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Reggeization of the fermion-fermion scattering amplitude in non-Abelian gauge theories

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We have calculated the fermion-fermion scattering amplitude for a non-Abelian gauge theory with SU(N) gauge symmetry in the limit of high energy with fixed momentum transfer through sixth order in the coupling constant. We have kept only the leading logarithmic terms in each order of perturbation theory. In order to avoid the infrared problem, the Higgs mechanism is invoked to give masses to the vector bosons of the theory. We find that the scattering amplitude exponentiates to a Regge form. This result is qualitatively different from an earlier published calculation.

I. INTRODUCTION

A considerable amount of work has been done on the theory of elastic two-body scattering in the limit of infinite s and finite t, where s is the center-of-mass energy squared and t is the four-momentum transfer squared in a collision. Qualitative forms of the scattering amplitude have been proposed based on the Regge model[1] and on the eikonal or diffraction model.[2] These models are built on extrapolations from nonrelativistic potential theory instead of relativistic first principles.

The only theoretical structure thus far developed which incorporates all the basic principles such as superposition, analyticity, and so on is quantum field theory. Because of this fact, explicit calculations based on relativistic field theories are of great interest. In such limits as s becoming infinite with t fixed, where the effects of resonances and other local properties should be smoothed out, one can hope to see general features suggested by field theories. Much work has already been done along this line. In most studies one keeps only the leading asymptotic term in each order of perturbation theory (the "leading ln" approximation). The hope is that the nonleading terms will not affect the qualitative conclusions.

The classic example of this approach is the analysis of "ladder" diagrams in ϕ^3 theory.[3] The Regge behavior has been shown to appear when the ladder diagrams in the t channel are summed. Many calculations of this nature have also been done for massive QED.[4] However, ϕ^3 theory or massive QED is not expected to be a realistic model for hadrons, because it does not take internal symmetries into account. Thus, it is important to look at the more realistic non-Abelian gauge theories.

In this paper we consider the scattering of two fermions for large s and fixed t for a class of non-Abelian gauge theories with SU(N) symmetry in which the vector mesons receive a mass μ through the mechanism of spontaneous symmetry breakdown. We find that to sixth order in the coupling constant the scattering amplitude exponentiates to a Regge form. Some time ago, Nieh and Yao[5] published the results of a similar calculation. Our results are qualitatively different from theirs. In extracting the behavior of individual diagrams as $s\to\infty$, we have used either the standard techniques in parameter space[6] or the infinite-momentum technique of Chang and Ma[7] or occasionally both. The main results of this paper have been reported in a short communication.[8]

An outline of our paper is as follows: In Sec. II we discuss the model we use, present the relevant notation and kinematics, and give the results for orders g^2 and g^4, where g is the coupling constant. In Sec. III we demonstrate some remarkable cancellations which occur in order g^6. In Sec. IV we calculate the leading behavior of the transition amplitude in order g^6. In Sec. V we consider the generalization of our results from SU(2) to SU(N) symmetry. And in Sec. VI we present our conclusions and summary. Six appendixes are included, where detailed analysis is performed or indicated.

II. NOTATION, KINEMATICS, THE MODEL, AND LOW-ORDER RESULTS

The components of a four-vector V_μ will be written as $(V_+, \vec{V}_\perp, V_-)$, which are related to the usual components by $V_\pm = V^0 \pm V^3$ and $\vec{V}_\perp = (V^1, V^2)$. The invariant product takes the form

$$V\cdot W = \tfrac{1}{2}V_+W_- + \tfrac{1}{2}V_-W_+ - \vec{V}_\perp\cdot\vec{W}_\perp\,.$$

The Dirac matrices in this representation have the following properties:

$$\{\gamma_+,\gamma_-\}=4\,,$$
$$\{\gamma_\pm,\vec{\gamma}_\perp\}=0\,,$$
$$\gamma_\pm{}^2=0\,.$$

We consider the scattering of two on-mass-shell fermions $p_1+p_2\to p_3+p_4$ in the limit $s\gg|t|,m^2,\mu^2$, where $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2<0$, m^2 is the fermion mass, and μ^2 is the vector-boson mass.

For convenience we choose (see Fig. 1)

$$p_1=P-\tfrac{1}{2}k\,,\quad p_2=P'+\tfrac{1}{2}k\,,$$
$$p_3=P+\tfrac{1}{2}k\,,\quad p_4=P'-\tfrac{1}{2}k\,,$$

where $k=(0,\vec{k}_\perp,0)$, and, as $s\to\infty$,

$$P=(\sqrt{s}\,,\vec{0},(m^2-\tfrac{1}{2}k^2)/\sqrt{s})+O((1/\sqrt{s}\,,\vec{0},[1/\sqrt{s}\,]^3))$$

and (2.1)

$$P'=((m^2-\tfrac{1}{2}k^2)/\sqrt{s}\,,\vec{0},\sqrt{s}\,)+O(([1/\sqrt{s}\,]^3,\vec{0},1/\sqrt{s}\,))\,.$$

Note that $t=k^2=-\vec{k}_\perp{}^2$, while $s=(P+P')^2$, and that $P^2+\frac{1}{2}k^2=m^2$ and $P'^2+\frac{1}{2}k^2=m^2$.

For concreteness we will restrict our considerations to the case of SU(2) gauge symmetry for the present. The generalization to SU(N) symmetry will be given in Sec. V. We start out with the usual Yang-Mills Langrangian and, in order to avoid the infrared problem, introduce a complex scalar doublet and invoke the Higgs mechanism to give masses to the vector mesons. This is a renormalizable model originally due to 't Hooft.[9] After the spontaneous symmetry breakdown, the vector mesons receive equal masses μ. In addition, a physical scalar particle with arbitrary mass M and two ghost particles appear. Fermions are then added to the theory and coupled to the massive vector mesons by the usual minimal-coupling assumption. The Feynman rules for this model are those of the usual Yang-Mills theory with the addition of a number of vertices and propagators associated with the scalar and ghost particles. However, as it turns out, for the purposes of our leading-ln calculation the only Feynman rules we will need in addition to the usual ones are the two illustrated in Fig. 2. The ghost particles do not enter to the leading ln. We work exclusively in the 't Hooft-Feynman gauge.

Let us define the invariant transition amplitude T by

$$\langle p_3,p_4|(S-1)|p_1,p_2\rangle=-iN(2\pi)^4\delta(p_1+p_2-p_3-p_4)T\,,$$

where N is a normalization constant. T may then be decomposed into an isospin-flip part (T^{f}) and an isospin-nonflip part (T^{nf}),

$$T=T^{\text{f}}(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}+T^{\text{nf}}\delta_{i_1,i_3}\delta_{i_2,i_4}\,,$$

where i_j is the isospin index for the particle with momentum p_j and where τ is a Pauli matrix. Lower case Latin indices will be used for isospin indices, while lower case Greek letters will be used for four-vector indices.

In lower order (g^2) we have the Born term, which is trivial,

$$T^{(2)}\equiv T_{\text{Born}}$$
$$=(g^2/8)\frac{1}{-t+\mu^2}\frac{s}{m^2}(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}\delta_{\lambda_1,\lambda_3}\delta_{\lambda_2,\lambda_3}\,,\tag{2.2}$$

where λ_j is the helicity of the particle with momentum p_j. Note that T_{Born} is proportional to $\delta_{\lambda_1,\lambda_3}\delta_{\lambda_2,\lambda_4}$. This is a simple consequence of the facts that, as $s\to\infty$,

$$\bar{u}_{\lambda_3}(P+\tfrac{1}{2}k)\gamma_+u_{\lambda_1}(P-\tfrac{1}{2}k)=\frac{\sqrt{s}}{m}\delta_{\lambda_1,\lambda_3}\tag{2.3}$$

and

$$\bar{u}_{\lambda_4}(P'-\tfrac{1}{2}k)\gamma_-u_{\lambda_2}(P'+\tfrac{1}{2}k)=\frac{\sqrt{s}}{m}\delta_{\lambda_2,\lambda_4}\,.\tag{2.4}$$

Because of Eqs. (2.3) and (2.4), one also finds in higher orders that the transition amplitude is

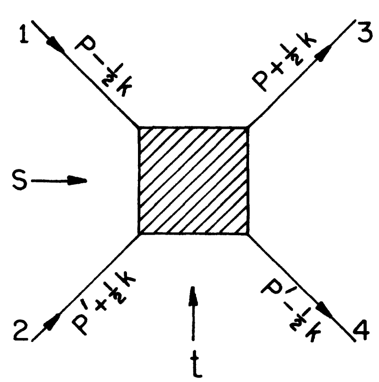

FIG. 1. Kinematics of the reaction $P_1+P_2\to P_3+P_4$.

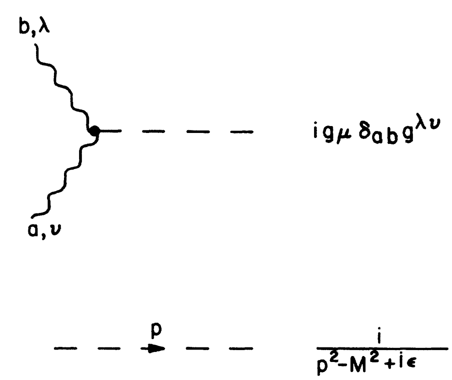

FIG. 2. Additional Feynman rules needed.

proportional to $\delta_{\lambda_1,\lambda_3}\delta_{\lambda_2,\lambda_4}$ as $s\to\infty$. Therefore, we shall henceforth omit the factor $\delta_{\lambda_1,\lambda_3}\delta_{\lambda_2,\lambda_4}$ with the understanding that we are always referring to the helicity-nonflip amplitude.

In order g^4 the leading diagrams are those numbered 1 and 2 in Fig. 3 (hereafter denoted 3.1 and 3.2). The amplitudes for these diagrams are identical to the amplitudes for the corresponding diagrams in QED,[7] except for the presence of Pauli matrices. Therefore, we simply state the answer, which is

$$T^{f}_{3.1-3.2} \equiv T^{f}_{3.1}+T^{f}_{3.2} = -(g^4/4)(2\pi)^{-4}\pi^2 \frac{s}{m^2}\ln(s/m^2)K(t) \tag{2.5}$$

and

$$T^{nf}_{3.1-3.2} \equiv T^{nf}_{3.1}+T^{nf}_{3.2} = -\tfrac{3}{4}i\pi g^4(2\pi)^{-4}\pi^2 \frac{s}{m^2}(\tfrac{1}{4})K(t)\,, \tag{2.6}$$

where

$$K(t) = \int_0^1 \frac{d\alpha_1 d\alpha_2\,\delta(\alpha_1+\alpha_2-1)}{(-t)\alpha_1\alpha_2+\mu^2-i\epsilon} = \frac{1}{\pi}\int \frac{d^2q_\perp}{[(\vec{q}+\frac{1}{2}\vec{k})_\perp^{\,2}+\mu^2-i\epsilon][(\vec{q}-\frac{1}{2}\vec{k})_\perp^{\,2}+\mu^2-i\epsilon]}\,. \tag{2.7}$$

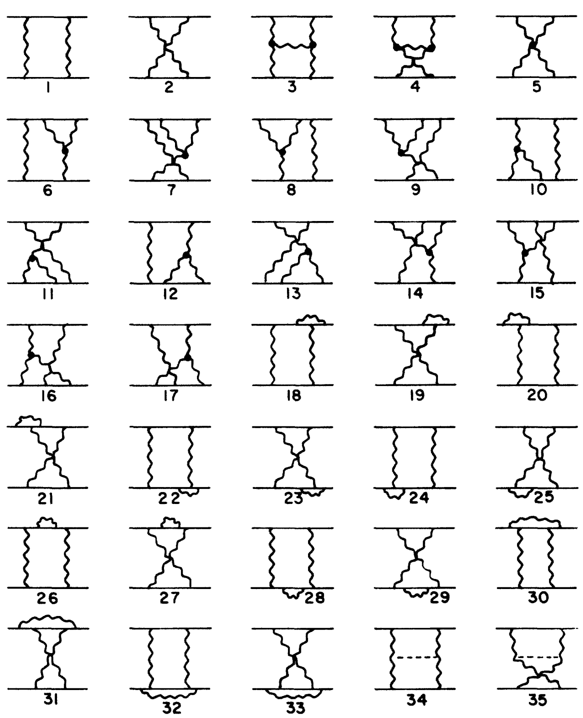

FIG. 3. The leading diagrams in $O(g^4)$ and $O(g^6)$.

The presence of Pauli matrices in the amplitudes $T_{3.1}$ and $T_{3.2}$ simply gives rise to numerical factors in $T^{f}_{3.1-3.2}$ and $T^{nf}_{3.1-3.2}$, which are derived in Appendix A. Note that $T^{nf}_{3.1-3.2}$ is smaller by a factor $\ln(s)$ than $T^{f}_{3.1-3.2}$. So $T^{nf}_{3.1-3.2}$ is negligible in the leading-ln approximation. Thus in order g^4 we have

$$T^{(4)} = -(g^4/4)(2\pi)^{-4}\pi^2\frac{s}{m^2}\ln(s/m^2)K(t)(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4} + O(s)\,. \tag{2.8}$$

Also, let us point out that, to the leading ln, $\ln(s/m^2)=\ln(s/s_0)$, where s_0 is an arbitrary constant.

III. CANCELLATIONS IN ORDER g^6

In order g^6 the leading diagrams are found to be those numbered 3–35 in Fig. 3. For each diagram in Fig. 3 there is an "isospin factor" most easily defined by an example. For diagram 3 the isospin factor is given by

$$(\tau_a\tau_b)_{i_1,i_3}(\tau_c\tau_d)_{i_2,i_4}\epsilon_{dbe}\epsilon_{ace}\,,$$

where

$$[\tau_a,\tau_b]=2i\epsilon_{abc}\tau_c\,.$$

The Pauli matrices are, of course, associated with the boson-fermion vertices, while the ϵ_{abc} come from the three- and four-boson vertices. These isospin factors are computed in Appendix A.

We will denote the sum of the diagrams labeled $m, m+1, \ldots, n-1, n$ in Fig. 3 by $T_{3.m-3.n}$. Of the 33 diagrams in $O(g^6)$ only eight need be computed. The remaining diagrams may be obtained from these eight by symmetry considerations. For example, diagrams 3.6 and 3.7 give identical amplitudes, while the amplitude for diagram 3.10 may be obtained from that of diagram 3.6 by the substitution $s\to u \sim -s$ along with a change in the isospin factor of the amplitude. In general the change in the isospin factor of a "crossed" diagram relative to an "uncrossed" diagram, e.g., diagram 3.7 vs diagram 3.6, results in an over-all minus sign in the isospin-flip amplitude of the crossed diagram relative to that of the uncrossed diagram, as seen in Appendix A. That is to say, one has

$$T^{f}_{\text{crossed}}(s) = -T^{f}_{\text{uncrossed}}(u) \approx -T^{f}_{\text{uncrossed}}(-s)\,, \tag{3.1}$$

while

$$T^{nf}_{\text{crossed}}(s) = T^{nf}_{\text{uncrossed}}(u) \approx T^{nf}_{\text{uncrossed}}(-s)\,. \tag{3.2}$$

Hence we see in general that if $T_{\text{uncrossed}}(s) \propto s\ln^n(s)$, then

$$T^{f}_{\text{uncrossed}}(s)+T^{f}_{\text{crossed}}(s) \propto s\ln^n(s)\,, \tag{3.3}$$

while

$$T^{\rm nf}_{\rm uncrossed}(s) + T^{\rm nf}_{\rm crossed}(s) \propto s\ln^{n-1}(s)\,. \qquad (3.4)$$

So for the sum of a given diagram and its $s \leftrightarrow u$ crossed counterpart the isospin-nonflip amplitude is less leading by a factor of ln$s$ than the isospin-flip amplitude.

For convenience, the amplitude for diagram number n in Fig. 3 will be written as $T_{3.n}(s) = G_{3.n}F_{3.n}(s)$, where $G_{3.n}$ is the isospin factor given in Appendix A. Then we have that

$$F_{\rm crossed}(s) = F_{\rm uncrossed}(u) \approx F_{\rm uncrossed}(-s)\,. \qquad (3.5)$$

First let us consider the "ladder" diagrams numbered 3, 4, and 5 in Fig. 3. In Fig. 4 we illustrate the labeling of the internal momenta for these diagrams. Individually the diagrams 3.3, 3.4, and 3.5 are known to go as s^2, but the s^2 behavior cancels in the sum of the three amplitudes. To obtain the leading behavior of the sum, it is best to add the amplitudes for the diagrams together before evaluating the integrals and to algebraically cancel out the terms which give rise to the s^2 behavior of the individual diagrams. Now to do this, we note that the s^2 behavior of the individual diagrams may be obtained correctly by making the approximations

$$\begin{aligned}\bar{u}_3(P+\tfrac{1}{2}k)\gamma_\nu(\not P+\not r+m)\gamma_\mu u_1(P-\tfrac{1}{2}k) &\approx \bar{u}_3(P+\tfrac{1}{2}k)\gamma_+(\not P+\not r)\gamma_+ u_1(P-\tfrac{1}{2}k)\delta_\mu^+\delta_\nu^+ \\ &= \frac{2\sqrt{s}}{m}(\sqrt{s}+r_+)\delta_\mu^+\delta_\nu^+ \end{aligned} \qquad (3.6)$$

and

$$\begin{aligned}\bar{u}_4(P'-\tfrac{1}{2}k)\gamma_\rho(\not P'+\not q+m)\gamma_\lambda u_2(P'+\tfrac{1}{2}k) &\approx \bar{u}_4(P'-\tfrac{1}{2}k)\gamma_-(\not P'+\not q)\gamma_- u_2(P'+\tfrac{1}{2}k)\delta_\rho^-\delta_\lambda^- \\ &= \frac{2\sqrt{s}}{m}(\sqrt{s}+q_-)\delta_\rho^-\delta_\lambda^- \end{aligned} \qquad (3.7)$$

in the numerators of the amplitudes $T_{3.3}$, $T_{3.4}$, and $T_{3.5}$. This is true because of the fact that in momentum space the s^2 behavior of $T_{3.3}$, $T_{3.4}$, or $T_{3.5}$ is due to a linear divergence of the integrals over the longitudinal momenta q_- and r_+, i.e., if one would use the infinite-momentum technique[7] to calculate the asymptotic behavior of $T_{3.3}$, $T_{3.4}$, or $T_{3.5}$, one would find that the amplitude is proportional to

$$\int_0^{\sqrt{s}} dq_-(\sqrt{s}+q_-)\int_0^{\sqrt{s}} dr_+(\sqrt{s}+r_+) \propto s^2\,.$$

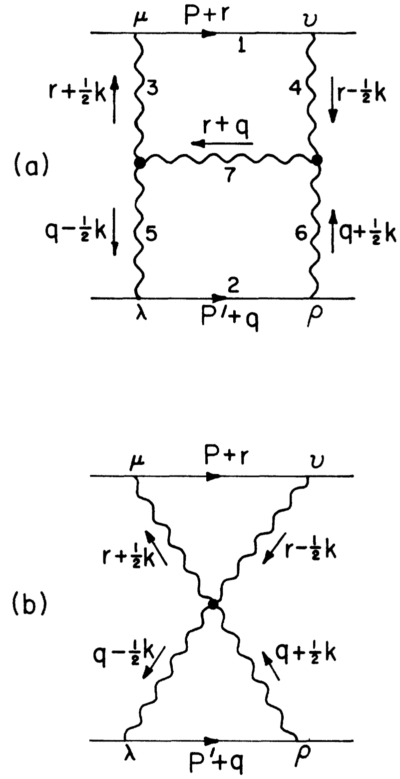

FIG. 4. Labeling of internal momenta, Feynman parameters, and Lorentz indices for diagrams 3 and 5 of Fig. 3.

We shall refer to the approximation (3.6) plus (3.7) as the $++--$ approximation. It corresponds to having boson-fermion couplings proportional to γ_+ along the upper fermion line through which a large P_+ flows, and having boson-fermion couplings proportional to γ_- along the lower fermion line through which a large P_- flows. Then in the $++--$ approximation $T_{3.3-3.5}$ is given by

$$T^{\rm f}_{3.3-3.5} = 2[F'_{3.3}(s) - F'_{3.3}(-s)] \qquad (3.8)$$

and

$$T^{\rm nf}_{3.3-3.5} = -6[F'_{3.3}(s) + F'_{3.3}(-s)], \qquad (3.9)$$

where

$$\begin{aligned}F'_{3.3}(s) = -\tfrac{1}{16}g^6\frac{s}{m^2}(2\pi)^{-8}\int d^4r\, d^4q(\sqrt{s}+q_-)(\sqrt{s}+r_+)\{2r^2+2q^2-3q_+q_- - 3r_+r_- - 3r_-q_+ - k^2-\mu^2\} \\ \times[(P+r)^2-m^2+i\epsilon]^{-1}[(P'+q)^2-m^2+i\epsilon]^{-1}[(q+r)^2-\mu^2+i\epsilon]^{-1} \\ \times[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1},\end{aligned} \qquad (3.10)$$

and where the factors 2 and -6 in (3.8) and (3.9), respectively, are isospin factors.

$F'_{3.3}(s)$ in Eq. (3.10) has been obtained by inserting $1=[(r+q)^2-\mu^2+i\epsilon]/[(r+q)^2-\mu^2+i\epsilon]$ in the amplitude for graph 3.5 and then adding that part of $T_{3.5}$ which has the same isospin structure as that of graph 3.3 to $T_{3.3}$. With the aid of Appendix A, and noting that $F_{3.4}(s)=F_{3.3}(-s)$, one then obtains the results (3.8), (3.9), and (3.10).

Notice that in the curly brackets in Eq. (3.10) there is no term proportional to q_-r_+. Such a term would give rise to s^2 behavior, but such terms have been canceled out by summing the diagrams 3.3–3.5.

To the leading ln we can make the further approximations in (3.10)

$$\sqrt{s}+q_-\approx\sqrt{s}$$

and (3.11)

$$\sqrt{s}+r_+\approx\sqrt{s}\,.$$

The combination of the $++--$ approximation plus the approximations (3.11) will be referred to as the "leading-particle approximation" for identification purposes. It corresponds to writing, e.g.,

$$\not{p}+\not{r}+m\approx\tfrac{1}{2}\sqrt{s}\,\gamma_-$$

and (3.12)

$$\not{p}'+\not{q}+m\approx\tfrac{1}{2}\sqrt{s}\,\gamma_+$$

in all of the numerators of the fermion propagators in a given amplitude. The validity of the leading-particle approximation for obtaining the leading behavior of a given diagram can most easily be seen in momentum space where the ln's are seen to arise from integrals of the form $\int_{1/\sqrt{s}}^{\sqrt{s}}dq_\pm/q_\pm=\ln s$. Replacing a factor of $\sqrt{s}$ in the numerator by a factor of $q_\pm$ would give an integral of the form

$$\int_{1/\sqrt{s}}^{\sqrt{s}}(q_\pm/\sqrt{s})dq_\pm/q_\pm\sim 1\,,$$

which is down by a factor of $\ln s$. In parameter space, the leading-particle approximation corresponds to neglecting terms of the form $q^s_\pm/\sqrt{s}$ relative to 1, where q^s is the constant "shift" term arising from the change of variables $q\to q'+q^s$ made to diagonalize the denominator of the parametric integral. Usually $q^s_\pm=\sqrt{s}\,a$, where a is a function of the Feynman parameters. Then $q^s_\pm/\sqrt{s}$ will be $\ll 1$ if $a\ll 1$ in the regions of integration in parameter space which contribute to the leading behavior. This is found to be the case. The important regions of integration always correspond to end-point contributions.

The value of $F'_{3.3}(s)$ to the leading ln as $s\to\infty$ may be obtained straightforwardly by standard techniques. In Appendix B we caclulate the value of $F'_{3.3}(s)$, and from Appendix B and Eqs. (3.8) and (3.9) we obtain

$$T^{\mathrm{f}}_{3.3-3.5}=-g^6(2\pi)^{-8}\pi^4\frac{1}{3!}\frac{s}{m^2}\ln^3(s/m^2)K(t)+O(s\ln^2 s) \tag{3.13}$$

and

$$T^{\mathrm{nf}}_{3.3-3.5}=-\tfrac{3}{2}i\pi g^6(2\pi)^{-8}\pi^4\frac{1}{2!}\frac{s}{m^2}\ln^2(s/m^2)K(t)+O(s\ln s)\,. \tag{3.14}$$

Note that the nonflip amplitude is less leading by a factor of $\ln s$ than the flip amplitude and is therefore negligible in the leading-ln approximation. However, we will find that the $s\ln^3 s$ behavior of the flip amplitude in Eq. (3.13) is canceled out by the contributions from the radiative correction graphs 3.6–3.17. Therefore, to show that the nonflip amplitude is negligible in $O(g^6)$ it will be necessary to show that the $s\ln^2 s$ behavior in Eq. (3.14) is canceled out by the contributions from the radiative correction graphs 3.6–3.17 in Fig. 3.

Let us consider first the diagrams 3.6–3.13. The labeling of the internal momenta for diagram 3.6 is illustrated in Fig. 5(a). One finds, using Appendix A and noticing that $F_{3.6}(s)=F_{3.7}(-s)=F_{3.8}(s)=F_{3.9}(-s)=F_{3.10}(s)=F_{3.11}(-s)=F_{3.12}(s)=F_{3.13}(-s)$, that

$$T^{\mathrm{f}}_{3.6-3.13}=4(-2i)(-2)[F_{3.6}(s)-F_{3.6}(-s)], \tag{3.15}$$

and that

$$T^{\mathrm{nf}}_{3.6-3.13}=4(-2i)(3)[F_{3.6}(s)+F_{3.6}(-s)], \tag{3.16}$$

where, in the leading-particle approximation,

$$F_{3.6}(s)=\frac{i}{32}g^6\frac{s^{5/2}}{m^2}(2\pi)^{-8}\int d^4q\,d^4r(2r_-+q_-)[(P-q)^2-m^2+i\epsilon]^{-1}[(P+r)^2-m^2+i\epsilon]^{-1}$$
$$\times[(P'+q)^2-m^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(q+r)^2-\mu^2+i\epsilon]^{-1}\,. \tag{3.17}$$

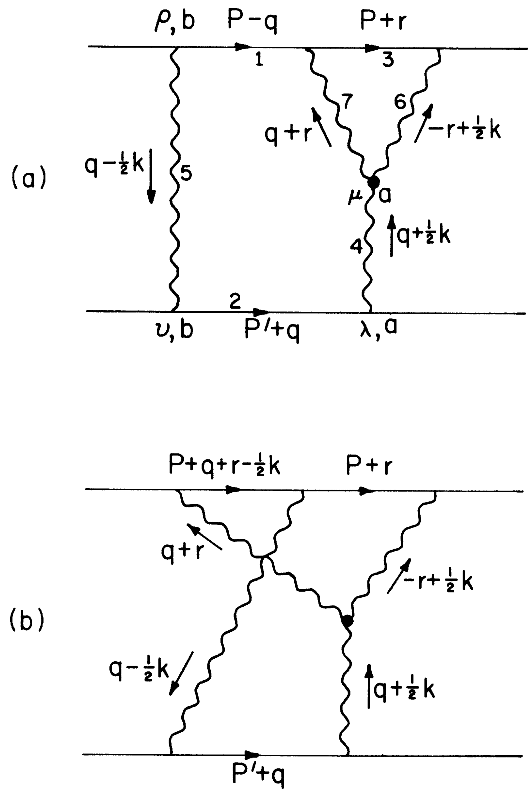

FIG. 5. Labeling of internal momenta, Feynman parameters, and Lorentz indices for diagrams 6 and 14 of Fig. 3.

We would like to point out that in the leading-particle approximation, which correctly gives the $s\ln^3 s$ behavior of $F_{3.6}(s)$, $F_{3.6}(s)$ contains no ultraviolet-divergent integration, and thus no renormalization is required. We would like to emphasize, however, that after the $s\ln^3 s$ has been shown to cancel out in $O(g^6)$ it will be essential to properly renormalize the radiative correction graphs in order to obtain the correct leading behavior in $O(g^6)$. In Appendix B, $F_{3.6}(s)$ is evaluated. After plugging its value into (3.15) and (3.16), we find

$$T^{\rm f}_{3.6-3.13} = g^6(2\pi)^{-8}\pi^4\frac{1}{3!}\frac{s}{m^2}\ln^3(s/m^2)K(t) + O(s\ln^2 s) \tag{3.18}$$

and

$$T^{\rm nf}_{3.6-3.13} = \tfrac{3}{4}i\pi g^6(2\pi)^{-8}\pi^4\frac{1}{2!}\frac{s}{m^2}\ln^2(s/m^2)K(t) + O(s\ln s)\,. \tag{3.19}$$

Upon comparing Eqs. (3.18) and (3.13) we see that

$$T^{\rm f}_{3.3-3.13} = O(s\ln^2 s)\,. \tag{3.20}$$

The $\ln^3 s$ behavior has been canceled out in the sum. Also, we see from (3.19) and (3.14) that

$$T^{\rm nf}_{3.3-3.13} = O(s\ln^2 s)\,. \tag{3.21}$$

Thus, if the isospin-nonflip amplitude is to be negligible in $O(g^6)$ as it was in $O(g^4)$, there will have to be an additional contribution to $T^{\rm nf}$ to cancel out the $\ln^2 s$ behavior of $T^{\rm nf}_{3.3-3.13}$. The necessary additional contribution comes from diagrams 3.14–3.17 in Fig. 3. These diagrams contribute only to the isospin-nonflip amplitude (see Appendix A), and one finds that

$$T^{\rm f}_{3.14-3.17} = 0 \tag{3.22a}$$

and

$$T^{\rm nf}_{3.14-3.17} = 2(6i)[F_{3.14}(s) + F_{3.14}(-s)]\,, \tag{3.22b}$$

where, in the leading-particle approximation,

$$\begin{aligned}F_{3.14}(s) = \frac{i}{32}g^6\frac{s^{5/2}}{m^2}(2\pi)^{-8}\int d^4q\,d^4r(2r_- + q_-)&[(P+q+r-\tfrac{1}{2}k)^2 - m^2 + i\epsilon]^{-1}[(P+r)^2 - \mu^2 + i\epsilon]^{-1}\\ &\times[(P'+q)^2 - m^2 + i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}\\ &\times[(q+r)^2 - \mu^2 + i\epsilon]^{-1}\,.\end{aligned} \tag{3.23}$$

See Fig. 5(b) for the labeling of the internal momenta.

We need not evaluate $F_{3.14}(s)$ explicitly, but can use the following device to obtain its value. Making the usual infinite-momentum technique approximations,[9] e.g.

$$[(P-q)^2 - m^2 + i\epsilon]^{-1} \approx (\sqrt{s})^{-1}\left[-q_- + \frac{q^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + O(1/s) + i\epsilon\right]^{-1}, \tag{3.24}$$

we write

$F_{3.6}(s)+F_{3.7}(s)+F_{3.14}(s)+F_{3.15}(s)+F_{3.8}(s)+F_{3.9}(s)$

$$= \frac{i}{32} g^6 \frac{s}{m^2}(2\pi)^{-8}\int d^4r\, d^4q(2r_- + q_-)[(q+r)^2 - \mu^2 + i\epsilon]^{-1}$$
$$\times[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}\left(q_+ + \frac{q^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}$$
$$\times\left\{\left(-q_- + \frac{q^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}\left(r_- + \frac{r^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}\right.$$
$$+\left[q_- + r_- + \frac{(q+r-\frac{1}{2}k)^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right]^{-1}\left(r_- + \frac{r^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}$$
$$\left.+\left[q_- + r_- + \frac{(q+r-\frac{1}{2}k)^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right]^{-1}\left(q_- + \frac{q^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}\right\} + (s \to u) + O(s\ln s)\,. \tag{3.25}$$

In the curly brackets in Eq. (3.25) we have the sum $\sum$ symbolically shown in Fig. 6,

$$\sum = \left(-q_- + \frac{q^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}\left(r_- + \frac{r^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1} + \left[q_- + r_- + \frac{(q+r-\frac{1}{2}k)^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right]^{-1}\left(r_- + \frac{r^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}$$
$$+\left[q_- + r_- + \frac{(q+r-\frac{1}{2}k)^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right]^{-1}\left(q_- + \frac{q^2}{\sqrt{s}} - \frac{\frac{1}{2}k^2}{\sqrt{s}} + i\epsilon\right)^{-1}. \tag{3.26}$$

Neglecting the $O(1/\sqrt{s}\,)$ terms one finds

$$\sum \approx (r_- + i\epsilon)^{-1}[(q_- + i\epsilon)^{-1} + (-q_- + i\epsilon)^{-1}] = (-2\pi i)\delta(q_-)(r_- + i\epsilon)^{-1}\,. \tag{3.27}$$

As is well known in QED studies[7] the presence of a $\delta(q_-)$ in an amplitude will in general reduce the amplitude by a factor of ln$s$. In fact, one easily sees by substituting Eq. (3.27) into Eq. (3.25) that

$$F_{3.6}(s)+F_{3.7}(s)+F_{3.14}(s)+F_{3.15}(s)+F_{3.8}(s)+F_{3.9}(s) = O(s\ln s)\,. \tag{3.18}$$

So from Eqs. (3.28), (3.22), (3.19), and (3.16), and the facts that $F_{3.7}(s)=F_{3.6}(-s)=F_{3.8}(-s)=F_{3.9}(s)$ and $F_{3.15}(s)=F_{3.14}(-s)$, we obtain

$$T^{\text{nf}}_{3.14-3.17} = T^{\text{nf}}_{3.6-3.13} + O(s\ln s)$$
$$= \tfrac{3}{4}\, i\pi g^6(2\pi)^{-8}\pi^4 \frac{1}{2!}\frac{s}{m^2}\ln^2(s/m^2)K(t) + O(s\ln s)\,. \tag{3.29}$$

Upon comparing Eqs. (3.29), (3.19), and (3.14), we see that

$$T^{\text{nf}}_{3.3-3.17} = O(s\ln s)\,. \tag{3.30}$$

The $\ln^2 s$ behavior has been canceled out in the sum. Thus we have demonstrated that the isospin-nonflip amplitude is less leading by a factor of ln$s$ than the isospin-flip amplitude in $O(g^6)$ and is therefore negligible in the leading-ln approximation.

Hence, we have shown in this section that

$$T^{(6)} \propto \ln^2 s(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4} + O(s\ln s)\,.$$

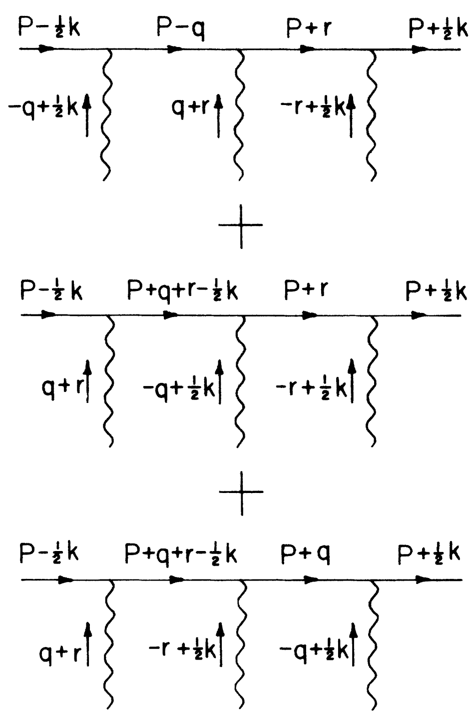

FIG. 6. Symbolic illustration of a sum occurring in Eq. (3.26).

IV. CALCULATION OF LEADING BEHAVIOR IN ORDER g^6

In Sec. III we saw some remarkable cancellations take place, which resulted in the conclusion that the transition amplitude in order g^6 was proportional to $s\ln^2 s$. Now we must calculate the coefficient of $s\ln^2 s$ in this order. To do this, we will first write the amplitudes for the graphs 3.3–3.13 in the leading-particle approximation [see Eq. (3.12)], add the amplitudes together before evaluating the integrals, and cancel out algebraically the terms which give rise to the $\ln^3 s$ behavior. After evaluating $T^{\mathrm{f}}_{3.3-3.13}$ in the leading-particle approximation, we must consider the contributions to the $\ln^2 s$ behavior which come from terms in $T^{\mathrm{f}}_{3.3-3.13}$ not given by the leading-particle approximation. In particular, we must properly consider the renormalization of the vertex correction graphs 3.6–3.13. Finally, we will have to take into account the contributions of the diagrams numbered 18–35 in Fig. 3. These diagrams also contribute $\ln^2 s$ to the isospin-flip amplitude.

A. Contribution from diagrams 3.3–3.13 in the leading-particle approximation

Consider the amplitude $F'_{3.3}(s)$ given in Eq. (3.10). The denominator of this expression is invariant under the combined interchanges $q_+ \leftrightarrow r_-$, $q_- \leftrightarrow r_+$, and $\vec{q}_\perp \leftrightarrow \vec{r}_\perp$, as follows from Eq. (2.1). Thus, in the first terms in the curly brackets in Eq. (3.10) we can replace q^2 by r^2 and q_+q_- by r_+r_-. Then from Eqs. (3.8), (3.10), (3.15), and (3.17) we find that, in the leading-particle approximation,

$$
\begin{aligned}
T^{\mathrm{f}}_{3.3-3.13} = -g^6 \frac{s^{5/2}}{m^2}(2\pi)^{-8}\int d^4r\, d^4q \left\{\frac{1}{\sqrt{s}}\,\tfrac{1}{4}(4r^2-6r_+r_- -3r_-q_+ -k^2-\mu^2)[(P-q)^2-m^2]+(2r_-+q_-)[(r+\tfrac{1}{2}k)^2-\mu^2]\right\} \\
\times[(P-q)^2-m^2+i\epsilon]^{-1}[(P+r)^2-m^2+i\epsilon]^{-1}[(P'+q)^2-m^2+i\epsilon]^{-1}[(q+r)^2-\mu^2+i\epsilon]^{-1} \\
\times[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}\,.
\end{aligned}
\tag{4.1}
$$

We shall refer to $T^{\mathrm{f}}_{3.3-3.13}$ evaluated in the leading-particle approximation as $A(s)$. In the curly brackets in Eq. (4.1), we have the sum

$$
\begin{aligned}
&\frac{1}{\sqrt{s}}\,\tfrac{1}{4}(4r^2-6r_+r_- -3r_-q_+ -k^2-\mu^2)[(P-q)^2-m^2]+(2r_-+q_-)[(r+\tfrac{1}{2}k)^2-\mu^2] \\
&\quad=[-\vec{r}_\perp^{\,2}-\tfrac{1}{2}r_+r_- -\tfrac{3}{4}r_-q_+ -\tfrac{1}{4}(-\vec{k}_\perp^{\,2}+\mu^2)][-q_-+O(1/\sqrt{s})]+(2r_-+q_-)(r_+r_- -\vec{r}_\perp^{\,2}-\vec{r}_\perp\cdot\vec{k}_\perp-\tfrac{1}{4}\vec{k}_\perp^{\,2}-\mu^2) \\
&\quad=q_-[\tfrac{3}{2}r_+r_- -\vec{r}_\perp\cdot\vec{k}_\perp-\tfrac{1}{4}(2\vec{k}_\perp^{\,2}+3\mu^2)+\tfrac{3}{4}r_-q_+]+2r_-[r_+r_- -\vec{r}_\perp^{\,2}-\vec{r}_\perp\cdot\vec{k}_\perp-\tfrac{1}{4}(\vec{k}_\perp^{\,2}+4\mu^2)]+O(1/\sqrt{s})\,.
\end{aligned}
\tag{4.2}
$$

The $O(1/\sqrt{s})$ terms can be shown not to contribute to the leading behavior of $A(s)$. Notice in Eq. (4.2) that a term in $q_-\vec{r}_\perp^{\,2}$ has been canceled out in the sum. Such a term would have given rise to $\ln^3 s$ behavior in $A(s)$. After combining Eq. (4.2) with Eq. (4.1), one can evaluate the leading behavior of $A(s)$ by standard techniques. This is done in Appendix C, with the result that

$$
\begin{aligned}
A(s) = -g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\,\frac{1}{2!}\ln^2(s/m^2) \\
\times[\tfrac{1}{2}K(t)-\tfrac{1}{2}(-t+\tfrac{3}{2}\mu^2)K^2(t)]\,.
\end{aligned}
\tag{4.3}
$$

B. Remaining contributions from diagrams 3.3–3.13

Next we must consider the contributions to the $\ln^2 s$ behavior of $T_{3.3-3.13}$ which are not given by the leading-particle approximation. First, there is a contribution from renormalization effects for the vertex correction graphs 3.6–3.13. Second, after one makes the shift of origin of the integration variables, e.g., $q \to q' + q^s$, then it may turn out that $q^s_\pm/\sqrt{s}$ is not small in the important regions of integration in α space, and so $q^s_\pm/\sqrt{s}$ will not be negligible compared to 1. So if we have a term in the numerator coming from the fermion propagators of the form $(1+q_+/\sqrt{s})(1+q_-/\sqrt{s})$, then we will make the approximation $(1+q_+/\sqrt{s})(1+q_-/\sqrt{s}) \approx 1+q_+/\sqrt{s}+q_-/\sqrt{s}$. That is, we retain linear terms such as $q^s_\pm/\sqrt{s}$, but ignore quadratic terms such as $q^s_+q^s_-/s$, relative to 1. The

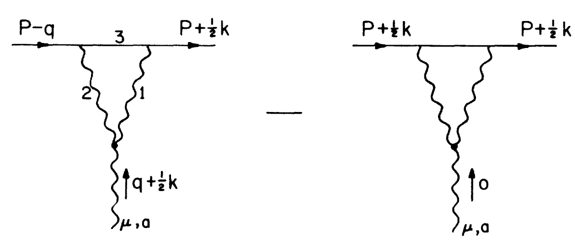

FIG. 7. Illustration of the renormalization of the vertex correction in Fig. 5.

quadratic terms such as $q_+^s q_-^s/s$ are always found to be small compared to 1 in the important regions of integration in parameter space.

Finally, we should also point out that if the important region of integration in parameter space corresponds to say ρ_1, ρ_2, ρ_3 small, and if say $q_+^s q_-^s \propto s\rho_1\rho_2\rho_3$, then a term in $q'_+ q'_-$ in the numerator of the integrand will be as important as a term in $q_+^s q_-^s$. This has already been seen to be the case for diagram 3.3 (see Appendix B), where the numerator term in $q'_+ q'_- + q_+^s q_-^s$ originates from the three-boson vertices.

Let us consider the radiative correction graphs 3.6–3.13. We renormalize the vertex correction by subtracting off the vertex with $q+\frac{1}{2}k=0$, as illustrated in Fig. 7. We can then write the exact renormalized vertex as

$$\tilde{V}_{\mu,a} = ig^3 \frac{\tau_a}{2}(2\pi)^{-4}\pi^2 \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\Big(\sum_i \alpha_i - 1\Big) \times \left\{ 6\gamma_\mu \ln\left[\frac{m^2\alpha_3{}^2 + \mu^2(\alpha_1+\alpha_2)}{f(P,q)}\right] - \frac{H_\mu(P,q)}{f(P,q)} + \frac{H_\mu(P,q=-\frac{1}{2}k)}{f(P,q=-\frac{1}{2}k)} \right\}, \tag{4.4}$$

where

$$f(P,q) = -(P-q)^2\alpha_2\alpha_3 - (q+\tfrac{1}{2}k)^2\alpha_1\alpha_2 + m^2\alpha_3(1-\alpha_1) + \mu^2(\alpha_1+\alpha_2) - i\epsilon \tag{4.5}$$

and

$$\begin{aligned} H_\mu(P,q) = {} & [(\alpha_2-2)(\not P - \not q) + (1+\alpha_1)(\not P + \tfrac{1}{2}\not k)][\alpha_2(\not P - \not q) + \alpha_1(\not P + \tfrac{1}{2}\not k) + m]\gamma_\mu \\ & + \gamma_\mu[\alpha_2(\not P - \not q) + \alpha_1(\not P + \tfrac{1}{2}\not k) + m][(1+\alpha_2)(\not P - \not q) + (\alpha_1 - 2)(\not P + \tfrac{1}{2}\not k)] \\ & - [2\alpha_2(\not P - \not q) + 2\alpha_1(\not P + \tfrac{1}{2}\not k) - 4m][(1-2\alpha_1)(P+\tfrac{1}{2}k)_\mu + (1-2\alpha_2)(P-q)_\mu]. \end{aligned} \tag{4.6}$$

Then in the numerator of the amplitude $T_{3.6}$ we will have the expression (see Fig. 5)

$$\bar{u}_3(P+\tfrac{1}{2}k)\tilde{V}_{\mu,a}(\not P - \not q + m)\gamma_\rho \tau_b u_1(P-\tfrac{1}{2}k) g^{\mu\lambda} g^{\rho\nu} \bar{u}_4(P'-\tfrac{1}{2}k)\gamma_\lambda \tau_a(\not P' + \not q + m)\gamma_\nu \tau_b u_2(P'+\tfrac{1}{2}k). \tag{4.7}$$

In this expression Eq. (4.7) we take the limit $\sqrt{s} \to \infty$, noting that in $\tilde{V}_{\mu,a}$ the large variable is $2P\cdot q \approx \sqrt{s}\, q_-$ so that in H_μ in Eq. (4.6) we need only keep the terms proportional to $2P\cdot q$. Then we find that the leading behavior in Eq. (4.7) comes from the term which has $\mu=\rho=+$ and $\lambda=\nu=-$. We then obtain for $T^{\rm f}_{3.6-3.13}$ the result that

$$\begin{aligned} T^{\rm f}_{3.6-3.13} \approx 8T^{\rm f}_{3.6} \approx {} & -ig^6(2\pi)^{-8}\pi^2 \frac{s^2}{m^2} \\ & \times \int d^4q \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\Big(\sum_i \alpha_i - 1\Big) \left\{ 3\ln\left[\frac{m^2\alpha_3{}^2 + \mu^2(\alpha_1+\alpha_2)}{f(P,q)}\right] - 2P\cdot q \, \frac{[\alpha_2\alpha_3 + (1-\alpha_3)(\frac{1}{2}-\alpha_2)]}{f(P,q)} \right\} \\ & \times \left(1 - \frac{q_+}{\sqrt{s}}\right)\left(1 + \frac{q_-}{\sqrt{s}}\right)[(P-q)^2 - m^2 + i\epsilon]^{-1}[(P'+q)^2 - m^2 + i\epsilon]^{-1} \\ & \times [(q+\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}, \end{aligned} \tag{4.8}$$

with $f(P,q)$ given in (4.5).

The expression (4.8) contains the leading-particle approximation to $T^{\rm f}_{3.6-3.13}$, which we have already considered. So we need to consider $T^{\rm f}_{3.6-3.13}$ as given in Eq. (4.8) minus $T^{\rm f}_{3.6-3.13}$ as given in the leading-particle approximation. We shall call this quantity $B(s)$. $B(s)$ can be shown to be given by the expression

$$\begin{aligned} B(s) = {} & -ig^6(2\pi)^{-8}\pi^2 \frac{s^2}{m^2} \int d^4q \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\Big(\sum_i \alpha_i - 1\Big) \\ & \times \left\{ 3\ln\left[\frac{m^2\alpha_3{}^2 + \mu^2(\alpha_1+\alpha_2)}{f(P,q)}\right]\left(1 - \frac{q_+}{\sqrt{s}}\right)\left(1 + \frac{q_-}{\sqrt{s}}\right) \right. \\ & \left. + \frac{2P\cdot q[\alpha_2\alpha_3 + (1-\alpha_3)(\frac{1}{2}-\alpha_2)]}{f(P,q)}\left(1 - \frac{q_+}{\sqrt{s}}\right)\left(1 + \frac{q_-}{\sqrt{s}}\right) + \frac{2P\cdot q(\frac{1}{2}-\alpha_2)}{f(P,q)} \right\} \\ & \times [(P-q)^2 - m^2 + i\epsilon]^{-1}[(P'+q)^2 - m^2 + i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1} \\ & \times [(q-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}. \end{aligned} \tag{4.9}$$

$B(s)$ given in Eq. (4.9) is evaluated in Appendix D with the result that

$$B(s)=g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)[2K(t)-I(t)]\,, \tag{4.10}$$

where

$$I(t)\equiv\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\,\delta\Big(\sum_i \alpha_i-1\Big)[(-t)\alpha_2\alpha_3+\mu^2(\alpha_2+\alpha_3)+m^2\alpha_1{}^2-i\epsilon]^{-1}\,. \tag{4.11}$$

Next, let us consider the ladder diagrams 3.3, 3.4, and 3.5. We have already considered $T_{3.3-3.5}$ in the leading-particle approximation. Now we must improve on the leading-particle approximation in order to obtain all the terms in $T_{3.3-3.5}$ which contribute to the $\ln^2 s$ behavior. We can write the exact amplitude for $T_{3.3-3.4}$ as (see Fig. 4)

$$\begin{aligned}T^{\mathrm{f}}_{3.3-3.4}=-(g^6/8)(2\pi)^{-8}\int d^4r\,d^4q\,M&[(P+r)^2-m^2+i\epsilon]^{-1}[(P'+q)^2-m^2+i\epsilon]^{-1}[(r+q)^2-\mu^2+i\epsilon]^{-1}\\&\times[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}\\&\times[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}-(s\to-s)\,,\end{aligned} \tag{4.12}$$

where

$$\begin{aligned}M=\bar{u}_3(P+\tfrac{1}{2}k)\gamma_\nu(\not{P}+\not{r}+m)\gamma_\mu u_1(P-\tfrac{1}{2}k)\bar{u}_4(P'-\tfrac{1}{2}k)\gamma_\rho(\not{P}'+\not{q}+m)\gamma_\lambda u_2(P'+\tfrac{1}{2}k)\\ \times\Big[\Big(2r+q+\frac{k}{2}\Big)^\lambda\Big(2r+q-\frac{k}{2}\Big)^\rho g^{\mu\nu}+\Big(2r+q+\frac{k}{2}\Big)^\lambda(k+q-r)^\mu g^{\nu\rho}-\Big(2r+q+\frac{k}{2}\Big)^\lambda\Big(2q+r+\frac{k}{2}\Big)^\nu g^{\mu\rho}\\ +\Big(\frac{k}{2}-2q-r\Big)^\mu\Big(2r+q-\frac{k}{2}\Big)^\rho g^{\lambda\nu}+\Big(\frac{k}{2}-2q-r\Big)^\mu(k+q-r)^\lambda g^{\nu\rho}-\Big(\frac{k}{2}-2q-r\Big)^\mu\Big(2q+r+\frac{k}{2}\Big)^\nu g^{\lambda\rho}\\ +(q-r-k)^\nu\Big(2r+q-\frac{k}{2}\Big)^\rho g^{\mu\lambda}+(q-r-k)(q-r+k)g^{\nu\rho}g^{\mu\lambda}-(q-r-k)^\rho\Big(2q+r+\frac{k}{2}\Big)^\nu g^{\mu\lambda}\Big]\,.\end{aligned} \tag{4.13}$$

Note that $T^{\mathrm{f}}_{3.4}(s)=-T^{\mathrm{f}}_{3.3}(-s)$. Now in the leading-particle approximation, and after canceling out the s^2 behavior as in Sec. III, we had in Eq. (3.10) the numerator factor

$$\mathrm{Num.}=\frac{s^2}{m^2}\{2r^2+2q^2-3q_+q_--3r_+r_--3r_-q_+-k^2-\mu^2\}\,. \tag{4.14}$$

As was noted in Appendix B the terms in r^2, q^2, q_+q_-, and r_+r_- in (4.14) contribute $\ln^3 s$ behavior in the amplitude for $T_{3.3}$. Then the crucial observation is that terms in M given in Eq. (4.13) proportional to either r^2, q^2, q_+q_-, or r_+r_- times either $q_\pm/\sqrt{s}$ or $r_\pm/\sqrt{s}$ will give contributions proportional to $\ln^2 s$. Hence these are the terms in M that we now need to ferret out. That these kinds of terms will give $\ln^2 s$ contributions can be seen by considering the momentum-space argument following Eq. (3.12). The basic approximation then is to write in place of Eq. (3.12)

$$\not{P}+\not{r}+m\approx\tfrac{1}{2}\sqrt{s}\,\gamma_-+\tfrac{1}{2}r_+\gamma_-+\tfrac{1}{2}r_-\gamma_+-\vec{r}_\perp\cdot\vec{\gamma}_\perp$$

and (4.15)

$$\not{P}'+\not{q}+m\approx\tfrac{1}{2}\sqrt{s}\,\gamma_++\tfrac{1}{2}q_+\gamma_-+\tfrac{1}{2}q_-\gamma_+-\vec{q}_\perp\cdot\vec{\gamma}_\perp\,.$$

And, because of Eqs. (2.3) and (2.4), we require that

$$\gamma_\nu(\not{P}+\not{r}+m)\gamma_\mu\propto\gamma_+$$

and (4.16)

$$\gamma_\rho(\not{P}'+\not{q}+m)\gamma_\lambda\propto\gamma_-$$

for specified ν, μ, ρ, and λ. Equations (4.16) need to be satisfied so that the amplitude will be proportional to s. Making the approximations (4.15), and taking into account the requirements of Eq. (4.16), we find that all the $\ln^2 s$ contributions to $T_{3.3-3.4}$ may be obtained by considering the seven terms in M obtained by putting in equation (4.13)

$$\begin{aligned}&(1)\ \mu=\nu=+,\quad \rho=\lambda=-\,,\\&(2)\ \mu=\nu=\perp,\quad \rho=\lambda=-\,,\\&(3)\ \mu=+,\ \nu=\perp,\quad \rho=\lambda=-\,,\\&(4)\ \mu=\perp,\ \nu=+,\quad \rho=\lambda=-\,,\\&(5)\ \mu=\nu=+,\quad \rho=\lambda=\perp\,,\\&(6)\ \mu=\nu=+,\quad \rho=-,\quad \lambda=\perp\,,\\&(7)\ \mu=\nu=+,\quad \rho=\perp,\quad \lambda=-\,.\end{aligned} \tag{4.17}$$

First, consider the term in $T^{\mathrm{f}}_{3.3-3.4}$ defined by the equation (1) in (4.17). This corresponds to the $++--$ approximation to $T^{f}_{3.3-3.4}$ discussed in Sec. III. In order to cancel out the s^2 behavior we must properly consider $T^{f}_{3.3-3.5}$ in the $++--$ approximation, which leads to Eqs. (3.8) and (3.10). Furthermore, we know that the $++--$ approximation includes the leading-particle approximation, which we have already considered. Therefore, what we must now consider is $T^{f}_{3.3-3.5}$ evaluated in the $++--$ approximation minus $T^{f}_{3.3-3.5}$ evaluated in the leading-particle approximation. We shall designate this difference as $C(s)$. $C(s)$ can be seen to be given by

$$C(s)=-(g^6/8)(2\pi)^{-8}\frac{s^2}{m^2}\int d^4r\,d^4q\left[\left(1+\frac{q_-}{\sqrt{s}}\right)\left(1+\frac{r_+}{\sqrt{s}}\right)-1\right]\{2r^2+2q^2-3q_+q_--3r_-q_+-k^2-\mu^2\}$$
$$\times[(P+r)^2-m^2+i\epsilon]^{-1}[(P'+q)^2-m^2+i\epsilon]^{-1}[(q+r)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}\,[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}-(s\leftrightarrow u)\,. \tag{4.18}$$

We can make the further approximation in (4.18) valid to the leading ln,

$$\left(1+\frac{q_-}{\sqrt{s}}\right)\left(1+\frac{r_+}{\sqrt{s}}\right)\approx1+\left(\frac{q_-}{\sqrt{s}}\right)+\left(\frac{r_+}{\sqrt{s}}\right). \tag{4.19}$$

In Appendix E $C(s)$ is evaluated with the result that

$$C(s)=g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)[\tfrac{1}{4}K(t)+I(t)]+O(s\ln s)\,. \tag{4.20}$$

Next we must consider the terms in $T^{f}_{3.3-3.4}$ defined by the sets of equations numbered (2)–(7) in (4.17) combined with Eqs. (4.12) and (4.13). These terms also contribute $s\ln^2 s$. We point out that $T^{f}_{3.5}$ only contributes in the $++--$ approximation. We shall designate these contributions (2)–(7) to $T^{f}_{3.3-3.4}$ as $D(s)$. $D(s)$ is evaluated in Appendix E with the result that

$$D(s)=-\tfrac{5}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)K(t)\,. \tag{4.21}$$

We have now taken into account all the contributions to the leading behavior of $T_{3.3-3.13}$. These contributions are $A(s)$ given in (4.3), $B(s)$ given in (4.10), $C(s)$ given in (4.20), and $D(s)$ given in (4.21). Summing these contributions, we obtain the result

$$\begin{aligned}T_{3.3-3.13}&=A(s)+B(s)+C(s)+D(s)+O(s\ln s)\\&=g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)\\&\quad\times[\tfrac{1}{2}K(t)+\tfrac{1}{2}(-t+\tfrac{3}{2}\mu^2)K^2(t)](\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}\\&\quad+O(s\ln s)\,.\end{aligned} \tag{4.22}$$

Note that in (4.22) the term in $I(t)$ has canceled out.

C. Contributions from diagrams 3.18–3.35

Now we must compute the contributions of the diagrams numbered 18–35 in Fig. 3. These diagrams are found to contribute $O(s\ln^2 s)$ to the isospin-flip amplitude, and as a consequence of Eqs. (3.1) and (3.2), the isospin-nonflip amplitude for these diagrams is only $O(s\ln s)$ and hence negligible.

Diagrams 3.18–3.33 are identical to the corresponding diagrams in QED except for isospin factors. Thus, since these diagrams have been computed in the QED case, we can obtain $T_{3.18-3.31}$ by consulting the literature,[10] and by using the results of Appendix A for the isospin factors. We would like to emphasize that in order to correctly obtain the contributions from the graphs 3.18–3.29 it is essential to properly renormalize the self-energy and vertex corrections, as we did for the vertex correction graphs 3.6–3.13. We find from Appendix A and Ref. 10 that

$$T_{3.18-3.25}=-\tfrac{1}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)\times K(t)(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}+O(s\ln s)\,, \tag{4.23}$$

$$T_{3.26-3.29}=-\tfrac{3}{8}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)\times K(t)(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}+O(s\ln s)\,, \tag{4.24}$$

and

$$T_{3.30-3.33}=\tfrac{1}{8}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)\times K(t)(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}+O(s\ln s)\,, \tag{4.25}$$

Finally, we compute the contributions of diagrams 3.34 and 3.35. The labeling of the internal momenta for these scalar ladders is the same as for the vector ladders as illustrated in Fig. 4(a). Then, with the Feynman rules for the scalar-boson–vector-boson vertex and for the scalar-boson propagator, as illustrated in Fig. 2, and recalling Eqs. (3.1) and (3.2), we obtain in the leading-particle approximation [see (3.12)]

$$T^{\mathrm{f}}_{3.34-3.35}=2T^{\mathrm{f}}_{3.34}+O(s\ln s)$$
$$=(g^6/4)(2\pi)^{-8}\frac{s^2}{m^2}\mu^2\int d^4r\,d^4q[(P+r)^2-m^2+i\epsilon]^{-1}[(P'+q)^2-m^2+i\epsilon]^{-1}$$
$$\times[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q+r)^2-M^2+i\epsilon]^{-1}\,. \tag{4.26}$$

In Appendix F $T^{\mathrm{f}}_{3.34-3.35}$ is evaluated and we find the result

$$T_{3.34-3.35}=-\tfrac{1}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)\mu^2K^2(t)(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}+O(s\ln s)\,. \tag{4.27}$$

Combining Eqs. (4.27), (4.25), (4.24), (4.23), (4.22), (3.30), and (3.22a) we finally obtain for the transition amplitude in order g^6 the result

$$T^{(6)}=T_{3.3-3.35}=\tfrac{1}{2}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)(-t+\mu^2)K^2(t)(\vec{\tau})_{i_1,i_3}\cdot(\vec{\tau})_{i_2,i_4}+O(s\ln s)\,. \tag{4.28}$$

This is the full leading sixth-order result.

V. GENERALIZATION TO SU(N)

Now we wish to indicate the generalization of the results of the preceding three sections to the case of SU(N) symmetry.

Let the λ_a ($a=1,\ldots,N^2-1$) be the $N\times N$ matrices, forming a representation of the group SU(N), which satisfy

$$\mathrm{Tr}(\lambda_a\lambda_b)=2\delta_{ab}\,,$$
$$[\lambda_a,\lambda_b]=2if_{abc}\lambda_c\,,$$
and
$$\{\lambda_a,\lambda_b\}=2d_{abc}\lambda_c+\frac{4}{N}\delta_{ab}1\,, \tag{5.1}$$

where repeated indices are summed. We may then, following the scheme of Grisaru, Schnitzer, and Tsao,[11] introduce N^2 complex scalar fields which develop a nonvanishing vacuum expectation value. Then, as seen in Ref. 11, the generalization of the Feynman rules that we need from the case of SU(2) symmetry to the case of SU(N) symmetry is accomplished by making the replacements $\tau_a\to\lambda_a$ in the fermion–vector-boson vertex, $\epsilon_{abc}\to f_{abc}$ in the three-vector-boson and four-vector-boson vertices, and $\delta_{ab}\to d_{abc}+(2/N)\delta_{ab}$ in the scalar-boson–vector-boson vertex. With these replacements, the only change in the amplitude of a given diagram numbered m in Fig. 3 is that an isospin factor $G_{3.m}(2)$ is replaced by an SU(N) gauge group factor $G_{3.m}(N)$. These factors $G_{3.m}(N)$ are derived in Appendix A, and hence using Appendix A we can generalize the results of the preceding sections to the SU(N) case by simply multiplying each amplitude $T_{3.m}$ ($m=1,2,\ldots,35$) as given in Secs. II, III, and IV by a factor $G_{3.m}(N)/G_{3.m}(2)$. Then we easily find that the net result in order g^4 of making the above replacements is that the amplitude is multiplied by $N/2$, i.e., Eq. (2.8) is replaced by

$$T^{(4)}=-(N/2)(g^4/4)(2\pi)^{-4}\pi^2\frac{s}{m^2}\ln(s/m^2)K(t)$$
$$\times(\lambda_a)_{i_1,i_3}(\lambda_a)_{i_2,i_4}+O(s)\,. \tag{5.2}$$

In order g^6, it can easily be seen that all of the cancellations which occur in this order go through for the SU(N) case just as for the SU(2) case with the obvious modifications. The net result is simply that the amplitude in order g^6 is multiplied by a factor $(N/2)^2$, i.e., Eq. (4.28) is replaced by

$$T^{(6)}=T_{3.3-3.35}$$
$$=(N/2)^2\tfrac{1}{2}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)$$
$$\times(-t+\mu^2)K^2(t)(\lambda_a)_{i_1,i_3}(\lambda_a)_{i_2,i_4}+O(s\ln s)\,. \tag{5.3}$$

VI. SUMMARY AND CONCLUSIONS

From Eqs. (2.2), (5.2), and (5.3) we have the result for the helicity-nonflip amplitude

$$T=T^{(2)}+T^{(4)}+T^{(6)}$$
$$=(g^2/8)\frac{1}{-t+\mu^2}\frac{s}{m^2}(\lambda_a)_{i_1,i_3}(\lambda_a)_{i_2,i_4}$$
$$\times[1-(N/2)(g^2/8\pi^2)(-t+\mu^2)K(t)\ln(s/m^2)+\tfrac{1}{2}(N/2)^2(g^2/8\pi^2)^2(-t+\mu^2)^2K^2(t)\ln^2(s/m^2)]\,. \tag{6.1}$$

This expression (6.1) contains what appears to be the first three terms of an exponential series in $(N/2)(g^2/8\pi^2)(-t+\mu^2)K(t)\ln(s/m^2)$. Hence, to the indicated order, the amplitude (6.1) may be rewritten as

$$T=(g^2/8)\frac{1}{-t+\mu^2}\left(\frac{s}{m^2}\right)^{1-F(t)}(\lambda_a)_{i_1,i_3}(\lambda_a)_{i_2,i_4}$$
$$=T_{\text{Born}}\left(\frac{s}{m^2}\right)^{-F(t)}, \qquad (6.2)$$

where $F(t)\equiv(N/2)(g^2/8\pi^2)(-t+\mu^2)K(t)$. The expression (6.2) is clearly of the Regge form. Thus, to order g^6 the results of perturbation theory suggest that the vector boson in renormalizable non-Abelian gauge theories with SU(N) symmetry lies on a Regge trajectory with a trajectory function

$$\alpha(t)\equiv 1-F(t)=1-(N/2)(g^2/8\pi^2)(-t+\mu^2)K(t). \qquad (6.3)$$

Note that the contributions of the scalar ladder diagrams numbered 34 and 35 in Fig. 3 are essential for Reggeization to occur. Thus, it appears that the entire apparatus of spontaneous symmetry breakdown is required to produce the delicate cancellations necessary for Reggeization to occur in non-Abelian gauge theories.

Notice that $\alpha(t)=1$ at $t=\mu^2$. Thus the vector boson lies on the trajectory. The real part of $\alpha(t)$ is illustrated in Fig. 8 for $(Ng^2/16\pi^2)=\frac{1}{2}$. As $|t|\to\infty$, $\alpha(t)\to-\infty$. At $t=0$, $\alpha(0)=1-(Ng^2/16\pi^2)$. When we approach $t=4\mu^2$ from values of $t<4\mu^2$, $\alpha(t)$ tends to $+\infty$. The imaginary part of $\alpha(t)$ has the simple form

$$\operatorname{Im}(\alpha(t))=2\pi(Ng^2/16\pi^2)\frac{t-\mu^2}{(t^2-4t\mu^2)^{1/2}}\times\theta(t-4\mu^2).$$

We should also point out that, in the leading-ln approximation, Eq. (6.2) may be replaced by

$$T=T_{\text{Born}}\left(\frac{s}{s_0}\right)^{-F(t)}$$
$$=(g^2/8)\frac{1}{-t+\mu^2}\frac{s_0}{m^2}\left(\frac{s}{s_0}\right)^{1-F(t)}, \qquad (6.4)$$

where s_0 is an arbitrary constant. This is due to the fact that $\ln(s/m^2)=\ln(s/s_0)+\ln(s_0/m^2)\approx\ln(s/s_0)$ to the leading ln.

It is also interesting to examine the infrared behavior of T. Letting $\mu^2\to0$ in (6.2), and using the fact that

$$K(t)\underset{\mu^2\to0}{\sim}\frac{2}{-t}\ln(-t/\mu^2), \qquad (6.5)$$

we find that

$$T\underset{\mu^2\to0}{\sim}(g^2/8)\left(\frac{s}{m^2}\right)^{1-A\ln(-t/\mu^2)\ln(s/m^2)}$$
$$=(g^2/8)\left(\frac{s}{m^2}\right)^{1-A\ln(-t/\Lambda^2)}\left(\frac{\mu^2}{\Lambda^2}\right)^{A\ln(s/m^2)}, \qquad (6.6)$$

where $A\equiv(N/2)(g^2/4\pi^2)$, and Λ^2 is an arbitrary constant $\Lambda^2\gg\mu^2$.

We see from (6.6) that as $\mu^2\to0$ the amplitude $T\to0$ as $(\mu^2)^{A\ln(s/m^2)}$. One might wonder whether, in analogy to QED, taking into consideration the emission of soft quanta would supply a multiplicative factor of $(\mu^2)^{-A\ln(s/m^2)}$ in Eq. (6.6). However, if the vector bosons in non-Abelian gauge theories are confined then there will be no such factor and the fermion-fermion transition amplitude will vanish for $\mu^2=0$, in the leading-ln approximation.

Additionally, we would like to mention the fact that the factor of N which occurs in the trajectory function as given in Eq. (6.3) is actually the Casimir operator C_N of the group SU(N) defined by

$$f_{abc}f_{abe}=C_N\delta_{ce}.$$

The idea that the elementary particles of a Lagrangian field theory might lie on Regge trajectories goes back to a series of papers by Gell-Mann *et al.*[12] Subsequently, Mandelstam[13] proposed certain criteria as sufficient conditions for Reggeization. These conditions are met for the vector bosons of a Yang-Mills-type field theory. A necessary condition for Reggeization is that the residue of the pole at $\alpha(t)=J$, where J is the spin of the elementary particle, should factorize into a product of factors depending only on initial- and final-state helicities, respectively. Grisaru, Schnitzer, and Tsao[11] have checked this factorization condition in the Born approximation for various two-body processes at $J=1$ for the identical SU(N) model that we have considered. They find that the factorization con-

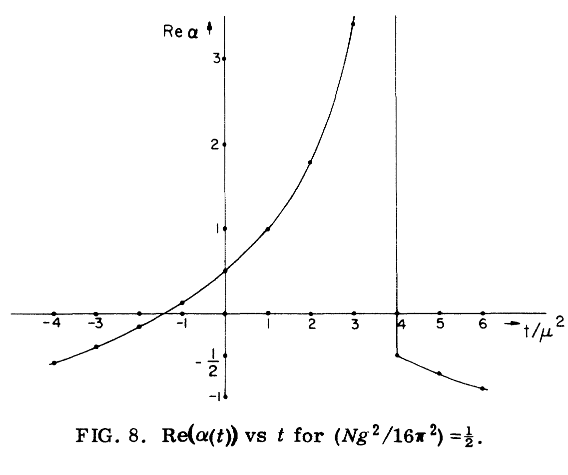

FIG. 8. $\mathrm{Re}(\alpha(t))$ vs t for $(Ng^2/16\pi^2)=\frac{1}{2}$.

dition is met in the Born approximation, and thus that the vector meson may lie on a Regge trajectory. This is equivalent to demonstrating Reggeization to the one-loop level in our leading-ln-type calculation.

While this manuscript was in preparation, we received a copy of a report by McCoy and Wu[14] in which they calculate the transition amplitude in order g^6 for the SU(2) case. Their calculation is done by putting in a transverse-momentum cutoff in the integrals, which at the end of the calculation is removed. This procedure is clearly dangerous, since it does not properly consider the renormalization of the vertex correction and self-energy graphs. However, it turns out that these authors obtain the same result in order g^6 for the SU(2) case as we do [Eq. (4.28)].

We have also received a copy of a report by Lipatov,[15] who calculates the fermion-fermion scattering amplitude to order g^6 using dispersion-relation techniques. He finds for the SU(2) case that the transition amplitude has the form as given in Eq. (6.1) with $N=2$.

Finally, we have also received a copy of a report by Nieh and Yao,[16] in which the transition amplitude for fermion-fermion scattering is calculated by a technique similar to ours. They, however, do not find the result (4.27), but rather find that in order g^6 the transition amplitude has terms in it proportional to $K(t)$ as well as terms proportional to $(-t+\mu^2)K^2(t)$. We do not know the reason for this discrepancy.

Cornwall and Tiktopoulos[17] have found results similar to ours for the case of high-energy, fixed-angle scattering.

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APPENDIX A

In this appendix we derive the gauge group factors $G_{3,m}(N)$ for SU(2) symmetry for the diagrams numbered m ($m=1,2,\ldots,35$) in Fig. 3. For the SU(2) symmetry case we called these factors isospin factors, and by putting $N=2$ in the results given below one can obtain the isospin factors used in the text.

In addition to the definitions given in Sec. V of the text, we will need the relations[18]

$$f_{abc}f_{abe}=N\delta_{ec},$$

$$d_{abc}d_{abe}=\frac{N^2-4}{N}\delta_{ec},$$

$$d_{abc}f_{abe}=0,$$

$$d_{aab}=0,$$

$$f_{abe}f_{cde}=\frac{2}{N}(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})+(d_{ace}d_{bde}-d_{bce}d_{ade}),$$

$$\lambda_a\lambda_a=2\frac{N^2-1}{N},$$

and

$$f_{abc}\lambda_a\lambda_b=iN\lambda_c.$$

For diagram 1 of Fig. 3 we have

$$\begin{aligned}
G_{3.1}&\equiv(\lambda_a\lambda_b)_{i_1,i_3}(\lambda_a\lambda_b)_{i_2,i_4}\\
&=\left(if_{abc}\lambda_c+d_{abc}\lambda_c+\frac{2}{N}\delta_{ab}1\right)_{i_1,i_3}\\
&\quad\times\left(if_{abe}\lambda_e+d_{abe}\lambda_e+\frac{2}{N}\delta_{ab}1\right)_{i_2,i_4}\\
&=-f_{abc}f_{abe}(\lambda_c)_{i_1,i_3}(\lambda_e)_{i_2,i_4}\\
&\quad+d_{abc}d_{abe}(\lambda_c)_{i_1,i_3}(\lambda_e)_{i_2,i_4}\\
&\quad+\frac{4}{N^2}(N^2-1)\delta_{i_1,i_3}\delta_{i_2,i_4}\\
&=\left(-N+\frac{N^2-4}{N}\right)(\lambda_c)_{i_1,i_3}(\lambda_c)_{i_2,i_4}\\
&\quad+\frac{4}{N^2}(N^2-1)\delta_{i_1,i_3}\delta_{i_2,i_4}.
\end{aligned}$$

From now on we will suppress the indices i_j:

$$\begin{aligned}
G_{3.2}&\equiv(\lambda_a\lambda_b)(\lambda_b\lambda_a)\\
&=\left(if_{abc}\lambda_c+d_{abc}\lambda_c+\frac{2}{N}\delta_{ab}1\right)\\
&\quad\times\left(if_{bae}\lambda_e+d_{bae}\lambda_e+\frac{2}{N}\delta_{ba}1\right)\\
&=(f_{abc}f_{abe}+d_{abc}d_{abe})(\lambda_c)(\lambda_e)+\frac{4}{N^2}(N^2-1)\delta\delta\\
&=\left(N+\frac{N^2-4}{N}\right)(\lambda_c)(\lambda_c)+\frac{4}{N^2}(N^2-1)\delta\delta,
\end{aligned}$$

$$\begin{aligned}
G_{3.3}&\equiv(\lambda_b\lambda_a)(\lambda_d\lambda_c)f_{cae}f_{bde}\\
&=-\tfrac{1}{4}(\lambda_b\lambda_a)([\lambda_e,\lambda_b][\lambda_a,\lambda_e])\\
&=-\tfrac{1}{4}(\lambda_b\lambda_a)(\lambda_e\lambda_b\lambda_a\lambda_e-\lambda_e\lambda_b\lambda_e\lambda_a-\lambda_b\lambda_e\lambda_a\lambda_e\\
&\qquad+\lambda_b\lambda_e\lambda_e\lambda_a).
\end{aligned}$$

Using

$$\lambda_e\lambda_a\lambda_e=\lambda_e\lambda_e\lambda_a+2if_{aeg}\lambda_e\lambda_g$$
$$=\left(2\frac{N^2-1}{N}-2N\right)\lambda_a$$

and

$$\lambda_e\lambda_b\lambda_a\lambda_e=\lambda_e\left(if_{bag}\lambda_g+d_{bag}\lambda_g+\frac{2}{N}\delta_{ba}\right)\lambda_e$$
$$=if_{bag}\left(2\frac{N^2-1}{N}-2N\right)\lambda_g$$
$$+d_{bag}\left(2\frac{N^2-1}{N}-2N\right)\lambda_g$$
$$+\frac{4}{N^2}(N^2-1)\delta_{ab}1$$
$$=\left(2\frac{N^2-1}{N}-2N\right)\lambda_b\lambda_a+4\delta_{ab}1,$$

we see that

$$G_{3.3}=-\tfrac{1}{4}(\lambda_b\lambda_a)(2N\lambda_b\lambda_a+4\delta_{ba}1)$$
$$=-\frac{N}{2}\left(-N+\frac{N^2-4}{N}\right)(\lambda_c)(\lambda_c)$$
$$-\frac{4}{N}(N^2-1)\delta\delta,$$

$$G_{3.4}=(\lambda_a\lambda_b)(\lambda_d\lambda_c)f_{cae}f_{bde}$$
$$=-\tfrac{1}{4}(\lambda_a\lambda_b)(2N\lambda_b\lambda_a+4\delta_{ba}1)$$
$$=-\frac{N}{2}\left(N+\frac{N^2-4}{N}\right)(\lambda_c)(\lambda_c)$$
$$-\frac{4}{N}(N^2-1)\delta\delta.$$

Diagram number 5 in Fig. 3 only enters into our calculation in the $++--$ approximation defined in the text. In this approximation, it can be seen directly from the Feynman rules that

$$G_{3.5}=G_{3.3}+G_{3.4},$$

$$G_{3.6}\equiv(\lambda_d\lambda_c\lambda_a)(\lambda_b\lambda_a)f_{cdb}$$
$$=-iN(\lambda_b\lambda_a)(\lambda_b\lambda_a)$$
$$=-iNG_{3.1},$$

$$G_{3.8}=-G_{3.10}=-G_{3.12}=G_{3.6},$$

$$G_{3.7}=(\lambda_d\lambda_c\lambda_a)(\lambda_a\lambda_b)f_{cdb}$$
$$=iNG_{3.2},$$

$$G_{3.9}=-G_{3.11}=-G_{3.13}=G_{3.7},$$

$$G_{3.14}\equiv(\lambda_c\lambda_a\lambda_d)(\lambda_b\lambda_a)f_{dcb}$$
$$=\frac{1}{2i}(\lambda_c\lambda_a[\lambda_c,\lambda_b])(\lambda_b\lambda_a)$$
$$=\frac{1}{2i}\left[\left(2\frac{N^2-1}{N}-2N\right)\lambda_a\lambda_b\right.$$
$$\left.-\left(2\frac{N^2-1}{N}-2N\right)\lambda_a\lambda_b-4\delta_{ab}1\right](\lambda_b\lambda_a)$$
$$=2i\delta_{ab}(1)(\lambda_b\lambda_a)$$
$$=\frac{4i}{N}(N^2-1)\delta\delta,$$

$$G_{3.15}=-G_{3.16}=-G_{3.17}=G_{3.14}.$$

In our calculation we do not need to know, for example, $G_{3.18}$ and $G_{3.19}$ separately, but only the difference $G_{3.18}-G_{3.19}$. This is because with $T_{3.18}\equiv G_{3.18}F(s)$, we have that $T_{3.19}=G_{3.19}F(-s)\approx-G_{3.19}F(s)$, so that $T_{3.18}+T_{3.19}\approx(G_{3.18}-G_{3.19})F(s)$. Hence for diagrams 18–35 we give only the differences $G_{3.n}-G_{3.n+1}$ that we require:

$$G_{3.18}-G_{3.19}=(\lambda_c\lambda_a\lambda_c\lambda_b)([\lambda_a,\lambda_b])$$
$$=\left(2\frac{N^2-1}{N}-2N\right)(\lambda_a\lambda_b)([\lambda_a,\lambda_b])$$
$$=\frac{-2}{N}(G_{3.1}-G_{3.2})$$
$$=4(\lambda_c)(\lambda_c),$$

$$G_{3.20}-G_{3.21}=G_{3.22}-G_{3.23}$$
$$=G_{3.24}-G_{3.25}$$
$$=G_{3.18}-G_{3.19},$$

$$G_{3.26}-G_{3.27}=(\lambda_a\lambda_c\lambda_c\lambda_b)([\lambda_a,\lambda_b])$$
$$=2\frac{N^2-1}{N}(G_{3.1}-G_{3.2})$$
$$=-4(N^2-1)(\lambda_c)(\lambda_c),$$

$$G_{3.28}-G_{3.29}=G_{3.26}-G_{3.27},$$

$$G_{3.30}-G_{3.31}=(\lambda_c\lambda_a\lambda_b\lambda_c)([\lambda_a,\lambda_b])$$
$$=\left(2\frac{N^2-1}{N}-2N\right)(G_{3.1}-G_{3.2})$$
$$=4(\lambda_c)(\lambda_c),$$

$$G_{3.32}-G_{3.33}=G_{3.30}-G_{3.31},$$

$$G_{3.34}-G_{3.35}=(\lambda_b\lambda_a)(\lambda_d\lambda_c)$$
$$\times\left[\frac{2}{N}(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})\right.$$
$$\left.+(d_{ace}d_{bde}-d_{bce}d_{ade})\right]$$
$$=(\lambda_b\lambda_a)(\lambda_d\lambda_c)f_{abe}f_{cde}$$
$$=-N^2(\lambda_c)(\lambda_c).$$

APPENDIX B

In this appendix we evaluate $F'_{3.3}(s)$ and $F_{3.6}(s)$ as $s\to\infty$ to the leading ln.

For $F'_{3.3}(s)$ we introduce Feynman parameters α_i with the internal propagators labeled as in Fig. 4(a). We combine the propagators with the Feynman parameter trick, shift the origins of integration, and evaluate the momentum integrals in the usual way. Then we obtain

$$F'_{3.3}(s)=+\tfrac{1}{16}g^6\frac{s^2}{m^2}(2\pi)^{-8}\pi^4\int_0^1 d\alpha_1\cdots d\alpha_7\,\delta\Big(\sum_i\alpha_i-1\Big)\times\left[\left(\frac{1}{J}+\frac{1}{L}\right)(\alpha_1\alpha_2\alpha_7 s+JLd)^{-2}+2(r_+^s r_-^s+q_+^s q_-^s)JL\,(\alpha_1\alpha_2\alpha_7 s+JLd)^{-3}\right]+O(s\ln^2 s)\,, \tag{B1}$$

where

$$J=\alpha_1+\alpha_3+\alpha_4+\alpha_7\,,$$

$$L=\alpha_2+\alpha_5+\alpha_6+\alpha_7-(\alpha_7{}^2/J)\,,$$

$$r_+^s=-\frac{\sqrt{s}\,\alpha_1}{J}-\frac{\alpha_1\alpha_7{}^2}{J^2L}\sqrt{s}\,,$$

$$r_-^s=\frac{\alpha_2\alpha_7}{JL}\sqrt{s}\,,$$

$$q_+^s=\frac{\alpha_1\alpha_7}{JL}\sqrt{s}\,,$$

$$q_-^s=-\frac{\sqrt{s}\,\alpha_2}{L}\,,$$

and

$$d=-[\vec{k}_\perp{}^2\alpha_5\alpha_6+\vec{k}_\perp{}^2\alpha_3\alpha_4+\mu^2-i\epsilon)+c\,.$$

The constant c in the last line is a complicated function of t, m^2, μ^2, and the α_i. We need not give c explicitly, however, since in the important regions of integration c is found to be ≈ 0. In Eq. (B1) the terms in the square brackets give $\ln^3 s$ behavior, and they come from terms in the numerator of Eq. (3.10) proportional to r^2 and q^2 and to r_+r_- and q_+q_-. One immediately sees in Eq. (B2) that the region $\alpha_1\approx\alpha_2\approx\alpha_7\approx 0$ will be important. But also one must take into account the effect of the "singular configurations" which occur when J or $L\approx 0$. Specifically, if we make the scaling transformations

$$\begin{aligned}&\alpha_2=\rho_1\alpha_2',\quad \alpha_5=\rho_1\alpha_5',\\&\alpha_6=\rho_1\alpha_6',\quad \alpha_7=\rho_1\alpha_7',\\&\alpha_2'+\alpha_5'+\alpha_6'+\alpha_7'=1\,,\end{aligned} \tag{B2}$$

and

$$\begin{aligned}&\alpha_1=\rho_2\alpha_1'',\quad \alpha_3=\rho_2\alpha_3''\,,\\&\alpha_4=\rho_2\alpha_4'',\quad \alpha_7'=\rho_2\alpha_7''\,,\\&\alpha_1''+\alpha_3''+\alpha_4''+\alpha_7''=1\,,\end{aligned} \tag{B3}$$

then the leading behavior of $F'_{3.3}(s)$ is obtained by considering the regions of integration where ρ_1, α_1'', α_2', and $\alpha_7''\approx 0$ and where ρ_2, α_1'', α_2', and $\alpha_7''\approx 0$. The result can then be straightforwardly shown to be

$$F'_{3.3}(s)=-\tfrac{1}{4}g^6(2\pi)^{-8}\pi^4\,\frac{1}{3!}\,\frac{s}{m^2}\ln^3\left(-\frac{s}{m^2}-i\epsilon\right)K(t)+O(s\ln^2 s)\,. \tag{B4}$$

To evaluate $F_{3.6}(s)$ we will, for illustrative purposes, use a "mixed" method combining parameter space and momentum-space techniques. We first evaluate the loop integral associated with the vertex correction in diagram 3.6. This we do using the Feynman parameter trick with the parameters α_i, $i=1,2,3$, associated with the internal propagators as shown in Fig. 5(a). Then performing the momentum integration in the usual way, we obtain

$$F_{3.6}(s)\approx(g^6/32)(2\pi)^{-8}\pi^2\frac{s^2}{m^2}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\,\delta(\alpha_1+\alpha_2+\alpha_3-1)\int d^4q\left[\frac{2P\cdot q(1-2\alpha_2)}{2P\cdot q\alpha_2\alpha_3+d}\right][(P-q)^2-m^2+i\epsilon]^{-1}\times[(P'+q)^2-m^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}\,, \tag{B5}$$

where

$$d=\mu^2(\alpha_1+\alpha_2)+m^2\alpha_3{}^2+\frac{k^2}{4}\,\alpha_2(\alpha_3-\alpha_1)-k\cdot q\alpha_1\alpha_2-q^2\alpha_2(\alpha_1+\alpha_3)-i\epsilon\,.$$

The value of $F_{3.6}(s)$ as $s\to\infty$ will be obtained by calculating the asymptotic value of the parametric integral in (B5) as $2P\cdot q\approx\sqrt{s}\,q_-\to\infty$, and then evaluating the remaining q integrals by the infinite-momentum technique.[7] The parametric integral gives

$$\int_0^1 \frac{d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1+\alpha_2+\alpha_3-1)2P\cdot q(1-2\alpha_2)}{2P\cdot q\alpha_2\alpha_3+d} \approx \frac{1}{2!}\ln^2(2P\cdot q - i\epsilon).$$

Then making the usual infinite-momentum technique approximations, e.g.,

$$[(P-q)^2-m^2+i\epsilon]^{-1}\approx(\sqrt{s})^{-1}\left[-q_-+\frac{q^2}{\sqrt{s}}+O\left(\frac{1}{s}\right)+i\epsilon\right]^{-1},$$

we obtain

$$F_{3,6}(s)\approx(g^6/32)(2\pi)^{-8}\pi^2\frac{s}{m^2}\tfrac{1}{2}\frac{1}{2!}\int dq_+dq_-d^2q_\perp \ln^2(\sqrt{s}\,q_- - i\epsilon)\left(-q_-+\frac{q^2}{\sqrt{s}}+i\epsilon\right)^{-1}$$
$$\times\left(q_++\frac{q^2}{\sqrt{s}}+i\epsilon\right)^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}. \qquad \text{(B6)}$$

Now if we integrate over dq_+ we find that the contour integral vanishes unless $0>q_->-\sqrt{s}$. With this restriction, we can then evaluate the integrals in the standard fashion,[7] obtaining

$$F_{3,6}(s)=-i(g^6/32)(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{3!}\ln^3\left(-\frac{s}{m^2}-i\epsilon\right)K(t)+O(s\ln^2 s). \qquad \text{(B7)}$$

APPENDIX C

In this appendix we evaluate $A(s)$ defined as $T_{3,3-3,13}$ in the leading-particle approximation. Combining Eqs. (4.1) and (4.2), we have

$$A(s)=-\tfrac{1}{2}g^6\frac{s^{5/2}}{m^2}(2\pi)^{-8}\int d^4r\,d^4q\{q_-[\tfrac{3}{2}r_+r_- - \vec{r}_\perp\cdot\vec{k}_\perp - \tfrac{1}{4}(2\vec{k}_\perp^{\,2}+3\mu^2)+\tfrac{3}{4}r_-q_+]$$
$$+2r_-[r_+r_- - \vec{r}_\perp^{\,2} - \vec{r}_\perp\cdot\vec{k}_\perp - \tfrac{1}{4}(\vec{k}_\perp^{\,2}+\mu^2)]\}$$
$$\times[(P-q)^2-m^2+i\epsilon]^{-1}[(P+r)^2-m^2+i\epsilon]^{-1}[(P'+q)^2-m^2+i\epsilon]^{-1}$$
$$\times[(q+r)^2-\mu^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1} \qquad \text{(C1)}$$

It is convenient to separate $A(s)$ into two parts. The first part, which we shall call $A^{(1)}(s)$, consists of the the piece of $A(s)$ which comes from the term in the curly brackets in (C1) proportional to $q_-r_+r_-$. The second part consists of the rest of $A(s)$ and is called $A^{(2)}(s)$.

$A^{(1)}(s)$ is most easily evaluated in momentum space. Making the usual infinite-momentum technique[7] approximations [see Eq. (3.24)], we write

$$A^{(1)}(s)\approx -g^6\frac{s^2}{m^2}(2\pi)^{-8}\tfrac{1}{4}\int dr_+dr_-d^2r_\perp dq_+dq_-d^2q_\perp\{\sqrt{s}\,q_-\tfrac{3}{2}(r_+r_-)\}(\sqrt{s})^{-3}\left(-q_-+\frac{q^2}{\sqrt{s}}+i\epsilon\right)^{-1}$$
$$\times\left(r_-+\frac{r^2}{\sqrt{s}}+i\epsilon\right)^{-1}\left(q_++\frac{q^2}{\sqrt{s}}+i\epsilon\right)^{-1}[(q+r)^2-\mu^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\approx +g^6\frac{s}{m^2}(2\pi)^{-8}\tfrac{3}{8}\int dr_+dr_-d^2r_\perp dq_+dq_-d^2q_\perp\{r_+\}\left(1+\frac{r^2}{\sqrt{s}\,r_-}\right)^{-1}\left(1-\frac{q^2}{\sqrt{s}\,q_-}\right)^{-1}$$
$$\times(q_++q^2/\sqrt{s}+i\epsilon)^{-1}[(q+r)^2-\mu^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}$$
$$\times[(r+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(r-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}. \qquad \text{(C2)}$$

Examining the contour integrals over dr_+ and dq_+, we find that the integrals give vanishing contribution unless $-\sqrt{s}<q_-<0$ and $0<r_-<-q_-$. Then evaluating the contour integrals we have

$$A^{(1)}(s)=g^6(2\pi)^{-8}\pi^3\frac{3}{2}\frac{s}{m^2}\int_{-\sqrt{s}}^{0}dq_-\int_0^{-q_-}dr_-\int d^2t_\perp d^2r_\perp\left(1+\frac{q_-}{\sqrt{s}}\right)^{-1}\left(1-\frac{\vec{r}_\perp{}^2}{\sqrt{s}\,r_-}\right)^{-1}\left(1+\frac{\vec{q}_\perp{}^2}{\sqrt{s}\,q_-}\right)^{-1}$$
$$\times[(\vec{r}+\vec{q})_\perp{}^2+\mu^2][(\vec{q}+\tfrac{1}{2}\vec{k})_\perp{}^2-i\epsilon]^{-1}\{r_-[(\vec{r}+\vec{q})_\perp{}^2+\mu^2]-(r_-+q_-)[(\vec{r}+\tfrac{1}{2}\vec{k})_\perp{}^2+\mu^2]\}^{-1}$$
$$\times[(\vec{q}-\tfrac{1}{2}\vec{k})_\perp{}^2+\mu^2-i\epsilon]^{-1}\{r_-(\vec{r}+\vec{q})_\perp{}^2+\mu^2]-(r_-+q_-)[(\vec{r}-\tfrac{1}{2}\vec{k})_\perp{}^2+\mu^2]\}^{-1}. \qquad \text{(C2}'\text{)}$$

The presence of the factor $(1-\vec{r}_\perp{}^2/\sqrt{s}\,r_-)^{-1}(1+\vec{q}_\perp{}^2/\sqrt{s}\,q_-)^{-1}$ in (C2) may be seen to provide cutoffs to the transverse integrals in (C2) when $\vec{r}_\perp{}^2>\sqrt{s}\,r_-$ and $\vec{q}_\perp{}^2>\sqrt{s}\,q_-$. So we may get the correct leading behavior of $A^{(1)}(s)$ by considering the regions of integration where $0>q_->-\epsilon\sqrt{s}$, $0<r_-<-q_-$, $0<\vec{r}_\perp{}^2<\epsilon\sqrt{s}\,r_-$, and $\vec{q}_\perp{}^2<\epsilon\sqrt{s}\,q_-$, and by putting $(1+q_-/\sqrt{s})^{-1}(1-\vec{r}_\perp{}^2/\sqrt{s}\,r_-)^{-1}(1+\vec{q}_\perp{}^2/\sqrt{s}\,q_-)^{-1}\approx 1$ in (C2). Here ϵ is restricted by $1\gg\epsilon\gg m/\sqrt{s}$.

With the approximations given in the preceding paragraph the integrals in (C2) are elementary and we obtain the result

$$A^{(1)}(s)=-g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\tfrac{1}{2}\ln^2(s/m^2)(-\tfrac{3}{2})K(t). \qquad \text{(C3)}$$

Next we evaluate $A^{(2)}(s)\equiv A(s)-A^{(1)}(s)$. Introducing Feynman parameters, combining the denominators, and making the usual shifts of the integration variables we find that

$$A^{(2)}(s)=-g^6(2\pi)^{-8}7!\,\frac{s^{5/2}}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_8\,\delta\Big(\sum_i\alpha_i=1\Big)\int d^4r'\,d^4q'\,N(r,q)[(q')^2R+(r')^2Q+c]^{-8}, \qquad \text{(C4)}$$

where

$$Q=\alpha_3+\alpha_6+\alpha_7+\alpha_8,$$
$$R=\alpha_1+\alpha_2+\alpha_4+\alpha_5+\alpha_8-(\alpha_8{}^2/Q),$$
$$c=-\frac{1}{R}\left\{P\alpha_1-P'\alpha_2+\frac{k}{2}(\alpha_4-\alpha_5)+\frac{\alpha_8}{Q}\left[P\alpha_3+\frac{k}{2}(\alpha_7-\alpha_6)\right]\right\}^2-\frac{1}{Q}\left[P\alpha_3+\frac{k}{2}(\alpha_7-\alpha_6)\right]^2+P^2(\alpha_1+\alpha_3)+P'^2\alpha_2$$
$$+\frac{k^2}{4}(\alpha_4+\alpha_5+\alpha_6+\alpha_7)-m^2(\alpha_1+\alpha_2+\alpha_3)-\mu^2(\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)+i\epsilon,$$
$$r=r'-\frac{1}{Q}\left[P\alpha_3+\frac{k}{2}(\alpha_7-\alpha_6)+q\alpha_8\right]\equiv r'+r^s-q\frac{\alpha_8}{Q},$$
$$q=q'+\frac{1}{R}\left\{P\alpha_1-P'\alpha_2+\frac{k}{2}(\alpha_4-\alpha_5)+\frac{\alpha_8}{Q}\left[P\alpha_3+\frac{k}{2}(\alpha_7-\alpha_6)\right]\right\}\equiv q'+q^s,$$

and

$$N(r,q)=q_-[-\vec{r}_\perp\cdot\vec{k}_\perp-\tfrac{1}{4}(2\vec{k}_\perp{}^2+3\mu^2)]+2r_-[r_+r_--\vec{r}_\perp{}^2-\vec{r}_\perp\cdot\vec{k}_\perp-\tfrac{1}{4}(\vec{k}_\perp{}^2+4\mu^2)]. \qquad \text{(C5)}$$

In order to avoid having to handle large unwieldy expressions, it is best to consider at this point what terms in $N(r,q)$ will contribute to the leading behavior ($s\ln^2 s$). To do this, consider the coefficient of s in c. This coefficient is $(\alpha_2/R)(\alpha_1+(\alpha_3\alpha_8/Q))$. The leading behavior of $A^{(2)}(s)$ as $s\to\infty$ will come from the regions of integration in α space where the coefficient of s is small. Now in $N(r,q)$, we will, after performing a Wick rotation, have terms in $(r')^2$, in $(q')^2$, and constant terms, i.e.,

$$N(r,q)=a_1(r')^2+a_2(q')^2+a_3+\text{odd in } r'+q'. \qquad \text{(C6)}$$

Therefore, it can be seen that after evaluating the r' and q' integrals we will have an expression of the form

$$A^{(2)}(s)\propto s^{5/2}\int_0^1 d\alpha_1\cdots d\alpha_8\,\delta\Big(\sum_i\alpha_i-1\Big)\{2(a_1R+a_2Q)[\alpha_2(\alpha_1Q+\alpha_3\alpha_8)s+QRd]^{-3}+3a_3Q^2R^2[\alpha_2(\alpha_1Q+\alpha_3\alpha_8)s+QRd]^{-4}\}$$

Let us make the transformations of variables as follows:

$$\alpha_1=\rho_1\alpha_1',\quad \alpha_3=\rho_1\alpha_3',\quad \alpha_6=\rho_1\alpha_6',\quad \alpha_7=\rho_1\alpha_7',\quad \alpha_8=\rho_1\alpha_8',\quad \alpha_1'+\alpha_3'+\alpha_6'+\alpha_7'+\alpha_8'=1,$$

and

$$\alpha_1' = \rho_2\alpha_1'' , \quad \alpha_2 = \rho_2\alpha_2'' , \quad \alpha_4 = \rho_2\alpha_4'' , \quad \alpha_5 = \rho_2\alpha_5'' , \quad \alpha_8 = \rho_2\alpha_8'' , \quad \alpha_1'' + \alpha_2'' + \alpha_4'' + \alpha_5'' + \alpha_8'' = 1 ,$$

and

$$\alpha_1'' = \rho_3\alpha_1''' , \quad \alpha_8'' = \rho_3\alpha_8''' , \quad \alpha_3''' + \alpha_8''' = 1 ,$$

and

$$\alpha_1''' = \rho_4\alpha_1'''' , \quad \alpha_3' = \rho_4\alpha_3'''' , \quad \alpha_1'''' + \alpha_3'''' = 1 ,$$

and

$$\rho_5 = \alpha_2'' . \tag{C7}$$

Then the leading behavior of $A^{(2)}(s)$ can be seen to come from the regions of integration where some of the ρ_i, $i=1,2,\ldots,5$ are small. The ρ_i, correspond to the "minimal t paths" when the effects of singular configurations are taken into account. It can be seen, in fact, that the leading behavior of $A^{(2)}(s)$ $(s\ln^2 s)$ comes from the regions of integration where some three of the ρ_i, say ρ_i, ρ_j, ρ_k $i,j,k=1,2,\ldots,5$ $(i\neq j\neq k)$ are small. Then, it can be shown that only terms in a_1R and a_2Q proportional to $\sqrt{s}\,\rho_i\rho_j\rho_k$ and terms in $a_3Q^2R^2 \propto (\sqrt{s}\,)^3\rho_i{}^2\rho_j{}^2\rho_k{}^2$ will contribute to the leading ln. Hence we need only keep terms in $N(r,q)$ which will behave as described in the preceding sentence for some $i,j,k=1,2,\ldots,5$ $(i\neq j\neq k)$. Then by analyzing the behavior of $N(r,q)$ in all of the above-mentioned important regions of integration one finds that $N(r,q)$ can be written as

$$N \approx (r')^2(-3\alpha_8/Q)q_-^s - 2r_+^s(q_-^s)^2(\alpha_8/Q)^2 + \vec{r}_\perp^{\,s}\cdot\vec{k}_\perp q_-^s - \tfrac{1}{4}(2\vec{k}_\perp^2 + 3\mu^2)q_-^s + \text{negligible}. \tag{C8}$$

Substituting (C8) into (C4), evaluating the momentum integrals, and considering all of the aforementioned important regions of integration in α space we find by straightforward but tedious analysis that

$$A^{(2)}(s) = -g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\tfrac{1}{2}\ln^2(s/m^2)[\,2K(t) - \tfrac{1}{2}(-t+\tfrac{3}{2}\mu^2)K^2(t)]\,. \tag{C9}$$

One finds, in fact, that the terms in $(r')^2(-3\alpha_8/Q)q_-^s$ and in $r_+^s(q_-^s)^2(\alpha_8/Q)^2$ in N as given in (C8) give contributions to the leading behavior from the region of integration in α space where ρ_1, ρ_4, and ρ_5 are small, while the term in $[\vec{r}_\perp^{\,s}\cdot\vec{k}_\perp - \frac{1}{4}(2\vec{k}_\perp{}^2+3\mu^2)]q_-^s$ gives a contribution from the region where ρ_3, ρ_4, and ρ_5 are small.

Combining (C9) and (C3) we obtain the result Eq. (4.3).

APPENDIX D

In this appendix we evaluate $B(s)$ which is given in Eq. (4.9).

We may separate $B(s)$ into two parts $B^{(1)}(s)$ and $B^{(2)}(s)$ with

$$\begin{aligned} B^{(1)}(s) \equiv -ig^6(2\pi)^{-8}\pi^2\frac{s^2}{m^2}\int d^4q\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\,\delta\Big(\sum_i\alpha_i - 1\Big) \\ \times\left\{3\ln\left[\frac{m^2\alpha_3{}^2+\mu^2(\alpha_1+\alpha_2)}{f(P,q)}\right] - 2P\cdot q\,\frac{(2\alpha_2\alpha_3-\frac{1}{2}\alpha_3)}{f(P,q)}\right\}\left(1-\frac{q_+}{\sqrt{s}}\right) \\ \times\left(1+\frac{q_-}{\sqrt{s}}\right)[(P-q)^2-m^2+i\epsilon]^{-1} \\ \times[(P'+q)^2-m^2+i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2-\mu^2+i\epsilon]^{-1}. \end{aligned} \tag{D1}$$

And, of course, $B^{(2)}(s)\equiv B(s)-B^{(1)}(s)$.

$f(P,q)$ is given by [see Eq. (4.5)]

$$f(P,q)\approx\alpha_2\alpha_3\sqrt{s}\,q_- - (P^2+q^2)\alpha_2\alpha_3 - (q+\tfrac{1}{2}k)^2\alpha_1\alpha_2 + m^2\alpha_3(1-\alpha_1) + \mu^2(\alpha_1+\alpha_2) - i\epsilon\,.$$

Then as $\sqrt{s}\to\infty$ we have that

$$\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\Big(\sum_i \alpha_i - 1\Big) \ln\left[\frac{m^2\alpha_3{}^2 + \mu^2(\alpha_1+\alpha_2)}{f(P,q)}\right] \approx -\tfrac{1}{2}\ln(\sqrt{s}\,q_- - i\epsilon)\,, \tag{D2}$$

and that

$$\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta\Big(\sum_i \alpha_i - 1\Big)\sqrt{s}\,q_- \frac{(2\alpha_2\alpha_3 - \frac{1}{2}\alpha_3)}{f(P,q)} \approx -\tfrac{1}{2}\ln(\sqrt{s}\,q_- - i\epsilon)\,. \tag{D3}$$

Then we have

$$B^{(1)}(s) = +ig^6(2\pi)^{-8}\pi^2\frac{s^2}{m^2}\int d^4q\Big(1-\frac{q_+}{\sqrt{s}}\Big)\Big(1+\frac{q_-}{\sqrt{s}}\Big)[\ln(\sqrt{s}\,q_- - i\epsilon)][(P-q)^2 - m^2 + i\epsilon]^{-1}$$
$$\times[(P'+q)^2 - m^2 + i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}\,. \tag{D4}$$

This expression (D4) may now be evaluated directly in momentum space in a way entirely analogous to the way $F_{3,6}(s)$ was evaluated in Appendix B. We find the result

$$B^{(1)}(s) = g^6(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\ln^2(s/m^2)K(t)\,. \tag{D5}$$

We should point out that the terms in $q_\pm/\sqrt{s}$ in (D4) may be neglected in evaluating $B^{(1)}(s)$. Now we need to evaluate $B^{(2)}(s)$, where

$$B^{(2)}(s) = ig^6(2\pi)^{-8}\pi^2\frac{s^2}{m^2}\int d^4q\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\delta\Big(\sum_i\alpha_i - 1\Big)\left[\frac{2P\cdot q(\frac{1}{2}-\alpha_2)}{f(P,q)}\right]\left[\Big(1-\frac{q_+}{\sqrt{s}}\Big)-1\right]$$
$$\times[(P-q)^2 - m^2 + i\epsilon]^{-1}[(P'+q)^2 - m^2 + i\epsilon]^{-1}[(q+\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}$$
$$\times[(q-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}\,. \tag{D6}$$

This expression can be seen to be equivalent to $T^f_{3.6-3.13}$ evaluated in the leading-particle approximation, but with a factor of 1 in the integrand replaced by $(1-q_+/\sqrt{s})(1-q_-/\sqrt{s})-1$. So we can write

$$B^{(2)}(s) = -g^6(2\pi)^{-8}\frac{s^{5/2}}{m^2}\int d^4q\,d^4r(2r_- + q_-)\left[\Big(1-\frac{q_-}{\sqrt{s}}\Big)\Big(1+\frac{q_-}{\sqrt{s}}\Big)-1\right]$$
$$\times[(P+r)^2 - m^2 + i\epsilon]^{-1}[(P-q)^2 - m^2 + i\epsilon]^{-1}$$
$$\times[(P'+q)^2 - m^2 + i\epsilon]^{-1}[(q+r)^2 - \mu^2 + i\epsilon]^{-1}$$
$$\times[(r-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}[(q-\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}$$
$$\times[(q+\tfrac{1}{2}k)^2 - \mu^2 + i\epsilon]^{-1}\,. \tag{D7}$$

We find in fact that to obtain the leading behavior of $B^{(2)}(s)$ it is sufficient to make the approximation in (D7) that

$$(2r_- + q_-)\left[\Big(1-\frac{q_+}{\sqrt{s}}\Big)\Big(1+\frac{q_-}{\sqrt{s}}\Big)-1\right] \approx (q_-)^2/\sqrt{s}\,. \tag{D8}$$

Then introducing Feynman parameters with the labelings indicated in Fig. 5(a), and evaluating the momentum integrals in the usual way, we obtain

$$B^{(2)}(s) \approx 2g^6(2\pi)^{-8}\pi^4\frac{s^3}{m^2}\int_0^1 d\alpha_1\ldots d\alpha_7\delta\Big(\sum_i\alpha_i - 1\Big)(\alpha_2/Y)^2XY[\alpha_2(\alpha_1X+\alpha_3\alpha_7)s+d]^{-3}\,, \tag{D9}$$

where

$$X = \alpha_3 + \alpha_6 + \alpha_7\,,$$

$$Y = \alpha_1 + \alpha_2 + \alpha_4 + \alpha_5 + \alpha_7 - (\alpha_7{}^2/X)\,,$$

and

$$d=-Y\left(P\alpha_3-\frac{k}{2}\alpha_6\right)^2-X[(\alpha_4-\alpha_5)+(\alpha_6\alpha_7/X)]^2\frac{k^2}{4}-X(P'\alpha_2)^2-X[P(\alpha_1+(\alpha_3\alpha_7/X))]^2$$
$$+XY\left[(\alpha_1+\alpha_3)P^2+\alpha_2(P')^2+(\alpha_4+\alpha_5+\alpha_6)\frac{k^2}{4}-m^2(\alpha_1+\alpha_2+\alpha_3)-\mu^2(\alpha_4+\alpha_5+\alpha_6+\alpha_7)\right]+i\epsilon\,. \tag{D10}$$

In obtaining $B^{(2)}(s)$ we have made the changes of variable

$$r\to r'-\frac{1}{X}\left(P\alpha_3-\frac{k}{2}\alpha_6\right)-q\frac{\alpha_7}{X} \tag{D11}$$

and

$$q\to q'-\frac{1}{Y}\left[-P\alpha_1+\alpha_2P'+(\alpha_4-\alpha_5)\frac{k}{2}-\frac{\alpha_7}{X}\left(P\alpha_3-\frac{k}{2}\alpha_6\right)\right]. \tag{D12}$$

If we make the transformations of variables

$$\alpha_1=\rho_1\alpha_1',\quad \alpha_3=\rho_1\alpha_3',\quad \alpha_6=\rho_1\alpha_6',$$

and

$$\alpha_7=\rho_1\alpha_7',\quad \alpha_1'+\alpha_3'+\alpha_6'+\alpha_7'=1\,,\quad \alpha_1'=\rho_2\alpha_1'',\quad \alpha_3'=\rho_2\alpha_3'',\quad \alpha_1''+\alpha_3''=1\,,$$

and

$$\alpha_1''=\rho_3\alpha_1''',\quad \alpha_7'=\rho_3\alpha_7''',\quad \alpha_1'''+\alpha_7'''=1\,,$$

then we find that the leading behavior of $B^{(2)}(s)$ as $s\to\infty$ comes from the region of integration where ρ_1, ρ_2, and ρ_3 are small. We then find the result

$$B^{(2)}(s)=-g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)I(t)\,, \tag{D13}$$

with $I(t)$ given as in Eq. (4.11).

The approximation (D8) can be justified by noting that the leading behavior of $B^{(2)}(s)$ must come from the regions of integration where some of the ρ_i ($i=1,2,3$) and possibly α_2 are small. Then by using Eq. (D11) and (D12) we can investigate the behavior of the neglected terms in Eq. (D8) in the various regions of integration just mentioned. We then find that the neglected terms in Eq. (D8) are indeed negligible to the leading ln.

Combining Eq. (D13) for $B^{(2)}(s)$ with Eq. (D5) for $B^{(1)}(s)$ we obtain the result (4.10).

APPENDIX E

In this appendix we evaluate $C(s)$ and $D(s)$ as defined in Sec. IV of the text.

$C(s)$ is given by Eqs. (4.18) and (4.19). We label the internal propagators with Feynman parameters as in Fig. 4(a), and make the shifts of origins in the loop integrals defined by

$$r=r'+r^s-q\frac{\alpha_7}{J}$$

and

$$q=q'+q^s\,,$$

where

$$r^s=-\frac{1}{J}\left[P\alpha_1+\frac{k}{2}(\alpha_3-\alpha_4)\right],$$

and

$$q^s=-\frac{1}{L}\left[\alpha_2P'+\tfrac{1}{2}k(\alpha_6-\alpha_5)-\frac{\alpha_7}{J}\left(P\alpha_1+\frac{k}{2}(\alpha_3-\alpha_4)\right)\right],$$

and

$$J=\alpha_1+\alpha_3+\alpha_4+\alpha_7\,,$$

and

$$L=\alpha_2+\alpha_5+\alpha_6+\alpha_7-\frac{\alpha_7{}^2}{J}. \tag{E1}$$

Then we find that

$$C(s)\approx-(g^6/4)(2\pi)^{-8}6!\frac{s^2}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_7\delta\left(\sum_i\alpha_i-1\right)$$
$$\times\int d^4r'd^4q'M(r,q)[(r')^2J+(q')^2L+c]^{-7}\,, \tag{E2}$$

where

$$C\equiv(\alpha_1\alpha_2\alpha_7 s/JL)+d$$

and

$$d\equiv\frac{k^2}{4}(\alpha_3+\alpha_4+\alpha_5+\alpha_6)-\frac{1}{J}\left[P\alpha_1+\frac{k}{2}(\alpha_3-\alpha_4)\right]^2-\frac{1}{L}\left[\frac{k}{2}(\alpha_6-\alpha_5)-\frac{\alpha_7}{J}\frac{k}{2}(\alpha_3-\alpha_4)\right]^2$$
$$-m^2(\alpha_1+\alpha_2)-\mu^2(\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7)+\alpha_1P^2+\alpha_2P'^2-\frac{\alpha_7{}^2}{J^2L}P^2\alpha_1{}^2-\frac{1}{J}P^2\alpha_1{}^2-\frac{1}{L}\alpha_2{}^2P'^2+i\epsilon\,, \tag{E3}$$

and where

$$M(r,q)=\left[\frac{1}{\sqrt{s}}(q'_-+q^s_-)+\frac{1}{\sqrt{s}}(r_++r^s_+)\right]\left[2(q'+q^s)^2+2\left(r'+r^s-q'\frac{\alpha_7}{J}-q^s\frac{\alpha_7}{J}\right)^2\right.$$
$$\left.-3(r'+r^s)_+(r'_-)-3(q'_+)(q+q^s)_-\right]+\text{negligible}$$
$$=\tfrac{1}{2}(q')^2(r^s_+/\sqrt{s})+\tfrac{1}{2}(r')^2(q^s_-/\sqrt{s})-\frac{1}{\sqrt{s}}(q^s_-+r^s_+)\left(q^s_+q^s_--r^s_+q^s_-\frac{\alpha_7}{J}\right)+\text{negligible}\,. \tag{E4}$$

In order to avoid a plethora of terms, we have kept in Eq. (E4) only the terms which are found to contribute to the leading behavior as $s\to\infty$. These terms can be seen to be of the kind described in the paragraph following Eq. (4.14) in the text.

Then evaluating the loop integrals in the usual way we find that

$$C(s)\approx-(g^6/4)(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_7\delta\left(\sum_i\alpha_i-1\right)$$
$$\times\left\{\left(\frac{\alpha_1+\alpha_2}{JL}\right)(\alpha_1\alpha_2\alpha_7 s+JLd)^{-2}+2s\left(\frac{\alpha_1}{J}+\frac{\alpha_2}{L}\right)\left(\frac{1}{J}+\frac{1}{L}\right)\alpha_1\alpha_2\alpha_7(\alpha_1\alpha_2\alpha_7 s+JLd)^{-3}\right\}. \tag{E5}$$

Now we make the changes of variables as in Eqs. (B2) and (B3) in Appendix B. Then the leading behavior of $C(s)$ in Eq. (E5) comes from the regions of integration where ρ_1, α''_1, and $\alpha''_7\approx 0$ and where ρ_2, α'_2, and $\alpha''_7\approx 0$. The result for $C(s)$ can then be seen to be Eq. (4.20).

Now we consider the evaluation of $D(s)$. $D(s)$ contains six terms corresponding to the six sets of equations numbered (2)–(7) in (4.17). These terms will be designated $D^i(s)$ ($i=2,3,\ldots,7$). First we consider $D^{(2)}(s)$. With $\mu=\nu=\perp$, $p=\lambda=-$ in (4.13), M assumes the form

$$M\to-\frac{1}{2}\frac{s^{3/2}}{m}(2r+q)_+{}^2r_-=-2\frac{s^{3/2}}{m}(r_+)^2r_-+\text{negligible}\,. \tag{E6}$$

Then introducing Feynman parameters and making the shifts of the origins of the loop momenta exactly as was done in the evaluation of $C(s)$ in this appendix, we find, after evaluating the loop integrals in the usual way, that

$$D^{(2)}(s)\approx -\tfrac{1}{2}g^6(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_7\delta\left(\sum_i\alpha_i-1\right)$$
$$\times\left[-2\frac{\alpha_1}{J^2}(\alpha_1\alpha_2\alpha_7 s+JLd)^{-2}+2s\frac{\alpha_1{}^2\alpha_2\alpha_7}{J^2}(\alpha_1\alpha_2\alpha_7 s+JLd)^{-3}\right]. \qquad \text{(E7)}$$

The leading behavior of $D^{(2)}(s)$ may be obtained by considering the region of integration where ρ_1, α_2', and $\alpha_7''\approx 0$, with the result that

$$D^{(2)}(s)=-\tfrac{1}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)K(t)+O(s\ln s). \qquad \text{(E8)}$$

$D^{(3)}(s)$ is obtained by setting $\mu=+$, $\nu=\perp$, $\rho=\lambda=-$ in (4.13). Then

$$M\to\frac{1}{2}\frac{s}{m}\bar{u}_3\vec{\gamma}_\perp(\vec{r}_\perp\cdot\vec{\gamma}^\perp)\gamma_+u_1\cdot\left[-(2q+r)_+\left(2\vec{q}+\vec{r}+\frac{\vec{k}}{2}\right)^\perp+(2r+q)_+(\vec{q}-\vec{r}-\vec{k})^\perp\right]$$
$$=\frac{3}{2}\frac{s^{3/2}}{m^2}(r_+)\vec{r}_\perp{}^2+\text{negligible}. \qquad \text{(E9)}$$

$D^{(4)}(s)$ is obtained by setting $\mu=\perp$, $\nu=+$, $\rho-\lambda=-$ in (4.13), and so

$$M\to\frac{3}{2}\frac{s^{3/2}}{m^2}(r_+)\vec{r}_\perp{}^2+\text{negligible}. \qquad \text{(E10)}$$

We find from (E9) and (E10) and by evaluating the loop integrals that

$$D^{(3)}(s)+D^{(4)}(s)\approx\tfrac{3}{4}g^6(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_7\delta\left(\sum_i\alpha_i-1\right)\left[\frac{\alpha_1}{J^2}(\alpha_1\alpha_2\alpha_7 s+JLd)^{-2}\right]. \qquad \text{(E11)}$$

The dominant region of integration is where ρ_1, α_2', and $\alpha_7''\approx 0$, and we find that

$$D^{(3)}(s)+D^{(4)}(s)=-\tfrac{3}{8}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)K(t)+O(s\ln s). \qquad \text{(E12)}$$

$D^{(5)}(s)$ is obtained by setting $\mu=\nu=+$, $\rho=\lambda=\perp$ in (4.13), and so

$$M\to -2\frac{s^{3/2}}{m^2}(q_-)^2q_+. \qquad \text{(E13)}$$

This leads to the result

$$D^{(5)}(s)\approx -\tfrac{1}{2}g^6(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_7\delta\left(\sum_i\alpha_i-1\right)\left[-2\frac{\alpha_2}{L^2}(\alpha_1\alpha_2\alpha_7 s+JLd)^{-2}\right.$$
$$\left.+2s\frac{\alpha_1\alpha_2{}^2\alpha_7}{L^2}(\alpha_1\alpha_2\alpha_7 s+JLd)^{-3}\right]. \qquad \text{(E14)}$$

The dominant region of integration is where ρ_1, α_1'', and $\alpha_7''\approx 0$, and we find that

$$D^{(5)}(s)=-\tfrac{1}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)K(t)+O(s\ln s). \qquad \text{(E15)}$$

$D^{(6)}(s)$ is obtained by setting $\mu=\nu=+$, $\rho=-$, $\lambda=\perp$ in (4.13) while $D^{(7)}(s)$ is obtained by setting $\mu=\nu=+$, $\rho=\perp$, $\lambda=-$ in (4.13). In both cases

$$M\to\frac{3}{2}\frac{s^{3/2}}{m^2}(r_-)\vec{r}_\perp{}^2+\text{negligible}. \qquad \text{(E16)}$$

With (E16) we obtain the expression

$$D^{(6)}(s)+D^{(7)}(s)\approx\tfrac{3}{4}g^6(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\int_0^1 d\alpha_1\cdots d\alpha_7\delta\left(\sum_i\alpha_i-1\right)\left[\frac{\alpha_2}{L^2}(\alpha_1\alpha_2\alpha_7 s+JLd)^{-2}\right] \qquad \text{(E17)}$$

The dominant region of integration is where ρ_2, α_1'', and $\alpha_7''\approx 0$, and we find the result

$$D^{(6)}(s)+D^{(7)}(s)=-\tfrac{3}{8}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)K(t)+O(s\ln s). \qquad \text{(E18)}$$

Summing the results (E18), (E15), (E12), and (E8) we obtain the result

$$D(s)=-\tfrac{5}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)K(t)+O(s\ln s)\,,$$

which is Eq. (4.21).

APPENDIX F

In this appendix we evaluate the contribution $T^{\mathrm{f}}_{3.\,34-3.\,35}$ from the graphs 34 and 35 in Fig. 3 as $s\to\infty$. The amplitude for $T^{\mathrm{f}}_{3.\,34-3.\,35}$ in the leading-particle approximation is given by Eq. (4.28). Introducing Feynman parameters with the same labeling of the propagators as for the vector ladder in Fig. 4(a), and evaluating the loop integrals in the usual way, we obtain

$$T^{\mathrm{f}}_{3.\,34-3.\,35}\approx 2T^{\mathrm{f}}_{3.\,34}\approx-\tfrac{1}{2}g^6(2\pi)^{-8}\pi^4\frac{s^2}{m^2}\mu^2\int_0^1 d\alpha_1\cdots d\alpha_7\,\delta\left(\sum_i\alpha_i-1\right)JL(s\alpha_1\alpha_2\alpha_7+JLd)^{-3}\,, \tag{F1}$$

where J, L, and d are as given in Eqs. (E1) and (E3) in Appendix E. The dominant region of integration is where α_1, α_2, and $\alpha_7\approx 0$, and we find, using the fact that

$$K^2(t)=\int_0^1\frac{d\alpha_1\cdots d\alpha_4\,\delta\left(\sum_i\alpha_i-1\right)(\alpha_1+\alpha_2)(\alpha_3+\alpha_4)}{\{(-t)[\alpha_1\alpha_2(\alpha_3+\alpha_4)+\alpha_3\alpha_4(\alpha_1+\alpha_2)]+\mu^2(\alpha_1+\alpha_2)(\alpha_3+\alpha_4)-i\epsilon\}^2}\,, \tag{F2}$$

that

$$T^{\mathrm{f}}_{3.\,34-3.\,35}=-\tfrac{1}{4}g^6(2\pi)^{-8}\pi^4\frac{s}{m^2}\frac{1}{2!}\ln^2(s/m^2)\mu^2K^2(t)\,,$$

which gives Eq. (4.27).

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