

## Hadronic models for the photoproduction of $\psi$ 's\*

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We examine the photoproduction of the  $\psi(3095)$  within the context of the assumption that the  $\psi$  contains a pair of new fundamental constituents. Using an inequality based on unitarity, we derive a lower limit on the cross section  $\sigma(\gamma p \rightarrow D\bar{D} + \text{anything}) \gtrsim 300$  nb where the  $D$ 's are hadrons carrying the new constituent bound to ordinary quarks. This suggests that it should be possible to detect  $D$ 's from their leptonic decays in  $\gamma$  beams. Comparing the unitarity relations for  $\gamma p \rightarrow \psi p$  and  $\psi p \rightarrow \psi p$ , we predict corrections to the vector-dominance hypothesis so that  $\sigma_{\text{tot}}(\psi p)$  is about a factor of two larger than expected. We discuss briefly the precision necessary for experiments on nuclear targets to test this prediction.

### I. INTRODUCTION

Recent measurements of  $\psi$  photoproduction at Cornell,<sup>1</sup> SLAC,<sup>2</sup> and Fermilab<sup>3</sup> have clarified and sharpened our understanding of the properties of the new particles. It is particularly significant that experimental measurements of the pseudo-elastic cross section

$$d\sigma/dt(\gamma p \rightarrow \psi p)$$

are larger than the upper limit implied by non-hadronic models for the  $\psi$  particles.<sup>4</sup> This supports the idea that the  $\psi$  particles experience the strong interactions, and in this paper we will adopt the most popular of the strong-interactions models for the new particles. We will assume that the  $\psi$  and  $\psi'$  are bound states of a quark-antiquark pair carrying a new quantum number. For convenience, we will call this quantum number charm but our results are more general than the usual SU(4) model.

The fundamental prediction of the generalized charm model for the  $\psi(3100)$  and  $\psi(3700)$  is the existence of new hadrons carrying a conserved quantum number. As this is being written, there is as yet no conclusive experimental evidence of these charmed hadrons but there are a number of indirect indications they might exist.<sup>5</sup> We intend to explore the implications for  $\psi$  photoproduction of the existence of these new particles. Our results are insensitive to the symmetry group in which the new particles are classified<sup>6</sup> and to their decay modes. The properties we do assume for these particles are as follows:

(1) They are massive. The meson masses begin in the range of 2 GeV. This value is approximately determined by the narrow width of the  $\psi(3100)$  and  $\psi(3700)$  and the location of the "threshold" rise in  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  at

around  $\sqrt{s} = 4$  GeV.

(2) They interact strongly with both the  $\psi$ 's and with ordinary hadrons subject to the restrictions of the Okubo-Zweig-Iizuka<sup>7</sup> (OZI) selection rules. These rules restrict the class of quark-line duality diagrams which can contribute to strong-interaction amplitudes. Basically, the constraint is that a quark-antiquark pair in the same hadron cannot annihilate each other.

(3) They carry an additive quantum number which is conserved in the strong interactions. They therefore decay either electromagnetically or weakly and are formed in pairs through the strong interactions.

Using these properties for charmed particles, we construct a model based on a peripheral approximation to unitarity for the imaginary part of the elastic  $\psi N$  amplitude. By looking at this model amplitude as a function of the mass of one of the external  $\psi$  legs, we conclude that the usual vector-dominance-model assumption<sup>8</sup> which neglects the off-mass-shell dependence of the scattering amplitude should be modified. Our best estimate of the quantitative value of the modification gives the result that the physical  $\psi N$  total cross section should be about a factor of two larger than that implied by the vector-dominance model. We point out that it is possible to test our results by measuring the  $A$  dependence of  $\psi$  photoproduction on nuclear targets or by measuring the  $q^2$  dependence in electroproduction experiments. Our model for the  $\psi N$  amplitude can be applied, with trivial modification, to the  $\phi N$  amplitude where we also find a correction factor which modifies the usual vector-dominance expression. Interestingly, this factor produces a value of  $\sigma_{\text{tot}}(\phi N)$  which agrees with the simple additive quark model.<sup>9</sup>

We also present an analysis based on unitarity which demonstrates the strong connection between

the photoproduction of the  $\psi$  and  $\psi'$  and the photoproduction of charmed particles. This relationship can be conveniently expressed in terms of lower bound on the product of the cross section for the production of charmed particles in a photon beam and the  $\psi N$  total cross section. Using our best estimate for  $\sigma^{\text{tot}}(\psi N)$ , we get

$$\sigma_{\gamma p \rightarrow \text{charm}} \gtrsim 300 \text{ nb}$$

at Fermilab and in the upper range of energies available at SLAC ( $E_{\text{LAB}} \gtrsim 18 \text{ GeV}$ ). From this value and estimates for the semileptonic and leptonic branching ratios of charmed particles<sup>10</sup> we can see that the decay of charmed states should provide a significant contribution to the inclusive cross section  $E d\sigma/d^3p(\gamma N \rightarrow e + \text{anything})$ .

One implication of the vector-dominance assumption which is not changed substantially by our modifications is that the ratio  $\sigma_{e1}(\psi N)/\sigma_{\text{tot}}(\psi N)$  must be a very small number (approximately 0.02–0.04). In this respect, the behavior of the  $\psi$  is anomalous since for all other known hadrons the ratio  $\sigma_{e1}(hN)/\sigma_{\text{tot}}(hN)$  is about  $\frac{1}{5}$ . We make some effort to understand the implications of this anomaly. We find, for example, that the application of simple ideas based on duality imply that the cross section for the diffractive breakup of the  $\psi$  into a pair of charmed particles should be small compared to the elastic cross section. Since this is just the opposite of what we would infer from the small ratio  $\sigma_{e1}(\psi N)/\sigma_{\text{tot}}(\psi N)$ , it would seem that it is impossible to naively extend dual models to the  $\psi$  in this manner.

The remainder of this paper is organized as follows: In Sec. II we discuss the available data on  $\psi$  photoproduction within the framework of the vector-dominance model and define a factor which measures the off-mass-shell behavior. We then briefly discuss how this factor can be determined experimentally from the  $A$  dependence of nuclear photoproduction. In Sec. III we present our model for the off-mass-shell behavior to calculate this factor. Section IV develops the inequality for  $\sigma_{\gamma p \rightarrow \text{charm}}$  and Sec. V contains a brief discussion of dual models. Finally, in Sec. VI, we summarize and present our conclusions.

## II. DATA ON $\psi$ PHOTOPRODUCTION AND THE VECTOR-DOMINANCE ASSUMPTION

We would like to discuss the data on  $\psi$  photoproduction in a framework which is compatible with the vector-dominance model<sup>8</sup> but which allows explicitly for the possibility that the coupling of a photon and a  $\psi$  may be different at  $q^2=0$  than at  $q^2=m_\psi^2$ . We therefore write the equation

$$d\sigma/dt(\gamma N \rightarrow \psi N) = \frac{\alpha}{4} \frac{4\pi}{\gamma_\psi^2} \lambda^2 [d\sigma/dt(\psi N \rightarrow \psi N)], \quad (2.1)$$

where

$$\gamma_\psi^2/4\pi = 2.8 \pm 0.3 \quad (2.2)$$

is determined from the SPEAR results on  $\Gamma(\psi \rightarrow e^+e^-)$ .<sup>11</sup>

The factor

$$\lambda = \frac{\gamma_\psi(m_\psi^2) A(\psi(q^2)N \rightarrow \psi N)|_{q^2=0}}{\gamma_\psi(0) A(\psi(q^2)N \rightarrow \psi N)|_{q^2=m_\psi^2}} \quad (2.3)$$

measures the variation of the photon- $\psi$  coupling,  $\gamma_\psi$ , with  $q^2$  and the off-mass-shell extrapolation of the invariant amplitude. We are making the assumption that this factor does not depend sensitively on  $s$  and  $t$ . We do not here try to distinguish the two, possibly related, dynamical origins for  $\lambda$  but we will have some comments on this later. As it stands, Eq. (2.1) is not terribly useful unless  $d\sigma/dt(\gamma N \rightarrow \psi N)$  and  $d\sigma/dt(\psi N \rightarrow \psi N)$  can be measured separately, or unless  $\lambda$  can be estimated in a particular model. The traditional application of Eq. (2.1) comes from the vector-dominance assumption  $\lambda=1$  which allows us to infer  $d\sigma/dt(\psi N \rightarrow \psi N)$  from a measurement of the photoproduction cross section. This assumption has been found to be valid in  $\rho$  and  $\omega$  photoproduction<sup>9</sup> but some hint of the necessity for corrections to it may be inferred from the trouble that naive vector dominance has with data on electroproduction.<sup>10,12</sup> In what follows, we will explicitly display the factor  $\lambda$  in most of the formulas.

If we, for simplicity, parameterize the spin-averaged amplitude for  $\psi N \rightarrow \psi N$  in the form

$$A(\psi N \rightarrow \psi N) = \sigma^{\text{tot}}(\psi N) e^{Bt/2} (i + \rho) q_{\psi N} \sqrt{s}, \quad (2.4)$$

where  $\rho = \text{Re}A/\text{Im}A$ ,  $q_{\psi N}$  is the  $\psi N$  c.m. momentum, and the optical theorem constraint at  $t=0$  is built in, we can then write

$$\frac{d\sigma}{dt}(\psi N \rightarrow \psi N) = \frac{[\sigma^{\text{tot}}(\psi N)]^2}{16\pi} (1 + \rho^2) e^{Bt}. \quad (2.5)$$

We can then use (2.1), (2.2), and (2.5) to infer the value of

$$\lambda(1 + \rho^2)^{1/2} \sigma_{\text{tot}}(\psi N) = \left[ \frac{4}{\alpha} \frac{\gamma_\psi^2}{4\pi} 16\pi e^{-Bt_{\text{max}}} \frac{d\sigma}{dt}(\gamma N \rightarrow \psi N)|_{t_{\text{max}}} \right]^{1/2}. \quad (2.6)$$

In Fig. 1, we have taken data from Cornell,<sup>1</sup> SLAC,<sup>2</sup> and Fermilab<sup>3</sup> and plotted this quantity. We have used the SLAC value for the slope of the differential cross section

$$B = 2.6 - 2.8 \text{ GeV}^{-2} \quad (2.7)$$

at Fermilab energies. The experimental papers

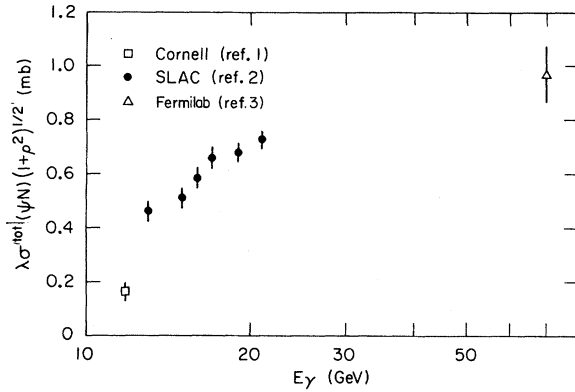


FIG. 1. The value of  $(1 + \rho^2)^{1/2} \lambda \sigma_{\text{tot}}(\psi N)$  determined from data on  $\psi$  photoproduction. The Cornell data are from Ref. 1, the SLAC data from Ref. 2, and the Fermilab data from Ref. 3.

should be consulted directly for details concerning the measurements but we note here that there may be some uncertainty in isolating the elastic component in experiments which were designed primarily as inclusive measurements.

In particular, the shallow slope inferred for the differential cross section at SLAC and Cornell may indicate a preferential contribution from inelastic channels to the measurements at large  $t$ . We might infer that the slope of the elastic cross section should be larger than that for a pointlike particle scattering from a nucleon, i.e.,

$$B \gtrsim \frac{d}{dt} \ln F^2(t) \Big|_{t=\langle t \rangle}, \quad (2.8)$$

where  $F(t)$  is the spin-averaged nucleon form factor and  $\langle t \rangle$  is the average value of  $t$  which reflects the range where the slope is measured. Inserting the dipole approximation for the nucleon form factor, Eq. (2.8) reads

$$B \gtrsim 5.6(1 - \langle t \rangle / 0.71)^{-1}, \quad (2.9)$$

and a measured value of  $B$  smaller than this is a strong indication that some of the data are from inelastic channels which have a shallower slope. We note here that the SLAC estimate of a 20% contamination from inelastic channels is made at  $t = t_{\text{max}}$ . The fraction of the data at more negative values of  $t$  which is inelastic may be much greater. We make no effort to correct for this possible underestimate of the slope but one place where the actual value of the slope is important is in the estimate of integrated elastic cross section.

Within the context of the vector-dominance framework, we can also infer the ratio of the elastic to total cross sections for  $\psi N$  scattering. Using Eq. (2.4), we can write

$$\frac{\sigma^{\text{el}}(\psi N)}{\sigma^{\text{tot}}(\psi N)} = \frac{1}{16\pi B} (1 + \langle \rho^2 \rangle) e^{B t_{\text{max}}} \sigma^{\text{tot}}(\psi N), \quad (2.10)$$

where we have averaged over the possible differences in the  $t$  dependence of the real and imaginary parts of the amplitude. Inserting into Eq. (2.6), we get

$$\begin{aligned} \frac{\sigma^{\text{el}}(\psi N)}{\sigma^{\text{tot}}(\psi N)} &= \frac{1}{B\lambda} \frac{(1 + \langle \rho^2 \rangle)}{(1 + \rho_{t_{\text{max}}}^2)^{1/2}} \\ &\times \left[ \frac{4}{\alpha} \frac{\gamma_\psi^2}{4\pi} \frac{e^{B t_{\text{max}}}}{16\pi} \frac{d\sigma}{dt}(\psi N \rightarrow \psi N) \Big|_{t_{\text{max}}} \right]^{1/2}. \end{aligned} \quad (2.11)$$

If we assume that, at high energies,  $\rho \rightarrow 0$ , we find that the data imply a small value for this ratio

$$\frac{\sigma^{\text{el}}(\psi N)}{\sigma^{\text{tot}}(\psi N)} \cong \frac{1}{\lambda} (1.3 \pm 0.4 \times 10^{-2}). \quad (2.12)$$

It is of interest that for all other known hadrons, the ratio of elastic to total cross sections turns out to be considerably higher

$$\frac{\sigma^{\text{el}}}{\sigma^{\text{tot}}}(\bar{p}p, \pi p, Kp, \rho p, \omega p, \phi p) \cong 0.1 - 0.2. \quad (2.13)$$

This means that if  $\lambda \cong 1$ , so that vector dominance is approximately correct, inelastic channels must play a significantly different role in  $\psi N$  collisions than in  $\pi N$  collisions. It is notable that analysis of the experimental data on  $\psi$  photoproduction suggests that it is not those inelastic channels containing  $\psi$ 's which contribute the bulk of the total cross section.

If we make the usual assumption that the  $\psi$  is a bound state of a quark-antiquark pair where the quark carries some new quantum number, application of the OZI selection rules suggests that the final states in  $\psi N$  collisions should usually contain these new quarks. Since (2.11) and (2.12) suggest the quarks are not bound to form a  $\psi$ , we can form the estimate

$$\sigma^{\text{tot}}(\psi N) \cong \sigma(\psi N \rightarrow D\bar{D} + \text{anything}), \quad (2.14)$$

where  $D$  and  $\bar{D}$  are used as generic names for particles carrying the new quantum number. Applying vector-dominance ideas to the photoproduction of these new particles, we would estimate

$$\begin{aligned} \sigma(\gamma p \rightarrow D\bar{D} + \text{anything}) &\cong \left( \frac{\sigma^{\text{tot}}(\psi N)}{\sigma^{\text{el}}(\psi N)} \right) \sigma(\gamma p \rightarrow \psi N) \\ &\cong \lambda 500 \text{ nb} \quad (E_\gamma \gtrsim 18 \text{ GeV}). \end{aligned} \quad (2.15)$$

We note that the identification of the  $\psi N$  total cross section with the cross section for the production of new particles is consistent with the fact

that the  $\psi$  photoproduction cross section appears to exhibit some threshold behavior in the region between  $\sqrt{s}=4.6$  and  $\sqrt{s}=5.5$ . It is only above the threshold for the OZI-rule-allowed inelastic channels that the elastic amplitude can develop its full magnitude and diffractive character. The energy dependence in Fig. 1 can be contrasted to the situation in  $\phi$  photoproduction where the data show remarkably little energy dependence from threshold to high energy.<sup>13</sup> We will discuss the cross section for the photoproduction of these hypothetical charmed particles at more length in Sec. IV. We will also return in Sec. V to the problem of determining under what circumstances we can estimate independently the ratio  $\sigma^{e1}/\sigma^{\text{tot}}$ .

#### The $A$ dependence of nuclear photoproduction

It is possible in principle through measurements of photoproduction on nuclear targets to obtain independent information on the size of  $\sigma^{\text{tot}}(\psi N)$ . It is interesting to see how precise the experimental measurements must be to detect the possible corrections to vector dominance. Using the Glauber scattering formalism<sup>14</sup> we can characterize the effective number of nucleons per nucleus in a  $\psi$ -photoproduction experiment

$$A_{\text{eff}} = \frac{1}{\sigma^{\text{tot}}(\psi N)} \int_0^\infty \{1 - \exp[-\sigma^{\text{tot}}(\psi N)T(b)]\} d^2b, \quad (2.16)$$

where

$$T(b) = A \int_{-\infty}^{+\infty} \rho(b, z) dz, \quad (2.17)$$

$\rho(b, z)$  is the nuclear density and  $A$  the nucleon number. In the limit that  $\sigma^{\text{tot}}(\psi N)$  is small, nuclear matter is transparent to  $\psi$ 's and  $A_{\text{eff}} \cong A$ .

We note that an experiment on a nuclear target which measures cross sections outside the coherent nuclear peak should measure cross sections

$$\frac{d\sigma^{\text{inc}}}{dt}(\gamma A \rightarrow \psi A) = A_{\text{eff}} \frac{d\sigma}{dt}(\gamma N \rightarrow \psi N), \quad (2.18)$$

while within the coherent peak, the cross section is proportional to  $(A_{\text{eff}})^2$ .

We can estimate the effect of  $\sigma^{\text{tot}}(\psi N)$   $= (1/\lambda)\sigma^{\text{tot}}_{\text{vector dominance}} \cong (1/\lambda)1 \text{ mb}$  on nuclear targets by using a simple hard-sphere model for the nuclear density:

$$T(b) = A \frac{3}{2\pi R^3} (R^2 - b^2)^{1/2}, \quad b < R \\ = 0, \quad b > R \quad (2.19)$$

where  $R = r_0 A^{1/3}$  (for  $r_0 = 1.12 \text{ fm}$ ) is the nuclear radius. Using (2.19) we can integrate (2.16) di-

rectly and write

$$A_{\text{eff}} = \frac{2\pi}{\sigma^{\text{tot}}(\psi N)} \frac{1}{\kappa^2} \left[ \frac{\kappa^2 R^2}{2} - \gamma(2, \kappa R) \right], \quad (2.20)$$

where

$$\kappa = A \sigma^{\text{tot}}(\psi N) \frac{3}{2\pi R^3} \\ = \frac{3}{2\pi r_0^3} \sigma^{\text{tot}}(\psi N) \quad (2.21)$$

is the inverse mean free path of  $\psi$ 's in nuclear matter and  $\gamma(a, x)$  is the incomplete  $\gamma$  function. Since  $\sigma^{\text{tot}}(\psi N)$  is assumed to be small, we can approximate for small  $\kappa R$

$$A^{\text{eff}} \cong A \left( 1 - \frac{3}{8} \kappa R \right) \\ = A \left[ 1 - \frac{9}{16\pi r_0^2} \sigma^{\text{tot}}(\psi N) A^{1/3} \right]. \quad (2.22)$$

For typical values of  $\sigma(\psi N)$ , we see the plot of  $A_{\text{eff}}/A$  in Fig. 2. We see that measurements of  $A_{\text{eff}}/A$  formed by taking the ratio of the incoherent nuclear cross section to the nucleon cross section,

$$\frac{d\sigma^{\text{inc}}(\psi A \rightarrow \psi A)}{A d\sigma(\psi N \rightarrow \psi N)} = \frac{A^{\text{eff}}}{A},$$

can distinguish  $\lambda = \frac{1}{2}$  from  $\lambda = 1$  if these are accurate to within 5%.

A more careful treatment of this ratio using realistic Wood-Saxon nuclear densities and phenomenological radii can be done but the calculation presented here gives a rough estimate of the importance of rescattering effects and establishes the feasibility of determining experimentally whether or not corrections are needed to vector dominance.

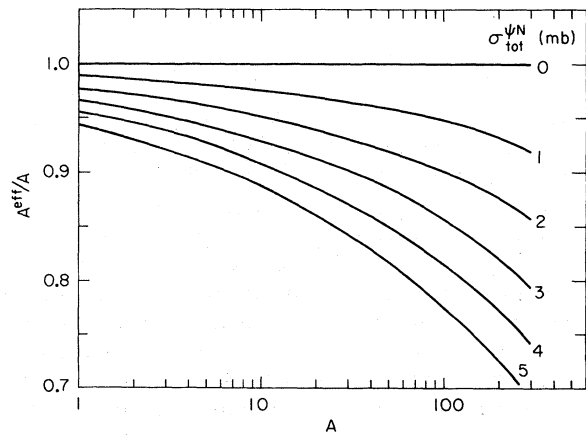


FIG. 2. The hard-sphere model for nuclear densities and the estimate of  $A_{\text{eff}}/A$  using Eq. (2.21) for different values of  $\sigma_{\text{tot}}(\psi N)$ .

### III. A MODEL FOR THE CORRECTIONS TO VECTOR DOMINANCE

We have indicated that, in view of the large range of  $q^2$  involved in the vector-dominance extrapolation, the factor  $\lambda$  defined in Eq. (2.3) may be substantially different from unity. We turn here to the problem of estimating this factor. We assume that the  $\psi$  couples strongly to particles carrying a new quantum number and, in agreement with the discussion in Sec. II, that channels containing these particles dominate the unitarity integral for the  $\gamma p \rightarrow \psi p$  and  $\psi p \rightarrow \psi p$  cross sections. We make a peripheral approximation to unitarity in order to evaluate this integral. Since unitarity is the operational principle in this calculation, only physical particles are involved.

#### Charmed particles and the peripheral approximation to unitarity

First we assume that we can approximately factorize the amplitude for the process  $\psi N \rightarrow M\pi + D\bar{D} + N$  in the form suggested by the diagram in Fig. 3(a):

$$A_{\psi N \rightarrow M\pi + D\bar{D} + N} \cong A_{\pi\psi \rightarrow D\bar{D}}(s_D, \theta_D) R(t_1) \times A_{\pi N \rightarrow M\pi + N}(\cdots k_i \cdots), \quad (3.1)$$

where  $A_{\pi\psi \rightarrow D\bar{D}}$  is the amplitude for a Reggeized  $\pi$  and a  $\psi$  to produce a  $D\bar{D}$  pair,  $R(t_1)$  is a Regge propagator, and  $A_{\pi N \rightarrow M\pi + N}$  is the amplitude for  $\pi N \rightarrow M\pi + N$ . This factorization is motivated by the success of the Amati-Bertocchi-Fubini-Stranghellini-Tonin type of multiperipheral model. We will discuss later the consequences of choosing another type of exchange. As usual, we neglect the dependence of the amplitudes on the

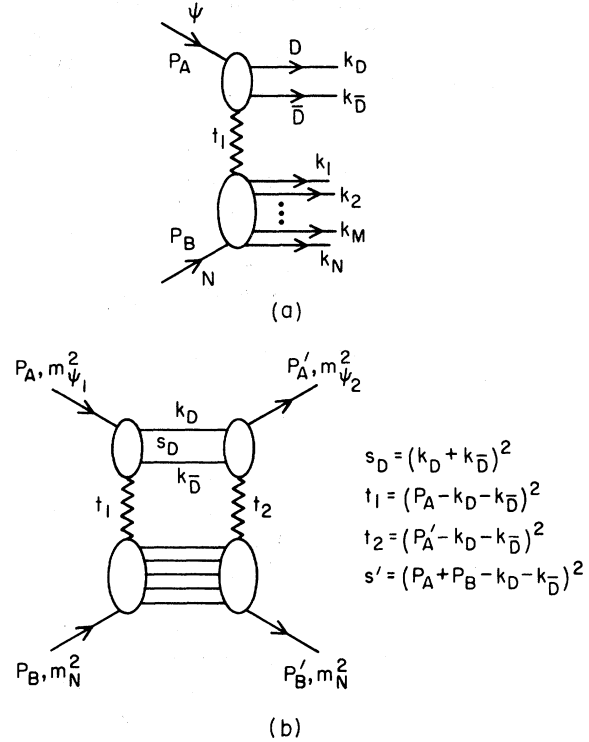


FIG. 3. Diagram (a) demonstrates the multiperipheral configuration assumed for the process  $\psi N \rightarrow D\bar{D} + m\pi + N$ . Diagram (b) labels the kinematic variables appropriate when we insert the diagram (a) into the unitarity expression for  $\psi_1 N \rightarrow \psi_2 N$ . We assume we can do this both for  $m_{\psi_1}^2 = 0$  (photoproduction) and  $m_{\psi_1}^2 = m_\psi^2$  (physical  $\psi N$  scattering).

(mass)<sup>2</sup> of the Reggeized pion.

The next step is to assume that this peripheral configuration of intermediate particles dominates the unitarity integral for  $\psi N \rightarrow \psi N$ , and write

$$\text{Disc} A_{\psi_1 N \rightarrow \psi_2 N} = \frac{1}{16\pi} \int \frac{d^4 k_D}{(2\pi)^3} \delta^{(+)}(k_D^2 - m_D^2) \frac{d^4 k_{\bar{D}}}{(2\pi)^3} \delta^{(+)}(k_{\bar{D}}^2 - m_{\bar{D}}^2) A_{\pi\psi_1 \rightarrow D\bar{D}}(s_D, \theta_D) A_{\pi\psi_2 \rightarrow D\bar{D}}^*(s_D, \theta_D) R(t_1) R(t_2) \times \frac{2\lambda(s', m_\pi^2, m_N^2)}{(2\pi)^4} \sigma_{\pi N}^{\text{tot}}(s'), \quad (3.2)$$

where

$$\lambda(x, y, z) = [x^2 + y^2 + z^2 - 2(xy + xz + yz)]^{1/2} \quad (3.3)$$

and the other kinematic variables are defined in Fig. 3(b). This peripheral approximation to the unitarity integral implements the constraints of the OZI selection rule and the assumption that those inelastic channels *not* directly involving a  $\psi$  dominate the total cross section. As before, we use  $D$  and  $\bar{D}$  as generic names for particles carrying the new type of quark bound to a  $u$ ,  $d$ , or  $s$  quark. Our result will *not* depend sensitively on the number of these new particles or the details of their spectroscopy. The only assumption we make here about the new particles is that they are heavy,

$$m_D \cong 2 \text{ GeV}. \quad (3.4)$$

We want to study the dependence of the discontinuity (3.4) as we continue one of the external  $\psi$ 's off mass

shell to the  $q^2$  of the photon. There are two separate types of dependence on  $m_{\psi_1}^2$  in Eq. (3.2). The amplitude  $\pi_1\psi_1 \rightarrow D\bar{D}$  can depend on the mass of the  $\psi$  and the region of integration can depend on  $m_{\psi_1}^2$  through the variation of the region of integration and

$$t_1^{\max} \cong \frac{-(s_D - m_{\psi_1}^2)(s' - m_N^2)}{s}. \quad (3.5)$$

If we parametrize

$$A_{\pi\psi(q^2) \rightarrow D\bar{D}} \cong Z(q^2) A_{\pi\psi \rightarrow D\bar{D}}, \quad (3.6)$$

with  $Z m_{\psi}^2 = 1$ , we can examine the second type of variation separately. Using the complete set of unitarity relations, we might hope to solve for both  $\lambda$  and  $Z$  but we will not attempt this here. After some manipulation, we can write

$$\begin{aligned} \text{disc } A_{\psi_1 N \rightarrow \psi_2 N} \propto Z(m_{\psi_1}^2) \int ds_D ds' \lambda(s_D, m_{\pi}^2, m_{\psi}^2) \sigma_{\pi\psi \rightarrow D\bar{D}}(s_D) R(t_1) R(t_2) \\ \times \lambda(s', m_{\pi}^2, m_N^2) \sigma_{\pi N}^{\text{tot}}(s') \theta(\ln s - \ln s' - \ln s_D - c) \\ \times \left[ \frac{d^4 K_D}{(2\pi)^3} \delta^{(*)}(s_D - K_D^2) \frac{d^4 K'}{(2\pi)^3} \delta^{(*)}(s' - K'^2) \delta^{(4)}(P_A + P_B - K' - K_D) \right]. \end{aligned} \quad (3.7)$$

The  $\theta$  function represents the constraint that the interval in rapidity space between the  $D$  and  $\bar{D}$  plus the interval spanned by the pions cannot be greater than the total rapidity available. At fixed  $s_D, s'$  the quantity in square brackets is merely the two-body phase-space integration with  $s_D$  and  $s'$  playing the role of masses. We can express this in terms of integrals over  $t_1$  and  $t_2$  and write

$$\begin{aligned} \text{disc } A_{\psi_1 N \rightarrow \psi_2 N} \propto Z m_{\psi_1}^2 \int ds_D ds' \lambda(s_D, m_{\pi}^2, m_{\psi}^2) \sigma_{\pi\psi \rightarrow D\bar{D}}(s_D) \lambda(s', m_{\pi}^2, m_N^2) \sigma_{\pi N}^{\text{tot}}(s') \theta(\ln s - \ln s' - \ln s_D - c) \\ \times \int dt_1 dt_2 R(t_1) R(t_2) / (-\Delta_4)^{1/2}, \end{aligned} \quad (3.8)$$

where  $\Delta_4$  is the symmetric Gram determinant defined by  $\Delta_4 = \Delta_4(s, t, t_1, t_2, m_{\psi_1}^2, m_{\psi_2}^2, m_N^2, s_D, s')$ ,

$$\Delta_4 = \begin{vmatrix} 2s & s + m_{\psi_1}^2 - m_N^2 & s + m_{\psi_2}^2 - m_N^2 & s + s_D - s' \\ s + m_{\psi_1}^2 - m_N^2 & 2m_{\psi_1}^2 & m_{\psi_1}^2 + m_{\psi_2}^2 - t & m_{\psi_1}^2 + s_D - t_1 \\ s + m_{\psi_2}^2 - m_N^2 & m_{\psi_1}^2 + m_{\psi_2}^2 - t & 2m_{\psi_2}^2 & m_{\psi_2}^2 + s_D - t_2 \\ s + s_D - s' & m_{\psi_1}^2 + s_D - t_1 & m_{\psi_2}^2 + s_D - t_2 & 2s_D \end{vmatrix}, \quad (3.9)$$

and the region of integration in  $t_1, t_2$  is the interior of the ellipse defined by the equation  $\Delta_4(t_1, t_2) = 0$ . If we make the approximation that the Reggeon propagators in (3.8) are exponentials,

$$R(t_1) R(t_2) = e^{b(t_1 + t_2)},$$

we can do the  $t$  integration in closed form. (See, for example, the discussion of Byckling and Kajantie.<sup>15</sup>) In the limit of large  $s$  and  $t = t_{\max}$  we can write

$$\int \frac{dt_1 dt_2 e^{b(t_1 + t_2)}}{[-\Delta_4(t_1, t_2)]^{1/2}} \cong \frac{\pi}{bs} \exp[b(t_1^{\max} + t_2^{\max})], \quad (3.10)$$

where  $t_{1,2}^{\max}$  are given by Eq. (3.5). In this limit, therefore, we have the expression

$$\begin{aligned} \lim_{\substack{s \rightarrow \infty \\ t = t_{\max}}} \text{disc } A_{\psi_1 N \rightarrow \psi_2 N} \propto Z(m_{\psi_1}^2) \int ds_D ds' \lambda(s_D, m_{\pi}^2, m_{\psi}^2) \sigma_{\pi\psi \rightarrow D\bar{D}}(s_D) \\ \times \lambda(s', m_{\pi}^2, m_N^2) \sigma_{\pi N}^{\text{tot}}(s') \frac{\pi}{bs} \exp \left[ -b \frac{(2s_D - m_{\psi_1}^2 - m_{\psi_2}^2)(s' - m_N^2)}{s} \right] \\ \times \theta(\ln s - \ln s' - \ln s_D - c). \end{aligned} \quad (3.11)$$

We now make the approximation that  $\sigma_{\pi N}^{\text{tot}}(s')$  is approximately constant over the entire region of integration so that

$$\sigma_{\pi N}^{\text{tot}}(s')\lambda(s', m_\pi^2, m_N^2) \cong (s' - m_N^2)\bar{\sigma}_{\pi N}^{\text{tot}}. \quad (3.12)$$

By using the high-energy expansion throughout the region of integration, we have made the assumption that this gives the *average* value of the integral. The  $s'$  integration runs from  $m_N^2$  to  $ds/s_D$ , where  $d$  is a parameter determined by the average transverse momenta in the multiperipheral chain. We can express the  $s'$  integration in terms of an incomplete  $\gamma$  function and write

$$\begin{aligned} \text{disc } A_{\psi_1 N \rightarrow \psi_2 N} \propto Z(m_{\psi_1}^2) \int ds_D \lambda(s_D, m_\pi^2, m_\psi^2) \sigma_{\pi \psi \rightarrow D \bar{D}}(s_D) \frac{\pi}{bs} \frac{s^2}{b^2(2s_D - m_{\psi_1}^2 - m_{\psi_2}^2)^2} \\ \times \gamma\left(2, db \frac{(2s_D - m_{\psi_1}^2 - m_{\psi_2}^2)}{s_D}\right). \end{aligned} \quad (3.13)$$

Since  $b$  determines the cutoff in  $t_1$  and  $t_2$  and  $d$  is related to the average transverse (mass)<sup>2</sup>, we expect the dimensionless product,  $db$ , to be a number  $\sim 1$  which is comparatively insensitive to our assumptions. If we denote

$$r = r(s_D) = (2s_D - m_{\psi_1}^2 - m_{\psi_2}^2)/s_D, \quad (3.14)$$

we have

$$\text{disc } A_{\psi_1 N \rightarrow \psi_2 N} \propto Z(m_{\psi_1}^2) \frac{\pi s}{b^3} \int ds_D \lambda(s_D, m_\pi^2, m_\psi^2) \sigma_{\pi \psi \rightarrow D \bar{D}}(s_D) s_D^{-2} [\gamma^{-2}(2, db r)]. \quad (3.15)$$

Fixing  $m_{\psi_2}^2 = m_\psi^2 = 9.6$  and varying  $s_D$  from  $4m_D^2 \cong 16$  to infinity, we see that  $r$  varies from

$$r(4m_D^2) = 1.4 - \frac{m_{\psi_1}^2}{16}, \quad (3.16)$$

$$r(\infty) = 2.$$

The quantity in curly brackets is therefore slowly varying over the region of integration, while

$$\lambda(s_D, m_\pi^2, m_\psi^2) \sigma_{\pi \psi \rightarrow D \bar{D}}(s_D) s_D^{-2} \propto s_D^{2\alpha_D - 3}, \quad (3.17)$$

where  $\alpha_D$  is the intercept of a typical charm-exchange trajectory which we take to be low,

$$\alpha_D < 0. \quad (3.18)$$

Using the mean-value theorem, we can therefore approximate

$$\begin{aligned} \text{Disc } A_{\psi_1 N \rightarrow \psi_2 N} \propto Z(m_{\psi_1}^2) \pi s [r_{\text{th}}^{-2} \gamma(2, db r_{\text{th}})] \\ \times \frac{(4m_D^2)^{2\alpha_D - 2}}{2 - 2\alpha_D}, \end{aligned} \quad (3.19)$$

and the ratio (2.3) can be given by

$$\begin{aligned} \lambda &\cong \frac{A(m_{\psi_1}^2 = 0)}{A(m_{\psi_2}^2 = m_\psi^2)} \\ &\cong \frac{(8m_D^2 - 2m_\psi^2)^2}{(8m_D^2 - m_\psi^2)^2} \frac{\gamma(2, db(2 - m_\psi^2/4m_D^2))}{\gamma(2, db(2 - m_\psi^2/2m_D^2))}, \end{aligned} \quad (3.20)$$

where we have neglected any variation in  $Z(m_\psi^2)$ . The value of the ratio as a function of the parameter  $db$  is plotted in Fig. 4. For reasonable ranges of the values of  $d$  and  $b$ , the factor  $\lambda$  is around  $\frac{1}{2}$ .

It is interesting to note that a similar value for the suppression factor  $\lambda$  has been calculated by Pumplin and Repko.<sup>16</sup> They treat the  $\psi$  as a non-relativistic bound state of the new quarks and their results depend on this detailed dynamical assumption.

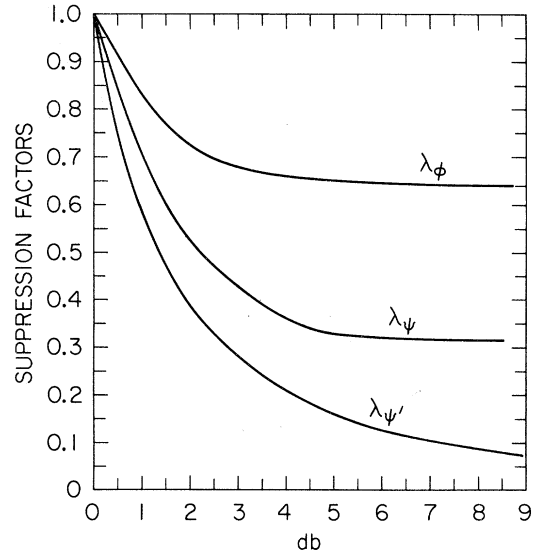


FIG. 4. The suppression factors for  $\gamma N \rightarrow \psi N$ ,  $\gamma N \rightarrow \phi N$ , and  $\gamma N \rightarrow \psi' N$  calculated in the peripheral unitarity model as described in the text. The parameter  $db$  is expected to be near unity since  $d$  measures the average  $p_T^2$  and  $b$  gives the falloff in  $t$  of the propagator but we display the dependence on this parameter to show how sensitive our results are to variations in the assumptions of the form of the matrix element.

B. The continuation in  $\phi$  photoproduction

We can do an analogous calculation for the continuations involved in  $\phi$  photoproduction. In that case, we write the expression

$$\text{disc } A_{\phi_1 N \rightarrow \phi_2 N} \propto Z(m_{\phi_1}^2) \frac{\pi S}{b^3} \int ds_K \lambda(s_K, m_{\pi}^2, m_{\phi}^2) \sigma_{\pi \psi \rightarrow K \bar{K}}(s_K) s_K^{-2} \times \left[ \frac{s_K^2}{(2s_K - m_{\phi_1}^2 - m_{\phi_2}^2)} \gamma \left( 2, db \frac{2s_K - m_{\phi_1}^2 - m_{\phi_2}^2}{s_K} \right) \right]. \quad (3.21)$$

The quantity in square brackets again varies slowly throughout the region of integration but the integrand vanishes at  $s_K = (m_{\phi} + m_{\pi})^2$  and attains its maximum value near  $s_K \cong 3m_{\phi}^2 = 3 \text{ GeV}^2$  and not at threshold. Again, we can estimate the variation with  $m_{\phi_1}^2$  by using the mean-value theorem to approximate the integral,

$$\lambda\phi = \frac{A(m_{\phi_1}^2 = 0)}{A(m_{\phi_1}^2 = m_{\phi}^2)} \cong \frac{(4m_{\phi}^2)^2}{(5m_{\phi}^2)^2} \frac{\lambda(2, db(\frac{5}{3}m_{\phi}^2))}{\lambda(2, db(\frac{4}{3}m_{\phi}^2))}. \quad (3.22)$$

The value of this suppression factor as a function of the parameter  $db$  is plotted in Fig. 7. We note that if we take  $db$  near its expected value of 1, we obtain a suppression factor  $\lambda_{\phi} \cong 0.8$ . This value of  $\lambda_{\phi}$  helps resolve a long-standing conflict between the quark model and vector dominance. Using the quark-model estimate

$$\sigma_{\text{tot}}(\phi p) = \sigma_{\text{tot}}(K^* p) + \sigma_{\text{tot}}(K^* n) - \sigma_{\text{tot}}(\pi^* p) \cong 13 \text{ mb} \quad (3.23)$$

we can use the analog of (2.1) and (2.6) to fit data on  $\phi$  photoproduction, and get<sup>9</sup>

$$\frac{1}{\lambda_{\phi}^2} \frac{\gamma_{\phi}^2}{4\pi} = 5.1 \pm 0.6. \quad (3.24)$$

Using the measured value of the  $\Gamma(\phi \rightarrow e^+ e^-)$  (Ref. 17), we have

$$\frac{\gamma_{\phi}^2}{4\pi} = 3.1 \pm 0.3 \quad (3.25)$$

or

$$\gamma_{\phi}^2 = 0.61 \pm 0.12, \quad \lambda_{\phi} = 0.78 \pm 0.08. \quad (3.26)$$

Data on  $\sigma^{\text{tot}}(\phi N)$  from the nuclear  $A$  dependence gives<sup>17</sup>

$$\sigma^{\text{tot}}(\phi N) = 12.0 \pm 3.9 \text{ mb}, \quad (3.27)$$

which provides some support for the quark-model result but is also consistent with  $\lambda_{\phi} = 1$ . We believe that the fact that our unitarity model apparently gives the right magnitude of the  $q^2$  de-

pendence in  $\phi$  photoproduction justifies its use in  $\psi$  photoproduction even though the range in  $q^2$  over which the extrapolation is made is much larger.

The expression (3.20) can also be used to give an estimate for the suppression in  $\psi'(3684)$  photoproduction if we again make the assumption that the cross section for  $\pi\psi' \rightarrow D\bar{D}$  peaks very near threshold. This factor also is plotted as a function of  $db$  in Fig. 4. Using SLAC data on  $\psi'$  photoproduction, we therefore estimate

$$\alpha_{\text{tot}}(\psi' N) \cong 1.5 \text{ (mb)}.$$

We note that the suppression factor calculated here has a similar physical origin to the factor calculated by Aviv *et al.*<sup>18</sup> in the context of a slightly different model. One important difference is that their suppression is much stronger for  $\phi$  photoproduction than for  $\psi$  photoproduction, while our approach is consistent with what is known about both systems.

If we modify our  $\pi$ -exchange model so that instead of (3.1) we let a Reggeon with intercept  $\alpha$  be exchanged, the largest correction to (3.20) involves the intercept in the form

$$\lambda \cong \frac{(8m_D^2 - 2m_{\psi}^2)^{2-2\alpha}}{(8m_D^2 - m_{\psi}^2)^{2-2\alpha}} \frac{\gamma(2-2\alpha, db(2 - m_{\psi}^2/4m_D^2))}{\gamma(2-2\alpha, db(2 - m_{\psi}^2/2m_D^2))}.$$

As the intercept,  $\alpha$ , approaches 1, the factor  $\lambda$  becomes identically 1 and there is no dependence on the mass of the external leg.

## IV. THE SCHWARZ INEQUALITY AND PHOTOPRODUCTION

There are some manipulations we can perform on photoproduction amplitudes without invoking directly the vector-dominance concept. One of these, which we will discuss here, illustrates the power of the OZI (Ref. 7) selection rules by deriving a connection between  $\psi$  photoproduction, the photoproduction of charmed particles, and  $\sigma^{\text{tot}}(\psi N)$ .

Suppose we begin with the unitarity relation, indicated in Fig. 5(a),

$$\begin{aligned}
& i[\langle \gamma p | A^+ | \psi p \rangle - \langle \psi p | A^- | \psi p \rangle] \\
& = - (2\pi)^4 \sum_m \langle \gamma p | A^+ | m \rangle \langle m | A^- | \psi p \rangle \delta^{(4)}(P_i - P_f).
\end{aligned} \tag{4.1}$$

In the forward direction, we can then use Hermitian analyticity and time reversal to write the left-hand side of (4.1) as the imaginary part of the nonflip amplitude,

$$\begin{aligned}
& 2 \operatorname{Im} A_{\gamma p \rightarrow \psi p}(s, t = t_{\max}) \\
& = (2\pi)^4 \sum_m \langle \gamma p | A^+ | m \rangle \langle m | A^- | \psi p \rangle \delta^{(4)}(P_i - P_f).
\end{aligned} \tag{4.2}$$

Now on the right-hand side of (4.1) and (4.2) we want to divide the sum over intermediate states into two parts reflecting our dynamical assumption that the  $\psi$  is a  $c\bar{c}$  state, where  $c$  and  $\bar{c}$  are quarks carrying a new additive quantum number. This separation is indicated in Fig. 5(b), where we isolate those states containing charmed quarks, either bound together in a  $\psi, \psi'$  or separated in the associated production of a pair of charmed hadrons

$$\begin{aligned}
& \sum \langle \gamma p | A^+ | m \rangle \langle m | A^- | \psi p \rangle \\
& = \sum_{\substack{m_c \\ (\text{charmed})}} \langle \gamma p | A^+ | m_c \rangle \langle m_c | A^- | \psi p \rangle \\
& + \sum_{\substack{m' \\ (\text{no charmed} \\ \text{quarks})}} \langle \gamma p | A^+ | m' \rangle \langle m' | A^- | \psi p \rangle,
\end{aligned} \tag{4.3}$$

where  $m_c$  denotes those states containing at least one  $c\bar{c}$  pair and  $m'$  are those intermediate states without charmed quarks. We now use the fact that the OZI (Ref. 7) selection rules suppress all the diagrams in the second sum. We do not have a perfect understanding of how big the suppression will be, so for the time being we simply parameterize it in the form

$$\begin{aligned}
& \sum_m \langle \gamma p | A^+ | m \rangle \langle m | A^- | \psi p \rangle \\
& \leq (1 + \epsilon) \sum_{m_c} \langle \gamma p | A^+ | m_c \rangle \langle m_c | A^- | \psi p \rangle,
\end{aligned} \tag{4.4}$$

where  $\epsilon$  is some parameter which bounds the possible violations of the OZI rules. We will not

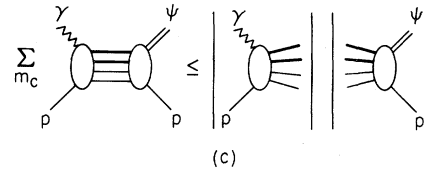
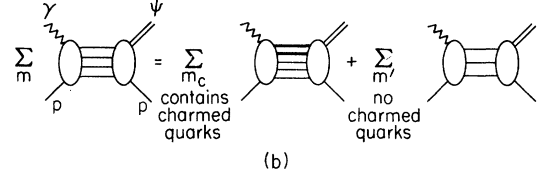
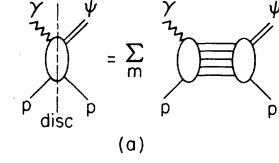


FIG. 5. Diagrams which indicate the steps leading up to the inequality (4.11). Diagram (a) illustrates the unitarity equation for  $\gamma p \rightarrow \psi p$ . Diagram (b) demonstrates the breakup of the sum over intermediate states in the unitarity equation into two parts. The first consists of those states which contain charmed quarks and are allowed by the OZI rules. The second set of intermediate states is suppressed by the OZI selection rules. Diagram (c) illustrates the application of the Schwarz inequality to the first sum.

discuss in detail the problem of estimating  $\epsilon$  but we believe it to be small. We now note that the right-hand side of (4.4) defines a scalar product in the Hilbert space of all those states containing charmed quarks. Since the photon is assumed to couple to charmed quarks with a typical electromagnetic coupling, we can include the state  $|\gamma p\rangle$  in the space. We can then use the Schwarz inequality to write

$$\sum_{m_c} \langle \gamma p | A^+ | m_c \rangle \langle m_c | A^- | \psi p \rangle \leq \| \langle \gamma p | A^+ \|_c \| A^- | \psi p \rangle \|_c, \tag{4.5}$$

where we have used the subscript  $c$  to denote the norm defined on the OZI-rule-allowed subspace. Combining (4.4) and (4.5) and inserting into (4.2), we have

$$2 \operatorname{Im} A_{\psi p \rightarrow \psi p}(s, t = t_{\max}) \leq (2\pi)^4 (1 + \epsilon) \| \langle \gamma p | A^+ \|_c \| A^- | \psi p \rangle \|_c \delta^{(4)}(P_f - P_i). \tag{4.6}$$

Taking the square of the absolute value of each side of (4.6), we have

$$4 (\operatorname{Im} A_{\gamma p \rightarrow \psi p})^2 \leq (2\pi)^8 (1 + \epsilon)^2 \left( \sum_{m_c} \langle \gamma p | A^+ | m_c \rangle \langle m_c | A^- | \gamma p \rangle \right) \left( \sum_{m_c} \langle \psi p | A^+ | m_c \rangle \langle m_c | A^- | \psi p \rangle \right) [\delta^4(P_f - P_i)]^2. \tag{4.7}$$

We can use the fact that momentum is conserved for each subset of particles to write

$$\delta^{(4)}(P_f - P_i)\delta^{(4)}(P_f - P_i) = \delta^{(4)}(P_f - P_{m_c})\delta^{(4)}(P_i - P_{m_c}) \quad (4.8)$$

so that we have

$$4[\text{Im}A_{\psi p \rightarrow \psi p}(s, t_{\max})]^2 \leq (1 + \epsilon)^2 [4q_s^{\gamma p} s^{1/2} \sigma_{\gamma p \rightarrow \text{charm}}(s)] [4q_s^{\psi p} s^{1/2} \sigma_{\psi p \rightarrow \text{charm}}(s)], \quad (4.9)$$

where  $q_s^{AB}$  is the c.m. momentum of the  $AB$  system. With this normalization

$$\frac{d\sigma}{dt}(\gamma p \rightarrow \psi p) = \frac{1}{64\pi s (q_s^{\gamma p})^2} [(\text{Re}A)^2 + (\text{Im}A)^2] \quad (4.10)$$

and so we write

$$16\pi \frac{d\sigma_{\gamma p \rightarrow \psi p}}{dt}(s, t = t_{\max}) \leq (1 + \epsilon)^2 (1 + \rho^2) \left( \frac{q_s^{\psi p}}{q_s^{\gamma p}} \right) \sigma_{\gamma p \rightarrow \text{charm}}(s) \sigma_{\psi p \rightarrow \text{charm}}(s), \quad (4.11)$$

where  $\rho = \text{Re}A(s, t_{\max})/\text{Im}A(s, t_{\max})$  and the ratio

$$\left( \frac{q_s^{\psi p}}{q_s^{\gamma p}} \right) = \frac{[s - (m_p + m_\psi)^2]^{1/2} [s - (m_p - m_\psi)^2]^{1/2}}{(s - m_p^2)}. \quad (4.12)$$

The cross sections  $\sigma_{\gamma p \rightarrow \text{charm}}$  and  $\sigma_{\psi p \rightarrow \text{charm}}$  include all those final states with particles containing charmed quarks. For the  $\psi p$  case we have reason to believe that

$$\sigma_{\text{tot}}^{\psi p}(s) \cong \sigma_{\psi p \rightarrow \text{charm}}(s) \quad (4.13)$$

since all other final states are forbidden by the OZI rules. For the photon

$$\sigma_{\gamma p \rightarrow \text{charm}}(s) = \sigma_{\gamma p \rightarrow \psi + x}(s) + \sigma_{\gamma p \rightarrow \psi' + x}(s) + \sigma_{\gamma p \rightarrow D\bar{D} + x}(s) + \dots, \quad (4.14)$$

where the  $D$ 's are charmed particles. We can use Eq. (4.11) in two ways. The first, and most straightforward application of the inequality, is that a measurement of the photoproduction of the hypothetical charmed particles can be made which will give a lower bound on  $\sigma_{\text{tot}}^{\psi p}(s)$ . This lower bound depends only on unitarity and on the OZI rules and is *completely independent of the vector-dominance hypothesis*. Measurement of charmed-particle photoproduction can therefore provide a significant bonus in that it can be combined with  $\psi$  photoproduction data to give an independent test of vector dominance.

In the absence of data on charmed-particle photoproduction we can apply (4.11) in the opposite way. Given a model for the  $\psi p$  total cross section we can see (4.11) to give a lower bound on the cross section for the production of charmed particles. If we take the vector-dominance result in Sec. II we can plug into the inequality and get

the lower limits on the quantity

$$(1 + \epsilon)^2 (1 + \rho^2)^{1/2} \sigma_{\gamma p \rightarrow \text{charm}}(s) \quad (4.15)$$

shown in Fig. 6. Under the assumption that the photoproduction amplitude is predominately imaginary and that  $\epsilon$  is small we can then estimate

$$\sigma_{\gamma p \rightarrow D\bar{D} + x} \gtrsim \lambda 500 \text{ nb} \quad (E_\gamma \gtrsim 20 \text{ GeV}). \quad (4.16)$$

This is an extremely large cross section when compared with  $\psi$  photoproduction and the lower range agrees with our previous estimate (2.16). It is still, of course, true that if the vector-dominance hypothesis underestimates  $\sigma^{\text{tot}}(\psi p)$  by a factor  $\lambda$  then we have overestimated  $\sigma_{\gamma p \rightarrow \text{charm}}$  by the same factor.

If we adopt the usual assumption that the new particles have a substantial branching ratio into leptons (in either purely leptonic or semileptonic decay modes) we can see that there should be a detectable contribution from these particles to the direct photoproduction of leptons. Using the lower limit (4.16) we can estimate at large- $p_T$  ratio of the  $e$ 's from  $D$  decay to the  $\pi$ 's. Letting  $B(D \rightarrow e)$  be the total leptonic branching ratio, we can estimate

$$R\left(\frac{e^\pm}{\pi^\pm}\right)_{p_T \gtrsim p_{T0}} \cong \frac{\sigma_{\gamma p \rightarrow D\bar{D}} B(D \rightarrow e^\pm) f(e^\pm \gtrsim p_{T0})}{\langle n \pi^\pm \rangle \sigma_{\text{tot}}^{\gamma p} \times f(\pi^\pm \gtrsim p_{T0})}, \quad (4.17)$$

where  $f(e^\pm \gtrsim p_{T0})$  and  $f(\pi^\pm \gtrsim p_{T0})$  are the fractions of  $e$ 's and  $\pi$ 's with transverse momentum greater

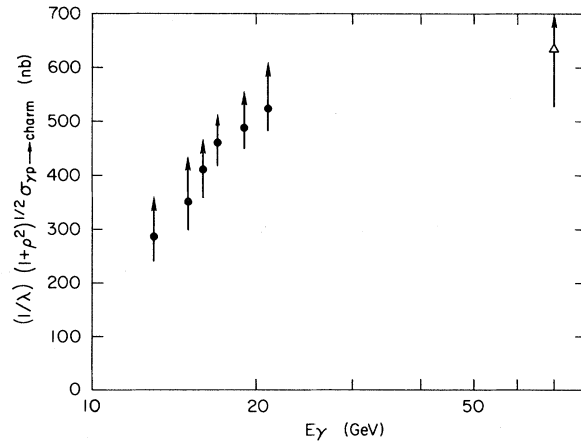


FIG. 6. The use of vector dominance and (4.11) to deduce a lower bound on  $\sigma_{\gamma p \rightarrow \text{charm}}$ .

than  $p_{T0}$ . Since the  $e$ 's are assumed to come from the decay of a massive  $D$ ,  $f(e^\pm > p_{T0})$  falls off slowly until  $p_{T0} > m_D/2$  while for  $\pi$ 's it falls off rapidly. Putting in a reasonable falloff for the pion spectrum we get

$$R\left(\frac{e^\pm}{\pi^\pm}\right)_{p_T \approx 1 \text{ GeV}} \cong 10^{-2} \times \lambda B(D \rightarrow e). \quad (4.18)$$

This is large compared to similar estimates for the contribution of charmed particles to this ratio in  $pp$  collisions since photons provide a rich source of the new quarks.

It is instructive to use the inequality (4.11) in a situation where we have better understanding of all facets of the result. Obviously we can go through entirely analogous arguments in the case of  $\phi p$  photoproduction to write the inequality

$$16\pi \frac{d\sigma_{\gamma p \rightarrow \phi p}}{dt}(s, t_{\max}) \leq (1 + \epsilon_s)^2 (1 + \rho^2) \sigma_{\gamma p \rightarrow \text{strange}}(s) \sigma_{\phi p \rightarrow \text{strange}}(s), \quad (4.19)$$

where we have used

$$\left(\frac{q_s^{\phi p}}{q_p^{\phi p}}\right) \cong 1 \quad \text{for } s \geq 10 \text{ GeV}^2. \quad (4.20)$$

In this case we can look at specific two-body reactions involving  $\phi$ 's in order to get an idea of how big the parameter  $\epsilon_s$  which measures the violation of the OZI selection rules can be. For example we can use crossing to write

$$\begin{aligned} \frac{d\sigma(\phi p \rightarrow K^+ \Lambda)}{d\sigma(\phi p \rightarrow \pi^- n)} &\cong \frac{d\sigma(K^- p \rightarrow \phi \Lambda)}{d\sigma(\pi^- p \rightarrow \phi n)} \\ &= 52 \pm 16, \end{aligned} \quad (4.21)$$

where the experimental value for the latter ratio is due to Ayres *et al.*<sup>19</sup> From this we can estimate  $\epsilon_s < \frac{1}{7}$ . Data on  $\phi$  photoproduction give<sup>9</sup>

$$\frac{d\sigma_{\gamma p \rightarrow \phi p}}{dt}(s, t_{\max}) = 2.6 \pm 0.5 \text{ } \mu\text{b/GeV}^2 \quad (4.22)$$

and a measurement of interference with Bethe-Heitler pairs has yielded at

$$\rho_s = \text{Re}A / \text{Im}A \leq 0.2 \text{ at } t = t_{\max} \quad (4.23)$$

so that using the quark-model result

$$\sigma_{\phi p \rightarrow \text{strange}} \cong 12 \text{ mb} \quad (4.24)$$

from vector-dominance arguments we get

$$4.2 \pm 0.8 \text{ } (\mu\text{b}) \leq (1 + \epsilon_s)^2 \sigma_{\gamma p \rightarrow \text{strange}}(s). \quad (4.25)$$

The production of strange particles by photons has been measured directly in a bubble-chamber experiment which gives<sup>20</sup>

$$\sigma_{\gamma p \rightarrow \text{strange}}(s) \cong 8.5 \text{ } \mu\text{b} \quad (s = 6-18 \text{ GeV}^2) \quad (4.26)$$

so the inequality fails by roughly a factor of 2 of being an equality.

*Note added in proof.* Naturally, the inequality can also be used in cases where the OZI rule is inoperative, for example, in  $\rho$  photoproduction, where the inequality reads as in (4.19) except that  $\phi$  is replaced by  $\rho$ , "strange" is replaced by "all," and  $\epsilon_s$  is set equal to zero. Using experimental data<sup>9</sup> the inequality leads to  $\sigma_{\rho p}(S \approx 18 \text{ GeV}^2) \leq 19 \text{ mb}$  and  $\sigma_{\rho p}(S \approx 9 \text{ GeV}^2) \leq 23 \text{ mb}$  if we use the value of  $\sim 0.2$  for  $\rho$  as estimated from Compton scattering. These limits for  $\sigma_{\rho p}(s)$  are well satisfied by estimates from nuclear photoproduction and vector dominance which give  $\sigma_{\rho p} \sim 26 \text{ mb}$ . Notice, however, that this number is relatively close to our bound, which can be understood as a vindication of the idea that the photon behaves predominantly like a  $\rho$  meson. This is because the equality is reached in precisely the case when they are identical and simply reduces (up to the  $\gamma_\rho$  coupling constant) to the conventional optical theorem for Compton scattering. Indeed, an equivalent way of expressing this is to say that the conventional method for determining  $\sigma$  from  $d\sigma/dt$  via the vector-dominance model always leads to a lower bound.

#### The inequality near threshold

It is interesting to consider the inequality (4.11) just above the  $\psi p$  threshold but below the  $\psi' p$  threshold. If we assume that the  $s$  wave dominates so that

$$\sigma_{\gamma p \rightarrow \psi p}(s) \cong 4\pi \frac{d\sigma_{\gamma p \rightarrow \psi p}}{d\Omega}(s, \theta = 0) \quad (4.27)$$

and the real-to-imaginary ratio does not change drastically in the available kinematic region, we can write the inequality (4.11) in the form

$$\frac{4\pi(\text{Im}A)^2}{(\text{Im}A)^2 + (\text{Re}A)^2} \leq (1 + \epsilon)^2 (q_s^{\psi p})^2 \sigma_{\psi p \rightarrow \psi p}, \quad (4.28)$$

where  $\text{Im}A$  and  $\text{Re}A$  are respectively the average values of the imaginary and real parts of the photoproduction cross section. We see that in this kinematic region the inequality suggests that the amplitude should be predominantly real. Only substantially above the available inelastic thresholds can the imaginary part of the amplitude dominate. In conjunction with typical estimates of the masses for charmed particles this should serve to warn against the assumption  $\rho \ll 1$  below  $E = 16 \text{ GeV}$ . This means that the  $\psi N$  cross section may be much lower than the values plotted in Fig. 1 in the Cornell-SLAC regimes.

#### V. THE RATIO $\sigma^{e1}(\psi p)/\sigma^{t01}(\psi p)$ IN THE FRAMEWORK OF DUAL MODELS

The analysis of the available data on  $\psi$  photoproduction within the context of the vector-dominance model has led to the conclusion

$$\sigma^{\text{el}}(\psi p)/\sigma^{\text{tot}}(\psi p) \ll 1. \quad (5.1)$$

If we, in addition, assume the validity of the Okubo-Zweig-Iizuka (OZI) selection rules for reactions involving  $\psi$ 's, we observe that the dominant inelastic channels must contain the quark-antiquark pair of the initial  $\psi$ . These quarks will usually appear in separate particles so the inequality (5.1) can be strengthened to read

$$\sigma^{\text{el}}(\psi p)/\sigma(\psi p \rightarrow D\bar{D} + \text{anything}) \ll 1. \quad (5.2)$$

Since the result (5.2) has important implications for charmed particle searches involving photon beams, it is interesting to see whether we can understand it from an independent line of reasoning. In this section we will examine the ratio (5.2) from the standpoint of some simple ideas based on duality.

There are many possible approaches to the duality properties of amplitudes involving  $\psi$ 's, such as, for example, the discussion of Finkelstein<sup>21</sup> and of Halzen and Kajantie.<sup>22</sup> Since we are interested in the relative normalization of elastic and inelastic cross sections, one straightforward application of duality involves starting with the triple-Regge expression for the reaction  $\psi p \rightarrow p + \text{anything}$ .

$$E \frac{d\sigma^{\text{T.R.}}}{d^3p}(\psi p \rightarrow p x) \sim \sum_{i,j} G_{ii,j}^{\psi}(t) (m^2)^{\alpha_j(0)-2\alpha_i(t)-1}. \quad (5.3)$$

As indicated in Fig. 7 we assume that the inelastic channels in this process are dominated by those containing a  $D\bar{D}$  pair.

We want to consider the relative normalization of the elastic and inelastic channels under the assumption that the triple-Regge expansion (5.3) gives a "semilocal" approximation to the elastic cross section,

$$\int_{m_{\phi^2-\Delta/2}}^{m_{\phi^2+\Delta/2}} dm^2 \frac{d\sigma^{\text{T.R.}}}{dt dm^2} \simeq \frac{d\sigma}{dt}(\psi p \rightarrow \psi p), \quad (5.4)$$

where  $\Delta$  is some (mass)<sup>2</sup> parameter which defines the size of the region which must be averaged. In the triple-Regge formalism we can apply finite-mass sum rules to the forward Reggeon  $\psi$  discontinuity indicated in Fig. 5(b) and our assumption (5.4) states that there is some value of  $\Delta$  which averages the  $\psi$  pole in the direct channel. In dual models with a universal slope for all Regge trajectories we would have  $\Delta \simeq 1/\alpha' \simeq 1 \text{ GeV}^2$ , but we are going to leave  $\Delta$  an arbitrary parameter. We therefore write

$$\frac{d\sigma}{dt}(\psi p \rightarrow \psi p) \simeq \left[ \sum_{ii,j} G_{ii,j}^{\psi}(t) (m_{\psi^2})^{\alpha_j(0)-2\alpha_i(t)-2} s^{2\alpha_i(t)-2} \right] \Delta. \quad (5.5)$$

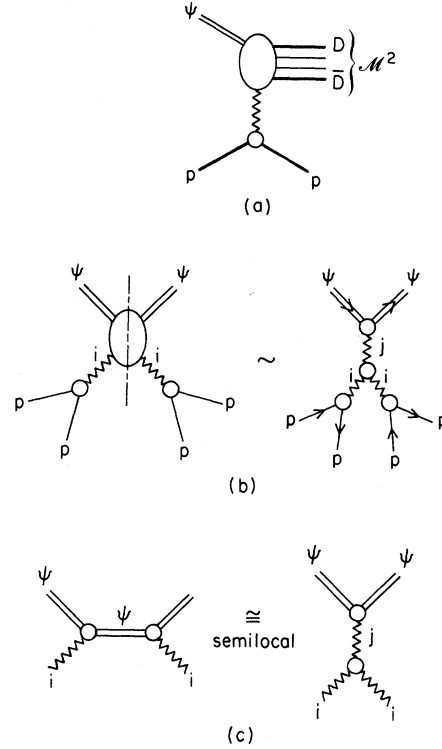


FIG. 7. Diagram (a) represents the leading contribution to  $\psi p \rightarrow p + \text{anything}$  incorporating the OZI rule. Diagram (b) gives the triple-Regge expression. Semilocal duality normalizes the inclusive and exclusive cross section as indicated in diagram (c).

At this point we make the additional assumption that the coupling to the protons in triple-Regge expansion, Eq. (5.3), is dominated by the Pomeron singularity. At large  $s$ , this makes sense for the small- $m^2$  piece of the cross section but we must keep in mind the fact that we might be neglecting contributions to charm production. For simplicity in comparing the elastic and the inelastic contributions we parametrize

$$G_{\Pi\Pi j}(t) \simeq G_{\Pi\Pi j} e^{Bt}, \quad (5.6)$$

$$\alpha_{\Pi}(t) = 1 + \alpha'_{\Pi}(t),$$

which allows us to integrate (5.5) over  $t$  and express

$$\sigma_{\text{el}}(s) = \sum_j \frac{G_{\Pi\Pi j} (m_{\psi^2})^{\alpha_j(0)-2}}{B + 2\alpha'_{\Pi} \ln(s/m^2)} \Delta. \quad (5.7)$$

To get the contribution to the inelastic diffractive cross section we must integrate in  $m^2$  from  $4m_D^2$  and in  $t$  up to

$$t_{\text{max}} \simeq -m_p^2 \left( \frac{m^2 - m_{\psi^2}}{s} \right). \quad (5.8)$$

We therefore write

$$\begin{aligned}\sigma_{\text{diff}}^{\text{inel}}(s) &\cong \sum_j \int_{4m_D^2}^s dm^2 \int_{-\infty}^{t_{\text{max}}} dt G_{\Pi\Pi j} e^{Bt} (m^2)^{\alpha_j(0)-2} (s/m^2)^{2\alpha_{\Pi}'t} \\ &\cong \sum_j G_{\Pi\Pi j} \int_{4m_D^2}^s dm^2 \frac{(m^2)^{\alpha_j(0)-2} \exp \left\{ -[B + 2\alpha_{\Pi}' \ln(s/m^2)] m_{\phi}^2 \left( \frac{m^2 - m_{\phi}^2}{s} \right) \right\}}{(B + 2\alpha_{\Pi}' \ln s/m^2)}.\end{aligned}\quad (5.9)$$

To get a rough estimate of the integral we can make the approximation  $\alpha_{\Pi}' \cong 0$  inside the integrand and neglect the  $t_{\text{max}}$  suppression in those regions of  $m^2$  where the integrand is large. We can then write

$$\sigma_{\text{diff}}^{\text{inel}}(s) \cong \sum_j G_{\Pi\Pi j} \frac{(4m_D^2)^{\alpha_j(0)-1}}{B(1-\alpha_j(0))}. \quad (5.10)$$

Within the spirit of these types of crude approximations we can assume that (5.7) and (5.10) are dominated by a single exchange. Making the approximation  $\alpha_{\Pi}' = 0$  in (5.7) we therefore have the estimate

$$\frac{\sigma^{\text{el}}(\psi p)}{\sigma_{\text{diff}}^{\text{inel}}(\psi p \rightarrow pX)} \cong \left( \frac{m_{\psi}^2}{4m_D^2} \right)^{\alpha_j(0)-2} \left( \frac{\Delta}{4m_D^2} \right) [1 - \alpha_j(0)]. \quad (5.11)$$

We are faced with something of a paradox in that our analysis of the total cross section has suggested that this ratio be small whereas the duality arguments give the ratio to be large unless the intercept of the exchange trajectory,  $\alpha_j(0)$ , is comparatively high. Since the  $\psi$  should decouple from the usual high-lying meson trajectories,  $\rho$ ,  $\omega$ ,  $A_2$ ,  $f$ , etc., this is a puzzle. Our analysis suggests that we have a fairly high-lying trajectory, which we will call " $f_{\text{charm}}$ " which couples substantially to charm. This is surprising. Recall that Carlson and Freund<sup>23</sup> within the context of the dual Pomeron model have predicted that any such trajectory must have a low intercept

$$\begin{aligned}\alpha_j(0) &\cong \alpha_{f_{\text{charm}}} \cong 1 - m_{\psi}^2 \\ &\cong -8.6\end{aligned}\quad (5.12)$$

in order to explain the size of the  $\psi$  coupling of the pomeron. We see therefore that our understanding of the duality properties of amplitudes involving  $\psi$ 's is far from complete.

Since we do not know exactly what percentage of charmed particles should be produced in the configuration of Fig. 7, the numerical estimates of  $\sigma^{\text{el}}(\psi p)/\sigma(\psi p \rightarrow \text{charm})$  do not serve to put a stringent bound on the ratio (5.11). It would seem that values of typical charm-trajectory intercepts used by Field and Quigg<sup>24</sup> or Barger and Phillips<sup>25</sup>

$$\alpha_{\text{charm}}(0) \cong -0.57 \quad (5.13)$$

are probably sufficiently high to assure consistency between the vector-dominance estimates of

$\sigma_{\text{tot}}(\psi N)$  and the duality estimates presented here.

For comparison, we may do an analogous calculation to estimate the ratio of the elastic scattering of the  $\phi$  meson to its inelastic breakup into  $K\bar{K}$  mesons

$$\frac{\sigma^{\text{el}}(\phi p)}{\sigma_{\text{diff}}^{\text{inel}}(\phi p \rightarrow pX)} \cong \left( \frac{m_{\phi}^2}{4m_K^2} \right)^{\alpha_i(0)-2} \left( \frac{\Delta}{4m_K^2} \right) [1 - \alpha_i(0)]. \quad (5.14)$$

Since  $m_{\phi}^2 \cong 4m_K^2 \cong \Delta \cong 1 \text{ GeV}^2$  this depends roughly on the intercept of the appropriate trajectory which can couple to  $\phi$ 's. With

$$\alpha_i(0) \cong \alpha_{f'}(0) \cong 0 \quad (5.15)$$

we get the reasonable estimate of unity for this ratio which is in agreement with what we know about  $\phi N$  scattering. This indicates that the arguments leading to (5.12) have some possibility of being correct.

Whatever values for the charm-production cross sections are measured experimentally, it seems clear that the study of the duality properties of amplitudes involving charmed particles and  $\psi$ 's will turn out to be very instructive. It may be, for example, that the  $\psi$  and  $\psi'$  are substantially different types of bound states than the  $\rho$ ,  $\omega$ ,  $\phi$  so that the usual duality ideas which are based on the existence of harmonic-oscillator potentials are invalid. This possibility is, in fact, suggested by the charmonium approach<sup>26</sup> to the  $\psi$  and  $\psi'$  where it is pointed out that the effective mass of the charmed quarks must be quite large. This large mass gives rise to a new distance scale in the bound-state problem. Whether or not some generalization of the duality concepts retains its validity in these models is uncertain.

## VI. CONCLUSIONS

If the currently attractive interpretation of the  $\psi(3095)$  and  $\psi(3684)$  as bound states of quarks carrying a new additive quantum number is correct, there must be new hadrons which are bound states of the new quarks and ordinary constituents. We have examined in some detail the consequences for photoproduction data of the existence of these new hadronic states. We find two results with significant experimental impact which are comparatively insensitive to the details of the spectro

scopy of the new states:

(1) The coupling of  $\psi p$  to new hadronic channels allows us to estimate the corrections to the usual vector-dominance formula for  $d\sigma/dt(\gamma p \rightarrow \psi p)$  in a simple and direct way. Doing this, we calculate that the total cross section for  $\psi p$  scattering should be about twice the value estimated from naive vector dominance. The validity of our calculation can be established by doing measurements with 5% accuracy on the photoproduction of  $\psi$ 's from nuclear targets. An interesting fact emerges in that we can apply our calculation in a straightforward way to  $\phi$  photoproduction where we predict a modification of the vector-dominance formula which gives agreement with the quark-model value of  $\sigma^{\text{tot}}(\phi p)$ . The same type of corrections to vector dominance can be found in simple models for the quark-antiquark bound state.

(2) Unitarity can be combined with the OZI selection rules to relate the "pseudoelastic" cross sec-

tion  $d\sigma/dt(\gamma p \rightarrow \psi p)$  to  $\sigma(\gamma p \rightarrow D\bar{D} + \text{anything})$  and  $\sigma_{\text{tot}}(\phi p)$  where  $D\bar{D}$  are used as generic names for the new hadrons. Available data and our estimates for the corrections to the vector-dominance expression can be combined to give a lower limit:

$$\sigma(\gamma p \rightarrow D\bar{D} + \text{anything}) \gtrsim 300 \text{ nb} \quad (E_{\text{lab}} \gtrsim 20 \text{ GeV}).$$

A cross section of this magnitude can be shown to contribute substantially to the rate of direct leptons if the  $D$ 's have a significant leptonic branching ratio.

In addition, we discuss the fact that analysis of photoproduction data leads to the conclusion that the ratio  $\sigma^{\text{el}}(\psi N)/\sigma^{\text{tot}}(\psi N)$  is a very small number. This observation is hard to understand within the usual duality framework which has been found useful in the description of hadronic dynamics. This suggests the possibility that the  $\psi$ 's will give a new and perhaps incisive perspective to dynamical problems.

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