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Symmetries, angular distributions in $\psi' \to \gamma\chi \to \gamma\gamma\psi$, and the interpretation of the χ(3400–3550) levels

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Recently a family of particles (to be denoted χ) has been discovered between 3400 and $\sim$ 3550 MeV. A formalism is presented for analysis of the decays $\psi'(3684) \to \gamma\chi \to \gamma\gamma\psi$ or $\gamma 0^- 0^-$. This formalism is sufficiently general to allow for transitions of several multipolarities. Previous treatments have been restricted to the assumption that the above transitions are purely $E1$ if $J^P(\chi) = 0^+$, 1^+, and 2^+. Here (with some model-dependent motivation) this restriction is removed. The predictions of specific heavy-quark models for the χ states are then discussed. On the basis of the single-quark-transition picture (the Melosh transformation), it is anticipated that the state $\chi(3410)$ will be identified as a $^3P_0$ $q\bar{q}$ level, and will be found to have $J^{PC} = 0^{++}$. It is also anticipated that the $^3P_2$ level should be fairly prominent.

I. INTRODUCTION

In addition to the narrow resonances $\psi(3095)$ [1] and $\psi'(3684)$,[2] there is good evidence for states of intermediate mass between 3400 and ~3550 MeV.[3] These will be denoted collectively as χ. Electromagnetic (cascade) decays of the type:

$$\psi' \to \gamma\chi, \tag{1}$$

$$\chi \to \gamma\psi, \tag{2}$$

have been identified. In addition, the states χ decay to an even number of pions. The identification of the spins and parities of the χ levels from such decays as (1) and (2) is a well-known problem encountered and solved in nuclear physics.[4] Here we present a simplified analysis for the case where ψ' is produced in $e\bar{e}$ annihilation and ψ decays into a lepton pair. We address ourselves primarily to three questions: (i) determination of the spins (and parities) of χ, (ii) determination of the helicity couplings (or equivalently photon multipolarities) in the processes (1) and (2), and (iii) classification of the observed χ states according to various models.

Some work has been done already on these problems. In particular, the expected angular correlations between the two successive photons in (1) and (2) and the angular distributions of the photons with respect to initial or final leptons have been stated under the assumption that the amplitudes in both (1) and (2) are dominated by $E1$ transitions [if $J^P(\chi) = 0^+, 1^+, 2^+$] or $M1$ transitions [if $J^P(\chi) = 0^-, 1^-, 2^-$].[5,6] The full distributions in all relevant angles have also recently been derived under the assumption of $E1$ or $M1$ dominance.[7] Such (nonintegrated) distributions are useful in cases of limited experimental acceptance.

In this paper we wish to go beyond the assumption of $E1$ or $M1$ dominance. This is useful on general grounds, since one can then make model-independent statements about spins based on angular distributions in reactions (1) and (2). Moreover, our experience with radiative transitions of lower-lying hadrons[8-10] suggests that such transitions are usually governed by several multipoles of comparable strength. This is expected to be true for the decays (1) and (2) as well if simple vector-dominance arguments hold.[11]

For example, if $J^P(\chi) = 0^+, 1^+, 2^+$, we shall argue that there may be reason to expect $M2$ (as well as $E1$) transitions in the decays (1) and (2). Certain models[11] specify the exact mixture of these two multipolarities. If the χ levels are $q\bar{q}$, $L=1$ states, one can conclude on very general grounds[9] that the 2^+ level will not be excited in Eq. (1) or decay in Eq. (2) via $E3$ transitions. All of these ideas lead to testable relations among helicity amplitudes.

The paper is organized as follows. The general angular distribution for the sequential processes (1) and (2) is given in Sec. II as a function of decay helicity amplitudes, which are also expressed in terms of multipoles. By integrating over various angles, one then obtains particularly simple expressions for partial distributions which can be used by themselves to extract spins and helicity amplitudes. Some of these may be useful in experiments in which only a single photon of known energy is observed. This may be either of the two photons in (1) or (2). Using the cascade (1)–(2) it is well known[12] that one cannot determine the parity of χ without measuring photon polarization. However, the hadronic decay modes of the χ can be used for this purpose: For example, if $J(\chi)=0$ or 2 and $\chi\to\pi^+\pi^-$ or K^+K^-, the parity of χ is clearly positive. Consequently, we present in Sec. III angular distributions for the cascade process

$$\psi'\to\gamma\chi\to\gamma\,\pi\pi \text{ or } \gamma K\bar{K}. \qquad (3)$$

The discovery of the decays (1) and (2) has given some support to the view that the ψ', ψ, and χ states are all composites of one[13,14] or more[15] heavy quarks and their corresponding antiquarks. In most quark models the ψ and ψ' are naturally assigned to $^3S_1$ states of the $q\bar{q}$ system and thus one expects accompanying $^1S_0$ ($J^P=0^-$) states. Furthermore, one expects orbitally excited $q\bar{q}$, $L=1$ and higher states. We shall consider in Sec. IV the possibilities that the χ particles are $^1S_0$ states, $^3P_{0,1,2}$ states, or a combination of the two [The $^1P_1$ states do not couple to $\gamma\psi$ or $\gamma\psi'$; they have the wrong charge-conjugation parity. They could be formed, however, by $\psi'\to 2\pi+(^1P_1)$ if sufficiently light, or by $(^1S_0)'\to\gamma+(^1P_1)$.] The transitions (1) and (2) are then amenable to a discussion within the single-quark-transition framework of the Melosh transformation.[9,16] Such transitions are governed by several reduced matrix elements in general; however, under certain circumstances these can be estimated using symmetry arguments from the lighter-quark examples.

Based on the (model-dependent) considerations of Sec. IV, and on the scanty present data, one can attempt to classify the χ(3400–3550) levels. The results of this attempt are presented in Sec. V.

Section VI summarizes our conclusions. The Appendix deals with an alternative formulation more suitable than that of Sec. II for discussing certain angular correlations.

II. ANGULAR DISTRIBUTIONS

To a good approximation, in the decay $\psi'\to\gamma'\chi\to\gamma'\gamma\psi$, the rest frames of the ψ', χ, and ψ coincide.[17] We shall consider this to be the case. Then in this frame we define the following.

z axis: direction of γ'.

x axis: defined by γ'-γ plane.

$\theta_{\gamma\gamma}$: angle between γ' and γ (Fig. 1).

θ', ϕ': polar and azimuthal angles of initial lepton (e^+ for definiteness). (Note that both initial and final lepton pairs will be collinear.) These angles are referred to the frame of Fig. 1.

θ, ϕ: polar and azimuthal angles of final lepton relative to a rotated frame in which γ defines the z axis and γ', γ define the x-z plane. In this frame the azimuthal angle of the photon γ' is $\phi=\pi$.

The joint angular distributions depend in a simple way on the helicity amplitudes characterizing the decays

$$\psi'(\lambda')\to\gamma'(\mu')+\chi(\nu') \qquad (4)$$

and

$$\chi(\nu)\to\gamma(\mu)+\psi(\lambda), \qquad (5)$$

with the relations

$$\lambda'=\mu'-\nu' \qquad (6)$$

and

$$\nu=\mu-\lambda \qquad (7)$$

holding among the helicities. The (real) helicity amplitudes for processes (4) and (5) may be denoted, respectively, by $B_{\nu'\mu'}$ ("before" the χ) and $A_{\nu\mu}$ ("after" the χ). (The order of the labels is somewhat unconventional.)

The helicity amplitudes for photon polarization μ (or μ') $=+1$ may be related to ones with μ (or μ') $=-1$ by parity.[18] Consequently, one may define

$$B_{\nu'}\equiv B_{\nu'1}=P_\chi(-1)^{J(\chi)}B_{-\nu',-1}, \qquad (8)$$

$$A_\nu\equiv A_{\nu 1}=P_\chi(-1)^{J(\chi)}A_{-\nu',-1}. \qquad (9)$$

Note that in Eqs. (8) and (9) angular momentum conservation [see Eqs. (6) and (7)] implies that $\nu'\geq 0$, $\nu\geq 0$. For example, if $J(\chi)=2$, the decays $\psi'\to\gamma+\chi$ are characterized by the three independent amplitudes B_2, B_1, and B_0, while the decays $\chi\to\gamma+\psi$ are described by A_2, A_1, and A_0.

The initial and final lepton directions act to analyze the polarizations of the ψ' and ψ, with the ψ' density matrix for unpolarized leptons[19] equal to

$$\rho^{(\lambda'\bar{\lambda}')}(\theta',\phi')=\epsilon_i^{*(\lambda')}\epsilon_j^{(\bar{\lambda}')}L^{ij}(\theta',\phi'), \qquad (10)$$

$$L^{ij}(\theta',\phi')\equiv\delta^{ij}-n^in^j, \qquad (11)$$

$$\hat{n}=(\sin\theta'\cos\phi',\sin\theta'\sin\phi',\cos\theta'). \qquad (12)$$

Here $\epsilon_i^{(\lambda')}$ is the usual polarization vector: $\epsilon^{(1)}=(-1/\sqrt{2},-i/\sqrt{2},0)$; $\epsilon^{(0)}=(0,0,1)$; $\epsilon^{(-1)}=-\epsilon^{(1)*}$. The explicit values of $\rho^{(\lambda'\bar{\lambda}')}(\theta',\phi')$ are shown in Table I. Equations (10)–(12) are just the general-

FIG. 1. Coordinate system for describing the decays $\psi'\to\gamma'\chi\to\gamma'\gamma\psi$.

TABLE I. Density-matrix elements $\rho^{(\lambda\tilde{\lambda})}(\theta,\phi)$, defined with $\mathrm{Tr}\rho = 2$.

$$\rho^{(11)}(\theta,\phi)=\frac{1+\cos^2\theta}{2}, \quad \rho^{(00)}(\theta,\phi)=\sin^2\theta,$$

$$\rho^{(10)}(\theta,\phi)=\frac{\sin\theta\cos\theta}{\sqrt{2}}e^{-i\phi}, \quad \rho^{(1-1)}(\theta,\phi)=\frac{\sin^2\theta}{2}e^{-2i\phi},$$

$$\rho^{(\tilde{\lambda}\lambda)}=\rho^{(\lambda\tilde{\lambda})*}=(-)^{\lambda+\tilde{\lambda}}\rho^{(-\lambda,-\tilde{\lambda})}$$

$$\rho^{(\lambda\tilde{\lambda})}=\sum_{\kappa=\pm1}\mathfrak{D}^1_{\lambda\kappa}(\phi,\theta,-\phi)\,\mathfrak{D}^{1*}_{\tilde{\lambda}\kappa}(\phi,\theta,-\phi)$$

ization to arbitrary lepton direction (θ', ϕ') of the fact that when the leptons are along the z axis, the tensor L^{ij} is of the form $\hat{x}\hat{x}+\hat{y}\hat{y}$, corresponding to an incoherent sum of $\lambda'=1$ and $\lambda'=-1$ ψ' polarizations. It is convenient to choose the frame in Eq. (12) such that the first photon γ' lies along the z axis and the second photon γ in the x-z plane with positive x component (Fig. 1).

Similarly, for the final lepton pair, one can write

$$\rho^{(\lambda\tilde{\lambda})}(\theta,\phi)=\epsilon_i^{*(\lambda)}\epsilon_j^{(\tilde{\lambda})}L_{ij}(\theta,\phi), \qquad (13)$$

$$L^{ij}(\theta,\phi)\equiv\delta^{ij}-m^i m^j, \qquad (14)$$

$$\hat{m}=(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta). \qquad (15)$$

In Eq. (15) the frame chosen is that with the second photon γ along the z axis and the first photon γ' with negative x component in the x-z plane.

One may now sum over unobserved photon polarizations and χ helicities to write a general joint angular distribution:

$$W(\theta',\phi',\theta_{\gamma\gamma},\theta,\phi)\propto\sum_{\substack{\nu'\tilde{\nu}';\,\mu'=\pm1\\ \nu\tilde{\nu};\,\mu=\pm1}}\rho^{(\mu'-\nu',\,\mu'-\tilde{\nu}')}(\theta',\phi')B_{|\nu'|}B_{|\tilde{\nu}'|}d^J_{-\nu'\nu}(\theta_{\gamma\gamma})d^J_{-\tilde{\nu}'\tilde{\nu}}(\theta_{\gamma\gamma})A_{|\nu|}A_{|\tilde{\nu}|}\rho^{*(\nu-\mu,\,\tilde{\nu}-\mu)}(\theta,\phi). \qquad (16)$$

The helicity amplitudes $B_{|\nu'|}$ and $A_{|\nu|}$ are those defined in Eqs. (8) and (9). Note that all references to the parity of χ has vanished at this point. Only by measuring the polarization of one of the photons can one determine the parity of the intermediate state in a γ-γ cascade.[4] We shall not discuss such difficult measurements here, though they would certainly be of use if they actually could be performed.

The expression (16) is quite suitable for a computational analysis. The d^J functions for $J\le 2$ are quoted, for example, in Ref. 20, while the density matrices ρ are noted in Table I. A maximum-likelihood fit can be performed by letting the amplitudes $A_{|\nu|}$ and $B_{|\nu'|}$, $0\le|\nu|, |\nu'|\le J$, be real free parameters. These parameters obey certain constraints if, for example, the transitions are pure $E1$ or are governed by higher symmetries. Some of these constraints will be mentioned below or in Sec. III. The full angular distributions (16) (rather than partially integrated versions) may be needed since most of the detection apparatus in e^+e^- experiments has limited acceptance.

For $J(\chi)=0$ the only nonvanishing amplitudes are A_0 and B_0, leading to an angular distribution[5-7]

$$W(\theta',\phi',\theta_{\gamma\gamma},\theta,\phi)\propto(1+\cos^2\theta')(1+\cos^2\theta). \qquad (17)$$

The θ' and θ dependences correspond to the requirement that γ' and γ be emitted with transverse polarizations. The spin-0 intermediate state leads to an absence of $\gamma\gamma$ correlation. When $J^P(\chi)=0^+$ (0^-) the transitions in Eqs. (1) and (2) must be pure $E1$ (pure $M1$). Equation (17) does not distinguish between these two possibilities.

The helicity amplitudes $A_{|\nu|}$ (and $B_{|\nu'|}$) are in definite ratios to one another for transitions of definite multipolarity. For general $J\equiv J(\chi)$ we write[21]

$$A^{(J)}_{|\nu|}=\sum_{J_\gamma}a^{(J)}_{J_\gamma}\left(\frac{2J_\gamma+1}{2J+1}\right)^{1/2}(J_\gamma,1,1,|\nu|-1\,|\,J,|\nu|), \qquad (18)$$

$$B^{(J)}_{|\nu'|}=\sum_{J'_\gamma}b^{(J)}_{J'_\gamma}\left(\frac{2J'_\gamma+1}{2J+1}\right)^{1/2}(J'_\gamma,1,1,|\nu'|-1\,|\,J,|\nu'|), \qquad (19)$$

where $a^{(J)}_{J_\gamma}$ and $b^{(J)}_{J'_\gamma}$ are (arbitrarily normalized) multipole amplitudes for the transitions (2) and (1), respectively. Specific $J\le2$ cases are listed in Table II. For pure $J_\gamma=1$ ($E1$ or $M1$) transitions, for example,

$$A_1^{(J=1)}=A_0^{(J=1)}, \qquad (20)$$

$$A_2^{(J=2)}=\sqrt{2}A_1^{(J=2)}=\sqrt{6}A_0^{(J=2)}, \qquad (21)$$

with similar expressions for the B's. The normalizations in (18) and (19) are chosen so that the transformation between helicity amplitudes and multipoles is orthogonal. Then

$$\Gamma(\chi\to\gamma+\psi)=\frac{2}{2J+1}\sum_{\nu\ge0}|A_\nu|^2 =\frac{2}{2J+1}\sum_{J_\gamma}|a^{(J)}_{J_\gamma}|^2. \qquad (22)$$

If $P(\chi)=+$, $a_1^{(J)}$ corresponds to $E1$, $a_2^{(J)}$ to $M2$, $a_3^{(J)}$ to $E3$, and so on.

Equation (16) may be discussed *in principle* when

TABLE II. Decomposition of helicity amplitudes for $\chi \to \gamma + \psi$ in terms of multipoles [Eq. (18)]. A similar decomposition holds for Eq. (19).

$J=0$

$$A_0^{(J=0)} = a_1^{(J=0)}.$$

$J=1$

$$A_1^{(J=1)} = (2)^{-1/2} a_1^{(J=1)} - (2)^{-1/2} a_2^{(J=1)},$$
$$A_0^{(J=1)} = (2)^{-1/2} a_1^{(J=1)} + (2)^{-1/2} a_2^{(J=1)}.$$

$J=2$

$$A_2^{(J=2)} = (\tfrac{3}{5})^{1/2} a_1^{(J=2)} - (3)^{-1/2} a_2^{(J=2)} + (15)^{-1/2} a_3^{(J=2)},$$
$$A_1^{(J=2)} = (\tfrac{3}{10})^{1/2} a_1^{(J=2)} + (6)^{-1/2} a_2^{(J=2)} - (\tfrac{8}{15})^{1/2} a_3^{(J=2)},$$
$$A_0^{(J=2)} = (\tfrac{1}{10})^{1/2} a_1^{(J=2)} + (2)^{-1/2} a_2^{(J=2)} + (\tfrac{6}{15})^{1/2} a_3^{(J=2)}.$$

integrated over some of its angles (though, as mentioned, this may not be possible *in practice* because of limited acceptance).

A. Integration over ϕ and ϕ'

This leads, respectively, to diagonal $\rho^{(\lambda\tilde{\lambda})}$ and $\rho^{(\lambda'\tilde{\lambda}')}$ and hence to a loss of information about the phase of the helicity amplitudes with respect to one another. Let us define

$$\beta_{\nu'}(\theta') \equiv \sum_{\mu'=\pm 1} \langle \rho^{(\mu'-\nu',\mu'-\tilde{\nu}')}(\theta',\phi')\rangle_{\phi'} B_{|\nu'|} B_{|\tilde{\nu}'|}$$
$$= \sum_{\mu'} \rho^{(\mu'-\nu',\mu'-\nu')}(\theta')(B_{|\nu'|})^2, \qquad (23)$$

$$\alpha_\nu(\theta) \equiv \sum_{\mu=\pm 1} \rho^{(\nu-\mu,\,\nu-\mu)}(\theta)(A_{|\nu|})^2. \qquad (24)$$

These quantities give the populations of various χ helicity states as a function of the angle θ' or θ: Hence, the initial or final dilepton pair can analyze these populations. Then

$$\langle W(\theta',\phi',\theta_{\gamma\gamma},\theta,\phi)\rangle_{\phi'\phi} \propto \sum_{\nu'\nu} \beta_{\nu'}(\theta')[d^J_{-\nu'\nu}(\theta_{\gamma\gamma})]^2 \alpha_\nu(\theta), \qquad (25)$$

an expression which involves only squares of helicity amplitudes.

B. Integration over all angles except $\theta_{\gamma\gamma}$

Since $\langle \rho^{(\lambda\tilde{\lambda})}(\theta,\phi)\rangle_{\theta,\phi} \propto \delta_{\lambda\tilde{\lambda}}$, one has

$$W(\theta_{\gamma\gamma}) \equiv \langle W(\theta',\phi',\theta_{\gamma\gamma},\theta,\phi)\rangle_{\theta'\phi'\theta\phi}$$
$$\propto \sum_{\nu\geq 0,\,\nu'\leq 0} [B_{|\nu'|}]^2 [d^J_{-\nu'\nu}(\theta_{\gamma\gamma})]^2 [A_{|\nu|}]^2 + (\theta_{\gamma\gamma} \to \pi - \theta_{\gamma\gamma}). \qquad (26)$$

[Use is made of identities quoted in Ref. 20 and of the identity

$$d^J_{\nu'\nu}(\theta) = (-1)^{J+\nu'} d^J_{\nu',-\nu}(\pi - \theta) \qquad (27)$$

in deriving this rule.] Specific cases are listed in Table III.

Expressions (27), (20), and (21) may be used to derive the $\gamma\gamma$ correlations quoted in Ref. 6 for pure $E1$ transitions.

For spin J the $\gamma\gamma$ correlation is in general a polynomial in $\cos\theta_{\gamma\gamma}$ of degree $2J$. Suppose, however, for $J=2$, *either* transition is pure $E1$. Then, by virtue of Eq. (21), the $\cos^4\theta_{\gamma\gamma}$ term *vanishes*. This result is understood on general grounds.[22] If (say) the first transition is pure $E1$, then

$$W^{(J=2)}(\theta_{\gamma\gamma}) \propto 3(A_2)^2(1+\cos^2\theta) + \tfrac{3}{2}(A_1)^2(3-\cos^2\theta) + (A_0)^2(5-3\cos^2\theta) \qquad (28)$$

or

$$W^{(J=2)}(\theta_{\gamma\gamma}) \propto 1 + \lambda\cos^2\theta, \quad -\tfrac{3}{5} \leq \lambda \leq 1. \qquad (29)$$

Note that the $\cos^4\theta_{\gamma\gamma}$ term vanishes in other cases as well as the pure-$E1$ case; these other cases all involve combinations of all three multipoles $E1$, $M2$, and $E3$.

For data of sufficiently high accuracy, the observation of a $\gamma\gamma$ correlation that required terms up to $\cos^4\theta_{\gamma\gamma}$ thus would have *two* implications: the existence of a χ state with $J \geq 2$ and [if $J^P(\chi) = 2^+$] the presence of non-$E1$ transitions both in $\psi' \to \gamma\chi$ and in $\chi \to \gamma\psi$.

TABLE III. Angular correlations in terms of helicity amplitudes $A_{|\nu|}$, $B_{|\nu'|}$.

$J=1$

$$W(\theta_{\gamma\gamma}) \propto (A_1B_1)^2 + 2(A_1B_0)^2 + 2(A_0B_1)^2 + \cos^2\theta_{\gamma\gamma}(2A_0{}^2 - A_1{}^2)(2B_0{}^2 - B_1{}^2)$$

$J=2$

$$W(\theta_{\gamma\gamma}) \propto (A_2B_2)^2 + 4(A_2B_1)^2 + 4(A_1B_2)^2 + 6(A_2B_0)^2 + 6(A_0B_2)^2 + 4(A_1B_1)^2 + 4(A_0B_0)^2$$
$$+ 6\cos^2\theta_{\gamma\gamma}[(A_2{}^2 - 2A_0{}^2)(B_2{}^2 - 2B_0{}^2) - 2(A_1{}^2 - 2A_0{}^2)(B_1{}^2 - 2B_0{}^2)]$$
$$+ \cos^4\theta_{\gamma\gamma}(A_2{}^2 - 4A_1{}^2 + 6A_0{}^2)(B_2{}^2 - 4B_1{}^2 + 6B_0{}^2)$$

C. Integration over all angles except θ (or θ')

The $d^J_{\nu'\nu}(\theta)$ have the property that

$$\int_0^\pi \sin\theta\, d\theta [d^J_{\nu'\nu}(\theta)]^2 = \frac{2}{2J+1}. \qquad (30)$$

Applying this to Eq. (24), one finds

$$W(\theta) \equiv \langle W(\theta', \phi', \theta_{\gamma\gamma}, \theta, \phi)\rangle_{\theta', \phi', \theta_{\gamma\gamma}, \phi} \propto \left(\sum_{\nu'} (B_{|\nu'|})^2\right) \sum_\nu \alpha_\nu(\theta), \qquad (31)$$

i.e.,

$$(J=0) \quad W(\theta) \propto 1 + \cos^2\theta, \qquad (32)$$

$$(J=1) \quad W(\theta) \propto \frac{1+\cos^2\theta}{2}(A_0)^2 + \sin^2\theta\,(A_1)^2, \qquad (33)$$

$$(J=2) \quad W(\theta) \propto \frac{1+\cos^2\theta}{2}[(A_2)^2 + (A_0)^2] + \sin^2\theta\,(A_1)^2. \qquad (34)$$

Similar expressions hold with $\theta \to \theta'$ and $A_\nu \to B_{\nu'}$.

D. Tests for spin J

We have already mentioned tests for spin J based on the highest power of $\cos^2\theta_{\gamma\gamma}$; under certain circumstances this power can be less then J. Another test involves the azimuthal dependences in Eq. (16). Let us integrate over θ'. Then the only ϕ'-dependent terms that survive come from $\rho^{(1,-1)}(\theta', \phi')$ and $\rho^{(-1,1)}(\theta', \phi')$. These terms can arise only when $\nu' = 0$ and $\tilde{\nu} = 2$ or $\nu' = 2$ and $\tilde{\nu}' = 0$, i.e., when $J \geq 2$. One will then obtain a distribution of the form $1 + \lambda \cos(2\phi')$. The B_2 and B_0 amplitudes are required to interfere to give this effect. A similar test is possible using the ϕ distribution integrated over θ.

Another test for spin J relies on the distribution in angle θ_2 of the second γ with respect to the initial $e\bar{e}$ beam direction, irrespective of the direction of the first γ. This distribution may be easier to measure than the $\gamma\gamma$ correlation mentioned above. It will be a polynomial of maximum order J in $\cos^2\theta_2$. The formulation of Eq. (16) is not directly suited to determining this distribution; an alternative method is presented in the Appendix A. One simple conclusion can be reached using the Cartesian-tensor formulation of Ref. 7. This is that *for pure* $E1$ *transitions,*

$$W^{(J=1)}(\theta_2) \propto 5 + \cos^2\theta_2. \qquad (35)$$

$$W^{(J=2)}(\theta_2) \propto 73 + 21\cos^2\theta_2. \qquad (36)$$

These are the *same* distributions as in the angle $\theta_{\gamma\gamma}$ for the pure $E1$ case. It will be seen in the Appendix, however, that the $\theta_{\gamma\gamma}$ and θ_2 distributions do *not* coincide *in general* (i.e., for mixed multipolarities).

In practice, limitations of statistics and acceptance probably will require fits to the full Eq. (16) to determine the spins of the χ particles. The tests we have suggested above are more suitable for determining the minimum spins.

III. DECAYS $\psi' \to \gamma\chi \to \gamma 0^- 0^-$

It appears that at least one of the χ levels, $\chi(3410)$, decays to $\pi\pi$ and/or $K\bar{K}$. This means that it has natural spin-parity, $J^P = 0^+, 1^-, 2^+, 3^-, \ldots$. The odd-$J$ states, with $J^{PC} = 1^{-+}, 3^{-+}, \ldots$, are forbidden by C invariance to decay to two pseudoscalar particles,[23] so we shall concentrate on the sequence $J^{PC}(\chi(3410)) = 0^{++}, 2^{++}, \ldots$. In such cases the final pair of mesons analyzes the polarization of the χ. Let us replace the second photon (γ) in Fig. 1 by the outgoing π^+ or K^+; the π^- or K^- will be going in the opposite direction. The angle $\theta_{\gamma\gamma}$ is replaced by $\theta_{\gamma P}$, the angle between the photon and the π^+. Then Eq. (16) is replaced by

$$W(\theta', \phi', \theta_{\gamma P}) \propto \sum_{\nu', \tilde{\nu}'; \mu' = \pm 1} \rho^{(\mu'-\nu', \mu'-\tilde{\nu}')}(\theta', \phi') B_{|\nu'|} B_{|\tilde{\nu}'|} d^J_{-\nu' 0}(\theta_{\gamma P}) d^J_{-\tilde{\nu}' 0}(\theta_{\gamma P}). \qquad (37)$$

If one integrates over $\theta_{\gamma P}$, one obtains the same information as in the $\gamma\gamma$ cascade case described above from the distribution in θ' and ϕ'. If one integrates (37) instead over θ' and ϕ', one obtains

$$W(\theta_{\gamma P}) \equiv \langle W(\theta', \phi', \theta_{\gamma P})\rangle_{\theta' \phi'} \propto \sum_{\nu'} [B_{|\nu'|}]^2 [d^J_{-\nu' 0}(\theta_{\gamma P})]^2. \qquad (38)$$

This expression may be used to determine both the spin of χ and the population of the various helicity amplitudes. For example, for $J = 2$,

$$W^{(J=2)}(\theta_{\gamma P}) \propto \tfrac{3}{8}(B_2)^2 \sin^4\theta_{\gamma P} + \tfrac{3}{2}(B_1)^2 \sin^2\theta_{\gamma P}\cos^2\theta_{\gamma P} + \tfrac{1}{4}(B_0)^2[3\cos^2\theta_{\gamma P} - 1]^2. \qquad (39)$$

If the transition $\psi' \to \gamma\chi$ is pure $E1$, for example,[5,6] this expression becomes

$$W^{(J=2)}(\theta_{\gamma P}, \text{pure } E1) \propto 5 - 3\cos^2\theta_{\gamma P}. \qquad (40)$$

Note that for $J=0$ the distribution (38) is

$$W^{(J=0)}(\theta', \phi', \theta_{\gamma P}) \propto 1+\cos^2\theta', \quad (41)$$

in analogy with Eq. (17).

IV. MODELS

Up to now what has been said follows primarily from angular momentum and parity conservation. For the remainder of this paper we examine the implications of various models and suggest tentative identifications for the χ states.

Let us review the properties[3] of the $\chi(3400-3550)$ states that we use as input to the models.

The study of the reaction

$$e^+e^- \to \psi' \to \gamma+\chi \to \gamma + \text{hadrons} \quad (42)$$

was carried out at SPEAR.[3] In the hadronic final states, a narrow peak is seen in the $\pi\pi$ and/or $K\bar{K}$, 4π, 6π, and $K\bar{K}\pi\pi$ mass spectra around 3410 MeV. In addition, the above final states, other than $\pi\pi$ and $K\bar{K}$, show a broad (~100-MeV wide) peak centered around 3530 MeV, which may actually consist of two unresolved peaks at slightly lower and higher masses.[3] At any rate the spectra appear to have few events above 3550 MeV that can be separated from direct $\psi' \to$ (hadrons) decays.

The reaction

$$e^+e^- \to \psi' \to \gamma+\chi \to \gamma+\gamma+\psi \quad (43)$$

has been studied as well.[3] Two groups of monochromatic photons are seen, one with $E_\gamma \sim 160$ MeV and one with $E_\gamma \sim 420$ MeV. It is impossible to tell *a priori* which photon was emitted first. However, if the state χ in reaction (43) corresponds to any of those seen in (42) it must be $\chi(\sim 3530)$, and hence the first photon in (43) would have had the lower energy. (In principle, the second photon would be expected to be slightly Doppler broadened in comparison with the first.)

In view of the above data, we speak of the states

$\chi(3400-3550)$

without committing ourselves to their exact number.

From the above properties one can first make some statements which are either totally model-independent or nearly so.

(*a*) *The χ states have $G=+$.* They decay to an even number of pions.

(*b*) *The χ states have charge-conjugation parity $C=+$.* They are formed by $\psi' \to \gamma+\chi$; both the ψ' and the γ have $C=-$.

(*c*) *The $\chi(3410)$ has $I=0$.* It decays to $K\bar{K}$ so $I \le 1$; it decays to $\pi\pi$ and $C(\pi\pi)=(-1)^I=+$, hence $I=0$.[23] The other χ states also probably have $I=0$ too, since they have even G and C and hence even I, and since the photon in $\psi' \to \gamma+\chi$ probably carries $|\Delta I| \le 1$ as in all other electromagnetic processes.

(*d*) *The $\chi(3410)$ has $J^{PC}=0^{++}, 2^{++}, 4^{++}, \ldots$.* It has natural spin-parity since it decays to two pseudoscalar mesons; since it decays to $\pi^+\pi^-$ and K^+K^-, and has even C, it cannot have odd J since $C(\pi^+\pi^-) = C(K^+K^-) = (-1)^J$.[23]

The types of models we shall consider here are based on the hypotheses of one[13,14] or more[15] species of heavy quarks. The ψ and ψ' are assumed to be neutral $^3S_1$ $q\bar{q}$, $L=0$ pairs in such models. Since the χ levels have $C=(-1)^{L+S}=+$, where L is the assumed $q\bar{q}$ relative angular momentum and S is the total spin, there are two low-lying possibilities for such states: either $L=1$ and $S=1$ (^3P_J levels, $J^{PC}=0^{++}$, 1^{++}, and 2^{++}) or $L=0$ and $S=0$ ($^1S_0$ levels, $J^{PC}=0^{-+}$). *At least the $\chi(3410)$ must then be a $q\bar{q}$, $L=1$ level* since it has natural spin-parity; its J^{PC} could be 0^{++} or 2^{++}. (In fact, we shall argue that 0^{++} is more likely.) The $\chi(3530)$ level or levels could be either $^1S_0$, ^3P_J, or a mixture of the two. At most one $^1S_0$ level around this mass is expected in models with one heavy quark[13,14] this level would be the hyperfine partner of the $^3S_1$ ψ'.

[Another $^1S_0$ level would be expected below the ψ' and could be the $X(2800)$ seen at DESY. (See Wiik, Ref. 3.)] In models with several heavy quarks,[15] it would be possible to have a pair of $^1S_0$ levels close together in mass; however, this does not seem to be the case with the observed $^3S_1$ levels. *If the $\chi(3530)$ is, in fact, composed of at least two levels, we suspect that at least one of these levels must be another ^3P_J $q\bar{q}$, $L=1$ state.* This suspicion is also supported by the relatively small mass splitting between the $\chi(3530)$ and the $\chi(3410)$: In general, $\vec{L}\cdot\vec{S}$ splittings both in the mesons and baryons seem to be relatively small.

The $q\bar{q}$, $L=1$ levels also include a $^1P_1$ state with $J^{PC}=1^{+-}$. This state can be formed by $\psi' \to (2\pi) + \chi(^1P_1)$ if it lies low enough in mass (below ~3400 MeV), and by $(^1S_0)' \to \gamma + \chi(^1P_1)$ if a $(^1S_0)$ level (the hyperfine partner of ψ') lies above it. Similarly, a $^1P_1$ level can decay to $\pi\pi\psi$ or to $\gamma+(^1S_0)$.

In what follows we shall make use of the single-quark–transition picture based on the Melosh transformation[16] and developed by Gilman and Karliner[9] and Hey and Weyers.[24] There is one simple test of the single-quark–transition picture that does not depend on whether the χ levels are composed of a single $q\bar{q}$ pair or mixtures of pairs of various quarks. If the χ is a $^3P_2$ state, i.e., if $J^P(\chi)=2^+$, and if ψ' and ψ are indeed $^3S_1$ levels, one predicts[9] that *the transitions $\psi' \to \gamma\chi$ and $\chi \to \gamma\psi$ contain no $E3$ contribution.* Indeed, since in such transitions $|\Delta L|=1$, and since single-quark transitions involve $|\Delta S| \le 1$, the photon can carry off at most two units of angular momentum. The absence of $E3$ transitions entails the relations[9]

$$A_1 = \frac{A_2}{2\sqrt{2}} + \frac{\sqrt{3}}{2} A_0, \tag{44}$$

$$B_1 = \frac{B_2}{2\sqrt{2}} + \frac{\sqrt{3}}{2} B_0, \tag{45}$$

among the helicity amplitudes describing, respectively, the decays $\chi(^3P_2) \to \gamma\psi$ and $\psi' \to \gamma\chi(^3P_2)$.

Most previous treatments of the decays $\psi' \to \gamma\chi$ and $\chi \to \gamma\psi$ have assumed $J_\gamma = 1$.[5-7] This corresponds to $M1$ transitions (the only ones possible) for $J^P(\chi) = 0^-$ and $E1$ transitions for $J^P(\chi) = 0^+, 1^+, 2^+$. There is no inconsistency *per se* with the Melosh transformation in such an assumption. Indeed, if heavy-quark magnetic moments are small, naive nonrelativistic quark models[5] argue that $E1$ transitions with no quark spin-flip will be the *only* important ones in $\psi' \to \gamma\chi(^3P_J) \to \gamma\gamma\psi$. This point of view is supported by the absence in current data of large $M1$ transitions: $\psi \to {}^1S_0 + \gamma$, $\psi' \to ({}^1S_0)' + \gamma$, $\psi' \to ({}^1S_0) + \gamma$, etc. indicating that such magnetic terms may in fact be small. However, they are possible in the single-quark-transition approach, and we shall argue that it may be premature to neglect them. Consequently, we present for comparison both the usual $E1$-dominance case and a special case in which $M2$ transitions play an important role.

The single-quark–transition theory[9,16,24] describes all transitions of the form

$$(q\bar{q},\ L=1) \to \gamma + (q\bar{q},\ L=0) \tag{46}$$

or

$$(q\bar{q},\ L=0) \to \gamma + (q\bar{q},\ L=1) \tag{47}$$

in terms of three reduced matrix elements. One of these changes L_z by a unit and has $\Delta W = 0$ ($W = W$ spin[25]); it is a pure electric dipole transition ($E1'$) and corresponds to convective motion of the quarks. In the language of Refs. 9 and 16, it transforms as $(8,1)+(1,8)$ under chiral SU(3)×SU(3). *This term is the only one present in the models of Refs.* 5 *and* 6. It is the only term present in the absence of the Melosh transformation.

In the single-quark–transition picture there are also two $\Delta W = 1$ contributions to the processes (46) and (47). One of these has $\Delta L_z = 0$ and transforms as $(3,\bar{3})$ or $(\bar{3},3)$; the other has $|\Delta L_z| = 1$ and transforms as $(8,1)-(1,8)$. These combine to give another electric dipole contribution $E1$ and a magnetic quadrupole contribution $M2$.[9] We shall argue that both of these additional contributions are important and must be included in describing the radiative decays (2) [and possibly also (1)]. Analyzing the decays (2) we shall find that these three reduced matrix elements are in a definite ratio to one another. In certain cases this will be true of the decays (1) as well.

There is one experimental situation in which $L=1 \to L=0$ radiative transitions can check the dominance of $E1'$, and this is in the decays of the $\underline{70}$, $L=1$ baryons. It is found [8-10] in this case that $M2$ contributions are quite significant as well. Similarly, the phototransitions ($\underline{56}$, $L=2$ ($\underline{56}$, $L=2) \to \gamma + (\underline{56}$, $L=0$) involve substantial $M3$ contributions in addition to $E2'$.

The transitions (46) and (47) have already been decomposed in terms of reduced matrix elements labeled by $(\Delta W, \Delta W_z)$ in the specific case where all the ^3P_J states have the same quark content.[11] This quark content need not even be the same as that of the ψ; only the over-all coefficient of proportionality will change. An equivalent decomposition for (46) in terms of multipole amplitudes is given in Table IV; a similar decomposition holds for (47).

Let us briefly review the theoretical arguments[11] for the presence of contributions in addition to $E1'$ in $^3P_J \to \gamma + \psi$. First, let the amplitude for $^3P_1 \to \gamma + \gamma$ be dominated by the ψ pole in one of the γ's corresponding to $^3P_1 \to \gamma + \psi$. By Yang's theorem,[26] a spin-1 particle cannot couple to two photons, so that the $\nu = 0$ amplitude in $^3P_1 \to \gamma + \psi$ must vanish. Then, by Table IV,

$$2E1' + E1 - M2 = 0. \tag{48}$$

TABLE IV. Matrix elements of the dipole operator D_+^{em} for $(^3P_J) \to \gamma_{\mu=1} + \psi$ in terms of multipole reduced matrix elements.[a] [A similar decomposition holds for $\psi' \to \gamma + (^3P_J)$.] The over-all scale is arbitrary.

State	ν	$E1'$	$E1$	$M2$
$^3P_2$	2	$\sqrt{6}$	$-\sqrt{6}/2$	$\sqrt{6}/2$
	1	$\sqrt{3}$	$-\sqrt{3}/2$	$-\sqrt{3}/2$
	0	1	$-1/2$	$-3/2$
$^3P_1$	1	$\sqrt{3}$	$\sqrt{3}/2$	$\sqrt{3}/2$
	0	$\sqrt{3}$	$\sqrt{3}/2$	$-\sqrt{3}/2$
$^3P_0$	0	$\sqrt{2}$	$\sqrt{2}$	0

[a] Relation to reduced matrix elements $D\,(\Delta W, \Delta W_z)$ of Ref. 11:

$$E1' \equiv D(0,0),$$
$$E1 \equiv D(1,1) - \tfrac{2}{3}\sqrt{2}\, D(1,0),$$
$$M2 \equiv D(1,1) + \tfrac{2}{3}\sqrt{2}\, D(1,0).$$

In the notation of Gilman and Karliner in Table III of the second of Refs. 9, $D(0,0) \propto (b)$, $D(1,1) \propto (c)$, $D(1,0) \propto (d)$.

A similar argument can be made at the level of $D(J^{PC}=1^{++})\to\gamma+\rho$ and extrapolated using U(4) or a higher symmetry to the heavy-quark sector. In either case the extrapolation is an extreme one. If it is at all valid, however, $E1'$ cannot be the only significant reduced matrix element. One might also expect $M2$ transitions to be important in analogy with the baryonic case mentioned above.

A second relation in Ref. 11 comes from the identification of the ΔL_z in electromagnetic transitions with that in pionic transitions via vector dominance. It has been shown that in pionic transitions of $\underline{35}$, $L=1$ mesons to the ground state, $\Delta L_z=1$ transitions dominate.[27] If vector dominance holds, one can show a similar result for electromagnetic transitions of $\underline{35}$, $L=1$ mesons to $\gamma+\pi$. If the appropriate higher symmetry is valid, this also holds for $\chi(^3P_J)\to\gamma+\psi$, and corresponds to the vanishing of $D(1,1)$ in Table IV, or

$$E1+M2=0. \tag{49}$$

Taken together, Eqs. (48) and (49) imply that the decay amplitudes A_ν for $^3P_J\to\gamma+\chi$ are in the ratios

$$\begin{aligned} J=2,\ \nu=2&:\ 2\sqrt{6},\\ \nu=1&:\ \sqrt{3},\\ \nu=0&:\ 0,\\ J=1,\ \nu=1&:\ \sqrt{3},\\ \nu=0&:\ 0,\\ J=0,\ \nu=0&:\ 0. \end{aligned} \tag{50}$$

The predictions of Eq. (50) are compared with those of strict $E1'$ dominance in Table V. If the ψ' is a radial excitation of the ψ, the only reduced matrix elements whose relative values can be predicted [i.e., to which Eqs. (48) and (49) should apply] are those for $\chi\to\gamma+\psi$; those for $\psi'\to\gamma+\chi$ cannot be related to known ones even using vector dominance. On the other hand, it is possible that the ψ' is the lowest $^3S_1$ level of a mixture of quarks orthogonal to that in the ψ if there is more than one species of heavy quark.[15] In this case the reduced matrix elements for $\psi'\to\gamma+\chi$ should also have the relative values predicted by Eqs. (48) and (49) *if all the χ levels are composed of the same $q\bar{q}$ mixtures*. Hence, we present two cases, one in which Eq. (26) applies only to $\chi\to\gamma+\psi$ and the second in which it applies to both transitions. (The estimates of Ref. 11 are based on the use of actual $\underline{35}$, $L=1$ pionic decays to evaluate reduced matrix elements, and, hence, differ somewhat from those given in Table V.)

TABLE V. Comparison of model involving strict $E1'$ dominance with model based on vector dominance (Ref. 11) for transitions $\psi'\to\gamma+\chi$ and $\chi\to\gamma+\psi$.

Strict $E1'$ dominance (Refs. 5–7)

$$\tilde\Gamma(2^+\to\gamma\psi)=\tilde\Gamma(1^+\to\gamma\psi)=\tilde\Gamma(0^+\to\gamma\psi),$$
$$\tfrac{1}{5}\tilde\Gamma(\psi'\to\gamma2^+)=\tfrac{1}{3}\tilde\Gamma(\psi'\to\gamma1^+)=\tilde\Gamma(\psi'\to\gamma0^+),$$
$$W(\theta_{\gamma\gamma},1^+)\propto 5+\cos^2\theta_{\gamma\gamma},$$
$$W(\theta_{\gamma\gamma},2^+)\propto 73+21\cos^2\theta_{\gamma\gamma},$$
$$W(\theta,1^+)\propto 3-\cos^2\theta=2+\sin^2\theta,$$
$$W(\theta,2^+)\propto 1+\tfrac{1}{13}\cos^2\theta,$$
$$W(\theta)=W(\theta').$$

Eq. (50) for $\chi\to\gamma+\psi$

$$\tilde\Gamma(2^+\to\gamma\psi)=\tfrac{27}{5}\tilde\Gamma(1^+\to\gamma\psi),\quad \tilde\Gamma(0^+\to\gamma\psi)=0,$$
$$W(\theta_{\gamma\gamma},1^+)=1+r\cos^2\theta_{\gamma\gamma},\quad r\equiv\frac{1-2B_0^2/B_1^2}{1+2B_0^2/B_1^2},$$
$$W(\theta,1^+)\propto 1-\cos^2\theta=\sin^2\theta,$$
$$W(\theta,2^+)\propto 1+\tfrac{3}{5}\cos^2\theta.$$

Eq. (50) for both $\psi'\to\gamma+\chi$ and $\chi\to\gamma+\psi$

In addition to above,

$$\tilde\Gamma(\psi'\to\gamma2^+)=9\tilde\Gamma(\psi'\to\gamma1^+),\quad \tilde\Gamma(\psi'\to\gamma0^+)=0,$$
$$W(\theta)=W(\theta'),$$
$$W(\theta_{\gamma\gamma},1^+)\propto 1+\cos^2\theta_{\gamma\gamma},$$
$$W(\theta_{\gamma\gamma},2^+)\propto 4\cos^4\theta_{\gamma\gamma}+93\cos^2\theta_{\gamma\gamma}+33.$$

For 0^+ angular distributions, see Eq. (17). Here $\tilde\Gamma\equiv\Gamma/$(phase space).

The suppression of the $^3P_0\to\gamma+\psi$ decay in Eq. (50) is notable. It comes from the fact that in the single-quark-transition theory there are *two* electric dipole contributions to $^3P_0$ electromagnetic decay, one ($E1'$) coming from the "convective" current and another ($E1$) from the magnetic moments of the quarks. The relations (48) and (49) imply that these two contributions exactly cancel one another. This goes against the grain of the naive nonrelativistic models mentioned earlier, which would predict $|E1'|\gg|E1|$. Fortunately, tests of these ideas will be possible. In the case of pure $E1'$ transitions, if phase-space factors are neglected, one finds

$$\Gamma(\psi'\to\gamma+{}^3P_2):\Gamma(\psi'\to\gamma+{}^3P_1):\Gamma(\psi'\to\gamma+{}^3P_0)=5:3:1\ \ (\text{pure }E1'), \tag{51}$$

and

$$\Gamma({}^3P_2\to\gamma+\psi)=\Gamma({}^3P_1\to\gamma+\psi)=\Gamma({}^3P_0\to\gamma+\psi)\ \ (\text{pure }E1'). \tag{52}$$

If, on the other hand, the relations (48)–(50) hold, the $^3P_2$ state is much more highly favored both in production and decay than would follow from Eqs. (51) and (52). This can be seen in Table V. For values of the reduced matrix elements based on *actual fits* to $\underline{35}$, $L=1$ meson decays, one obtains instead of the values in Table V the predictions

$$\tilde{\Gamma}(^3P_2\to\gamma+\psi):\tilde{\Gamma}(^3P_1\to\gamma_1\psi):\tilde{\Gamma}(^3P_0\to\gamma+\psi) \simeq (35 \text{ to } 40):3:1 \quad \text{(model of Ref. 11)} \tag{53}$$

where $\tilde{\Gamma}\equiv\Gamma/(\text{phase space})$. If the same ratios of reduced matrix elements held for the processes (1), one would obtain similar predictions for $\psi'\to\gamma+{}^3P_J$, but multiplied by the statistical weight $2J+1$. Such extreme ratios probably can be ruled out already by the observed signals in $\chi(3410)\to$hadrons.[3,23]

According to Table V, one of the best spin and multipolarity tests occurs for the $^3P_1$ state with $J^{PC}=1^{++}$. For this state, $W(\theta)$ is expected to be peaked around $\theta=90°$ (rather than at 0° and 180° as for $J=0$ or $J=2$). Moreover, the degree of peaking is fairly sensitive to admixtures of $M2$ transitions.

V. ATTEMPTS AT CLASSIFICATION

The preceding discussion allows us to guess at certain properties of the observed χ levels and to predict where remaining states may lie. The results of this exercise are summarized in Fig. 2. The specific points of interest in Fig. 2 are the following:

1. The $\chi(3410)$ has $J^{PC}=0^{++}$. This prediction is based on several circumstances. The primary reason for this assignment is the *absence or weakness of the transition $\chi(3410)\to\gamma+\psi$,* in comparison with the observed $\chi(3530)\to\gamma+\psi$ rate. Otherwise, on the basis of the observed 0^-0^- mode of the $\chi(3410)$,[3] one would have been unable to decide between the 0^{++} and 2^{++} alternatives mentioned in Sec. III.

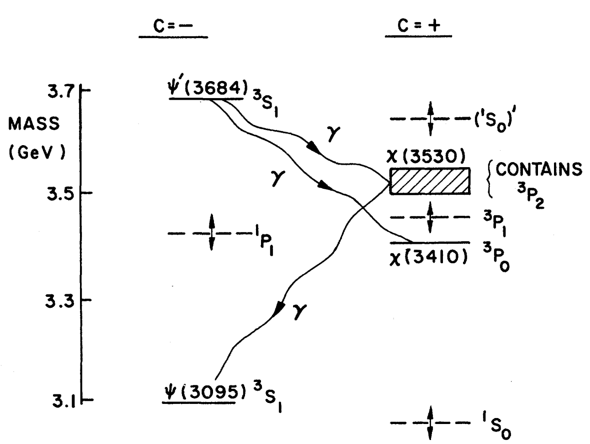

FIG. 2. Possible classification of observed χ levels and prediction of remaining states. Not shown is the $X(2.8)$ mentioned by Wiik in Ref. 3, produced in $\psi\to\gamma+X(2.8)$ and a possible candidate for the lowest $^1S_0$ level. It is also possible (see Wiik, Ref. 3) that $\chi(3410)\to\gamma+\psi$.

This argument assumes, of course, that the $\chi(3410)$ and $\chi(3530)$ are produced roughly equally in $\psi'\to\gamma+\chi$[28]; they do seem to contribute roughly equally to the four-pion and six-pion signals.[3] The absence or weakness of the decay $\chi(3410)\to\gamma+\psi$ is suggested by data taken at SPEAR: See Feldman, Ref. 3. Essentially no events of this type are found as compared with at least 50 events consistent with $\chi(3510)\to\gamma+\psi$. [By contrast, the DASP group (see Wiik, Ref. 3) sees a signal of two events in the former process and six in the latter.] A simpler argument that the $\chi(3410)$ should have $J^{PC}=0^{++}$ is that it is consistent with being the lowest-lying χ object, if one puts hadronic and electromagnetic decay information together. (In the "light" $q\bar{q}$, $L=1$ hadrons, the 0^{++} states also lie lowest.)

The 2^{++} state, wherever it lies, should also have an observed 0^-0^- decay mode. Why is this not observed? A possible answer lies once more within the realm of the Melosh transformation.[27] We have already mentioned that the $\underline{35}$, $L=1$ meson transitions to pions and $\underline{35}$, $L=0$ mesons are dominated by a $(3,\overline{3})$, $\Delta L_z=1$ term. Let us assume decays of $\chi(0^{++},2^{++})$ proceed by mixing of these states with small amounts of $\underline{35}$, $L=1$ (ordinary light-quark) states followed by decays of these states obeying the usual Okubo-Iizuka-Zweig[29] selection rule. Whatever the light-quark compositions of the χ states, let them be the *same* for the 0^{++} and 2^{++} states. Then in the limit of $(3,\overline{3})$, $\Delta L_z=1$ dominance, one finds

$$\frac{\tilde{\Gamma}(0^{++}\to\pi\pi)}{\tilde{\Gamma}(2^{++}\to\pi\pi)}=\frac{\tilde{\Gamma}(0^{++}\to K\overline{K})}{\tilde{\Gamma}(2^{++}\to K\overline{K})}=10\,. \tag{54}$$

Since only a total of 11 events ($\pi^+\pi^-$ and K^+K^-) are observed at SPEAR in the $\chi(3410)$ decay,[3] the failure to observe another peak in the 0^+0^- spectrum is not evidence against a 2^{++} state at present levels of statistics. In fact, recent data from DESY (Wiik, Ref. 3) contain one candidate each for $\chi(3410)\to0^-0^-$ and $\chi(3510)\to0^-0^-$.

2. The $\chi(3530)$ band probably contains a state with $J^{PC}=2^{++}$. It is only this band at present which is responsible for the cascade process $\psi'\to\gamma\chi\to\gamma\gamma\psi$. The (model-dependent) arguments of Sec. IV indicate that it is the $^3P_2$ state whose decay to $\gamma\psi$ should be most prominent.

3. A $J^{PC}=1^{++}$ state should exist in the range 3.4–3.55 GeV. This state is expected as a $^3P_1$ $q\bar{q}$ state if the $^3P_0$ and $^3P_2$ states are identified as above. It has been argued[30] that the hadronic decay of this state should be suppressed. This occurs since a color singlet with spin 1 cannot decay to two massless colored gluons. Depending on the hadronic branching ratios of the $^3P_2$ and $^3P_0$ states, this argument could lead one to expect the $^3P_1$ rather than the $^3P_2$ to dominate the cascade process. [Note the unfavorable ratio, however, for $^3P_1 \to \gamma + \psi$, quoted in Eq. (53).] Naively one might expect the $^3P_1$ to lie between the $^3P_0$ and the $^3P_2$ (if the mass formula has the usual $\vec{L}\cdot\vec{S}$ splitting).[31]

4. *There should exist $^1S_0$ and $^1P_1$ states as shown in Fig. 2.* One puzzle in the present approach is the smallness of the observed $\psi' \to \gamma\chi$ widths.[3,32] A number of schemes[5,14] have predicted these widths to be an order of magnitude greater. While the present symmetry approach makes no prediction for the absolute magnitude of these widths, one nonetheless feels uneasy that they have turned out to be so small. One possibility, suggested for example in Refs. 30 and 33, is that the spatial overlap of the ψ' with $\gamma\chi$ simply is small for some dynamical reason or other.[34] Another possibility, valid only in models with more than one heavy quark, is that ψ' contains quarks other than those in the χ states or in ψ. Hence, the χ states would be orbital excitations of the ψ, while the ψ' would have its own family of orbital excitations χ' lying above the ψ'. A small overlap between the quarks in ψ' and those in χ or ψ would then give the small decay rates for $\psi' \to \gamma\chi$, while the transitions $\chi \to \gamma\psi$ would proceed with "normal" rates.

VI. SUMMARY

We have reviewed methods for determining the spins and parities of the states χ observed in $\psi' \to \gamma + \chi$, $\chi \to$ (hadrons or $\gamma + \psi$).[35] These methods apply to cases in which the photon does not necessarily have the lowest possible multipolarity, and we have presented possible reasons for the existence of such mixed-multipolarity transitions. On the basis of specific estimates for the relative strengths of such transitions (and for certain hadronic χ decays), we have attempted a preliminary quark-model classification of the observed χ levels. We conclude that the most plausible quark-model assignment for $\chi(3410)$ is $^3P_0$ and that within the $\chi(3530)$ peak there is a $^3P_2$ state.

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APPENDIX: ANGULAR DISTRIBUTION OF THE SECOND PHOTON

In this Appendix we discuss in more detail the angular distribution of the second photon with respect to the electron-positron beam direction. This distribution allows the determination of the spin J of the intermediate χ particle. The advantage of this method over the photon-photon correlation measurements lies in the requirement that only the second photon has to be detected. By not observing the first photon one is integrating over its angles θ_1, ϕ_1. We discuss in detail the simpler case $J(\chi)=1$ and only quote the results for $J(\chi)=2$, where the corresponding formulas are more tedious.

We use as Z axis the incident positron direction, and an arbitrarily chosen X axis which is redundant. With respect to the Z axis the ψ' particle is produced in the incoherent superposition of $J_Z = \pm 1$ states. The two states have identical decay distributions since parity is conserved. We shall therefore consider only the state $J_Z = +1$ in what follows.

After the decay of the ψ' particle with the emission of a right-handed (R) photon at θ_1, ϕ_1, the χ particle is left in a coherent superposition of its states (quantized with respect to the Z axis):

$$|\chi\rangle = R_1|11\rangle + R_0|10\rangle + R_{\bar{1}}|1\bar{1}\rangle = \sum_m R_m|1m\rangle\,, \tag{A1}$$

where the coefficients R_m are functions of the angles θ_1, ϕ_1 and of the helicity amplitudes B_0, B_1 (we use the notation $-1 \equiv \bar{1}$):

$$R_m = B_0 \mathfrak{D}^{*1}_{11}\mathfrak{D}^1_{m0} + B_1 \mathfrak{D}^{*1}_{10}\mathfrak{D}^1_{m\bar{1}}\,. \tag{A2}$$

The arguments of all $\mathfrak{D}$ functions are the angles $(\phi_1, \theta_1, -\phi_1)$ or $(\phi_1, \theta_1, 0)$.

Similarly after the emission of a left-handed (L) photon the coefficients L_m describing the state of the χ particle are

$$L_m = \overline{B}_0 \mathfrak{D}^{*1}_{1\bar{1}}\mathfrak{D}^1_{m0} + \overline{B}_1 \mathfrak{D}^{*1}_{10}\mathfrak{D}^1_{m1}\,, \tag{A3}$$

where the helicity amplitudes $\overline{B}_m$ refer to left-handed photon emission.

Using the coefficients R_m, L_m we can compute the total rate $W(\theta_2)$ for observing a second photon at θ_2 independently of its polarization and of the polarization and angles of the first photon:

$$W(\theta_2) = |A_1|^2 \left\langle \left| \sum_m R_m \mathfrak{D}^{1*}_{m1} \right|^2 + \left| \sum_m L_m \mathfrak{D}^{1*}_{m1} \right|^2 + \left| \sum_m R_m \mathfrak{D}^{1*}_{m\bar{1}} \right|^2 + \left| \sum_m L_m \mathfrak{D}^{1*}_{m\bar{1}} \right|^2 \right\rangle_{\theta_1,\phi_1}$$
$$+ 2|A_0|^2 \left\langle \left| \sum_m R_m \mathfrak{D}^{1*}_{m0} \right|^2 + \left| \sum_m L_m \mathfrak{D}^{1*}_{m0} \right|^2 \right\rangle_{\theta_1,\phi_1} \quad \text{(A4)}$$

where we have suppressed the dependence of $\mathfrak{D}^1_{mm'}(\phi_2, \theta_2, -\phi_2)$ on its variables. In Eq. (A4) the angular brackets represent integration over all angles ϕ_1, θ_1 of the first photon.

In evaluating Eq. (A4) one has to perform angular averages of expressions of the type $\langle R_m^* R_n + L_m^* L_n \rangle$. These vanish unless $m = n$ by integration over ϕ_1. Taking account of this, one may write Eq. (A4) as

$$W(\theta_2) = \langle R_1^* R_1 + L_1^* L_1 + R_{\bar{1}}^* R_{\bar{1}} + L_{\bar{1}}^* L_{\bar{1}} \rangle [\,|A_1|^2 (|d^1_{11}|^2 + |d^1_{1\bar{1}}|^2) + 2|A_0|^2 |d^1_{10}|^2]$$
$$+ \langle R_0^* R_0 + L_0^* L_0 \rangle (2|A_1|^2 |d^1_{01}|^2 + 2|A_0|^2 |d^1_{00}|^2)\,. \quad \text{(A5)}$$

Replacing the rotation functions $d^1_{mm'}$ by their explicit representations, we obtain for the angular distribution of the second photon $W(\theta_2)$ of a spin-1 object χ

$$W^{(J=1)}(\theta_2) = C_1[\tfrac{1}{2}|A_1|^2(1+\cos^2\theta_2) + |A_0|^2(1-\cos^2\theta_2)] + C_0[\,|A_1|^2(1-\cos^2\theta_2) + 2|A_0|^2\cos^2\theta_2]\,. \quad \text{(A6)}$$

Equation (A6) shows that the distribution $W(\theta_2)$ is typically quadratic in $\cos\theta_2$ for a spin-1 intermediate state χ. It is interesting that the terms in $\cos^2\theta_2$ are all multiplied by the same bilinear combination $(2|A_0|^2 - |A_1|^2)$. This same combination multiplies the term in $\cos^2\theta_{\gamma\gamma}$, as may be seen in Table III. Thus, if $2|A_0|^2 = |A_1|^2$ the distribution in θ_2 will be isotropic, as will be the distribution in $\theta_{\gamma\gamma}$ even though the spin of χ is unity.

The coefficients C_i in (A6) may be obtained by explicit angular integration using the expressions (A2) and (A3). We find

$$C_1 \equiv \langle R_1^* R_1 + L_1^* L_1 + R_{\bar{1}}^* R_{\bar{1}} + L_{\bar{1}}^* L_{\bar{1}} \rangle = \tfrac{4}{15}(3B_0^2 + 3B_1^2 - B_1B_0)\,,$$
$$C_0 \equiv \langle R_0^* R_0 + L_0^* L_0 \rangle = \tfrac{4}{15}(2B_0^2 + 2B_1^2 + B_1B_0)\,. \quad \text{(A7)}$$

As seen from these equations $W(\theta_2)$ depends also on interference terms between the helicity amplitudes B_1, B_0. It should also be noted that in the $E1$-$E1$ case when $B_0 = B_1$ and $A_0 = A_1$ one obtains an angular distribution in θ_2 of the same form as the corresponding $\theta_{\gamma\gamma}$ distribution (see Table III or V):

$E1$-$E1$ case for $J = 1$,

$$W(\theta_2) \propto 5 + \cos^2\theta_2\,. \quad \text{(A8)}$$

The same computations can be carried through for the case $J = 2$. The analog of Eq. (A4) has an extra set of terms proportional to the helicity amplitude $|A_2|^2$, in which the first index of the $\mathfrak{D}$ functions is ± 2. The analog of Eq. (A6) reads

$$W(\theta_2) = C_2[\tfrac{1}{8}|A_2|^2(1 + 6\cos^2\theta_2 + \cos^4\theta_2) + \tfrac{1}{2}|A_1|^2(1-\cos^4\theta_2) + \tfrac{3}{4}|A_0|^2\sin^4\theta_2]$$
$$+ C_1[\tfrac{1}{2}|A_2|^2(1-\cos^4\theta_2) + \tfrac{1}{2}|A_1|^2(1 - 3\cos^2\theta_2 + 4\cos^4\theta_2) + 3|A_0|^2\sin^2\theta_2\cos^2\theta_2]$$
$$+ C_0[\tfrac{3}{4}|A_2|^2\sin^4\theta_2 + 3|A_1|^2\sin^2\theta_2\cos^2\theta_2 + \tfrac{1}{2}|A_0|^2(3\cos^2\theta - 1)^2]\,, \quad \text{(A9)}$$

where the coefficients C_i are bilinear forms in B_1, B_0, B_2 obtained by angular integration over θ_1, ϕ_1 of forms of type $(R_i^* R_i + L_i^* L_i)$. These are given below. Equation (A9) shows that typically the decay distribution of a $J = 2$ particle has terms up to $\cos^4\theta_2$. It is interesting that in all terms of (A9) the $\cos^4\theta_2$ has as coefficient the bilinear form $(|A_2|^2 - 4|A_1|^2 + 6|A_0|^2)$ which is the same as the A coefficient of the $\cos^4\theta_{\gamma\gamma}$ distribution given in Table III. Thus if $|A_2|^2 - 4|A_1|^2 + 6|A_0|^2$ vanishes, the distributions $W(\theta_2)$ as well as $W(\theta_{\gamma\gamma})$ will only be quadratic in $\cos\theta_2$ (or $\cos\theta_{\gamma\gamma}$, respectively). This happens in particular if the transition $\chi \to \gamma\psi$ is pure $E1$ so that $A_2 : A_1 : A_0 = \sqrt{6} : \sqrt{3} : 1$. Similarly, it is sufficient for the transition $\psi' \to \gamma\chi$ to be pure $E1$ in order for the $\cos^4\theta_2$ term to vanish. The explicit expression for the coefficients C_i in (A9) is given below:

$$\begin{aligned} C_2 &\equiv \langle R_2^* R_2 + L_2^* L_2 + R_{\bar 2}^* R_{\bar 2} + L_{\bar 2}^* L_{\bar 2}\rangle \\ &= \tfrac{8}{105}[6B_0{}^2 + 8B_1{}^2 + 8B_2{}^2 - \sqrt{3}B_0B_1 + 3\sqrt{2}B_1B_2 + 2\sqrt{6}B_0B_2], \\ C_1 &\equiv \langle R_1^* R_1 + L_1^* L_1 + R_{\bar 1}^* R_{\bar 1} + L_{\bar 1}^* L_{\bar 1}\rangle \\ &= \tfrac{4}{105}[15B_0{}^2 + 13B_1{}^2 + 13B_2{}^2 + \sqrt{3}B_0B_1 - 3\sqrt{2}B_1B_2 - 2\sqrt{6}B_0B_2], \\ C_0 &\equiv \langle R_0^* R_0 + L_0^* L_0\rangle \\ &= \tfrac{4}{105}[8B_0{}^2 + 6B_1{}^2 + 6B_2{}^2 + \sqrt{3}B_0B_1 - 3\sqrt{2}B_1B_2 - 2\sqrt{6}B_0B_2]. \end{aligned} \tag{A10}$$

It may be checked, using the expressions (A10), that under the assumption of $E1$-$E1$ dominance $B_2{:}B_1{:}B_0 = \sqrt{6}:\sqrt{3}:1 = A_2{:}A_1{:}A_0$ the angular distribution $W(\theta_2)$ takes the form

$$W(\theta_2) = 73 + 21\cos^2\theta_2. \tag{A11}$$

which is analogous to the distribution in the angle $\theta_{\gamma\gamma}$, given in Table V.

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