Comments and Addenda

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High-energy amplitudes of Yang-Mills theory in the eighth order*

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We give the asymptotic forms of the eighth-order amplitudes for both fermion-fermion scattering and vector-vector scattering in the Yang-Mills theory with an isospin-1/2 Higgs boson, in the limit $s\to\infty$ with t fixed.

Recently, McCoy and Wu[1,2] calculated, in the sixth order, the high-energy amplitude for fermion-fermion scattering in the Yang-Mills theory with an isospin-$\frac{1}{2}$ Higgs boson.[3] We have now completed the eighth-order calculations of this amplitude for both fermion-fermion scattering and vector-meson–vector-meson scattering.

We express the non-spin-flip amplitude[4] for fermion-fermion scattering as

$$\mathfrak{M}_{ff}=2^{-4}m^{-2}(F_0-\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}F_1), \tag{1}$$

and the non-spin-flip amplitude for vector-meson–vector-meson scattering as

$$\mathfrak{M}_{VV}=[\tfrac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta}G_0+\tfrac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta}-\delta_{\alpha\delta}\delta_{\beta\gamma})G_1 +\tfrac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}-\tfrac{2}{3}\delta_{\alpha\beta}\delta_{\gamma\delta})G_2]. \tag{2}$$

In the above, g is the coupling constant, m is the mass of the fermion, s is the center-of-mass energy squared, $\vec{\tau}^{(i)}$ are the Pauli matrices for the isospin of the fermion, and α and γ (β and δ) are the isospin indices of the incoming (outgoing) vector mesons. The invariant amplitudes F_n and G_n are so chosen that they are of isospin $I=n$ in the t channel, where $t=-\vec{\Delta}^2$, with $\vec{\Delta}$ the momentum transfer.

We have obtained, in the eighth order, the asymptotic form of both the real part and the imaginary part of the invariant amplitudes, in the limit $s\to\infty$ with t fixed. We shall give, for completeness, the results of the second–eighth orders here. The amplitudes F_1 and G_1 are simplest. Up to the eighth order, the leading terms of F_1 and G_1 form the first four terms in the perturbation series of a Regge pole term:

$$F_1\sim\tfrac{1}{2}G_1\sim\frac{g^4}{\Delta^2+\lambda^2}\,s^{\alpha_1}(1-e^{-i\pi\alpha_1}), \tag{3}$$

where

$$\alpha_1=1-g^2(\vec{\Delta}^2+\lambda^2)\int\frac{d^2q_\perp}{(2\pi)^3}\frac{1}{(\vec{q}_\perp{}^2+\lambda^2)[(\vec{\Delta}-\vec{q}_\perp)^2+\lambda^2]}\,. \tag{4}$$

Notice that the Regge pole α_1 passes through the vector-meson pole [$\alpha_1=1$ and the coefficient of s^{α_1} in (3) blows up at $t=\lambda^2$], is of $I=1$, and has odd signature. This suggests that α_1 is the Reggeization of the vector meson.[1,5]

The other invariant amplitudes are more complex:

$$F_0\sim\frac{3i\pi s}{4}g^4\left[\frac{a}{\Delta^2+\frac{5}{4}\lambda^2}+\left(-\frac{a^2}{\Delta^2+\frac{5}{4}\lambda^2}+4A\right)g^2\ln s+\left(\frac{a^3}{\Delta^2+\frac{5}{4}\lambda^2}-8aA+4B\right)g^4\frac{(\ln s)^2}{2!}\right], \tag{5}$$

$$G_0\sim4i\pi sg^4\left[\frac{a}{\Delta^2+\frac{5}{4}\lambda^2}+\left(-\frac{a^2}{\Delta^2+\frac{5}{4}\lambda^2}+4A\right)g^2\ln s+\left(\frac{a^3}{\Delta^2+\frac{5}{4}\lambda^2}-8aA+4B\right)g^4\frac{(\ln s)^2}{2!}\right], \tag{6}$$

and

$$G_2 \sim 2i\pi g^4\left[\frac{b}{\Delta^2+2\lambda^2}+\left(\frac{b^2}{\Delta^2+2\lambda^2}-4A\right)g^2\ln s+\left(\frac{b^3}{\Delta^2+2\lambda^2}-8bA+8B\right)g^4\frac{(\ln s)^2}{2!}\right]. \tag{7}$$

In the above

$$a=(2\Delta^2+\tfrac{5}{2}\lambda^2)\int\frac{d^2q_\perp}{(2\pi)^3}\frac{1}{(\vec{q}_\perp{}^2+\lambda^2)[(\vec{\Delta}-\vec{q}_\perp)^2+\lambda^2]}, \tag{8}$$

$$b=(\Delta^2+2\lambda^2)\int\frac{d^2q_\perp}{(2\pi)^3}\frac{1}{(\vec{q}_\perp{}^2+\lambda^2)[(\vec{\Delta}-\vec{q}_\perp)^2+\lambda^2]}, \tag{9}$$

$$A=\int\frac{d^2q_{1\perp}d^2q_{2\perp}}{(2\pi)^6}\frac{1}{(\vec{q}_{1\perp}{}^2+\lambda^2)(\vec{q}_{2\perp}{}^2+\lambda^2)[(\vec{\Delta}-\vec{q}_{1\perp}-\vec{q}_{2\perp})^2+\lambda^2]}, \tag{10}$$

and

$$B=\int\frac{d^2q_{1\perp}d^2q_{2\perp}d^2q_{3\perp}}{(2\pi)^9}\frac{1}{(\vec{q}_{1\perp}{}^2+\lambda^2)(\vec{q}_{2\perp}{}^2+\lambda^2)(\vec{q}_{3\perp}{}^2+\lambda^2)}$$
$$\times\left\{\frac{1}{(\Delta-\vec{q}_{1\perp}-\vec{q}_{2\perp}-\vec{q}_{3\perp})^2+\lambda^2}+\frac{(\vec{q}_{1\perp}+\vec{q}_{3\perp})^2+\lambda^2}{[(\vec{q}_{1\perp}+\vec{q}_{2\perp}+\vec{q}_{3\perp})^2+\lambda^2][(\vec{q}_{1\perp}+\vec{q}_{3\perp}+\vec{\Delta})^2+\lambda^2]}\right\}. \tag{11}$$

We close by making comparisons of the high-energy behavior in Yang-Mills theory with that in QED. (i) Just as in QED, all integrals over the transverse momenta are convergent, and all $\ln s$ factors come from integrations over the phase space of the longitudinal momenta. This means that the energy dependence of cross sections is a consequence of the creation of pionization products[1]; (ii) Just as in QED, the convergence of integrations over transverse momenta is a result of spectacular cancellations among sets of diagrams. Such cancellations appear to depend on the group properties of gauge theories only,[6] and are not restricted to the SU(2) group treated here. Neither do they depend on the existence of the Higgs bosons, which are, however, convenient to have in order to avoid the complication of infrared divergences. (iii) Unlike the situation in QED, the largest term in each perturbation order is real. It is also of $I=1$ in the t channel. In the eighth order, for example, F_1 is of the order of $s(\ln s)^3$, while F_0 is of the order of $is(\ln s)^2$; (iv) This means that, in *each* perturbation order, $\mathrm{Re}\mathfrak{M}/\mathrm{Im}\mathfrak{M}$ approaches infinity as $s\to\infty$, and the cross section for a charge-exchange reaction is equal to four times the elastic cross section. Such behaviors are contradictory to well-founded experimental facts. It is therefore gratifying that such large terms in F_1 appear to cancel one another as we sum over *all* perturbation orders. More precisely, the 2nd–8th order results suggest that these terms add up to the Regge pole term in (3), with $\alpha_1<1$ in the physical region of the s channel. This is to be compared with the situation in QED, where the real part of the elastic scattering amplitude due to the exchange of a photon does not Reggeize—a behavior sometimes taken to indicate a shortcoming of quantum electrodynamics as a model of strong interactions[7]; (v) There are terms in F_0, G_0, and G_2 which can also be summed into Regge pole terms. Specifically, the first terms in each perturbation order in (5) and (6) are summed into s^{-a}, and the first terms in each perturbation order in (7) are summed into s^b. Further work on this point is in progress.

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[1]B. McCoy and T. T. Wu, Phys. Rev. Lett. 35, 604 (1975).

[2]The first published results of the sixth-order and the eighth-order amplitudes for fermion-fermion scattering were given by H. T. Nieh and Y.-P. Yao, Phys. Rev. Lett. 32, 1074 (1974). Their sixth-order result is in disagreement with that of McCoy and Wu. We have gone through the sixth-order calculations and are in agreement with McCoy and Wu.

[3]For the Feynman rules in this theory, see G. 't Hooft and M. Veltman, in *Renormalization of Yang-Mills Fields and Applications to Particle Physics, proceedings of a Colloquium, Marseille, 1972*, edited by C. P. Korthals-Altes (C. N. R. S., Marseille, France, 1972).

[4]In the high-energy limit, the leading terms of the asymptotic amplitude are of non-spin-flip type.

[5]M. Grisaru, H. Schnitzer, and H. Tsao, Phys. Rev. Lett. 30, 811 (1973).

[6]P. Yeung, Phys. Rev. D (to be published).

[7]F. Low, Phys. Rev. D 12, 163 (1975).