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Unified gauge theories with right-handed currents and heavy fermions*

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Gauge models with heavy fermions and right-handed currents are discussed based on the gauge groups $SU(2)_A \times U(1) \times SU(4)'$, $SU(2)_A \times SU(2)_B \times SU(4)'$, and $SU(4) \times SU(4)'$ and are constructed so as to lead to the $\Delta I = 1/2$ rule. $SU(4) \times SU(4)'$ is advocated as the ultimate unifying gauge group of nature, and it is shown how at various stages of spontaneous breakdown both the $SU(2)_A \times SU(2)_B \times SU(4)'$ and $SU(2)_A \times U(1) \times SU(4)'$ groups manifest themselves. We also show that CP violation takes an interesting complexion in these models and leads to exactly the relations $\eta_{+-} \simeq \eta_{00}$ in $K_L \to 2\pi$ decays. Furthermore, we show that the magnitude of CP violation is related to gauge interactions that violate the heavy quark degeneracy.

I. INTRODUCTION

The experimental and theoretical developments of recent months have very clearly highlighted the need for extra degrees of freedom in hadron physics. (a) On the theoretical side, a reconciliation between the absence of $\Delta S = 1$ neutral currents and construction of renormalizable models[1] of weak and electromagnetic interactions requires one to postulate an extra quark degree of freedom[2] (called charm quark, χ). Of course, from a purely aesthetic point of view, if one demanded lepton-quark symmetry,[3] this extra degree of freedom would be very desirable indeed. Coupled with the postulate of an extra color degree of freedom[4] of quarks, one obtains a beautiful 4×4 structure for the basic building blocks of matter, the fermions.

(b) On the experimental side, first, one has the recently discovered narrow resonances,[5] $\psi(3095)$ and $\psi'(3684)$, which could be thought of as bound states of the above charm quark[6] χ or even of further degrees of freedom.[7] Secondly, the value of ratio $R = \sigma(e^+e^- \to X)/\sigma(e^+e^- \to \mu^+\mu^-)$ has been observed[8] at SPEAR to be around 6 at center-of-mass energy 5–6 GeV, whereas naive-parton-model estimates would suggest a value of $\frac{10}{3}$, thus obviating the need for new fermionic degrees of freedoms (i.e., new quarks and/or new leptons).

It is, therefore, of considerable interest to study gauge models of weak and electromagnetic interactions, which incorporate these new degrees of freedom, and to try to understand the observed hadronic symmetries and selection rules of nature in terms of these models. In this article, we will focus our attention on one class of such models, where the number of extra degrees of freedom is dictated by lepton-quark symmetry and the theoretical requirement of renormalizability of a superunified theory constructed out of them.

The assumption of an intrinsic lepton-hadron symmetry leads us to consider a sixteenfold set of fermions consisting of three color quartets of quarks and the four known leptons ν, e^-, μ^-, and ν'.[9,10] If we then want to describe all forces of nature in terms of one basic gauge coupling constant, we are led to postulate the existence of another sixteenfold set of heavier fermions (called heavy fermions) to avoid triangle anomalies to preserve renormalizability of the super-unified theory.[11] An interpretation of the new particles in terms of this model can be given in a number of ways.[7,12] We will then study the gauge models based on this 32-fold set of fermions. As is well known,[9,13] the strong interactions will be generated using the "color" degree of freedom. The remaining eightfold "valence" degrees of freedom will be used to construct models for weak interactions. At this point, one can adopt any one of a number of ways to construct models of weak and electromagnetic interactions.[14] The only constraints are that the low-energy weak interaction involving known hadrons must be of $V - A$ type and that processes involving $\Delta S = 1$ neutral currents as well as $\Delta S = 2$ transitions must be highly suppressed compared to normal first-order weak processes such as μ decay or β decay. Furthermore, right-handed current interactions involving observed hadrons and leptons must also be suppressed. It is well known how extra fermionic degrees of freedom help in the construction of models that satisfy the above constraints, and all this can of course be done using only the left-handed currents and without ever activating the right-handed ones, as is done, for example, using $SU(2)_L \times U(1)$ gauge groups. Interesting phenomena can, however, arise if we let the right-handed currents play an active role in these models. This is exemplified in the references of footnote 10, which use either left-right symmetric gauge groups $SU(2)_L \times SU(2)_R \times SU(4)'$ or just plain $SU(2)_L \times SU(2)'_R \times SU(4)'$ gauge groups for generating weak interactions. These models demonstrate in an interesting manner the origin of parity and *CP* violation in gauge theories as well as the origin of chiral and isospin symmetries and how gauging of the right-handed currents is essential to achieve this purpose. However, it turns out that in all these models the right-handed currents are highly suppressed at low energies and therefore, are not accessible in present experiments.

There is, however, another class of models, first written down in 1972 by the author,[15] where it was suggested that not only do the right-handed currents play an active role in model building but they also appear with a comparable strength with the known weak currents at low energies. The model was originally suggested as a way to accommodate *CP* violation in gauge theories and has been further analyzed in a recent paper by De Rújula, Georgi, and Glashow[16] and shown to possess a number of other desirable properties such as the $\Delta I=\frac{1}{2}$ rule, to predict dileptons of the same charge, $\mu^-\mu^-$, etc. Generalizations of this model have been considered in subsequent papers.[17-21] Our aim in this article will be to propose and analyze unified gauge models which use right-handed currents of the type suggested in Ref. 15 and which involve the 32-fold set of basic fermionic constituents listed above. We will analyze gauge models based on gauge groups $SU(2)\times U(1)$, $SU(2)_A\times SU(2)_B\times SU(4)'$, and $SU(4)\times SU(4)'$. In particular, we comment on the neutrino interactions in these models involving both neutral- and charged-current interactions, $\Delta S=2$ transitions, nonleptonic decays of charmed particles, recent events from Kolar gold mine,[22] etc.

We then study the question of *CP* violation within these theories and speculate on the origin of other hadronic symmetries in the context of these models. Finally, we comment on some speculations on the possible existences of subquarks at the next level of elementarity in the hierarchy of particle physics.[23]

II. GAUGE MODELS WITH CHARM-CHANGING RIGHT-HANDED CURRENTS

Before presenting the new gauge models, we would like to present our notation. As mentioned in the Introduction, we will work with a sixteen-fold set of fermions denoted as[9,10]

$$\begin{bmatrix} \mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \mathcal{P}_d\equiv\nu \\ \mathcal{N}_a & \mathcal{N}_b & \mathcal{N}_c & \mathcal{N}_d\equiv e^- \\ \lambda_a & \lambda_b & \lambda_c & \lambda_d\equiv\mu^- \\ \chi_a & \chi_b & \chi_c & \chi_d\equiv\nu' \end{bmatrix} \quad (1)$$

and a heavy counterpart to these denoted by a prime. The subscripts *a*, *b*, *c*, etc. stand for "color." The conventional hadronic weak current[24] is given in terms of the basic quarks as

$$J_\mu=\sum_{a=1}^{3}\bar{\mathcal{P}}_{aL}\gamma_\mu(\mathcal{N}_{aL}\cos\theta_C+\lambda_{aL}\sin\theta_C)\,. \quad (2)$$

This current manifests itself at low energy with strength G_F. An additional current that appears with equal strength in conventional gauge theories is the so-called GIM (Glashow-Iliopoulos-Maiani) current J'_μ, i.e.,

$$J'_\mu=\sum_{a=1}^{3}\bar{\chi}_{aL}\gamma_\mu(-\mathcal{N}_{aL}\sin\theta_C+\lambda_{aL}\cos\theta_C)\,. \quad (3)$$

This current, essential for avoiding $\Delta S=1$, neutral current, has several other experimentally testable predictions.[25] It is possible to introduce another fermion current[15] within the gauge-theory framework, which will be the subject of this article. It is

$$J^C_\mu=\sum_{a=1}^{3}\bar{\chi}_{aR}\gamma_\mu\mathcal{N}_{aR}\,. \quad (4)$$

The immediate implications of this current[16] are the following:

(a) It gives rise to the $\Delta I=\frac{1}{2}$ rule for $\Delta C=0$ nonleptonic processes since the effective $\Delta S=1$, $\Delta C=0$ Hamiltonian in these models is of the form (dropping color index)

$$\approx\frac{G_F}{\sqrt{2}}\bar{\chi}_L\gamma_\mu\lambda_L\,\bar{\mathcal{N}}_R\gamma_\mu\chi_R+\text{H.c}\,. \quad (5)$$

There are of course, induced $\bar{\mathcal{N}}\lambda$-type terms due to higher-order graphs. The important point is that this Hamiltonian is purely $\Delta I=\frac{1}{2}$ and the absence of Cabibbo suppression therefore causes this to dominate over other terms in the weak Hamiltonian which contain both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ parts. If one takes into account the anomalous dimension of the operator[18] the extent of enhancement can be as big as sixty times over the $\Delta I=\frac{3}{2}$ part.

(b) This current causes large $D_0-\bar{D}_0$ mixing where D^0 is a $\chi\bar{\mathcal{P}}$ composite, thereby giving rise to dilepton final states of the type $\mu^-\mu^-$ along with the ones[25] of the type $\mu^-\mu^+$ due to the production of charm particles (in this case D^0) in ν-N scattering at large energies. It has been claimed in the literature[18] that the new right-handed currents may imply a large $\Delta S=2$ transition. We have demonstrated that[19] such fears are unfounded and are presumably nonexistent. We will present our argument below. For purposes of subsequent discussion, we would like to state here that the new right-handed current implies the following effective $\Delta S=2$ Lagrangian:

$$\mathcal{L}^{\text{eff}}_{\Delta S=2}=G^{\Delta S=2}[4\bar{\lambda}(1-\gamma_5)\mathfrak{N}\bar{\lambda}(1-\gamma_5)\mathfrak{N} +\bar{\lambda}\sigma_{\mu\nu}(1-\gamma_5)\mathfrak{N}\bar{\lambda}\sigma_{\mu\nu}(1-\gamma_5)\mathfrak{N}] +\text{H.c.} \quad (5')$$

The magnitude of $G^{\Delta S=2}$ depends on the detailed assignment of fermions to representations of the gauge group. The dispute, however centers on how large the K^0-$\bar{K}^0$ transition matrix element due to the above operator becomes.

We now present three kinds of models, each kind employing a separate gauge group. The gauge groups are

(a) $SU(2)\times U(1)\times SU(3)'$,
(b) $SU(2)_A\times SU(2)_B\times SU(4)'$,
(c) $SU(4)\times SU(4)'$.

Model (a): $SU(2)\times U(1)\times SU(3)'$. Historically, the right-handed currents of Eq. (4) were first introduced within an SU(2)×U(1) framework.[15] The assignment of fermions to the gauge group so that one ensures the cancellation of triangle anomalies are as follows:

$$\begin{pmatrix}\mathcal{P}_L\\ \mathfrak{N}_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_L\\ \lambda_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_R\\ \mathfrak{N}_R(\phi)\end{pmatrix},$$
$$\begin{pmatrix}\nu_L\\ e_L^-\end{pmatrix},\quad \begin{pmatrix}\nu_L'\\ \mu_L^-\end{pmatrix},\quad \begin{pmatrix}E_R^0\\ e_R^-\end{pmatrix}, \quad (6)$$

where

$$\mathfrak{N}_R(\phi)=\mathfrak{N}_R\cos\phi+i\lambda_R\sin\phi$$

and

$$\lambda_R(\phi)=i\mathfrak{N}_R\sin\phi+\lambda_R\cos\phi\,.$$

These fermions are assigned to doublets under the SU(2) group, whereas the remaining fermions, i.e., $E_L^0, \mathcal{P}_R, \mu_R^-, \lambda_R(\phi)$ are assigned to the singlet representation. E^0 is a heavy lepton introduced to make the theory free of triangle anomalies. It is necessary only if we are not within the 32-fold fermion framework. With the 32-fold fermions, there exists a variety of possibilities, the simplest of which is to let the heavier counterparts of these fermions couple with the same gauge bosons with opposite chirality. However, we present another alternative, which turns out to be useful in getting a desirable theory of *CP* violation (as discussed in Sec. VI). The assignments go as follows: The doublets are

$$\begin{pmatrix}\mathcal{P}_L\\ \mathfrak{N}_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_L\\ \lambda_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_R\\ \mathfrak{N}_R(\phi)\end{pmatrix},\quad \begin{pmatrix}\chi_R'\\ \lambda_R(\phi)\end{pmatrix}, \quad (7)$$
$$\begin{pmatrix}\nu_L\\ e_L^-\end{pmatrix},\quad \begin{pmatrix}\nu_L'(\alpha)\\ \mu_L^-\end{pmatrix},\quad \begin{pmatrix}E_R^0\\ e_R^-\end{pmatrix},\quad \begin{pmatrix}M_R^0\\ M_R^-\end{pmatrix},$$

where

$$\nu_L'(\alpha)=\nu_L'\cos\alpha+M_L^0\sin\alpha$$

and (8)

$$M_L^0(\alpha)=-\nu_L'\sin\alpha+M_L^0\cos\alpha$$

and the remaining fermions (i.e., $\mathcal{P}_{L,R}', \mathcal{P}_R, \chi_L', \lambda_{L,R}', \mathfrak{N}_{L,R}', M_L^0(\alpha), \mu_R^-, M_L^-, E_L^0, E_{L,R}^-$) are singlets under SU(2) with U(1) quantum numbers appropriately chosen for both cases to match the charge ($Q=T_3+\frac{1}{2}Y$). In case (i), lepton-quark symmetry is destroyed by the introduction of the E^0 fermion; however, in case (ii), baryon-lepton unification is achieved by gauging the SU(4)′ group of three colors and the lepton number as in Ref. 9. Most of the experimental implications of this model have already been discussed.[15,16,19] Here, we would like to point out its implications for neutral-current phenomena[19] and *CP* violation in a subsequent section.

The model, however, has several disadvantages. First of all, due to the presence of the Abelian U(1) group, the quantization of electric charge of the elementary fermions remains unexplained. Besides, one has three independent gauge couplings, a feature which is less than desirable since our ultimate aim is unification of all interactions.

Model (b): $SU(2)_A\times SU(2)_B\times SU(4)'$. This model has its origin in the left-right symmetric theories described in Refs. 9 and 10 and has the following appealing features: (i) It provides an understanding of electric charge quantization. (ii) By imposing the discrete symmetry ($A \leftrightarrow B$) on the Lagrangian prior to spontaneous breaking,[10] it is possible to reduce the number of gauge couplings to only two (i.e., $g_A=g_B$ and f). (iii) The theory is parity- and *CP*-invariant prior to spontaneous breaking. The spontaneous breaking can then be arranged[10] such that $M_B \gg M_A$. This implies that at low energies the *B*-type currents are suppressed (by a factor M_A^2/M_B^2) as compared to the *A*-type currents. However, at energies of the order of 10^4 GeV, both *A*- and *B*-type currents appear with equal strength and a whole new regime of particle phenomena opens up. The assignment of fermions to the representations of this group can be done in several ways. The two models we present here seem consistent with present experiments but vary only insofar as their prediction for the strength of $\Delta S=2$ transitions are concerned.

Case (i): The assignment of quarks is as follows[19]:

$$\begin{pmatrix}\mathcal{P}_L\\ \mathfrak{N}_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_L\\ \lambda_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_R\\ \mathfrak{N}_R(\phi)\end{pmatrix},\quad \begin{pmatrix}\chi'_R\\ \lambda_R(\phi)\end{pmatrix};\quad (\tfrac{1}{2},0,4) \tag{9}$$

$$\begin{pmatrix}\mathcal{P}_R\\ \mathfrak{N}'_R\end{pmatrix},\quad \begin{pmatrix}\mathcal{P}'_R\\ \lambda'_R\end{pmatrix},\quad \begin{pmatrix}\mathcal{P}'_L\\ \mathfrak{N}'_L\end{pmatrix},\quad \begin{pmatrix}\chi'_L\\ \lambda'_L\end{pmatrix};\quad (0,\tfrac{1}{2},4).$$

Case (ii): An alternative assignment, which also yields the $\Delta I=\frac{1}{2}$ rule for nonleptonic decays, is

$$\begin{pmatrix}\mathcal{P}_L\\ \mathfrak{N}_L(\theta_C)\end{pmatrix},\quad \begin{pmatrix}\chi_L\cos\alpha+\mathcal{P}'_L\sin\alpha\\ \lambda_L(\theta_C)\end{pmatrix},$$
$$\begin{pmatrix}\chi_R\cos\beta+\mathcal{P}'_R\sin\beta\\ \mathfrak{N}_R(\phi)\end{pmatrix},\quad \begin{pmatrix}\chi'_R\\ \lambda_R(\phi)\end{pmatrix};\quad (\tfrac{1}{2},0,4) \tag{10}$$
$$\begin{pmatrix}\mathcal{P}_R\\ \mathfrak{N}'_R\end{pmatrix},\quad \begin{pmatrix}-\chi_R\sin\beta+\mathcal{P}'_R\cos\beta\\ \lambda'_R\end{pmatrix},$$
$$\begin{pmatrix}-\mathcal{P}'_L\cos\alpha+\chi_L\sin\alpha\\ \mathfrak{N}'_L\end{pmatrix},\quad \begin{pmatrix}\chi'_L\\ \lambda'_L\end{pmatrix};\quad (0,\tfrac{1}{2},4).$$

It is easy to see that for small α and β, the model yields the $\Delta I=\frac{1}{2}$ rule for nonleptonic decays. However, the difference between these two cases is in their prediction for $G^{\Delta S=2}$: Case (i) predicts

$$G^{\Delta S=2}\approx\frac{G_F^2M_\chi^2}{16\pi^2}\cos^2\theta_C\,, \tag{11}$$

whereas in case (ii) one has

$$G^{\Delta S=2}\approx\frac{G_F^2}{16\pi^2}(M_\chi\cos\alpha\cos\beta+M_{\mathcal{P}'}\sin\alpha\sin\beta)^2\,. \tag{12}$$

It is therefore possible in case (ii) to have the strength of the $\Delta S=2$ transitions reduced by choosing the expression in parentheses in Eq. (12), i.e.,

$$(M_\chi\cos\alpha\cos\beta+M_{\mathcal{P}'}\sin\alpha\sin\beta)\,,$$

small enough. The assignment of leptons to this gauge group is done as follows:

$$\begin{pmatrix}\nu_L\\ e_L^-\end{pmatrix},\quad \begin{pmatrix}\nu'_L(\alpha)\\ \mu_L^-\end{pmatrix},\quad \begin{pmatrix}E_R^0\\ e_R^-\end{pmatrix},\quad \begin{pmatrix}M_R^0\\ M_R^-\end{pmatrix};\quad (\tfrac{1}{2},0,4)$$

and (13)

$$\begin{pmatrix}\nu_R\\ E_R^-\end{pmatrix},\quad \begin{pmatrix}M_L^0(\alpha)\\ M_L^-\end{pmatrix},\quad \begin{pmatrix}\nu'_R\\ \mu_R^-\end{pmatrix},\quad \begin{pmatrix}E_L^0\\ E_L^-\end{pmatrix};\quad (0,\tfrac{1}{2},4).$$

We allow for a mixing between the muon neutrino and the heavy lepton M^0 to explain the observations of Krishnaswamy *et al.*[22] The value of $\alpha\approx10^{-3}$ to be consistent with μ-decay experiments and this suppression seems essential to understand the results of Ref. 22.[26]

Model (c): SU(4)×SU(4)′ group. This model will embody the ultimate unification of all forces, since by demanding the invariance of the Lagrangian prior to spontaneous breaking under the discrete symmetry that transforms unprime ↔ prime [SU(4) ↔ SU(4)′], one gets only one gauge coupling g describing all forces of nature (except gravitation). There is, of course, charge quantization in this theory.[27] The assignment of quarks to this group are as follows:

$$\begin{bmatrix}\mathcal{P}_L\\ \mathfrak{N}_L(\theta_C)\\ \mathfrak{N}'_L(\theta_C)\\ \mathcal{P}'_L\end{bmatrix},\quad \begin{bmatrix}\chi_L\\ \lambda_L(\theta_C)\\ \lambda'_L(\theta_C)\\ \chi'_L\end{bmatrix},\quad \begin{bmatrix}\chi_R\\ \mathfrak{N}_R(\phi)\\ \mathfrak{N}'_R(\phi)\\ \mathcal{P}_R\end{bmatrix},\quad \begin{bmatrix}\chi'_R\\ \lambda_R(\phi)\\ \lambda'_R(\phi)\\ \mathcal{P}'_R\end{bmatrix}. \tag{14}$$

An important problem here is to arrange the spontaneous breaking in such a way that only a subset of the currents manifest themselves with strength G_F, all the rest being highly suppressed. This purpose is achieved by choosing Higgs multiplets $\phi(15,1)$ and $\phi(6,1)$ [the indices within the parentheses denote the representation content under SU(4)×SU(4)′] to have the following vacuum expectation values:

$$\langle\phi(15,1)\rangle\equiv\begin{bmatrix}a&&&\\&-a&&\\&&-a&\\&&&a\end{bmatrix} \tag{15}$$

$$\langle\phi(6,1)\rangle\equiv d\begin{bmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \tag{16}$$

One first breaks the SU(4)×SU(4)′ group down to SU(2)×SU(2)×SU(4)′ generated by $(T_2^1, T_1^2, T_1^1-T_2^2)$ and $(T_4^3, T_3^4, T_3^3-T_4^4)$. $\langle\phi(15,1)\rangle$ then breaks the group down to a U(1)×U(1)×SU(4)′ which can further be broken to leave us only with a U(1) corresponding to electromagnetism. The rest of the breaking may be achieved by choosing Higgs mesons like $\Sigma(4,4^*)$ and giving them a vacuum expectation value

$$\langle \Sigma(4,4^*)\rangle = \begin{bmatrix} a_1 & & & \\ & a_1 & & \\ & & a_1 & \\ & & & a_4 \end{bmatrix} . \quad (17)$$

We now choose

$$d, a_4 \gg a, a_1 . \quad (18)$$

This ensures that only gauge mesons W_2^1, W_1^2, and $W_1^1 - W_2^2$ remain the lightest of the SU(4) gauge bosons; the others have masses proportional to g^2d^2 and $g^2a_4^2$ and are, therefore, much heavier than these. So the currents corresponding those degrees of freedom manifest themselves only at very high energies (i.e., 10^4 GeV or higher). An interesting point to note is that $\phi(6,1)$ breaks the gauge group $SU(4)\times SU(4)'$ down to the gauge group $SU(2)_A\times SU(2)_B\times SU(4)'$ of model (b). Therefore, the predictions of this model are the same as those of model (b) at low energies.

Finally, we would like to remark that both the $SU(2)_A\times SU(2)_B\times SU(4)'$ and $SU(2)\times U(1)\times SU(4)'$ groups arise in successive stages of spontaneous breaking of $SU(4)\times SU(4)'$. Therefore, evidence in favor of either of the gauge groups could really be evidence in favor of the ultimate unifying gauge group $SU(4)\times SU(4)'$. The successive stages of breaking are provided as follows:

$$SU(4)\times SU(4)' \quad (\text{coupling } g)$$

$$\downarrow \langle \phi(6,1)\rangle$$

$$SU(2)_A\times SU(2)_B\times SU(4)' \quad (g_A = g_B = f)$$

$$\downarrow \langle \phi(15,1)\rangle = \begin{bmatrix} a & & & \\ & a & & \\ & & a & \\ & & & -3a \end{bmatrix}$$

$$SU(2)\times U(1)\times SU(4)' \quad (g = g' = f) .$$

III. NEUTRINO INTERACTIONS

In this section, we will briefly present the implications of the new right-handed currents for charged- as well as neutral-current interaction of neutrinos: The details of charged-current interactions have been studied in a recent paper by Barger, Weiler, and Phillips,[28] who find that, with a suitable amount of antiquark components, the neutrino anomaly for $x<0.1$ can be explained. It may also be that the recently observed[29] dimuons ($\mu^+\mu^-$) in $\bar{\nu}$ scattering are due to the antiquark components in the nucleon since they also arise in the region of small x. Moreover, in all the models we have presented, the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}\approx \frac{1}{4}$ after the charm threshold has been reached. If, however, in model (b) $m_A \approx m_B$, then the ratio $\sigma^{\bar{\nu}}/\sigma^{\nu}\approx 1$ at energies around 100 to 200 GeV.

Now, coming to the neutral-current interactions,[30] the most interesting situation arises in model (a), where one has a two-parameter description of the neutral-current interaction, the parameters being

$$\tan\theta_W = g'/g \quad \text{and} \quad \epsilon = (m_{W^+}/m_Z \cos\theta_W)^2 . \quad (19)$$

If we choose $\epsilon\approx 1$, it essentially becomes a one-parameter situation and the agreement with available leptonic and hadronic data is good. As has been discussed in Ref. 19, leptonic scattering data, i.e., Gargamelle data, the two $\bar{\nu}_\mu e^-$ events, and the Reines reactor experiments restrict the angle θ_W for this model as follows:

$$0.62 < \sin^2\theta_W < 0.73 . \quad (20)$$

For arbitrary ϵ, this equation becomes

$$0.12 < \epsilon(-\tfrac{1}{2} + \sin^2\theta_W) < 0.23 . \quad (21)$$

Hadronic data[31] require that

$$\sin^2\theta_W < 0.7 . \quad (22)$$

Thus, there is still a range of values for θ_W consistent with the data. In contrast, note that for the Weinberg-Salam model there is only one value, i.e., $\sin^2\theta_W \simeq 0.36$, for which there is agreement with data. So, more experimental work on the leptonic and hadronic neutral currents will be decisive in testing model (a) for right-handed currents.

As far as models (b) and (c) are concerned, the hadronic neutral current has the general isospin structure

$$g[(V_\mu^3 + A_\mu^3) - (V_\mu^0 - A_\mu^0)]Z_\mu + gK_\mu S_\mu^0 \quad (23)$$

and therefore the effective interaction will depend on the masses of Z and S. We do not pursue this any further in this article. However, one point we would like to stress is the presence of an axial-vector isosinglet neutral current, which is absent in most gauge models. This could be checked by elastic scattering of neutrinos and antineutrinos on isosinglet targets such as deuterons.

Finally, as has already been noted in Ref. 16, one will observe dileptons of the type $\mu^-\mu^-$ in neutrino scattering (and $\mu^+\mu^+$ in antineutrino scattering) due to the mixing of $D_0 - \bar{D}_0$. The relative ratio is

$$\frac{\sigma(\mu^-\mu^-)}{\sigma(\mu^-\mu^+)} \simeq \left(\frac{\Delta m_{D_1-D_2}}{\Gamma_{D_1}}\right) \approx 1 . \quad (24)$$

IV. $\Delta C=1$ NONLEPTONIC DECAYS

The structure of the $\Delta C=1$ nonleptonic Hamiltonian has a considerably different SU(3) structure in our model than in the conventional GIM model. This has an important bearing on the question of the K/π ratio in e^+e^- annihilation. These decays have been analyzed in Ref. 32 within the GIM model, where it is found that the nonleptonic Hamiltonian transforms like 20- and 84-dimensional representations under SU(4). Assuming that the 84-dimensional representation is suppressed [like the 27-plet in the case of SU(3)] one finds that $H_{\rm nl}^{\Delta C=1}$ transforms like the 6-dimensional representation of SU(3). The decays of the charmed particles D^+, D^0, and F^+ have been studied in the context of this model in Ref. 32.

On the other hand, with the new right-handed currents, there appear[33] 15- and 20-dimensional as well as 84-dimensional representations of SU(4) in $H_{\rm nl}$. Now, the SU(3) representations $H_{\rm nl}^{\Delta C=1}$ are of the following type (using conventional tensor notation):

$$H_{\rm nl}^{\Delta C=1} \approx \tfrac{1}{4}\{[6]^{22}\cos^2\theta_C + 2[6]^{23}\cos\theta_C\sin\theta_C + [6]^{33}\sin^2\theta_C\}_{LL} + \tfrac{1}{4}\{-[6]^{23}\cos\theta_C + [6]^{33}\sin\theta_C + \tfrac{3}{2}[3^*]_1\cos\theta_C\}_{RL} \qquad (25)$$

where L and R stand for left- and right-handed currents. The detailed tables in this case are given in Ref. 33.

The analysis of Ref. 33 has an interesting implication for the experiment of Boyarski *et al.*[34] and its subsequent analysis by Einhorn and Quigg.[35] The point is that the experiment of Boyarski *et al.* implies that

$$\frac{\Gamma(D^0\to K^-\pi^+)}{\Gamma(D^0\to \text{all})} \lesssim 2.9\% . \qquad (26)$$

If the GIM model is taken to be correct, this immediately implies that (using the analysis of Ref. 32)

$$\frac{\Gamma(D^0\to PP)}{\Gamma(D^0\to \text{all})} \lesssim 8.6\% , \qquad (27)$$

where P stands for pseudoscalar meson. This value seems rather low considering the amount of phase space available and seems to cast doubt on the validity of the GIM current. On the other hand, from our analysis with the extra right-handed current we find that the upper limit (26) implies, under plausible assumptions about the various reduced matrix elements, that

$$\frac{\Gamma(D^0\to PP)}{\Gamma(D^0\to \text{all})} \lesssim 30\% , \qquad (28)$$

a value which is more reasonable.

V. K_L-K_S MASS DIFFERENCE

In this section, we will discuss the issue of K_L-K_S mass difference in theories with right-handed currents. The effective $\Delta S=2$ Lagrangian in such models is presented in Eq. (5a). In most models

$$G^{\Delta S=2} \approx G_F^2 m_\chi^2/16\pi^2 .$$

Note the absence of the Cabibbo suppression factor $\sin^2\theta_C$. It is precisely this absence that has prompted the conjecture that the K_L-K_S mass difference may be large in this model. An actual evaluation of the K_L-K_S mass difference involves a whole series of hadronic intermediate states and is in general hard to perform. However, the matter is simplified by keeping only the vacuum intermediate state.[36] Whereas this estimate may provide a very rough estimate (presumably correct only to within one order of magnitude), to use this estimate to say that the $\Delta M_{K_L-K_S}$ mass difference is fifty times larger if $G^{\Delta S=2}\approx G_F^2 m_\chi^2/16\pi^2$ may be stretching things a bit too far. In fact, in Ref. 19, we have first shown that the contributions of the scalar and pseudoscalar terms oppose each other in sign. Secondly, we show that the π^0 intermediate state, which arises because of the scalar term in Eq. (5a), makes a contribution which is as large as the vacuum contribution where the pseudoscalar part contributes. Thus, the vacuum estimate is at best uncertain and could be wrong by orders of magnitude. More quantitatively,[19]

$$\left|\frac{\mathfrak{M}^{\rm vac}_{K^0-\bar K^0}}{\mathfrak{M}^{\pi^0}_{K^0-\bar K^0}}\right| \simeq 2\times 10^3 \times\left(\frac{m_K^2 m_\pi^2}{m_0^4}\right)\left(1-\frac{m_K^2}{m_0^2}\right)^2\left(\frac{m_\lambda - m_{\mathfrak{N}}}{m_\lambda + m_{\mathfrak{N}}}\right)^2 , \qquad (29)$$

where m_0 is the characteristic mass which parameterizes the q^2 dependence of the K_{l3} form factors; $m_{\mathfrak{N}}$ and m_λ are quark masses. So, if for example we choose $m_0^2 \simeq 2.1\ \text{GeV}^2$ (the low-energy experiments allow m_0^2 to lie between 1.5 and 2.1 GeV^2), and if $m_{\mathfrak{N}} \simeq m_\lambda/10$, then the above ratio is 1, which leads to an almost exact cancellation between vacuum and π^0 contributions. From this, we conclude that the question of the K_L-K_S mass difference in these models is largely an open one at present and certainly does not impose any constraint on model building. However, should it turn out on exact calculation that $\mathfrak{M}_{K^0-\bar K^0}$ is well approximated by the vacuum contribution, then we will have to look to models of the type of model (b(ii)) [see Eqs. (10) and (12)] to describe weak interactions.

VI. *CP* VIOLATION

In this section, we will address ourselves the question of *CP* violation[37] in these gauge models. As mentioned earlier, the original gauge model with right-handed currents[15] was suggested to explain *CP* violation. The basic strategy remains the same; however, more interesting consequences for *CP* violation can be obtained within the framework of heavy quarks.

Crucial to the question of *CP* is the phase i ($e^{i\pi/2}$) in $\mathfrak{N}_R(\phi)$ and $\lambda_R(\phi)$, i.e.,

$$\begin{aligned}\mathfrak{N}_R(\phi) &= \mathfrak{N}_R\cos\phi + i\lambda_R\sin\phi\,,\\ \lambda_R(\phi) &= i\mathfrak{N}_R\sin\phi + \lambda_R\cos\phi\,.\end{aligned} \tag{30}$$

We assume that all the rest of the fermion fields do not have complex mixings. We see that, in all the models suggested [Eqs. (7), (9), (10), etc.], one has two doublets having same representation content as follows:

$$\begin{pmatrix}\chi_R\\ \mathfrak{N}_R(\phi)\end{pmatrix}, \quad \begin{pmatrix}\chi'_R\\ \lambda_R(\phi)\end{pmatrix}. \tag{31}$$

Therefore, if we write down the effective $\Delta C=0$ *CP*-violating nonleptonic Hamiltonian, we find that the dominant term is the following:

$$H_{\rm nl}^{(-)} \approx i(G_F/\sqrt{2})\times\sin\phi\cos\phi(\bar\lambda_R\gamma_\mu\chi_R\bar\chi_R\gamma_\mu\mathfrak{N}_R - \bar\lambda_R\gamma_\mu\chi'_R\bar\chi'_R\gamma_\mu\mathfrak{N}_R)\,. \tag{32}$$

This has the following immediate implications:

(a) *CP*-violating $\Delta S=1$ transitions are purely $\Delta I=\frac{1}{2}$. Therefore, this coupled with the $\Delta I=\frac{1}{2}$ rule for *CP*-conserving decays in these models implies immediately that

$$\eta_{+-}\simeq\eta_{00}\,. \tag{33}$$

This is consistent with experiments.

(b) The magnitude of η_{+-} is given roughly by

$$\eta_{+-}\simeq\sin\phi\left(\frac{m_{\chi'}-m_\chi}{m_\chi}\right). \tag{34}$$

So, if χ' and χ are nearly degenerate, with their mass difference arising out of gauge interactions, then one has a link between the magnitude of *CP* violation and χ-χ' symmetry breaking. In this case, in order for *CP* violation to vanish entirely in the limit of $m_\chi = m_{\chi'}$, the GIM mechanism must be implemented through a different charm quark $\mathcal{P}'$ (rather than χ), i.e., instead of

$$\begin{pmatrix}\chi_L\\ \lambda_L(\theta_C)\end{pmatrix}$$

doublet in Eq. (9), we must have

$$\begin{pmatrix}\mathcal{P}'_L\\ \lambda_L(\theta_C)\end{pmatrix},$$

where

$$\begin{pmatrix}\chi_L\\ \mathfrak{N}'_L\end{pmatrix}$$

becomes the new B doublet. We could then choose ϕ to be maximal, i.e., $\pi/4$, so that the entire *CP* violation comes from χ-χ' mass splitting. If, however, $m_{\chi'}\gg m_{\chi'}$ then ϕ must be very small ($\approx 10^{-4}$–10^{-5}).

(c) Coming to the question of the electric dipole moment (edm) of the neutron, d_n, we find that the edm of the λ quark $d_\lambda \approx eG_F\sin\phi\cos\theta_C\, m_\chi$ and therefore, a rough estimate[38] for d_n would give

$$\begin{aligned}d_n &\approx \frac{eG_F\sin\phi\, m_\chi}{10}\\ &\approx 10^{-24}\ e\,\text{cm}\,.\end{aligned} \tag{35}$$

This is also roughly consistent with present experiments. Further details and other phenomenological consequences of this model will be the subject of a forthcoming paper.[27]

VII. CONCLUSION AND OUTLOOK

In conclusion, we would like to stress that the new charm-changing right-handed currents have very interesting theoretical as well as experimental implications. On the theoretical side, one has an understanding of (a) the $\Delta I=\frac{1}{2}$ rule for nonleptonic *CP*-conserving decays, (b) the $\Delta I=\frac{1}{2}$ rule for nonleptonic *CP*-violating decays and consequently one obtains the relation $\eta_{+-}\simeq\eta_{00}$ for $K_L\to 2\pi$ decays as well as $\eta_{+-0}\approx\eta_{000}$ for $K_S\to 3\pi$ decays. On the experimental side, we have stressed that the neutrino experiments involving both charged and neutral currents will be crucial in testing the various kinds of models. In particular, if $\sigma^{\bar\nu}/\sigma^\nu$ remains near its present value of 0.33 ± 0.08 even in the range of neutrino energies $E_\nu\sim 200$ GeV, the models of Refs. 12, 17, 18, and 20 will be incompatible with experiments and only the models (a), (b), and (c) of Ref. 19 and the present paper will be acceptable. We have then argued that the $\Delta S=2$ transitions do not provide any meaningful constraints on model building. We also exhibit the character of $\Delta C=1$ nonleptonic decays[33] with new currents.

Finally, we have tried to work within a framework with a 32-fold set of fermions and have argued that the ultimate unification of all matter and forces may take place within an $SU(4)\times SU(4)'$ gauge theory framework. However, other gauge groups such as $SU(2)_A \times SU(2)_B \times SU(4)'$ or $SU(2)\times U(1)\times SU(4)'$ may manifest themselves at various stages of the spontaneous breakdown of the $SU(4)\times SU(4)'$ group. With a 32-fold set of fermions, the question of course arises as to what is the value of the ratio

$$R=\frac{\sigma(e^+e^-\to \text{hadrons})}{\sigma(e^+e^-\to \mu^+\mu^-)}$$

at and above the present SPEAR energies. This question relies heavily on the nature of the quark mass spectrum. Our discussion on CP violation would seem to indicate that at least one of the heavier quarks (χ') must lie near the conventional charm quark χ. It has been argued in Ref. 12 that if we choose the mass matrix for the quarks as

$$\mathfrak{M}_{q'-q^0}=\begin{matrix} & q_L' & q_L \\ q_R' & m_{q'} & \Delta \\ q_R & \Delta' & 0\end{matrix}\;,$$

where q' and q denote symbolically the heavy and light quarks, respectively, one would expect

$$m_q \simeq \frac{\Delta\Delta'}{m_{q'}}\,, \tag{36}$$

i.e., heavy quarks must have an "inverted" mass spectrum. In such a case $m_{\mathcal{P}'}$, $m_{\mathfrak{N}'}$, and $m_{\lambda'}$ are expected to be much bigger than the other five quark masses. At present energies, therefore, the quark contribution to R should be around $\frac{14}{3}$. We will similarly expect (using quark-lepton symmetry) that only one of the heavy leptons is excited at present SPEAR energies (giving rise to the $\mu^{\mp}e^{\mp}$ events). Thus, we predict $R\simeq 5$–6 at present energies, in agreement with observations. The asymptotic value of R can, however, be as big as 9.

What are further avenues of research? One promising line is the idea that there may exist subquarks of which the 32-fold set of fermions considered in this article are bound states.[23] For example, one may contemplate four fermions F and five spin-0 bosons [four bosons for four colors (C) and one (H) for generating heavy quarks out of the FC bound states) as the subquarks out of which all our 32 fermions are made.

Then of course, there is a question of trying to understand the hadronic symmetries within this model and to think of relations between the light and heavy quarks. For example, we found that a nonvanishing CP-violating amplitude is proportional to $(M_\chi - M_{\chi'})$; then to understand the origin of CP-violation we may ask: Is there any approximate degeneracy between any of the heavy quarks and the χ quarks? If so, then the interaction that violates this symmetry may also be responsible for CP violation.

Finally, we would like to point out that, for most considerations in this paper, one can interchange $n_R(\phi)$ and $\lambda_R(\phi)$. As was noted in Ref. 15 and also as is clear from Sec. VI, the nature of CP violation remains unalteres as a result of this and so do the considerations on the unification of coupling constants and neutral- and charged-current neutrino interactions. The only difference is that now renormalization group[18] arguments have to be invoked to understand the $\Delta I=\frac{1}{2}$ rule. Furthermore, in this case, the questions raised about the validity of the scheme of Ref. 15 from a naive current-algebra study of nonleptonic decays may also be avoided.[39]

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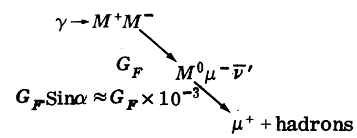

We of course assume that $m_{M^+} > m_{M^0}$. See A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 35, 628 (1975).

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(a) As noted by these authors, there could be a considerable contribution of the $I=\frac{3}{2}$ part with strength G_F or $G_F \sin\theta_C$ coming from sources other than products of weak currents, such as (i) electromagnetism, or (ii) the Higgs part of the Lagrangian in gauge models. The presence of either of these will invalidate the arguments of the above authors.

(b) Secondly, as noted by De Rújula, Georgi, and Glashow,[20] one ought not to ignore the final-state interactions in $K \to 3\pi$ decays.