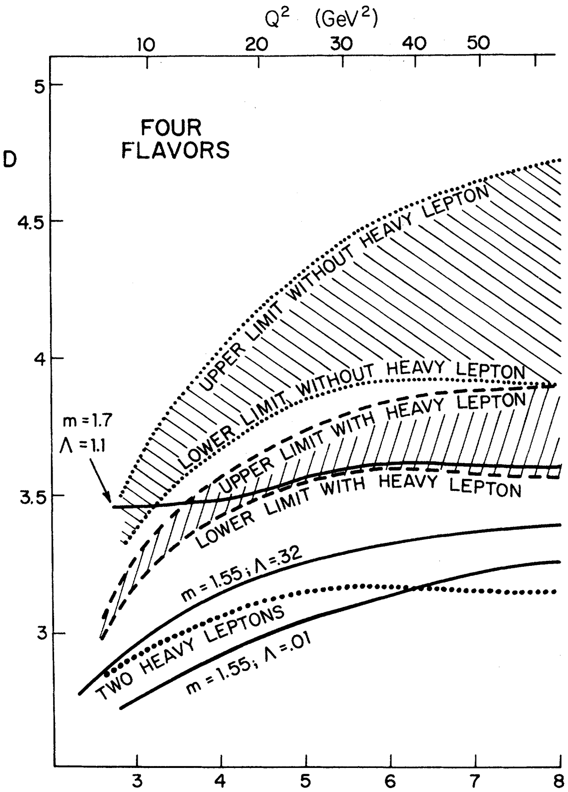

FIG. 1. "Experimental" results for $D(Q)$ in the four-flavor model with and without heavy leptons. Theoretical predictions are also shown, Λ and m are in GeV.

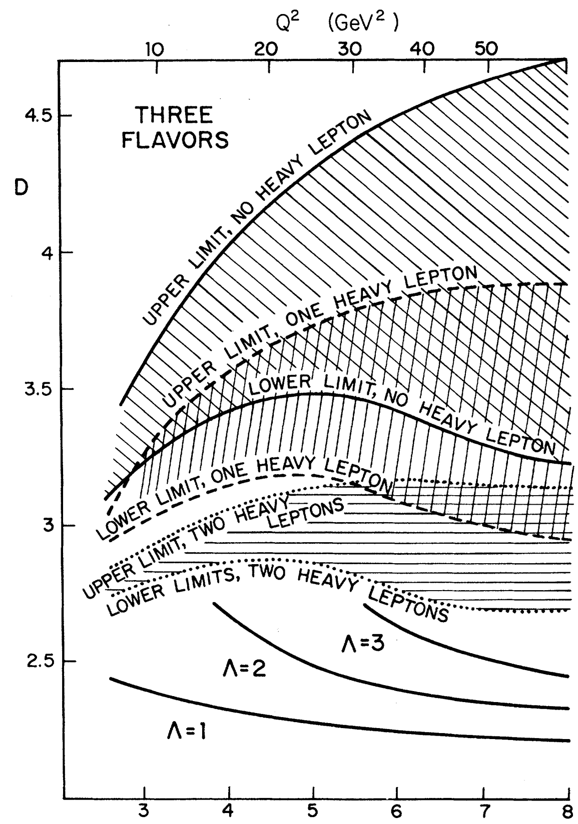

FIG. 2. "Experimental" results for $D(Q)$ in the three-flavor model with and without heavy leptons. Theoretical predictions are also shown, Λ and m are in GeV.

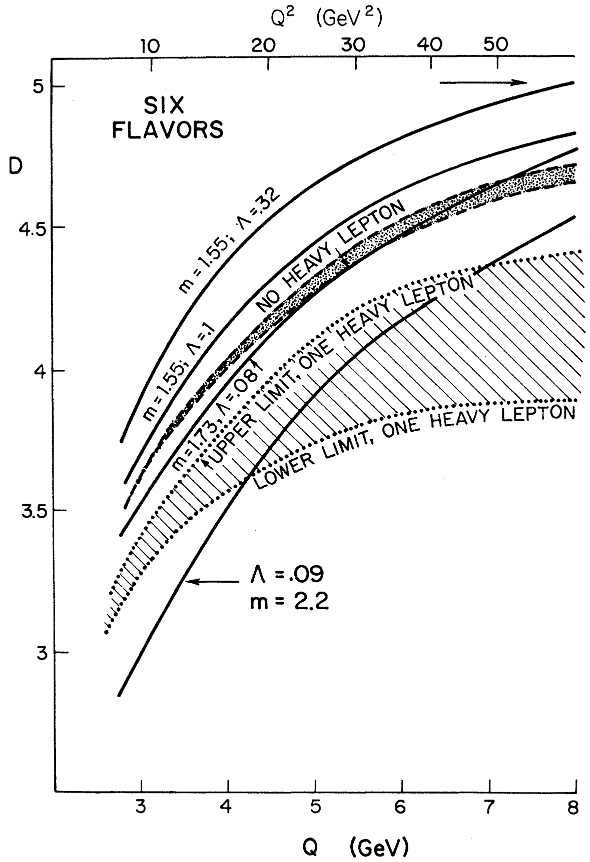

FIG. 3. "Experimental" results for $D(Q)$ in the six-flavor model with and without heavy leptons. Theoretical predictions are also shown, Λ and m are in GeV.

$$D(Q)=\sum_i q_i^2\left(1+\frac{\overline{g}(Q)^2}{4\pi^2}+O(\overline{g}^4)\right), \tag{4}$$

where $\overline{g}(Q)$ is the effective coupling constant

$$\frac{\overline{g}(Q)^2}{4\pi}=\frac{12\pi}{33-2n}\,\frac{1}{\ln(Q^2/\Lambda^2)}. \tag{5}$$

In (4) and (5), q_i are the quark charges and Λ is the single parameter which characterizes the effective coupling. This prediction is not very useful for comparison with existing data because it ignores the mass of the heavy quark (or quarks) responsible (we believe) for the excitement between $W=3.0$ and 4.5 GeV. Indeed, the most striking feature of the "data" in Figs. 1, 2, and 3, more or less independent of asymptotic details and heavy-lepton subtractions, is its fast rise from $Q=3$ to 6 GeV. This is a clear signal that the heavy-quark masses cannot be ignored in this region. Fortunately, it is not difficult to include the effects of quark masses in the calculation.

Assume for simplicity that those quarks in the color SU(3) gauge theory whose effects are accessible below $Q=8$ GeV (see Ref. 13) are of only two kinds: light quarks, with masses sufficiently small that they can be taken to be zero with negligible error for $Q>3$ GeV; and heavy quarks which are approximately degenerate and characterized by a mass m. Our renormalization prescription is as follows[5]: We make all wave-function and coupling-constant renormalizations at a Euclidean momentum $p^2=-M^2$; but we renormalize the heavy-quark masses so that m is the position of the pole in heavy-quark propagators. This makes sense to any finite order in perturbation theory, even if "free" quarks do not exist. The coupling constant defined by our prescription depends on the renormalization point M. If we change M, we can find a new gauge coupling constant g and a suitable rescaling of the fields such that the theory is the same with the same value of m. Thus, we expect the Green's functions Γ of the theory to satisfy a Callan-Symanzik equation of the form

$$\left[M\frac{\partial}{\partial M}+\beta(g,m/M)\frac{\partial}{\partial g}+\sum_i\gamma_i(g,m/M)\right]\Gamma=0.$$

Here, in contrast to the situation with only massless quarks, the functions β and γ_i depend not only on g, but on the dimensionless quantity m/M.

The function D of Eq. (1a) satisfies a Callan-Symanzik equation with $\gamma=0$:

$$\left[M\frac{\partial}{\partial M}+\beta(g,m/M)\frac{\partial}{\partial g}\right]D(Q/M,m/M,g)=0$$
$$=\left[-Q\frac{\partial}{\partial Q}-m\frac{\partial}{\partial m}+\beta(g,m/M)\frac{\partial}{\partial g}\right]D(Q/M,m/M,g). \tag{6}$$

It is easy to see that (6) is satisfied for

$$D(Q/M,m/M,g)=D(1,m/Q,\overline{g}(Q/M,m/M,g)), \tag{7a}$$

where

$$Q\frac{\partial}{\partial Q}\overline{g}=\beta(\overline{g},m/Q)\quad\text{and}\quad\overline{g}(1,m/M,g)=g. \tag{7b}$$

If $\bar{g}$ is small for some Q of interest, we can calculate $D(1,m/Q,\bar{g})$ and $\beta(\bar{g},m/Q)$ to some order in perturbation theory and use (7b) to find $\bar{g}$ and therefore D for all Q for which $\bar{g}$ is small.

We quote here the following results:

$$D(1,m/Q,\bar{g})=\left(\sum_{\substack{\text{light}\\ \text{quarks}}} q^2\right)\left(1+\frac{\bar{g}(Q)^2}{4\pi^2}+O(\bar{g}^4)\right)+\left(\sum_{\substack{\text{heavy}\\ \text{quarks}}} q^2\right)\left[F_1\left(\frac{m^2}{Q^2}\right)+\frac{\bar{g}(Q)^2}{4\pi^2}F_2\left(\frac{m^2}{Q^2}\right)+O(\bar{g}^4)\right]. \tag{8}$$

The function F_1 is

$$F_1(x)=1-6x+\frac{12x^2}{(1+4x)^{1/2}}\ln\left(\frac{(1+4x)^{1/2}+1}{(1+4x)^{1/2}-1}\right). \tag{9}$$

The function F_2 has been calculated by Källén[14] and is rather complicated. Here we use an approximate form obtained via the dispersion relation from Schwinger's approximation[15] to the second-order contribution to R in the timelike region. It is

$$F_2(x)\cong\left(\frac{3}{4}-\frac{\pi^2}{2}\right)F_1(x)+\frac{\pi^2}{2}+\frac{1}{4}+\left(\frac{2\pi^2}{3}-1\right)x-\frac{4\pi^2x}{1+4x}+x\left[\frac{5\pi^2}{3}-\frac{1}{2}-4x\left(\frac{2\pi^2}{3}-1\right)\right]\ln\left(1+\frac{1}{4x}\right). \tag{10}$$

The β function is

$$\beta(\bar{g},m/Q)=-\frac{\bar{g}^3}{16\pi^2}\left[11-\tfrac{2}{3}n_L-\tfrac{2}{3}n_HF_1\left(\frac{m^2}{Q^2}\right)\right]+O(\bar{g}^5), \tag{11}$$

where n_L (n_H) is the number of light (heavy) quarks triplets [again we assume a color SU(3) gauge group]. Ignoring the terms of $O(\bar{g}^5)$, we can integrate (7b) to obtain

$$\frac{1}{\bar{g}^2}=\frac{1}{g^2}+\frac{1}{16\pi^2}\left[(11-\tfrac{2}{3}n_L)\ln\frac{Q^2}{M^2}-\tfrac{2}{3}n_H\int_{M^2}^{Q^2}\frac{dz}{z}F_1\left(\frac{m^2}{z}\right)\right]. \tag{12}$$

The integration in (12) can be done explicitly, but it is easier and adequate for our purposes to integrate the approximate form

$$F_1\left(\frac{m^2}{Q^2}\right)\cong\frac{Q^2}{Q^2+5m^2}. \tag{13}$$

which is accurate to within a few percent. We then obtain

$$\frac{\bar{g}^2}{4\pi}\cong\frac{4\pi}{\dfrac{16\pi^2}{g^2}+(11-\tfrac{2}{3}n_L)\ln\dfrac{Q^2}{M^2}-\tfrac{2}{3}n_H\ln\dfrac{Q^2+5m^2}{M^2+5m^2}}$$

$$=\frac{12\pi}{(33-2n_L)\ln\dfrac{Q^2}{\Lambda^2}-2n_H\ln\dfrac{Q^2+5m^2}{\Lambda^2+5m^2}}, \tag{14}$$

where Λ (which is a function of g and M) is the single parameter which determines the effective coupling constant. The form (14) for the effective coupling constant is a bit more complicated than the analogous form in a fully massless theory, but the meaning should be clear. For $\Lambda, Q\ll m$, the term involving n_H drops out. The heavy quarks are irrelevant, as we would expect from the Appelquist-Carazzone theorem.[13] It is only for $Q\gg m$ that we see the full effect of the heavy-quark contribution to β and $\bar{g}$.

IV. COMPARISON OF MODELS AND EXPERIMENT

We have two free continuous parameters (m and Λ) and a discrete parameter (the number N of produced heavy leptons) to vary in an attempt to make a model agree with the data. For definiteness, we call a fit "reasonable" if $N\le 2$, 1 GeV $\le m\le 3$ GeV and $\Lambda\lesssim 1$ GeV. The last condition is necessary to explain the early onset of scaling in electroproduction within the same class of models (see discussion below).

In Fig. 2, theoretical curves for $\Lambda=1$, 2, and 3 GeV are given and compared with the data (with $N=0$, 1, or 2) in the "old" charmless model in which $n_L=3$, $n_H=0$. The theoretical curves reach their asymptotic values logarithmically from above [as in Eq. (4) with no quark-mass dependence]. No fit is possible for reasonable values of the parameters. *Models with fewer than four flavors are excluded whether or not heavy leptons are being produced.*

Back to Fig. 1, we compare the data with the "standard" model ($n_L=3$, $n_H=1$). Marginally reasonable fits are obtained for $N=1$, $\Lambda\sim 1.1$ GeV and a mass of the charmed quark $m\sim 1.7$ GeV. Here we see the logarithmic dependence of $\bar{g}$ at small $Q\lesssim 5$ GeV competing with the "threshold effect" of the heavy-quark mass to give a prediction which is flatter than the data. However, since with this value of Λ, $\bar{g}(3\text{ GeV})^2/4\pi\cong 0.6$, perturbation theory may be misleading in this region. "Reasonable" fits are also obtained for $N=2$, $\Lambda\sim 0.01$ GeV, and $m\sim 1.55$ GeV. *The standard model with (only one kind of) charm is acceptable provided one to two types of heavy leptons are being produced at present energies.*

Recent work by many authors points toward the theoretical appeal of a class of models with six quark flavors.[16] The three "extra" quarks have charges $\frac{2}{3}$, $\frac{2}{3}$, and $-\frac{1}{3}$. In Fig. 3 we compare the data with these six-quark models, assuming that the three heavy-quark types all have mass $\sim m$ and are being excited at present energies. "Reasonable" fits are possible only if no heavy leptons are being produced. They occur in the range of parameters $m=1.5$ to 1.8 GeV for rather small $\Lambda=60$ to 100 MeV. *Six-quark models are acceptable provided no heavy leptons are being produced at present energies.*

Besides the four-quark and six-quark models we have discussed, there are various intermediate possibilities which yield reasonable fits. Addition or deletion of a heavy quark with charge $-\frac{1}{3}$ makes only small changes in the parameters of the best fit. For example, adding such a quark to the "standard" model, we obtain a five-quark model with heavy-quark charges $\frac{2}{3}$ and $-\frac{1}{3}$. This model gives a good fit with $m=1.55$ GeV, $\Lambda=0.45$ GeV, and $N=1$. Similarly, we can delete a charge $-\frac{1}{3}$ quark from the six-quarks to obtain a different five-quark model which fits the data for $N=0$.[16]

It is easy to generalize our results [Eqs. (7)–(14)] to models in which there are several quark-mass scales. We will not discuss this further except to note the obvious fact that very heavy quarks, $m \gg$ a few GeV, have a negligible effect for $Q \ll m$, so we can ignore possible heavy quarks too heavy for their effects to be observable below $Q \simeq 8$ GeV.

V. INCLUSIVE ELECTRON AND NEUTRINO DEEP-INELASTIC SCATTERING

For each model which "fits" the e^+e^- data we determine an allowed range of values of the scale parameter Λ. With Λ determined, we can predict, for each model, the deviations from scaling in any of the combinations of electroproduction structure functions $F_2(ep)-F_2(en)$, and the neutrino scattering structure functions $F_2(\nu)-F_2(\bar{\nu})$ or $xF_3(\nu \text{ or } \bar{\nu})$ on any target.[3] This is done in terms of the function at some fixed Q^2 around which the data approximately scale. We refer to the above structure functions or differences of structure functions generically as $F(\omega, Q^2)$, where ω is the conventionally defined scaling variable $\omega=2\nu/Q^2$. Unfortunately, we do not have the theoretical tools to predict quantitatively the deviations from scaling in any other structure function, say for muon scattering on an iron target. Qualitatively, however, the scaling violations for any structure function should be similar, in magnitude and direction, to those for $F(\omega, Q^2)$ (except at large ω).

As an example of scaling violations in deep-inelastic scattering, we study the "standard" four-flavor model assuming one type of heavy lepton is being produced at SLAC. We borrow from previous work[3] the predictions for $F(\omega, Q^2)/F(\omega, Q^2=5\text{ GeV}^2)$ at different values of Λ. Figures 4(a)-4(b) show this ratio for $\Lambda=1$ and 0.32 GeV. The Q^2 dependence for $\Lambda=1$ GeV is much stronger than the Q^2 dependence (of a different

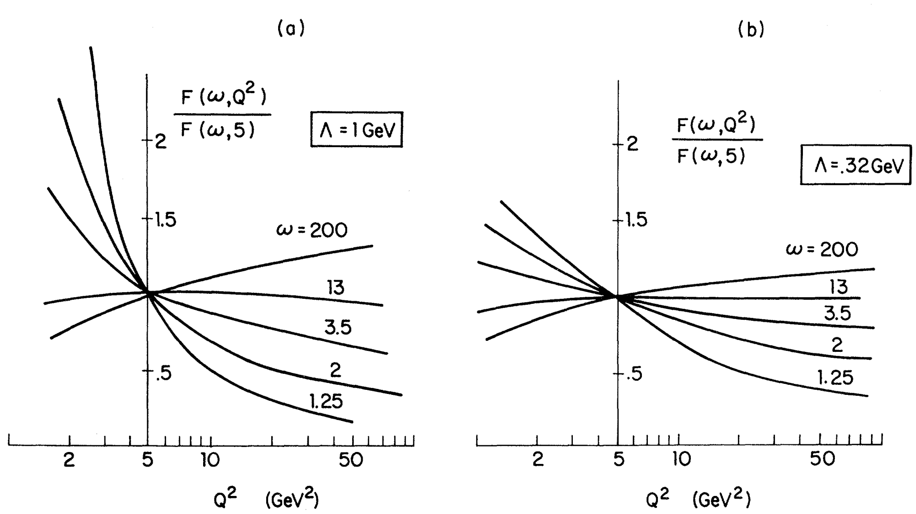

FIG. 4. Predicted fractional deviations from scaling in the four-flavor model for $m=1.55$ GeV.

structure function) observed in the μ-iron scattering. The Q^2 dependence for $\Lambda = 0.32$ GeV agrees qualitatively with the μ-iron data. On the other hand we see in Fig. 1, that $\Lambda = 0.32$ GeV gives a prediction for $D(Q)$ in e^+e^- annihilation which is systematically one standard deviation below the "data." If the standard model with one heavy lepton is correct, the actual Λ is probably somewhere between 1 and 0.32 GeV.

VI WIDTH OF $J(3.095)$

The knowledge of the heavy-quark masses and the scale parameter Λ of the underlying quark and gluon field theory are not enough, at present, to predict the hadronic width Γ of J in a rigourous fashion. But a naive nonrelativistic bound-state picture exists where the decay occurs via the short-distance annihilation of the charmed quark pair into three gluons.[17] Assuming that the relevant quark-gluon coupling constant for this process is $\alpha_s(W = M(J)) = \bar{g}(M(J))^2/4\pi$, this naive approach predicts the ratio of hadronic and leptonic widths to be

$$\gamma \equiv \frac{\Gamma(J \to \text{hadrons})}{\Gamma(J \to e^+e^-)} = \frac{5(\pi^2 - 9)\alpha_s{}^3(J)}{18\pi^2}.$$

In the conventional four-flavor theory with $\Lambda = 0.32$ GeV and $m = 1.55$ GeV, we predict $\gamma = 42$, to be compared with $\gamma(\text{exp}) = 12 \pm 3$. Thus, in this model, we "understand" the narrow width of J (ψ) to within a factor of 3. For $\Lambda = 1$ GeV, perturbation theory would not make sense at these energies and we lose all predictive power. In the six-quark model with $\Lambda = 0.1$ GeV, and $m = 1.55$ GeV we would predict $\gamma = 13$, a surprising coincidence.

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