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High-energy behavior of non-Abelian gauge theories

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This paper is a detailed account of a study in perturbation theory of the high-energy behavior of non-Abelian gauge theories. We calculate the fermion-fermion scattering amplitude up to sixth order in the coupling constant in the high-energy limit $s \to \infty$ with fixed t, in the approximation of keeping only the leading logarithmic terms. Our results indicate that the high-energy behavior of non-Abelian gauge theories are complicated, and quite different from the known behaviors of other field theories studied so far.

I. INTRODUCTION

The high-energy behavior of hadron-hadron scattering amplitudes has been under intensive study, both theoretically and experimentally, for almost two decades. On the theoretical side, one approach is to study the high-energy behavior of the scattering amplitude in various renormalizable field theories. The study is made through perturbation calculation in terms of Feynman diagrams, making high-energy approximations at each order of perturbation. The basic assumption of this approach is that the highly complicated physical hadrons can actually be described by the highly idealized field theories. Another assumption that is usually made and is technical in nature is the validity of the high-energy approximation of keeping leading terms at each order of perturbation. Notwithstanding these uncertainties, studies of this kind are useful in that they may shed light on the general features of high-energy hadron scattering, or they may serve as a basis for selecting field-theoretic models for hadrons. However, one should not lose sight of the uncertainties in interpreting the results of these studies.

Among the prominent field theories that have been extensively studied are the ϕ^3 theory and quantum electrodynamics (QED). The ϕ^3 theory is the simplest field theory and many interesting results have been obtained in this theory.[1] In the past few years, QED has been intensively studied by Cheng and Wu.[2] Their results are extremely interesting. First, a rising total cross section, actually saturating the Froissart bound,[3] is predicted in general agreement with the result of the Pisa–Stony Brook experiment at CERN.[4] Second, QED is a gauge theory, and there is increasing evidence that gauge theories may be of fundamental physical importance.

An immediate important question is whether the salient features of QED are also possessed by the potentially more realistic (for hadrons) non-Abelian gauge theories.[5] Additional impetus for studying the high-energy behavior of non-Abelian gauge theories comes from recent activities in constructing renormalizable models for weak and electromagnetic interactions,[6] as well as the observation that non-Abelian gauge theories are asymptotically free[7] and therefore might provide a basis for understanding Bjorken scaling in deep-inelastic electron scattering. These are the motivations for our present study.

Some of the results here were published in a letter,[8] modified by an addendum.[9] The present article is a detailed account of the sixth-order calculation. We will report part of the eighth-order calculation in a subsequent article.

The sixth-order results indicate that the high-energy behavior of non-Abelian gauge theories is quite different from all the other theories studied so far. In fact, the non-Abelian theories have a much richer structure.

We recall that in ϕ^3 theory or QED, in each order, the dominant diagrams at high energy are of multiperipheral type.[1,2] In other words, in the t-channel effectively only two-particle cuts are included. In non-Abelian gauge theories multiple-particle-exchange effects enter. Besides, renormalization effects are important.

In view of the complexity involved, we have chosen to study SU(2) as the internal-symmetry group. This way we do not have to consider scalar-fermion-fermion and gauge scalar-vector-vector couplings. The technique used, however, is general enough to encompass all such modifications if necessary.

Specifically, we study in the sixth order the high-energy limit of the near forward scattering of two fermions. If we denote by s the square of the total energy in the center-of-mass system, by $-t$ the square of momentum transfer, and by T^{nf} and T^{f} the isospin-nonflip and isospin-flip amplitudes, respectively, we find that the important regions from which the dominant terms arise are as follows:

(1) vertex and self-energy renormalization,

$$T^{nf} = (-60\pi i \ln s) A K_1,$$
$$T^f = -20(\ln^2 s - i\pi \ln s) A K_1; \tag{1.1}$$

(2) ladder with scalar production,

$$T^{nf} = (3\pi i \ln s) A \mu^2 (K_1)^2,$$
$$T^f = 2(\ln^2 s - i\pi \ln s) A \mu^2 (K_1)^2; \tag{1.2}$$

(3) radiative diagrams and ladder with vector-meson production,

$$T^{nf} = [48 - 18(\mu^2 - \tfrac{2}{3}t) K_1] \pi i \ln s A K_1,$$
$$T^f = [16 - 6(\mu^2 - \tfrac{2}{3}t) K_1] (\ln^2 s - i\pi \ln s) A K_1; \tag{1.3}$$

(4) 3-vector-meson exchange in the t channel,

$$T^{nf} = (12\pi i \ln s) A K_2,$$
$$T^f = 0; \tag{1.4}$$

where

$$A = (16\pi^2)^{-2} (g/2)^4 g^2 s m^{-2},$$
$$K_1 = \int d\alpha_1 d\alpha_2 \delta(1 - \alpha_1 - \alpha_2) \frac{1}{-t\alpha_1\alpha_2 + \mu^2},$$
$$K_2 = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \times [-t\alpha_1\alpha_2\alpha_3 + \mu^2(\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3)]^{-1}.$$

To the leading order in $\ln s$, the isospin-nonflip amplitude T^{nf} is purely imaginary, and the isospin-flip amplitude T^f is purely real. The imaginary part of T^f is given in the above equations, and there is no other contribution to the first order of $\ln s$.

The plan of this paper is as follows: In Sec. II, we shall define the model. Notations will be developed. Fourth-order results are reported in Sec. III.

In Sec. IV, we shall do some preliminary work to cancel out unwanted terms, which would otherwise mask the true high-energy behavior of the theory in this order.

In Sec. V, we delve into the problem of extracting the dominant behavior of the terms prepared in Sec. IV.

Discussions follow in Sec. VI.

Three appendices are attached, where some rather detailed analysis is either presented or indicated.

II. LAGRANGIAN AND KINEMATICS

For simplicity, we take isospin as the internal-symmetry group. We have both an isodoublet of fermions and an isodoublet of bosons. The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g\vec{A}_\mu \times \vec{A}_\nu)^2 - |(\partial_\mu - ig\tfrac{1}{2}\vec{\tau}\cdot\vec{A}_\mu)\varphi|^2 - \bar{\mu}^2|\varphi|^2 - \tfrac{1}{2}h|\varphi|^4$$
$$- \bar{\psi}\gamma_\mu\left(\frac{1}{i}\partial^\mu - \frac{g}{2}\vec{\tau}\cdot\vec{A}^\mu\right)\psi - m\bar{\psi}\psi. \tag{2.1}$$

After making a change of variable and invoking vacuum instability (owing to $\bar{\mu}^2 < 0$), we write

$$\varphi = \frac{1}{\sqrt{2}}(\lambda + \chi + i\vec{\tau}\cdot\vec{\zeta})\begin{pmatrix}0\\1\end{pmatrix}. \tag{2.2}$$

where $\lambda/\sqrt{2}$ is the vacuum expectation value of φ. Then the interaction density is

$$\mathcal{L}_{int} = -\frac{g}{2}(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu)\cdot\vec{A}^\mu\times\vec{A}^\nu - \frac{g^2}{4}(\vec{A}_\mu\times\vec{A}_\nu)\cdot(\vec{A}^\mu\times\vec{A}^\nu) + \frac{g}{2}\bar{\psi}\gamma_\mu\vec{\tau}\psi\cdot\vec{A}^\mu - \frac{g}{2}\vec{A}^\mu\cdot(\vec{\zeta}\partial_\mu\chi - \chi\partial_\mu\vec{\zeta})$$
$$-\frac{g}{2}\vec{A}^\mu\cdot(\vec{\zeta}\times\partial_\mu\vec{\zeta}) - \frac{g^2}{2}(\chi^2 + \vec{\zeta}^2)\vec{A}_\mu\cdot\vec{A}^\mu - \frac{g}{2}\mu\chi\vec{A}_\mu\cdot\vec{A}^\mu - \frac{h}{8}(\vec{\zeta}^2 + \chi^2)^2 - \frac{h}{2}\lambda\chi(\vec{\zeta}^2 + \chi^2), \tag{2.3}$$

where $\mu = g\lambda/2$ is the mass of the physical mode of A_μ. χ has a mass of $\sqrt{h}\,\lambda$. We specialize to the Feynman-'t Hooft gauge, then $\vec{\zeta}$ has a "mass" μ. The Faddeev-Popov loops are generated by the additive effective Lagrangian

$$-i\,\mathrm{Tr}\ln\left(1 + \frac{\tfrac{1}{2}\mu g\chi + \vec{t}\cdot(g\vec{A}^\mu\ \partial_\mu + \mu\tfrac{1}{2}g\vec{\zeta})}{-\partial^2 + \mu^2}\right), \tag{2.4}$$

where

$$(t_j)_{ik} = \epsilon_{ijk} \tag{2.5}$$

are the structure constants.

We write the S-matrix elements as

$$\langle f|(S-1)|i\rangle = iN(2\pi)^4\delta(p_i - p_f)T. \tag{2.6}$$

N is the usual wave-function normalization.

We shall use the following kinematic notation (see Fig. 1):

$$p_1 = p - \tfrac{1}{2}k, \quad p_3 = p + \tfrac{1}{2}k,$$
$$p_2 = p' + \tfrac{1}{2}k, \quad p_4 = p' - \tfrac{1}{2}k,$$

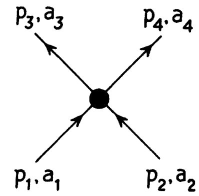

FIG. 1. Schematic drawing of fermion-fermion scattering.

$$p^\mu = ((\vec{p}^{\,2} + \tfrac{1}{4}\vec{k}^2 + m^2)^{1/2}, 0, 0, |\vec{p}|),$$
$$p'^\mu = ((\vec{p}^{\,2} + \tfrac{1}{4}\vec{k}^2 + m^2)^{1/2}, 0, 0, -|\vec{p}|),$$
$$k^\mu = (0, k_x, k_y, 0),$$
$$|\vec{k}| = (k_x{}^2 + k_y{}^2)^{1/2},$$
$$g^{\mu\nu} = (-1, 1, 1, 1),$$
$$s = -(p_1 + p_2)^2, \quad t = -(p_1 - p_3)^2,$$

and

$$s + t + u = 4m^2.$$

The Dirac equation is

$$(m + \gamma \cdot p) u(p) = 0$$

or (2.7)

$$\bar{u}(p)(m + \gamma \cdot p) = 0.$$

We shall consider the limit $s \to \infty$ while $t \leq 0$ is finite.

Furthermore, we decompose the T matrix into isospin-flip and isospin-nonflip amplitudes, i.e.,

$$T = \delta_{a_1 a_3} \delta_{a_2 a_4} T^{\text{nf}} + \vec{\tau}_{a_1 a_3} \cdot \vec{\tau}_{a_2 a_4} T^f. \tag{2.8}$$

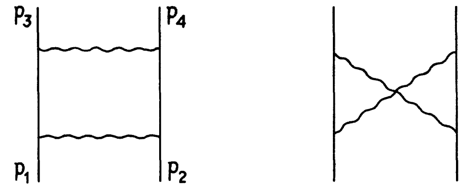

FIG. 2. Graphs which contribute in fourth order to dominant behavior.

III. FOURTH-ORDER CALCULATION

There are two fourth-order diagrams (Fig. 2) which contribute to the dominant high-energy behavior. The extraction of their contributions is exactly the same as in massive QED. We delete the details;

$$T_{(a)}^{4\text{th}}(s) = \frac{1}{16\pi^2}\left(\frac{g}{2}\right)^4 \left(\frac{-s}{m^2}\right) \ln(-s) K_1 \times \bar{u}(p_3) \tau_{a_1} \tau_{a_2} u(p_1) \bar{u}(p_4) \tau_{a_1} \tau_{a_2} u(p_2), \tag{3.1}$$

$$T_{(b)}^{4\text{th}}(s) = \frac{1}{16\pi^2}\left(\frac{g}{2}\right)^4 \left(\frac{s}{m^2}\right) \ln s K_1 \times \bar{u}(p_3) \tau_{a_1} \tau_{a_2} u(p_1) \bar{u}(p_4) \tau_{a_2} \tau_{a_1} u(p_2). \tag{3.2}$$

IV. SIXTH-ORDER CALCULATION–ALGEBRAIC REARRANGEMENT

The extraction of leading high-energy behavior in non-Abelian gauge theories is complicated by cancellation among graphs. As is well known, to extract the first few leading terms for a graph is laborious; and to see them cancel away is, to say the least, discouraging. A better approach, and in fact one more likely to lead to correct answers, is to arrange terms among graphs so that the cancellation occurs algebraically in the integrands, preferably before the momentum integrations are performed. This we will do in the following. It may not be obvious at first sight why we choose to arrange terms in the manner we have done. However, we will explain as we go on.

A. Ladder amplitudes

We assign momenta to internal lines as indicated in graphs (1), (2), and (3) of Fig. 3. The corresponding amplitudes are

$$iT^{(1)} = i\left(\frac{g}{2}\right)^4 g^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \bar{u}(p_3) \gamma^{\mu_3} \tau_{a_3} \frac{1}{m + \gamma \cdot (p_1 - k_1)} \gamma^{\mu_1} \tau_{a_1} u(p_1) \bar{u}(p_4) \gamma^{\mu_2} \tau_{a_2} \frac{1}{m + \gamma \cdot (p_4 - k_2)} \gamma^{\mu_4} \tau_{a_4} u(p_2)$$
$$\times \epsilon_{a_1 a_2 a_5} \epsilon_{a_5 a_4 a_3} N_{\mu_1 \mu_2 \mu_3 \mu_4} \frac{1}{k_1{}^2 + \mu^2} \frac{1}{(k + k_1)^2 + \mu^2} \frac{1}{(k_1 - k_2)^2 + \mu^2} \frac{1}{k_2{}^2 + \mu^2} \frac{1}{(k + k_2)^2 + \mu^2}. \tag{4.1}$$

$$iT^{(2)} = i\left(\frac{g}{2}\right)^4 g^2 \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \bar{u}(p_3) \gamma^{\mu_3} \tau_{a_3} \frac{1}{m + \gamma \cdot (p_1 - k_1)} \gamma^{\mu_1} \tau_{a_1} u(p_1) \bar{u}(p_4) \gamma^{\mu_4} \tau_{a_4} \frac{1}{m + \gamma \cdot (p_2 + k_2)} \gamma^{\mu_2} \tau_{a_2} u(p_2)$$
$$\times \epsilon_{a_1 a_2 a_5} \epsilon_{a_5 a_4 a_3} N_{\mu_1 \mu_2 \mu_3 \mu_4} \frac{1}{k_1{}^2 + \mu^2} \frac{1}{(k + k_1)^2 + \mu^2} \frac{1}{(k_1 - k_2)^2 + \mu^2} \frac{1}{k_2{}^2 + \mu^2} \frac{1}{(k + k_2)^2 + \mu^2}, \tag{4.2}$$

where

$$N_{\mu_1\mu_2\mu_3\mu_4}=N^a_{\mu_1\mu_2\mu_3\mu_4}+N^b_{\mu_1\mu_2\mu_3\mu_4}+N^c_{\mu_1\mu_2\mu_3\mu_4}, \tag{4.3}$$

$$N^a_{\mu_1\mu_2\mu_3\mu_4}=(2k_2-k_1)_{\mu_1}(2k_2-k_1+k)_{\mu_3}\,g_{\mu_2\mu_4}+(2k_1-k_2)_{\mu_2}(2k_1-k_2+k)_{\mu_4}\,g_{\mu_1\mu_3}, \tag{4.4a}$$

$$\begin{aligned}N^b_{\mu_1\mu_2\mu_3\mu_4}=&(-k_1-k_2-2k)_{\mu_2}(2k_2-k_1)_{\mu_1}\,g_{\mu_3\mu_4}+(-k_1-k_2-2k)_{\mu_1}(2k_1-k_2)_{\mu_2}\,g_{\mu_3\mu_4}+(2k_2-k_1)_{\mu_1}(2k_1-k_2+k)_{\mu_4}\,g_{\mu_2\mu_3}\\&+(2k_1-k_2)_{\mu_2}(2k_2-k_1+k)_{\mu_3}\,g_{\mu_1\mu_4}+(-k_2-k_1)_{\mu_3}(2k_1-k_2+k)_{\mu_4}\,g_{\mu_1\mu_2}+(-k_2-k_1)_{\mu_4}(2k_2-k_1+k)_{\mu_3}\,g_{\mu_1\mu_2},\end{aligned} \tag{4.4b}$$

$$N^c_{\mu_1\mu_2\mu_3\mu_4}=(-k_1-k_2)\cdot(-k_1-k_2-2k)\,g_{\mu_1\mu_2}\,g_{\mu_3\mu_4}, \tag{4.4c}$$

and

$$\begin{aligned}iT^{(3)}=&\,i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar u(p_3)\gamma^{\mu_3}\tau_{a_3}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_1}\tau_{a_1}u(p_1)\bar u(p_4)\gamma^{\mu_2}\tau_{a_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma^{\mu_4}\tau_{a_4}u(p_2)\\&\times[\epsilon_{a_1a_4a_5}\epsilon_{a_2a_3a_5}(g_{\mu_1\mu_2}g_{\mu_3\mu_4}-g_{\mu_1\mu_3}g_{\mu_2\mu_4})+\epsilon_{a_3a_1a_5}\epsilon_{a_4a_2a_5}(g_{\mu_3\mu_4}g_{\mu_1\mu_2}-g_{\mu_3\mu_2}g_{\mu_1\mu_4})\\&+\epsilon_{a_3a_4a_5}\epsilon_{a_1a_2a_5}(g_{\mu_3\mu_1}g_{\mu_2\mu_4}-g_{\mu_3\mu_2}g_{\mu_1\mu_4})]\\&\times\frac{1}{k_1^2+\mu^2}\,\frac{1}{(k+k_1)^2+\mu^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,.\end{aligned} \tag{4.5}$$

Let us look at $iT^{(3)}$. This amplitude cannot give any $\ln s$ factor, since the left half of the graph has completely independent internal integration from the right. In fact, the factors $\epsilon_{a_1a_4a_5}\epsilon_{a_2a_3a_5}g_{\mu_1\mu_2}g_{\mu_3\mu_4}$ and $\epsilon_{a_3a_4a_5}\epsilon_{a_1a_2a_5}g_{\mu_3\mu_2}g_{\mu_1\mu_4}$ give rise to s^2 terms while the other terms together give at most s.

Thus, we write it as

$$iT^{(3)}\cong iT_1^{(3)}+iT_2^{(3)}, \tag{4.6}$$

where

$$\begin{aligned}iT_1^{(3)}=&\,i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar u(p_3)\gamma^{\mu_1}\tau_{a_3}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\tau_{a_1}u(p_1)\bar u(p_4)\gamma_{\mu_2}\tau_{a_2}\\&\times\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_4}u(p_2)\\&\times\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\frac{1}{k_1^2+\mu^2}\,\frac{1}{(k+k_1)^2+\mu^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,,\end{aligned} \tag{4.7}$$

and

$$\begin{aligned}iT_2^{(3)}=&\,i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar u(p_3)\gamma^{\mu_1}\tau_{a_3}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\tau_{a_1}u(p_1)\bar u(p_4)\gamma_{\mu_1}\tau_{a_4}\frac{1}{m+\gamma\cdot(p_2+k_2)}\gamma_{\mu_1}\tau_{a_2}u(p_2)\\&\times\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\frac{1}{k_1^2+\mu^2}\,\frac{1}{(k+k_1)^2+\mu^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,.\end{aligned} \tag{4.8}$$

We have interchanged $\mu_1 \leftrightarrow \mu_2$ in one of the bilinears of $iT_{1,2}^{(3)}$. This does not affect the coefficients of the s^2 terms. Also a change of labels and dummy integration variables has been made in $iT_2^{(3)}$.

We turn to work on $iT^{(1)}$ and $iT^{(2)}$ so that terms contributing to s^2 behavior cancel out the corresponding ones in $iT^{(3)}$.

The first thing to notice is that $N^a_{\mu_1\mu_2\mu_3\mu_4}$ [Eq. (4.4a)] contributes up to power of $\ln s$ $O(1)$ to $iT^{(1),(2)}$ [Eqs. (4.1) and (4.2)]. This we neglect.

For $N^b_{\mu_1\mu_2\mu_3\mu_4}$[Eq. (4.4b)] we do the following manipulation: Clearly, in order to obtain $\ln s$ factors for the amplitudes, both p_1 (or p_3) and p_2 (or p_4) must appear somewhere in the seven denominator factors before we carry out the internal integration. Because of this, we realize that we are allowed to drop terms with $(k_1)_{\mu_1}$ and $(k_2)_{\mu_2}$, while replacing $(k_1)_{\mu_3}$ by $-k_{\mu_3}$ and $(k_2)_{\mu_4}$ by $-k_{\mu_4}$. This is a consequence of applying the Dirac equation and what we said just a moment ago. For example, if we have $(k_1)_{\mu_1}$, then after contracting with γ^{μ_1}, the relevant combination in $iT^{(1),(2)}$ is

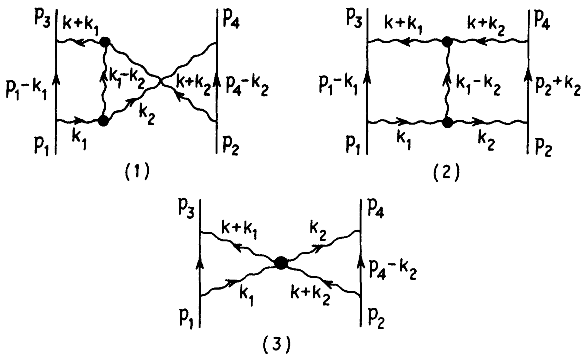

FIG. 3. Ladder graphs in sixth order.

$$\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma\cdot k_1 u(p_1)$$
$$= -\frac{1}{m+\gamma\cdot(p_1-k_1)}(m+\gamma\cdot p_1-\gamma\cdot k_1)\,u(p_1)$$
$$= -u(p_1)\,. \tag{4.9}$$

This we do not need to consider any further, since p_1 has been completely eliminated from the denominators. As another example, suppose we have $(k_1)_{\mu_3}$, then the relevant combination is

$$\bar{u}(p_3)\gamma\cdot k_1\frac{1}{m+\gamma\cdot(p_1-k_1)}$$
$$= -\bar{u}(p_3)[m+\gamma\cdot(p_1-k_1)+\gamma\cdot k]\frac{1}{m+\gamma\cdot(p_1-k_1)}$$
$$= -\bar{u}(p_3)\left[1+\gamma\cdot k\frac{1}{m+\gamma\cdot(p_1-k_1)}\right]$$
$$\cong -\bar{u}(p_3)\gamma\cdot k\frac{1}{m+\gamma\cdot(p_1-k_1)}. \tag{4.10}$$

In short $(k_1)_{\mu_3}\to -k_{\mu_3}$.

After we make all these substitutions and simplifications, we have

$$\begin{aligned} iT_b^{(1)}\cong\; & i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\big(-4\bar{u}(p_3)\{\gamma^{\mu_3}[m-\gamma\cdot(p_1-k_1)]\gamma\cdot k_2-\gamma\cdot k_2[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_3}\}\tau_{a_3}\tau_{a_1}u(p_1)\\ &\times\bar{u}(p_4)\{\gamma\cdot k_1[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_3}-\gamma_{\mu_3}[m-\gamma\cdot(p_4-k_2)]\gamma\cdot k_1\}\tau_{a_2}\tau_{a_4}u(p_2)\\ &-4\bar{u}(p_3)\gamma^{\mu_3}[m-\gamma\cdot(p_1-k_1)]\gamma\cdot k_2\tau_{a_3}\tau_{a_1}u(p_1)\\ &\times\bar{u}(p_4)\{\gamma\cdot k[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_3}-\gamma_{\mu_3}[m-\gamma\cdot(p_4-k_2)]\gamma\cdot k\}\tau_{a_2}\tau_{a_4}u(p_2)\\ &-4\bar{u}(p_3)\{\gamma^{\mu_3}[m-\gamma\cdot(p_1-k_1)]\gamma\cdot k-\gamma\cdot k[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_3}\}\tau_{a_3}\tau_{a_1}u(p_1)\\ &\times\bar{u}(p_4)\gamma\cdot k_1[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_3}\tau_{a_2}\tau_{a_4}u(p_2)\\ &+4\bar{u}(p_3)\gamma\cdot k[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_3}\tau_{a_3}\tau_{a_2}u(p_1)\bar{u}(p_4)\gamma_{\mu_3}[m-\gamma\cdot(p_4-k_2)]\gamma\cdot k\tau_{a_2}\tau_{a_4}u(p_2)\big)\\ &\times\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\frac{1}{m^2+(p_1-k_1)^2}\frac{1}{k_2^2+\mu^2}\frac{1}{m^2+(p_4-k_2)^2}\\ &\times\frac{1}{k_1^2+\mu^2}\frac{1}{(k+k_1)^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}. \end{aligned}\tag{4.11}$$

and a similar expression for $iT_b^{(2)}$. $iT_b^{(1),(2)}$ here are of course $iT^{(1),(2)}$, respectively, when $N_{\mu_1\mu_2\mu_3\mu_4}$ is due to $N^b_{\mu_1\mu_2\mu_3\mu_4}$ [Eqs. (4.1)–(4.4)]. We will show in Appendix A that $iT_b^{(1),(2)}$ do not contribute to $O(s\ln^2 s)$. Note the special combinations in the first three terms of $iT_b^{(1)}$.

$N^c_{\mu_1\mu_2\mu_3\mu_4}$ [Eq. (4.4c)] is now taken up. It will lead to s^2 terms for $iT^{(1),(2)}$. Thus, instead of treating it alone, we find it advantageous to combine $iT_c^{(1),(2)}$ with $iT_{1,2}^{(3)}$ to effect cancellation of this unwanted behavior. We then consider

$$\begin{aligned} iT^{(1,3)}&\equiv iT_c^{(1)}+iT_1^{(3)}\\ &=i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_3}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)\bar{u}(p_4)\gamma_{\mu_2}\tau_{a_2}\tau_{a_4}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\\ &\quad\times\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\frac{1}{k_1^2+\mu^2}\frac{1}{(k+k_1)^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}\\ &\quad\times[2k_1^2+2k_2^2+2k\cdot(k_1+k_2)+\mu^2], \end{aligned}\tag{4.12}$$

where we have multiplied and divided $iT_1^{(3)}$ by $(k_1-k_2)^2+\mu^2$ to arrive at this form. Now we write,

$$2k_1^2+2k_2^2+2k\cdot(k_1+k_2)+\mu^2=\{(k_1^2+\mu^2)+[(k+k_1)^2+\mu^2]\}+\{(k_2^2+\mu^2)+[(k+k_2)^2+\mu^2]\}-3\mu^2-2k^2 \tag{4.13}$$

and define

$$iT^{(1,3)}=iT^{(1,3)}(k_1)+iT^{(1,3)}(k_2)+iT^{(1,3)}(\mu^2,t), \tag{4.14}$$

where, e.g., $iT^{(1,3)}(k_1)$ is the piece due to $[(k_1^2+\mu^2)+(k+k_1)^2+\mu^2]$ in Eq. (4.13), i.e.,

$$iT^{(1,3)}(k_1)=i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_3}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)$$
$$\times\bar{u}(p_4)\gamma_{\mu_2}\tau_{a_2}\tau_{a_4}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)$$
$$\times\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\left[\frac{1}{k_1^2+\mu^2}+\frac{1}{(k+k_1)^2+\mu^2}\right]\frac{1}{(k_1-k_2)^2+\mu^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}. \tag{4.15}$$

Similarly,

$$iT^{(1,3)}(k_2)=i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_3}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)$$
$$\times\bar{u}(p_4)\gamma_{\mu_2}\tau_{a_2}\tau_{a_4}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)$$
$$\times\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\frac{1}{k_1^2+\mu^2}\,\frac{1}{(k+k_1)^2+\mu^2}\,\frac{1}{(k_1-k_2)^2+\mu^2}\left[\frac{1}{k_2^2+\mu^2}+\frac{1}{(k+k_2)^2+\mu^2}\right], \tag{4.16}$$

and

$$iT^{(1,3)}(\mu^2,t)=-3i\left(\frac{g}{2}\right)^4 g^2(\mu^2+\tfrac{2}{3}k^2)\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_3}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)$$
$$\times\bar{u}(p_4)\gamma_{\mu_2}\tau_{a_2}\tau_{a_4}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}\frac{1}{k_1^2+\mu^2}\,\frac{1}{(k+k_1)^2+\mu^2}$$
$$\times\frac{1}{(k_1-k_2)^2+\mu^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}. \tag{4.17}$$

If we are to evaluate the high-energy behavior of $iT^{(1,3)}(k_1)$ [Eq. (4.15)] and $iT^{(1,3)}(k_2)$ [Eq. (4.16)] we would find that it is $O(s\ln^3 s)$. We do not want to do that, however. We prefer to cancel these amplitudes out with parts of the radiative graphs.

Before we turn to this task, we may remark that it is obvious we should do the same thing to $iT_c^{(2)}$ and $iT_2^{(3)}$, i.e., we define

$$iT^{(2,3)}=iT_c^{(2)}+iT_2^{(3)}=iT^{(2,3)}(k_1)+iT^{(2,3)}(k_2)+iT^{(2,3)}(\mu^2), \tag{4.18}$$

where, e.g.,

$$iT^{(2,3)}(k_1)=i\left(\frac{g}{2}\right)^4 g^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_3}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)$$
$$\times\bar{u}(p_4)\gamma_{\mu_1}\tau_{a_4}\tau_{a_2}\frac{1}{m+\gamma\cdot(p_2+k_2)}\gamma_{\mu_2}u(p_2)$$
$$\times\epsilon_{a_1a_2a_3}\epsilon_{a_5a_4a_3}\left[\frac{1}{k_1^2+\mu^2}+\frac{1}{(k+k_1)^2+\mu^2}\right]\frac{1}{(k_1-k_2)^2+\mu^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}. \tag{4.19}$$

B. Radiative graphs

We will consider only those radiative graphs due to three-vector coupling in this subsection. The rest will be included later.

There are altogether twelve of them in this category (see Figs. 4 and 5). However, we need to consider only two, since graphs (6) and (7) are related to graphs (4) and (5), respectively, by $s-u$ crossing, plus proper change of isospin matrices. Graphs (8) and (9) are the time-reversed diagrams of graphs (4) and (6), respectively. Finally, graphs (4′)–(9′) are the left-right reflected graphs of the corresponding unprimed ones.

With the momenta as assigned in graph (4), the amplitude is

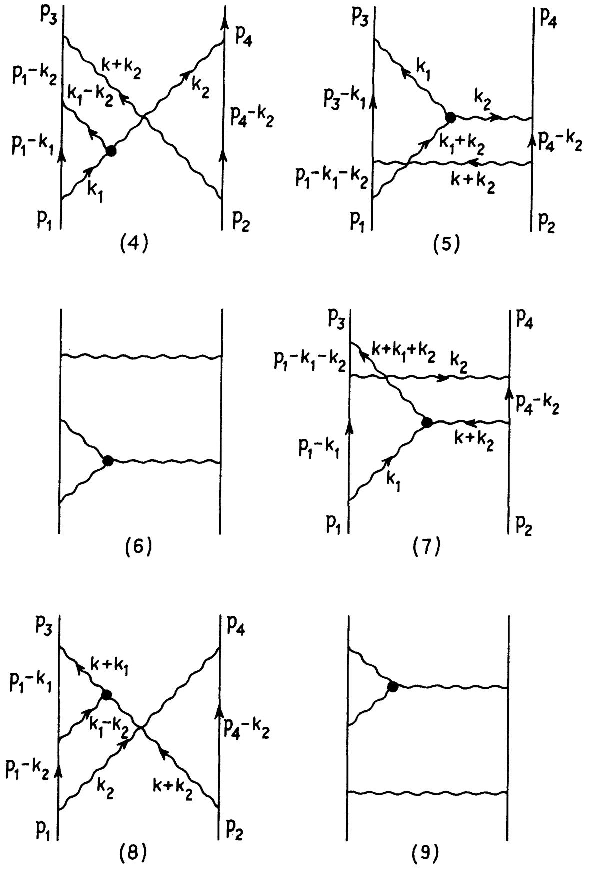

FIG. 4. Three-vector radiative graphs.

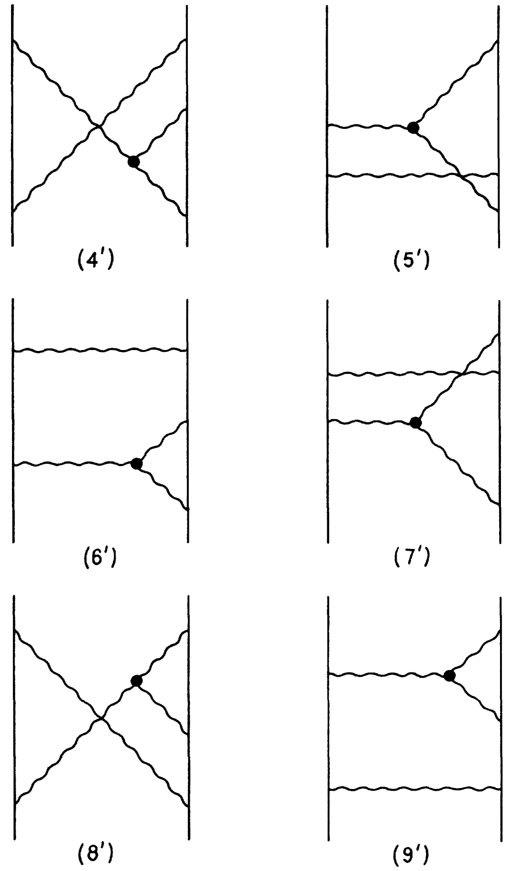

FIG. 5. Left-right mirrors of graphs in Fig. 4.

$$iT^{(4)}=-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}\gamma^{\mu_3}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_4}u(p_1)$$
$$\times\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma^{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)$$
$$\times\epsilon_{a_2a_3a_4}[-g_{\mu_3\mu_2}(k_1-2k_2)_{\mu_4}-g_{\mu_2\mu_4}(k_1+k_2)_{\mu_3}+g_{\mu_4\mu_3}(2k_1-k_2)_{\mu_2}]$$
$$\times\frac{1}{k_1^{\,2}+\mu^2}\,\frac{1}{(k_1-k_2)^2+\mu^2}\,\frac{1}{k_2^{\,2}+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,. \tag{4.20}$$

There are basically two kinds of terms in the three-vector vertex: the ones with k_2 and ones with k_1. We can show that the former ones give $O(s\ln^3 s)$ to $iT^{(4)}$, while the latter after renormalization give $O(s\ln^2 s)$. We then define

$$iT^{(4)}=iT_1^{(4)}+iT_2^{(4)}, \tag{4.21}$$

where $iT_{1,2}^{(4)}$ are obtained by retaining only the $k_{1,2}$-dependent terms, respectively, of the three-vector vertex, i.e.,

$$iT_1^{(4)}=-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}$$
$$\times\{-\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)]\gamma^{\mu_2}[m-\gamma\cdot(p_1-k_1)]\gamma\cdot k_1-\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)]\gamma\cdot k_1[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_2}$$
$$+2\gamma^{\mu_1}k_1^{\mu_2}[m-\gamma\cdot(p_1-k_2)]\gamma^{\lambda}[m-\gamma\cdot(p_1-k_1)]\gamma_\lambda\}u(p_1)$$
$$\times\epsilon_{a_2a_3a_4}\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)$$
$$\times\frac{1}{m^2+(p_1-k_2)^2}\,\frac{1}{m^2+(p_1-k_1)^2}\,\frac{1}{k_1^{\,2}+\mu^2}\,\frac{1}{(k_1-k_2)^2+\mu^2}\,\frac{1}{k_2^{\,2}+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,, \tag{4.22}$$

$$iT_2^{(4)} = -\left(\frac{g}{2}\right)^5 g \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}$$
$$\times\left[2\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}\gamma^{\mu_2}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma\cdot k_2-\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}\gamma\cdot k_2\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\right.$$
$$\left.-\gamma^{\mu_1}k_2^{\mu_2}\frac{1}{m+\gamma\cdot(p_1-k_2)}\gamma^{\lambda}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma_\lambda\right]u(p_1)$$
$$\times\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\epsilon_{a_2a_3a_4}\frac{1}{k_1^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}. \tag{4.23}$$

We introduce the following expression:

$$i\overline{T}^{(4)} = \left(\frac{g}{2}\right)^5 g \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\epsilon_{a_2a_3a_4}$$
$$\times\frac{1}{k_1^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}. \tag{4.24}$$

It will turn out that this is a useful quantity to consider in order to cancel out the $O(s\ln^3 s)$ behavior in the ladder. Thus, we calculate the difference between $iT_2^{(4)}$ and $i\overline{T}^{(4)}$, Eqs. (4.23) and (4.24). We reexpress one of the fermion propagators in $i\overline{T}^{(4)}$ as

$$\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}u(p_1)=\bar{u}(p_3)\left\{\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}[m+\gamma\cdot(p_1-k_2)]\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\right\}u(p_1)$$
$$=\bar{u}(p_3)\left\{-\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}\gamma\cdot k_2\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\right.$$
$$\left.+\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}(m+\gamma\cdot p_1)\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\right\}u(p_1). \tag{4.25}$$

Then

$$iT_2^{(4)}-i\overline{T}^{(4)}=-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}\epsilon_{a_2a_3a_4}$$
$$\times\left\{\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)](m+\gamma\cdot p_1)[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_2}\right.$$
$$-2\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)]\gamma\cdot k_2[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_2}$$
$$+2\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)]\gamma^{\mu_2}[m-\gamma\cdot(p_1-k_1)]\gamma\cdot k_2$$
$$\left.-\gamma^{\mu_1}k_2^{\mu_2}[m-\gamma\cdot(p_1-k_2)]\gamma^{\lambda}[m-\gamma\cdot(p_1-k_1)]\gamma_\lambda\right\}$$
$$\times u(p_1)\,\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)$$
$$\times\frac{1}{k_1^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}\frac{1}{m^2+(p_1-k_2)^2}\frac{1}{m^2+(p_1-k_1)^2}. \tag{4.26}$$

The noticeable features of the terms in the curly brackets are (a) the first term has a projection operator $(m+\gamma\cdot p_1)$, which can be shown to suppress its high-energy behavior, (b) the last three factors of the third term are in the reversed order of those of the second term. This combination also creates high-energy suppression. And, finally, (c) the fourth term involves a contraction, which is not a good way to realize large high-energy behavior. In fact, by paralleling the analysis to be pursued in Sec. V, we can show that $iT_2^{(4)}-i\overline{T}^{(4)}\sim O(s\ln s)$. This will be further discussed in Appendix B. Then

$$iT^{(4)}=iT_1^{(4)}+i\overline{T}^{(4)}+i(T_2^{(4)}-i\overline{T}^{(4)})$$
$$\cong iT_1^{(4)}+i\overline{T}^{(4)}+O(s\ln s). \tag{4.27}$$

We now manipulate the expression for graph (5), the amplitude of which is

$$iT^{(5)} = -\left(\frac{g}{2}\right)^5 g\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_3}\tau_{a_3}\frac{1}{m+\gamma\cdot(p_3-k_1)}\gamma^{\mu_1}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1-k_2)}\gamma^{\mu_4}\tau_{a_4}u(p_1)$$
$$\times\bar{u}(p_4)\gamma^{\mu_2}\tau_{a_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_1}u(p_2)$$
$$\times\epsilon_{a_2a_3a_4}\left[-g_{\mu_3\mu_2}(k_1-k_2)_{\mu_4}-g_{\mu_2\mu_4}(2k_2+k_1)_{\mu_3}+g_{\mu_4\mu_3}(2k_1+k_2)_{\mu_2}\right]$$
$$\times\frac{1}{(k_1+k_2)^2+\mu^2}\,\frac{1}{k_1{}^2+\mu^2}\,\frac{1}{k_2{}^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,. \tag{4.28}$$

We introduce the following two expressions:

$$i\overline{T}_a^{(5)} = -\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\tau_{a_3}\tau_{a_1}\tau_{a_4}u(p_1)\epsilon_{a_2a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)$$
$$\times\frac{1}{k_1{}^2+\mu^2}\,\frac{1}{(k_1-k_2)^2+\mu^2}\,\frac{1}{k_2{}^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}$$
$$= -\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1'}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1'-k_2)}\gamma^{\mu_2}\tau_{a_3}\tau_{a_1}\tau_{a_4}u(p_1)\epsilon_{a_2a_3a_4}$$
$$\times\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)\frac{1}{k_1'^2+\mu^2}\,\frac{1}{(k_1'+k_2)^2+\mu^2}\,\frac{1}{k_2{}^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}$$
$$(k_1'=k_1-k_2)\,, \tag{4.29}$$

and

$$i\overline{T}_b^{(5)} = -\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_2}\frac{1}{m+\gamma\cdot(p_3-k_1)}\gamma^{\mu_1}\tau_{a_3}\tau_{a_1}\tau_{a_4}u(p_1)\epsilon_{a_2a_3a_4}$$
$$\times\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)\frac{1}{k_1{}^2+\mu^2}\,\frac{1}{(k_1+k_2)^2+\mu^2}\,\frac{1}{k_2{}^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,. \tag{4.30}$$

$i\overline{T}_{a,b}^{(5)}$ are used to juxtapose according to

$$iT^{(5)} = (iT^{(5)} - i\overline{T}_a^{(5)} - i\overline{T}_b^{(5)}) + i\overline{T}_a^{(5)} + i\overline{T}_b^{(5)}\,, \tag{4.31}$$

where, if we multiply and divide the first bilinear form of $i\overline{T}_a^{(5)}$ and $i\overline{T}_b^{(5)}$, respectively, by $m+\gamma\cdot(p_3-k_1)$ and $m+\gamma\cdot(p_1-k_1-k_2)$, we obtain

$$iT^{(5)} - i\overline{T}_a^{(5)} - i\overline{T}_b^{(5)}$$
$$= -\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}N_{(5)}^{\mu_1\mu_2}\epsilon_{a_2a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)$$
$$\times\frac{1}{m^2+(p_3-k_1)^2}\,\frac{1}{m^2+(p_1-k_1-k_2)^2}\,\frac{1}{m^2+(p_4-k_2)^2}\,\frac{1}{k_1{}^2+\mu^2}\,\frac{1}{(k_1+k_2)^2+\mu^2}\,\frac{1}{k_2{}^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,, \tag{4.32}$$

with

$$N_{(5)}^{\mu_1\mu_2} = \bar{u}(p_3)\{2\gamma^{\mu_2}[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma\cdot k_2 - 2\gamma\cdot k_2[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma^{\mu_2}$$
$$-[m+\gamma\cdot(p_3-k_1)][m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma^{\mu_2}$$
$$-\gamma^{\mu_2}[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)][m+\gamma\cdot(p_1-k_1)]$$
$$+k_2{}^{\mu_2}\gamma^\lambda[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma_\lambda - \gamma^{\mu_2}[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma\cdot k_1$$
$$-\gamma\cdot k_1[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma^{\mu_2} + 2k_1^{\mu_2}\gamma^\lambda[m-\gamma\cdot(p_3-k_1)]\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_1-k_2)]\gamma_\lambda\}$$
$$\times\tau_{a_3}\tau_{a_1}\tau_{a_4}u(p_1)\,. \tag{4.33}$$

We will show in Appendix C that

$$iT^{(5)} - i\overline{T}_a^{(5)} - i\overline{T}_b^{(5)} \cong O(s\ln s)\,. \tag{4.34}$$

Now we see the purpose of interlacing $iT^{(4),(5)}$ with the overbarred quantities. For example,

$$i\overline{T}^{(4)}+i\overline{T}_a^{(5)}=-\left(\frac{g}{2}\right)^5\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}[\tau_{a_3},\tau_{a_1}]\tau_{a_4}u(p_1)$$
$$\times\epsilon_{a_2a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_1)\frac{1}{k_1^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}, \tag{4.35}$$

which cancels out the term with $1/(k_1^2+\mu^2)$ in the square brackets of $iT^{(1,3)}(k_1)$ [Eq. (4.15)], upon using the commutation relation

$$[\tau_{a_3},\tau_{a_1}]=2i\epsilon_{a_3a_1a_5}\tau_{a_5} \tag{4.36}$$

and relabelling some isospin indices.

Similarly, for graphs (7) and (8) we introduce

$$i\overline{T}_a^{(7)}=-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1-k_2)}\gamma^{\mu_2}\tau_{a_3}\tau_{a_2}\tau_{a_4}u(p_1)$$
$$\times\epsilon_{a_1a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)\frac{1}{k_1^2+\mu^2}\frac{1}{(k+k_1+k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}$$
$$=-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1'}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1')}\gamma^{\mu_2}\tau_{a_3}\tau_{a_2}\tau_{a_4}u(p_1)\epsilon_{a_1a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)$$
$$\times\frac{1}{(k+k_1')^2+\mu^2}\frac{1}{(k_1'-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}\qquad(k_1+k_2=k_1'), \tag{4.37}$$

$$i\overline{T}_b^{(7)}=-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_2}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_1}\tau_{a_3}\tau_{a_2}\tau_{a_4}u(p_1)$$
$$\times\epsilon_{a_1a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)\frac{1}{k_1^2+\mu^2}\frac{1}{(k+k_1+k_2)+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}, \tag{4.38}$$

and

$$i\overline{T}^{(8)}=\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\tau_{a_3}\tau_{a_4}\tau_{a_2}u(p_1)$$
$$\times\epsilon_{a_1a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)\frac{1}{(k+k_1)^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}. \tag{4.39}$$

We can show that

$$iT^{(7)}-i\overline{T}_a^{(7)}-i\overline{T}_b^{(7)}\cong O(s\ln s), \tag{4.40}$$

and

$$iT_2^{(8)}-i\overline{T}^{(8)}\cong O(s\ln s), \tag{4.41}$$

where $iT_2^{(8)}$ is that part of $iT^{(8)}$ in which the three-vector vertex has only k_2 dependence. Besides,

$$i\overline{T}_a^{(7)}+i\overline{T}^{(8)}=\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_1}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma^{\mu_2}\tau_{a_3}[\tau_{a_4},\tau_{a_2}]u(p_1)$$
$$\times\epsilon_{a_1a_3a_4}\bar{u}(p_4)\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)$$
$$\times\frac{1}{(k+k_1)^2+\mu^2}\frac{1}{(k_1-k_2)^2+\mu^2}\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}, \tag{4.42}$$

which cancels out the other part of $iT^{(1,3)}(k_1)$ (Eq. (4.15) with $1/[(k+k_1)^2+\mu^2]$ in the square brackets).

It is clear by now that we can introduce $i\overline{T}^{(6)}$ and $i\overline{T}^{(9)}$ in the same manner. Together with $i\overline{T}_b^{(5)}$ and $i\overline{T}_b^{(7)}$, they will cancel out $iT^{(2,3)}(k_1)$. $i\overline{T}^{(4')}$, $i\overline{T}_{a,b}^{(5')}$, $i\overline{T}^{(6')}$, $i\overline{T}_{a,b}^{(7')}$, $i\overline{T}^{(8')}$, and $i\overline{T}^{(9')}$ similarly defined will cancel out $iT^{(1,3)}(k_2)$ and $iT^{(2,3)}(k_2)$.

All in all, we find that

$$iT^{(1)}+iT^{(2)}+iT^{(3)}+iT^{(4)}+\cdots+iT^{(9)}+iT^{(4')}+\cdots+iT^{(9')}$$

$$=iT^{(1,3)}(\mu^2,t)+iT^{(2,3)}(\mu^2,t)+iT_1^{(4)}+iT_1^{(6)}+iT_1^{(8)}+iT_1^{(9)}+iT_1^{(4')}+iT_1^{(6')}+iT_1^{(8')}+iT_1^{(9')}+O(s\ln s)\,. \quad (4.43)$$

We may remind our reader that the subscript 1 in $iT_1^{(4)\cdots(9')}$ means that we retain only the k_1- (and possibly k-) dependent part of the three-vector vertex.

V. SIXTH-ORDER CALCULATION; EXTRACTION OF LEADING HIGH-ENERGY BEHAVIOR

A. Scalar χ production

The graphs we first consider are shown in Fig. 6. We have

$$iT^{(10)}=\frac{1}{i}\left(\frac{g}{2}\right)^4(g\mu)^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_2}\tau_{a_2}[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_1}\tau_{a_1}u(p_1)\bar{u}(p_4)\gamma_{\mu_1}\tau_{a_1}[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_2}\tau_{a_2}u(p_2)$$

$$\times\frac{1}{m^2+(p_1-k_1)^2}\,\frac{1}{k_1^2+\mu^2}\,\frac{1}{(k+k_1)^2+\mu^2}\,\frac{1}{(k_1-k_2)^2+m_\chi^2}\,\frac{1}{k_2^2+\mu^2}\,\frac{1}{(k+k_2)^2+\mu^2}\,\frac{1}{m^2+(p_4-k_2)^2}\,. \quad (5.1)$$

We may approximate, at high energy,

$$\bar{u}(p_3)\gamma^{\mu_2}[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_1}u(p_1)\cong 2p^{\mu_1}p^{\mu_2}/m$$

and (5.2)

$$\bar{u}(p_4)\gamma_{\mu_1}[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_2}u(p_2)\cong 2p'_{\mu_1}p'_{\mu_2}/m\,.$$

We introduce α parameters for this amplitude in the same way as we would for graph (1) (see Appendix A). We have

$$iT^{(10)}\cong i\left(\frac{1}{16\pi^2}\right)^2\left(\frac{g}{2}\right)^4 g^2\mu^2\frac{s^2}{m^2}I^{(10)}$$

$$\times\bar{u}(p_3)\tau_{a_2}\tau_{a_1}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2)\,, \quad (5.3)$$

where

$$I^{(10)}=2\int\prod_{i=1}^{7}d\alpha_i\,\delta\left(1-\sum_{j=1}^{7}\alpha_j\right)\frac{\lambda_1\lambda_2}{D_{(10)}{}^3} \quad (5.4)$$

with

$$D_{(10)}=\alpha_1\alpha_2\alpha_7 s+d_{(10)}\,,$$

$$d_{(10)}=k^2[\alpha_3\alpha_4(\alpha_5+\alpha_6)+\alpha_5\alpha_6(\alpha_3+\alpha_4)]$$

$$+\mu^2(\alpha_3+\alpha_4+\alpha_5+\alpha_6)\lambda_1\lambda_2\,,$$

and

$$\lambda_1\lambda_2=(\alpha_3+\alpha_4)(\alpha_5+\alpha_6)\,.$$

Simple integrations yield

$$I^{(10)}\cong\frac{1}{2}\frac{\ln^2 s}{s}$$

$$\times\int d\alpha_3 d\alpha_4 d\alpha_5 d\alpha_6\,\delta(1-\alpha_3-\alpha_4-\alpha_5-\alpha_6)$$

$$\times\frac{(\alpha_3+\alpha_4)(\alpha_5+\alpha_6)}{d_{(10)}{}^2}\,. \quad (5.5)$$

We then scale the variables according to

$$\alpha_3=\rho_1\alpha'_3,\quad \alpha_4=\rho_1\alpha'_4\,,$$

and

$$\alpha_5=\rho_2\alpha'_5,\quad \alpha_6=\rho_2\alpha'_6\,,$$

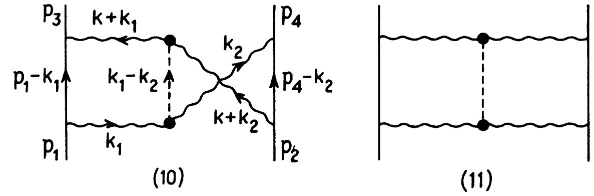

FIG. 6. Ladder graphs with physical scalar production.

then

$$I^{(10)} \cong \frac{1}{2}\frac{\ln^2 s}{s}\int d\rho_1 d\rho_2 \delta(1-\rho_1-\rho_2)\int d\alpha_3' d\alpha_4' \delta(1-\alpha_3'-\alpha_4') \times \int d\alpha_5' d\alpha_6' \delta(1-\alpha_5'-\alpha_6') \frac{1}{[k^2(\rho_1\alpha_3'\alpha_4'+\rho_2\alpha_5'\alpha_6')+\mu^2(\rho_1+\rho_2)]^2}$$

$$\cong \frac{1}{2}\frac{\ln^2 s}{s} K_1^2 , \qquad (5.6)$$

with

$$K_1 \equiv \int d\alpha_1 d\alpha_2 \delta(1-\alpha_1-\alpha_2)\frac{1}{k^2\alpha_1\alpha_2+\mu^2} . \qquad (5.7)$$

Consequently,

$$iT^{(10)} \cong i\left(\frac{1}{16\pi^2}\right)^2\left(\frac{g}{2}\right)^4 g^2\mu^2\left(\frac{s}{m^2}\right)\tfrac{1}{2}\ln^2 s K_1^2 \bar{u}(p_3)\tau_{a_2}\tau_{a_1}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2) . \qquad (5.8)$$

The contribution of graph (11) is obtained by $s \leftrightarrow -s$ and a proper change of τ-matrix ordering,

$$iT^{(11)} \cong i\left(\frac{1}{16\pi^2}\right)^2\left(\frac{g}{2}\right)^4 g^2\mu^2\left(-\frac{s}{m^2}\right)\tfrac{1}{2}\ln^2(-s) K_1^2 \bar{u}(p_3)\tau_{a_2}\tau_{a_1}u(p_1)\bar{u}(p_4)\tau_{a_2}\tau_{a_1}u(p_2) . \qquad (5.9)$$

B. Evaluation of $iT^{(1,3)}(\mu,^2 t)$ and $iT^{(2,3)}(\mu,^2 t)$

The extraction of the leading behavior of $iT^{(1,3)}(\mu^2, t)$ is almost identical to that in the previous subsection, with proper changes of coefficients and isospin structure. We find

$$iT^{(1,3)}(\mu^2,t) \cong i\left(\frac{1}{16\pi^2}\right)^2 3\left(\frac{g}{2}\right)^4 g^2(\mu^2+\tfrac{2}{3}k^2)\left(\frac{s}{m^2}\right)\tfrac{1}{2}\ln^2 s K_1^2 \bar{u}(p_3)\tau_{a_3}\tau_{a_1}u(p_1)\bar{u}(p_4)\tau_{a_2}\tau_{a_4}u(p_2)\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3} , \qquad (5.10)$$

and

$$iT^{(2,3)}(\mu^2,t) \cong i\left(\frac{1}{16\pi^2}\right)^2 3\left(\frac{g}{2}\right)^4 g^2(\mu^2+\tfrac{2}{3}k^2)\left(\frac{-s}{m^2}\right)\tfrac{1}{2}\ln^2(-s) K_1^2 \bar{u}(p_3)\tau_{a_3}\tau_{a_1}u(p_1)\bar{u}(p_4)\tau_{a_4}\tau_{a_2}u(p_2)\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3} . \qquad (5.11)$$

C. Renormalization due to fermion-fermion-vector coupling

The graphs which fall into this category are the ones shown in Figs. 7 and 8. Those in Figs. 8 are left-right reflections of the ones in Fig. 7. These graphs, except for trivial isospin factors, were considered before in massive quantum electrodynamics.[10]

Take graph (12) to start. Strictly speaking, it has nothing to do with renormalization. We include it here only for cataloging convenience. We write

$$iT^{(12)} = \frac{1}{i}\left(\frac{g}{2}\right)^2\int\frac{d^4k_2}{(2\pi)^4}(iT^{(12)}_{\text{left}})^{\mu_1\mu_2}\bar{u}(p_4)\gamma_{\mu_1}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_2+k_2)}\gamma_{\mu_2}\tau_{a_2}u(p_2)\frac{1}{k_2^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2} , \qquad (5.12)$$

where

$$(iT^{(12)}_{\text{left}})^{\mu_1\mu_2} = \left(\frac{g}{2}\right)^4\int\frac{d^4k_1}{(2\pi)^4}\bar{u}(p_3)\gamma^{\mu_3}\tau_{a_3}\frac{1}{m+\gamma\cdot(p_3-k_1)} \times\gamma^{\mu_1}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_1-k_2)}\gamma^{\mu_2}\tau_{a_2}\frac{1}{m+\gamma\cdot(p_1-k_1)}\gamma_{\mu_3}\tau_{\mu_3}u(p_1)\frac{1}{k_1^2+\mu^2}$$

$$= \left(\frac{g}{2}\right)^4 3!\int dx_1dx_2dx_3dx_4\delta\left(1-\sum_{i=1}^4 x_i\right)\int\frac{d^4k_1}{(2\pi)^4}\frac{N_{12}^{\mu_1\mu_2}}{d_{(12)}^{\;4}} , \qquad (5.13)$$

with

$$d_{(12)} = x_1[m^2+(p_1-k_1)^2]+x_2[m^2+(p_3-k_1)^2]+x_3[m^2+(p_1-k_1-k_2)^2]+x_4(k_1^2+\mu^2)$$

$$= \bar{k}_1^2+b_{(12)}^{\;2} ,$$

$$b_{(12)}^{\;2} = (x_1+x_2+x_3)^2m^2+x_2(x_1+x_3)k^2+x_3(2x_2+x_4)k\cdot k_2-2x_3x_4p\cdot k_2+x_3(1-x_3)k_2^2+x_4\mu^2 ,$$

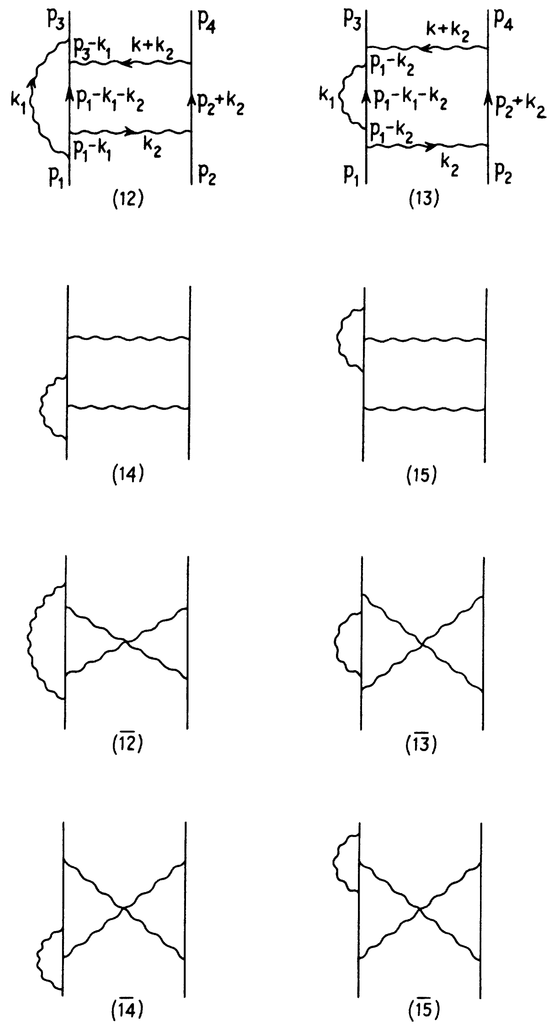

FIG. 7. Vector-fermion-fermion radiative graphs.

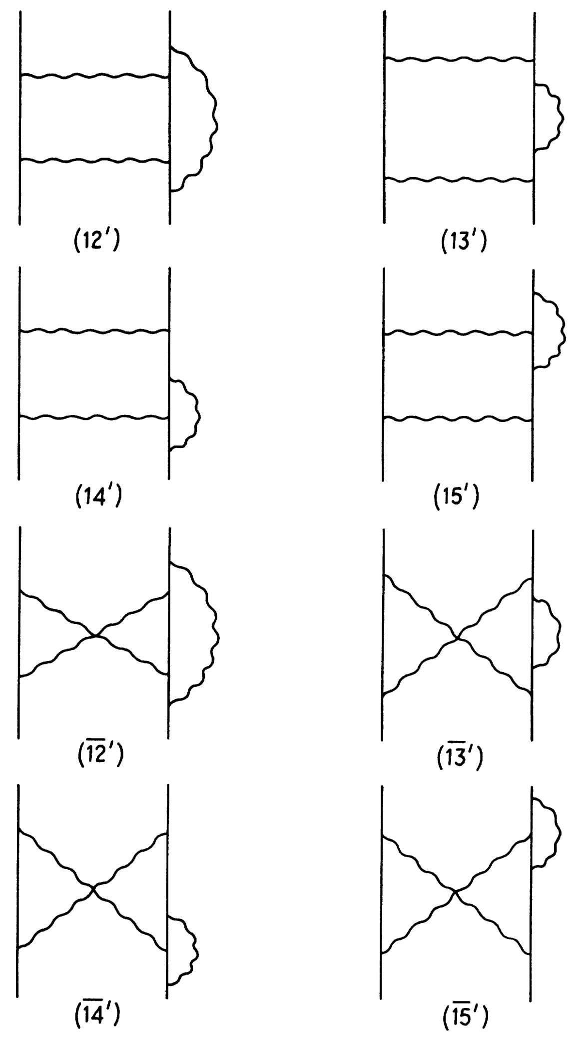

FIG. 8. Left-right mirrors of graphs in Fig. 7.

and

$$\bar{k}_1 = k_1 - [(x_1 + x_3)p_1 + x_2 p_3 - x_3 k_2] .$$

After rationalizing the fermion propagators, the leading behavior for $N_{(12)}^{\mu_1\mu_2}$ is obtained by retaining only the $\gamma \cdot \bar{k}_1$ term for those two lines adjacent to the external fermions, i.e.,

$$N_{(12)}^{\mu_1\mu_2} \cong \frac{2p^{\mu_1}p^{\mu_2}}{m}\, \bar{u}(p_3)\tau_{a_3}\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_1)\bar{k}_1{}^2 x_4 ,$$

where an average has been performed. Then after the $\bar{k}_1$ integration

$$(iT_{\text{left}}^{(12)})^{\mu_1\mu_2} \cong \frac{i}{16\pi^2}\left(\frac{g}{2}\right)^4 \frac{2p^{\mu_1}p^{\mu_2}}{m}\, 2\int dx_1dx_2dx_3dx_4\,\delta\left(1-\sum_{i=1}^{4} x_i\right) x_4 \frac{1}{b_{(12)}{}^2}\,\bar{u}(p_3)\tau_{a_3}\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_1) . \tag{5.14}$$

Inserting this into Eq. (5.12), we obtain

$$iT^{(12)} \cong \left(\frac{g}{2}\right)^6 \left(\frac{1}{16\pi^2}\right) \frac{2p^{\mu_1}p^{\mu_2}}{m} \bar{u}(p_3)\tau_{a_3}\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_1)$$

$$\times 2 \int dx_1 dx_2 dx_3 dx_4 \delta\left(1-\sum_{i=1}^{4} x_i\right) x_4 \int \frac{d^4k_2}{(2\pi)^4} \bar{u}(p_4)\gamma_{\mu_1}\tau_{a_1}[m-\gamma\cdot(p_2+k_2)]\gamma_{\mu_2}\tau_{a_2}u(p_2)$$

$$\times 3! \int d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 \delta\left(1-\sum_{j=1}^{4}\alpha_j\right)\frac{1}{D_{(12)}{}^4} \tag{5.15}$$

when

$$D_{(12)} = \alpha_1 b_{(12)}{}^2 + \alpha_2[m^2+(p_2+k_2)^2] + \alpha_3[(k+k_2)^2+\mu^2] + \alpha_4(k_2{}^2+\mu^2)$$

$$\cong \bar{k}_2{}^2 - \alpha_1\alpha_2 x_3 x_4 s + \alpha_3\alpha_4 k^2 + \mu^2 \quad (\bar{k}_2 \cong k_2 + \alpha_3 k),$$

in which we have made use of the observation that the important region of integration is $\alpha_1 \sim \alpha_2 \sim x_3 \sim 0$ to simplify.

A straightforward integration leads to

$$iT^{(12)} \cong i\left(\frac{g}{2}\right)^6 \left(\frac{1}{16\pi^2}\right)^2 \left(\frac{-s}{m^2}\right) \tfrac{1}{2}\ln^2(-s) K_1 \bar{u}(p_3)\tau_{a_3}\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_1)\,\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2). \tag{5.16}$$

We turn our attention to graph (13). This requires a mass renormalization. It is clear that the point of subtraction has no effect on the most leading term. We choose to subtract at the physical mass of the fermion. Then

$$iT^{(13)} = \left(\frac{g}{2}\right)^4 \int \frac{d^4k_2}{(2\pi)^4} \bar{u}(p_3)\gamma^{\mu_1}\tau_{a_1}\frac{1}{m+\gamma\cdot(p_1-k_2)}[-\Sigma_{\rm ren}(p_1-k_2)]\frac{1}{m+\gamma\cdot(p_1-k_2)}\gamma^{\mu_2}\tau_{a_2}u(p_1)$$

$$\times \bar{u}(p_4)\tau_{a_1}\gamma_{\mu_1}\frac{1}{m+\gamma\cdot(p_2+k_2)}\gamma_{\mu_2}\tau_{a_2}u(p_2)\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}, \tag{5.17}$$

in which

$$\Sigma_{\rm ren}(p_1-k_2) = [m+\gamma\cdot(p_1-k_2)]^2\left(\frac{g}{2}\right)^2\frac{1}{8\pi^2}(\tau_{a_3})^2$$

$$\times \int_0^1 dx_1 dx_2 \delta(1-x_1-x_2)x_1x_2 \int_0^1 dz \frac{m(1+x_2)+[\gamma\cdot(p_1-k_2)-m](1-x_2)\left[1-\dfrac{2m^2x_2(1+x_2)z}{m^2x_2{}^2+\mu^2x_1}\right]}{m^2x_2{}^2+[(p_1-k_2)^2+m^2]x_1x_2z+\mu^2x_1} \tag{5.18}$$

is the renormalized mass operator. We introduce α parameters and write

$$iT^{(13)} = \left(\frac{g}{2}\right)^6 \frac{1}{8\pi^2}\int_0^1 dz \int dx_1 dx_2 \delta(1-x_1-x_2)x_1x_2$$

$$\times \int \frac{d^4k_2}{(2\pi)^4} N^{\mu_1\mu_2}_{(13)}\bar{u}(p_4)\gamma_{\mu_1}\tau_{a_1}[m-\gamma\cdot(p_2+k_2)]\gamma_{\mu_2}\tau_{a_2}u(p_2)$$

$$\times 3! \int d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 \delta\left(1-\sum_{i=1}^{4}\alpha_i\right)\frac{1}{D_{(13)}{}^4}. \tag{5.19}$$

where

$$D_{(13)} = \alpha_1\{m^2x_2{}^2+[(p_1-k_2)^2+m^2]x_1x_2z+\mu^2x_1\}$$

$$+\alpha_2[(k+k_2)^2+\mu^2]+\alpha_3(k_2{}^2+\mu^2)+\alpha_4[(p_2+k_2)^2+m^2]$$

$$\cong \bar{k}_2{}^2 - \alpha_1\alpha_4x_1x_2zs + \alpha_2\alpha_3k^2+\mu^2, \quad \bar{k}_2 \cong k_2+\alpha_2 k$$

and

$$N^{\mu_1\mu_2}_{(13)} \cong \frac{2p^{\mu_1}p^{\mu_2}}{m} x_1\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_3}\tau_{a_2}u(p_1).$$

After carrying out the $\bar{k}_2$ integration, we arrive at

$$iT^{(13)} \cong \frac{i}{8\pi^2}\frac{1}{16\pi^2}\left(\frac{g}{2}\right)^6 \frac{s^2}{m^2}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_3}\tau_{a_2}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2)$$

$$\times \int dz \int dx_1 dx_2 \delta(1-x_1-x_2)x_1{}^2 x_2 \int d\alpha_2 d\alpha_3 \delta(1-\alpha_2-\alpha_3) \int d\alpha_1 d\alpha_4 \frac{1}{(-\alpha_1\alpha_4 x_1 x_2 z s + k^2\alpha_2\alpha_3+\mu^2)^2}$$

$$\cong i\left(\frac{1}{16\pi^2}\right)^2\left(\frac{g}{2}\right)^6 \frac{-s}{m^2}\tfrac{1}{2}\ln^2(-s)K_1\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_3}\tau_{a_2}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2)\,. \tag{5.20}$$

Graphs (14) and (15) can be treated similarly. (We subtract the vertex correction at zero momentum transfer when the fermions are on shell.) Let us not overload the reader with details. Suffice it to write down the results

$$iT^{(12)}+iT^{(13)}+iT^{(14)}+iT^{(15)} \cong iT^{(12')}+iT^{(13')}+iT^{(14')}+iT^{(15')}$$

$$\cong i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2\frac{-s}{m^2}\tfrac{1}{2}\ln^2(-s)K_1\bar{u}(p_3)[\tau_{a_1},\tau_{a_3}][\tau_{a_3},\tau_{a_2}]u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2)\,. \tag{5.21a}$$

and

$$iT^{(\overline{12})}+iT^{(\overline{13})}+iT^{(\overline{14})}+iT^{(\overline{15})} = iT^{(\overline{12}')}+iT^{(\overline{13}')}+iT^{(\overline{14}')}+iT^{(\overline{15}')}$$

$$\cong i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2\frac{s}{m^2}\tfrac{1}{2}\ln^2 s K_1\bar{u}(p_3)[\tau_{a_1},\tau_{a_3}][\tau_{a_3},\tau_{a_2}]u(p_1)\bar{u}(p_4)\tau_{a_2}\tau_{a_1}u(p_2)\,. \tag{5.21b}$$

D. Evaluation of $iT_1^{(4)},\dots,iT_1^{(9')}$

Let us return to Eq. (4.43). We already picked out the dominant terms of $iT^{(1,3)}(\mu^2)$ and $iT^{(2,3)}(\mu^2)$, as they are given by Eqs. (5.10) and (5.11). We need now to work on $iT_1^{(4)}$, etc.

We draw attention to Eq. (4.22). There are two ways in which $iT_1^{(4)}$ can give $O(s\ln^2 s)$ terms. One way is due to vertex renormalization; this can be seen, because if we retain the $k_1{}^2$ term in the curly brackets, we will have a divergent integral. The second way is when we shift k_1 to carry out its integration; the shifted part turns out to give a non-negligible contribution.

We write

$$iT_1^{(4)} = -\left(\frac{g}{2}\right)^5 g \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}2!\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)(N_{(4)}^{\mu_1\mu_2}/d_{(4)}{}^3)\epsilon_{a_2a_3a_4}$$

$$\times\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\frac{1}{m^2+(p_1-k_2)^2}\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}\,. \tag{5.22}$$

where

$$d_{(4)} = x_1[m^2+(p_1-k_1)^2]+x_2[(k_1-k_2)^2+\mu^2]+x_3(k_1{}^2+\mu^2)$$
$$=\bar{k}_1{}^2+b_{(4)}{}^2\,.$$
$$k_1=\bar{k}_1+x_1p_1+x_2k_2\,,$$
$$b_{(4)}{}^2=-2x_1x_2p_1\cdot k_2+x_1{}^2m^2+x_2(1-x_2)k_2{}^2+(x_2+x_3)\mu^2\,,$$

and

$$N_{(4)}^{\mu_1\mu_2}=\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}\{-\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)]\gamma^{\mu_2}[m-\gamma\cdot(p_1-k_1)]\gamma\cdot k_1$$
$$-\gamma^{\mu_1}[m-\gamma\cdot(p_1-k_2)]\gamma\cdot k_1[m-\gamma\cdot(p_1-k_1)]\gamma^{\mu_2}$$
$$+2\gamma^{\mu_1}k_1^{\mu_2}[m-\gamma\cdot(p_1-k_2)]\gamma^{\lambda}[m-\gamma\cdot(p_1-k_1)]\gamma_\lambda\}u(p_1)\,.$$

It is natural to split

$$N_{(4)}^{\mu_1\mu_2}=N_{(4)}^{\mu_1\mu_2}(k_1=\bar{k}_1)+N_{(4)}^{\mu_1\mu_2}(k_1=x_1p_1+x_2k_2)\,, \tag{5.23}$$

where $N_{(4)}^{\mu_1\mu_2}(k_1=\bar{k}_1)$ is obtained by equating k_1 to $\bar{k}_1$ and $N_{(4)}^{\mu_1\mu_2}(k_1=x_1p_1+x_2k_2)$ is obtained by equating k_1 to $x_1p_1+x_2k_2$. Note that because of the $\bar{k}_1$ integration, there is no cross term between $\bar{k}_1$ and $x_1p_1+x_2k_2$.

It is easy to show

$$N_{(4)}^{\mu_1\mu_2}(k_1=\bar{k}_1)\cong\frac{2p^{\mu_1}p^{\mu_2}}{m}3\bar{k}_1{}^2\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}u(p_1) \tag{5.24}$$

which indicates that a subtraction is necessary to render a finite result. We choose, consistently as before, to subtract at the point when $k_2=0$ and when the fermions are on shell; i.e., instead of Eq. (5.22), what we should be calculating is the subtracted quantity,

$$\begin{aligned}(iT_1^{(4)})_{\text{sub}}=&-\left(\frac{g}{2}\right)^5 g\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,2!\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)\\&\times\{N_{(4)}^{\mu_1\mu_2}(k_1=k_1-\bar{k}_1)/d_{(4)}{}^3\\&-\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_1}\tau_{a_3}\tau_{a_4}[m-\gamma\cdot(p_1-k_2)](-6x_1{}^2m^2+3\bar{k}_1{}^2)\gamma^{\mu_2}u(p_1)/\bar{d}_{(4)}{}^3\}\\&\times\epsilon_{a_2a_3a_4}\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\\&\times\frac{1}{m^2+(p_1-k_2)^2}\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2},\end{aligned} \tag{5.25}$$

with

$$\bar{d}_4=\bar{k}_1{}^2+\bar{b}_{(4)}{}^2$$

and

$$\bar{b}_{(4)}{}^2=b_{(4)}{}^2|_{k_2=0}=x_1{}^2m^2+(x_2+x_3)\mu^2$$

For identification purposes, we will call the part due to $N_{(4)}^{\mu_1\mu_2}(k_1=\bar{k}_1)$ and the subtraction the renormalization part, i.e.,

$$\begin{aligned}(iT_1^{(4)})_{\text{ren}}=&-\left(\frac{g}{2}\right)^5 g\int\frac{d^4\bar{k}_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\,2!\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)\\&\times\{N_{(4)}^{\mu_1\mu_2}(k_1=\bar{k}_1)/d_{(4)}{}^3\\&-\bar{u}(p_3)\gamma^{\mu_1}\tau_{a_1}\tau_{a_3}\tau_{a_4}[m-\gamma\cdot(p_1-k_2)](-6x_1{}^2m^2+3\bar{k}_1{}^2)\gamma^{\mu_2}u(p_1)/\bar{d}_{(4)}{}^3\}\epsilon_{a_2a_3a_4}\\&\times\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\frac{1}{m^2+(p_1-k_2)^2}\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}.\end{aligned} \tag{5.26}$$

The steps taken to extract the dominant term of Eq. (5.26) are slight variations of those for $iT^{(13)}$. We quote only the result,

$$(iT_1^{(4)})_{\text{ren}}\cong-\left(\frac{1}{16\pi^2}\right)^2 3\left(\frac{g}{2}\right)^5 g\frac{s}{m^2}\,\tfrac{1}{2}\ln^2 sK_1\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}u(p_1)\bar{u}(p_4)\tau_{a_2}\tau_{a_1}u(p_2)\epsilon_{a_2a_3a_4}. \tag{5.27}$$

We tackle $N_{(4)}^{\mu_1\mu_2}(k_1=x_1p_1+x_2k_2)$ due to the shift in k_1, which appears in

$$\begin{aligned}(iT_1^{(4)})^{\text{shift}}=&-\left(\frac{g}{2}\right)^5 g\frac{i}{16\pi^2}\int\frac{d^4k_2}{(2\pi)^4}\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)[N_{(4)}^{\mu_1\mu_2}(k_1=x_1p_1+x_2k_2)/b_{(4)}{}^2]\\&\times\epsilon_{a_2a_3a_4}\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}\frac{1}{m+\gamma\cdot(p_4-k_2)}\gamma_{\mu_1}u(p_2)\frac{1}{m^2+(p_1-k_2)^2}\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}.\end{aligned} \tag{5.28}$$

of which the k_1 integration has been performed.

Combining denominators through the introduction of α parameters, we have

$$\begin{aligned}(iT_1^{(4)})^{\text{shift}}=&-\left(\frac{g}{2}\right)^5 g\frac{i}{16\pi^2}\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)\\&\times 4!\int d\alpha_1d\alpha_2d\alpha_3d\alpha_4d\alpha_5\delta\left(1-\sum_{i=1}^{5}\alpha_i\right)\\&\times\int\frac{d^4k_2}{(2\pi)^4}N_{(4)}^{\mu_1\mu_2}(k_1=x_1p_1+x_2k_2)|\epsilon_{a_2a_3a_4}\bar{u}(p_4)\tau_{a_2}\tau_{a_1}\gamma_{\mu_2}[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_1}u(p_2)\frac{1}{D_{(4)}{}^5},\end{aligned} \tag{5.29}$$

where

$$D_{(4)}=\alpha_1[m^2+(p_1-k_2)^2]+\alpha_2[(k+k_2)^2+\mu^2]+\alpha_3[m^2+(p_4-k_2)^2]+\alpha_4(k_2{}^2+\mu^2)+\alpha_5 b_{(4)}{}^2$$
$$=[\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5x_2(1-x_2)]\bar{k}_2{}^2+[\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5x_2(1-x_2)]^{-1}B_{(4)}{}^2,$$
$$B_{(4)}{}^2=(s-2m^2-k^2)(\alpha_1+x_1x_2\alpha_5)\alpha_3$$
$$+m^2[(\alpha_1+x_1x_2\alpha_5)^2+\alpha_3{}^2+\alpha_5x_1{}^2][\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5x_2(1-x_2)]$$
$$+\mu^2[\alpha_2+\alpha_4+\alpha_5(x_2+x_3)][\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5x_2(1-x_2)]+k^2\alpha_2(\alpha_4+\alpha_5x_2x_3),$$

and

$$k_2=\bar{k}_2+\frac{p_1(\alpha_1+x_1x_2\alpha_5)+p_4\alpha_3-k\alpha_2}{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5x_2(1-x_2)}.$$

The high-energy behavior is controlled by the conditions $\alpha_3\cong 0$ and/or $\alpha_1+x_1x_2\alpha_5\cong 0$. After a detailed analysis similar to that in Appendix B, we come to the conclusion that

$$N_{(4)}^{\mu_1\mu_2}(k_1=x_1p_1+x_2k_2)$$
$$\cong\frac{-8p^{\mu_1}p^{\mu_2}p_1\cdot p_4x_2\alpha_3}{\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5x_2(1-x_2)}\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}u(p_1). \tag{5.30}$$

This in fact is due to the x_2k_2 part of the shift, whence, after k_2 integration,

$$(iT_1^{(4)})^{\rm shift}\cong-\left(\frac{g}{2}\right)^5 g\left(\frac{i}{16\pi^2}\right)^2 4\frac{s^3}{m^2}$$
$$\times\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}u(p_1)\epsilon_{a_2a_3a_4}$$
$$\times\bar{u}(p_4)\tau_{a_2}\tau_{a_1}u(p_2)I_{(4)}, \tag{5.31}$$

with

$$I_{(4)}=\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)$$
$$\times\int d\alpha_1d\alpha_2d\alpha_3d\alpha_4d\alpha_5\delta\left(1-\sum_{i=1}^5\alpha_i\right)\frac{x_2\alpha_3}{(B_{(4)}{}^2)^3}. \tag{5.32}$$

A simple change of variables

$$\alpha_1=\alpha_1'x_1x_2,\quad \alpha_1'+\alpha_5=\rho \tag{5.33}$$

and elementary integrations give

$$I_{(4)}\cong\frac{1}{2!}\frac{1}{2!}\frac{1}{s^2}\ln^2 s\,K_1 \tag{5.34}$$

which leads to

$$(iT^{(4)})^{\rm shift}\cong 2\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2\frac{s}{m^2}\tfrac{1}{2}\ln^2 s\,K_1$$
$$\times\bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_4}u(p_1)\epsilon_{a_2a_3a_4}$$
$$\times\bar{u}(p_4)\tau_{a_2}\tau_{a_1}u(p_2). \tag{5.35}$$

The total contribution to $iT_1^{(4)}$ is, of course, the sum of Eqs. (5.27) and (5.35), i.e.,

$$iT_1^{(4)}=(iT_1^{(4)})_{\rm ren}+(iT_1^{(4)})^{\rm shift}. \tag{5.36}$$

The contributions of graphs (6), (8), and (9) can be obtained from Eq. (5.36) by simple changes, as remarked in the introduction to Sec. II B. What is necessary to complete the analysis of the radiative graphs of Fig. 4 is graph (5) [and graph (7), which is obtainable from graph (5) by $s\leftrightarrow u$ and rearranging isospin matrices].

We concluded in Eqs. (4.34) and (4.43) that if we are only interested in $O(s\ln^2 s)$, then we do not need to be concerned with graphs (5) and (7) at all. However, as it turns out, the leading terms of the isospin nonflip amplitude are purely imaginary and of order $s\ln s$. For graphs (4), (6), (8) and (9), owing to their being planar, we automatically obtain this order of accuracy as a bonus (we emphasize, for the imaginary part of the isospin nonflip and the real part of the flip amplitudes only). We are therefore induced to obtain also the $s\ln s$ terms for graphs (5) and (7).

Needless to say, it is less laborious if we can devise a trick to accomplish this feat. The pertinent observation is this: We can correctly obtain the dominant terms for graphs (4)–(9) by splitting them into halves. The $s\to\infty$, t finite limit can be reached by first evaluating the left halves with $k_2{}^2$ held finite but $p_1\cdot k_2$ tending large. After that the halves are joined and evaluated in the desired $s\to\infty$, t finite limit.

Now, if there were no isospin we could repeat the gauge-invariance argument as used in massive QED to show that the most leading terms of the left halves cancel out when we add up, e. g., graphs (4), (5), and (9). Since the dominant terms of the left halves lead to the dominant terms of the whole amplitudes, we conclude that if we replace all the τ matrices by 1, the leading high-energy terms of graphs (4), (5), and (9), should add up to zero. Thus, we have

$$(iT_1^{(9)})_{\rm ren}\cong-3\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2\frac{-s}{m^2}\tfrac{1}{2}\ln^2(-s)K_1\bar{u}(p_3)\tau_{a_3}\tau_{a_4}\tau_{a_2}u(p_1)\epsilon_{a_1a_3a_4}\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2), \tag{5.37}$$

$$(iT^{(9)})^{\text{shift}} \cong 2\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2 \frac{-s}{m^2}\,\tfrac{1}{2}\ln^2(-s)K_1\bar{u}(p_3)\tau_{a_3}\tau_{a_4}\tau_{a_2}u(p_1)\epsilon_{a_1a_3a_4}\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2)\,, \tag{5.38}$$

and thereupon,

$$(iT_1^{(5)})_{\text{ren}} \equiv (iT^{(5)} - i\overline{T}_a^{(5)} - i\overline{T}_b^{(5)})_{\text{ren}}$$

$$\cong 3\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2\left[\frac{s}{m^2}\,\tfrac{1}{2}\ln^2 s + \frac{-s}{m^2}\,\tfrac{1}{2}\ln^2(-s)\right] K_1\bar{u}(p_3)\tau_{a_3}\tau_{a_2}\tau_{a_4}u(p_1)\epsilon_{a_1a_3a_4}\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2) \tag{5.39}$$

and

$$(iT_1^{(5)})^{\text{shift}} \equiv (iT^{(5)} - i\overline{T}_a^{(5)} - i\overline{T}_b^{(5)})^{\text{shift}}$$

$$\cong -2\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2\left[\frac{s}{m^2}\,\tfrac{1}{2}\ln^2 s + \frac{-s}{m^2}\,\tfrac{1}{2}\ln^2(-s)\right] K_1\bar{u}(p_3)\tau_{a_3}\tau_{a_2}\tau_{a_4}u(p_1)\epsilon_{a_1a_3a_4}\bar{u}(p_4)\tau_{a_1}\tau_{a_2}u(p_2)\,. \tag{5.40}$$

Altogether, then

$$(iT_1^{(4)} + iT_1^{(5)} + iT_1^{(6)} + iT_1^{(7)} + iT_1^{(8)} + iT_1^{(9)})_{\text{ren}} = (iT_1^{(4')} + iT_1^{(5')} + iT_1^{(6')} + iT_1^{(7')} + iT_1^{(8')} + iT_1^{(9')})_{\text{ren}}$$

$$\cong 3\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2 K_1\,\bar{u}(p_3)\{[\tau_{a_3},\tau_{a_1}]\tau_{a_4}\epsilon_{a_2a_3a_4} + \tau_{a_3}[\tau_{a_2},\tau_{a_4}]\epsilon_{a_1a_3a_4}\}u(p_1)$$

$$\times\, u(p_4)\left[\frac{-s}{m^2}\,\tfrac{1}{2}\ln^2(-s)\,\tau_{a_1}\tau_{a_2} + \frac{s}{m^2}\tfrac{1}{2}\ln^2 s\,\tau_{a_2}\tau_{a_1}\right]u(p_2) \tag{5.41}$$

and

$$(iT_1^{(4)} + iT_1^{(5)} + iT_1^{(6)} + iT_1^{(7)} + iT_1^{(8)} + iT_1^{(9)})^{\text{shift}} = (iT_1^{(4')} + iT_1^{(5')} + iT_1^{(6')} + iT_1^{(7')} + iT_1^{(8')} + iT_1^{(9')})^{\text{shift}}$$

$$\cong -2\left(\frac{g}{2}\right)^5 g\left(\frac{1}{16\pi^2}\right)^2 K_1\bar{u}(p_3)\{[\tau_{a_3},\tau_{a_1}]\tau_{a_4}\epsilon_{a_2a_3a_4} + \tau_{a_3}[\tau_{a_2},\tau_{a_4}]\epsilon_{a_1a_3a_4}\}u(p_1)$$

$$\times\,\bar{u}(p_4)\left[\frac{-s}{m^2}\tfrac{1}{2}\ln^2(-s)\tau_{a_1}\tau_{a_2} + \frac{s}{m^2}\,\tfrac{1}{2}\ln^2 s\,\tau_{a_2}\tau_{a_1}\right]u(p_2)\,. \tag{5.42}$$

Note that Eqs. (5.38) and (5.39) are consistent with Eq. (4.34).

E. Three-vector exchange

Our arduous analysis will come to an end upon examining effects due to three-vector-meson exchange in the t channel. The graphs are shown in Fig. 9. It is easy to see that graphs (16′), (17′), and (18′) can be obtained from (16), (17), and (18), respectively, by twisting the left fermion line. This corresponds to proper $s \leftrightarrow u$ plus some rearrangment of τ matrices. Besides, graph (17) is the time reversed diagram of graph (18). Thus, we actually need to consider only two graphs.

These amplitudes were in fact analyzed in massive QED.[11] We shall therefore delete the details. The following results are obtained:

$$iT^{(16)} \cong 2i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2\frac{-s}{m^2}\ln(-s)I^{(16)}\bar{u}(p_3)\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_1)\,\bar{u}(p_4)\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_2)\,, \tag{5.43}$$

$$iT^{(16')} \cong 2i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2\frac{-s}{m^2}\ln s\, I^{(16)}\bar{u}(p_3)\tau_{a_3}\tau_{a_2}\tau_{a_1}u(p_1)\bar{u}(p_4)\,\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_2)\,, \tag{5.44}$$

$$iT^{(17)} \cong -2i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2 \frac{s}{m^2} \ln s I^{(17)} \bar{u}(p_3)\tau_{a_1}\tau_{a_3}\tau_{a_2}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_2), \tag{5.45}$$

$$iT^{(18)} \cong -2i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2 \frac{s}{m^2} \ln s I^{(17)} \bar{u}(p_3)\tau_{a_2}\tau_{a_1}\tau_{a_3}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_2). \tag{5.46}$$

$$iT^{(17')} \cong 2i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2 \frac{-s}{m^2} \ln(-s) I^{(17)*} \bar{u}(p_3)\tau_{a_2}\tau_{a_3}\tau_{a_1}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_2), \tag{5.47}$$

and

$$iT^{(18')} \cong 2i\left(\frac{g}{2}\right)^6\left(\frac{1}{16\pi^2}\right)^2 \frac{-s}{m^2} \ln(-s) I^{(17)*} \bar{u}(p_3)\tau_{a_3}\tau_{a_1}\tau_{a_2}u(p_1)\bar{u}(p_4)\tau_{a_1}\tau_{a_2}\tau_{a_3}u(p_2), \tag{5.48}$$

where

$$I^{(16)} = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3) \ln \frac{(\alpha_1+\alpha_2)(\alpha_2+\alpha_3)}{\alpha_2^{\,2}} \frac{1}{\alpha_1\alpha_2\alpha_3 k^2 + [\alpha_2(\alpha_1+\alpha_3)+\alpha_1\alpha_3]\mu^2} \tag{5.49}$$

and

$$I^{(17)} = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3) \left[\ln \frac{\alpha_1\alpha_2}{\alpha_3(\alpha_1+\alpha_2)} - i\pi\right] \frac{1}{\alpha_1\alpha_2\alpha_3 k^2 + [\alpha_3(\alpha_1+\alpha_2)+\alpha_1\alpha_2]\mu^2}. \tag{5.50}$$

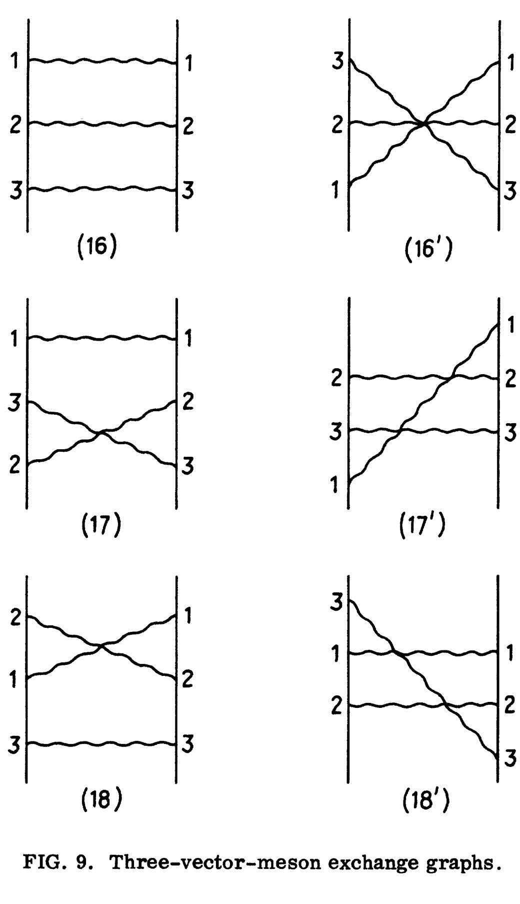

FIG. 9. Three-vector-meson exchange graphs.

VI. SUMMARY AND DISCUSSION

If we combine Eqs. (5.21a), (5.21b), and (5.41), we obtain the first result (1.1). Equations (5.8) and (5.9) are combined to give the result (1.2), while the result (1.3) is the sum of Eqs. (5.10), (5.11), and (5.42). Equations (5.43)–(5.48) lead to (1.4).

The amplitudes in the sixth order are obtained by combining (1.1)–(1.4):

$$T^{\mathrm{nf}}_{(6\mathrm{th})} = [-12 - 15(\mu^2 - \tfrac{4}{5}t)K_1]\pi i \ln s A K_1 + 12\pi i \ln s A K_2, \tag{6.1}$$

$$T^{\mathrm{f}}_{(6\mathrm{th})} = [-4 - 4(\mu^2 - t)K_1](\ln^2 s - i\pi \ln s) A K_1 \tag{6.2}$$

These are the final results of our sixth-order calculation.

In the process of preparing this manuscript, we learned of an independent calculation by McCoy and Wu,[12] which was performed by using an approximation scheme somewhat different from ours. In this paper, we have adopted the more traditional approach of extracting the high-energy behavior in the Feynman α space. In the approach of McCoy and Wu, which may be called the "infinite-momentum approximation", the internal-momentum variables are first discarded in comparison with the infinite momenta of the fermions, and then integrations are carried out by imposing a transverse-momentum cutoff. In contrast, we have considered the contributions of these internal-momentum variables, and kept the leading terms. Roughly speaking, the calculation of McCoy and Wu corresponds to taking the infinite-momentum limit before performing internal-momentum integrations and renormalization whereas our calculation corresponds to performing the integrations and renormalizations first and then taking the high-energy limit.

The result of McCoy and Wu is somewhat different from ours. Theirs corresponds to dropping the terms linear in K_1 in (6.1) and (6.2). Although we realize that the two different calculational schemes do not necessarily lead to identical results, since they correspond to two different limits, we are nonetheless somewhat puzzled by the apparent difference in the two results. It is known that the two different calculational schemes indeed lead to identical results in the case of QED. Since we had the benefit of knowing the result of McCoy and Wu during the final drafting of the present manuscript, we have made efforts to check our calculation. Difference, however, still persists.

The infinite-momentum technique has its advantage in its simplicity (relative to the conventional technique using the α parameters), and is therefore better suited for carrying out higher-order calculations. However, it is less understood and is perhaps subjected to question concerning its rigor. If the conventional method yields identical results as the infinite-momentum method in low-order calculations, then justification is provided for the infinite-momentum method, which can then be used, with more confidence, in higher-order calculations. Unfortunately, our calculation (using the conventional method) does not agree with that of McCoy and Wu. It is of importance that another independent calculation using the conventional method should be carried out to settle this question.

Another reason for our detailed discussion above has to do with Reggeization. If the results obtained by McCoy and Wu are accepted, then the isospin-flip amplitude Reggeizes and the vector mesons lie on the corresponding trajectory. This was demonstrated by Grisaru, Schnitzer, and Tsao[13] at the one-loop level in our language. In fact, a recent report by Lipatov[14] for the vector-meson–vector-meson scattering also gives such a behavior. This last calculation is done dispersively, and corresponds to the sixth order conventionally. Unfortunately, we are not sufficiently fluent in this approach to be able to make a meaningful comment on the exact correspondence between the dispersive calculation and ours.

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APPENDIX A

We want to show that $iT_b^{(1),(2)}$ do not contribute to $O(s\ln^2 s)$. To be specific, we discuss Eq. (4.11). We introduce α parameters to diagonalize and complete squares for the internal momenta. Then Eq. (4.11) takes on the form

$$iT_b^{(1)} = i\left(\frac{g}{2}\right)^4 g^2 6! \int \prod_{i=1}^{7} d\alpha_i\, \delta\left(1-\sum_{j=1}^{7}\alpha_j\right) \int \frac{d^4\bar{k}_1}{(2\pi)^4}\frac{d^4\bar{k}_2}{(2\pi)^4}\frac{N_{(1)}}{D_{(1)}{}^7}\,\epsilon_{a_1a_2a_5}\epsilon_{a_5a_4a_3}, \tag{A1}$$

where

$$\begin{aligned} D_{(1)} &= \alpha_1[m^2+(p_1-k_1)^2]+\alpha_2[m^2+(p_4-k_2)^2]+\alpha_3[(k+k_1)^2+\mu^2]+\alpha_4(k_1{}^2+\mu^2) \\ &\quad +\alpha_5[(k+k_2)^2+\mu^2]+\alpha_6(k_2{}^2+\mu^2)+\alpha_7[(k_1-k_2)^2+\mu^2] \\ &= \lambda_1\bar{k}_1{}^2+\lambda_2\bar{k}_2{}^2+\frac{1}{\lambda_1\lambda_2}\{\alpha_1\alpha_2\alpha_7 s+[(\alpha_1+\alpha_3+\alpha_4+\alpha_7)\alpha_2{}^2+(\alpha_2+\alpha_5+\alpha_6+\alpha_7)\alpha_1{}^2-2\alpha_1\alpha_2\alpha_7]m^2 \\ &\quad +[\alpha_3\alpha_4(\alpha_2+\alpha_5+\alpha_6)+\alpha_5\alpha_6(\alpha_1+\alpha_3+\alpha_4)+\alpha_7(\alpha_3+\alpha_5)(\alpha_4+\alpha_6)-\alpha_1\alpha_2]k^2 \\ &\quad +(\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7)\lambda_1\lambda_2\mu^2\}\ . \end{aligned} \tag{A2}$$

k's are related to $\bar{k}$'s by

$$k_1 = \cos\theta\,\bar{k}_1 + \sin\theta\,\bar{k}_2 + k_1^{\text{shift}} ,$$

$$k_1^{\text{shift}} = \frac{1}{\lambda_1\lambda_2}\{(\alpha_2+\alpha_5+\alpha_6+\alpha_7)\alpha_1 p_1 + \alpha_2\,\alpha_7 p_4 - [\alpha_3(\alpha_2+\alpha_5+\alpha_6+\alpha_7)+\alpha_5\alpha_7]k\} ,$$

$$k_2 = -\sin\theta\,\bar{k}_1 + \cos\theta\,\bar{k}_2 + k_2^{\text{shift}} , \tag{A3}$$

and

$$k_2^{\text{shift}} = \frac{1}{\lambda_1\lambda_2}\{(\alpha_1+\alpha_3+\alpha_4+\alpha_7)\alpha_2 p_4 + \alpha_1\,\alpha_7 p_1 - [\alpha_5(\alpha_1+\alpha_3+\alpha_4+\alpha_7)+\alpha_3\alpha_7]k\} . \tag{A4}$$

θ here is the angle of rotation to diagonalize the internal integrations. All we need to know about it are the properties

$$\left(\frac{1}{\lambda_1}\cos^2\theta + \frac{1}{\lambda_2}\sin^2\theta\right) = \frac{\alpha_2+\alpha_5+\alpha_6+\alpha_7}{\lambda_1\lambda_2} ,$$

$$\left(\frac{1}{\lambda_2}\cos^2\theta + \frac{1}{\lambda_1}\sin^2\theta\right) = \frac{\alpha_1+\alpha_3+\alpha_4+\alpha_7}{\lambda_1\lambda_2} , \tag{A5}$$

$$\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\sin\theta\cos\theta = -\frac{\alpha_7}{\lambda_1\lambda_2} ,$$

$$\lambda_1\lambda_2 = (\alpha_1+\alpha_3+\alpha_4)(\alpha_2+\alpha_5+\alpha_6) + \alpha_7(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6) . \tag{A6}$$

$N_{(1)}$ in Eq. (A1) is the numerator in the curly brackets of Eq. (4.11).

The denominator of Eq. (A2) tells us that the dominant high-energy behavior of $IT_b^{(1)}$ is controlled by $\alpha_1 \sim \alpha_2 \sim \alpha_7 \sim 0$, perhaps not simultaneously. In any event, it is necessary for us to pick up s^2 from the numerator $N_{(1)}$ in order to attain a behavior $s\ln^2 s$ for the amplitude, since we will inevitably pick up a $1/s$ factor from the denominator after all the integrations. Let us then take a close look at the numerator. Consider the first term of $N_{(1)}$ with all isospin matrices suppressed

$$N_{(1)}^{\text{1st term}} \cong -4\,\bar{u}(p_3)[\gamma^{\mu_3}\gamma\cdot(p_1-k_1)\gamma\cdot k_2 - \gamma\cdot k_2\,\gamma\cdot(p_1-k_1)\,\gamma^{\mu_3}]u(p_1)$$
$$\times\,\bar{u}(p_4)\,[\gamma\cdot k_1\,\gamma\cdot(p_4-k_2)\gamma_{\mu_3} - \gamma_{\mu_3}\gamma\cdot(p_4-k_2)\,\gamma\cdot k_1\,]u(p_2) . \tag{A7}$$

First of all, we cannot have all k's replaced by linear combinations of $\bar{k}$'s as given in Eqs. (A3) and (A4), because we simply do not have s^2 for the numerator this way.

Therefore, at most a pair of k's is replaced by $\bar{k}$'s. However, they should not be adjacent to each other, such as $\bar{u}(p_3)(\gamma^{\mu_3}\gamma\cdot\bar{k}_1\,\gamma\cdot\bar{k}_1 - \gamma\cdot\bar{k}_1\,\gamma\cdot\bar{k}_1\,\gamma^{\mu_3})u(p_1)$ or $\bar{u}(p_3)(\gamma^{\mu_3}\gamma\cdot\bar{k}_1\,\gamma\cdot\bar{k}_2 - \gamma\cdot\bar{k}_2\,\gamma\cdot\bar{k}_1\,\gamma^{\mu_3})u(p_1)$. They vanish either identically or after average.

Hence, at most one of the k's in each of the bilinear forms of Eq. (A7) can be replaced by $\bar{k}$'s. Then there are the following possibilities:

(i) $$N_{(1)}^{\text{1st term}} \sim -(\bar{k}^2)\bar{u}(p_3)[\,\gamma^{\mu_3}\gamma\cdot(p_1-k_1^{\text{shift}})\gamma^{\mu_4} - \gamma^{\mu_4}\gamma\cdot(p_1-k_1^{\text{shift}})\gamma^{\mu_3}\,]u(p_1)$$
$$\times\,\bar{u}(p_4)\,[\gamma_{\mu_4}\gamma\cdot(p_4-k_2^{\text{shift}})\,\gamma_{\mu_3} - \gamma_{\mu_3}\,\gamma\cdot(p_4-k_2^{\text{shift}})\gamma_{\mu_3}u(p_2) , \tag{A8}$$

where $(\bar{k}^2)$ denotes some correct combinations of $\bar{k}_1{}^2$ or $\bar{k}_2{}^2$ with θ. To obtain s^2 for Eq. (A8), we must have $k_1^{\text{shift}} \sim p_1$ and $k_2^{\text{shift}} \sim p_4$ of Eqs. (A3) and (A4). Pushing $\gamma\cdot p_1$ and $\gamma\cdot p_4$ through to act on the appropriate spinors, we see that we can at most achieve sk^2. This is negligible.

(ii) $$N_{(1)}^{\text{1st term}} \sim (\bar{k}^2)\,\bar{u}(p_3)\,[\,\gamma^{\mu_3}\gamma^{\mu_4}\gamma\cdot k_2^{\text{shift}} - \gamma\cdot k_2^{\text{shift}}\,\gamma^{\mu_4}\gamma^{\mu_3}]\,u(p_1)$$
$$\times\bar{u}(p_4)\,[\,\gamma_{\mu_4}\,\gamma\cdot(p_4-k_2^{\text{shift}})\,\gamma_{\mu_3} - \gamma_{\mu_3}\gamma\cdot(p_4-k_2^{\text{shift}})\gamma_{\mu_4}]u(p_2) . \tag{A9}$$

In this case, since γ^{μ_3} and γ^{μ_4} stand next to each other in the first factor, we have no way of obtaining s^2. Likewise,

$$N_{(1)}^{\text{1st term}} \sim (\bar{k}^2)\bar{u}(p_3)\,[\,\gamma^{\mu_3}\gamma\cdot(p_1-k_1^{\text{shift}})\gamma^{\mu_4} - \gamma^{\mu_4}\gamma\cdot(p_1-k_1^{\text{shift}})\gamma^{\mu_3}\,]u(p_1)$$
$$\times\,\bar{u}(p_4)\,[\,\gamma\cdot k_1^{\text{shift}}\,\gamma_{\mu_4}\gamma_{\mu_3} - \gamma_{\mu_3}\gamma_{\mu_4}\,\gamma\cdot k_1^{\text{shift}}]u(p_2) \tag{A10}$$

can be neglected.

$$\text{(iii)}\ N_{(1)}^{\text{1st term}} \sim -(\bar{k}^2)\bar{u}(p_3)[\gamma^{\mu_3}\gamma^{\mu_4}\gamma\cdot k_2^{\text{shift}} - \gamma\cdot k_2^{\text{shift}}\gamma^{\mu_4}\gamma^{\mu_3}]u(p_1) \times \bar{u}(p_4)[\gamma\cdot k_1^{\text{shift}}\gamma_{\mu_4}\gamma_{\mu_3} - \gamma_{\mu_3}\gamma_{\mu_4}\gamma\cdot k_1^{\text{shift}}]u(p_2)\ . \quad \text{(A11)}$$

This time, two sets of γ_{μ_3} and γ_{μ_4} stand next to each other to tame the high-energy behavior.

Therefore, all the k's in Eq. (A7) must be due to shifts. In that case, k_2^{shift} in $\bar{u}(p_3)\cdots u(p_1)$ must be proportional p_2 or k, and k_1^{shift} in $\bar{u}(p_4)u(p_2)$ must be proportional p_1 or k, otherwise they act on spinors and give a small numerator. We notice in fact, that p_2 and p_1 so obtained are accompanied by α_2 and α_1, respectively, as seen in Eqs. (A3) and (A4). What amounts to is a behavior $\sim\alpha_1\alpha_2(p_1\cdot p_4)^2k^2$, which is negligible.

An analysis of this kind can be carried out for the second term of $N_{(1)}$,

$$N_{(1)}^{\text{2nd term}} \cong -4\bar{u}(p_3)\gamma^{\mu_3}\gamma\cdot(p_1-k_1)\gamma\cdot k_2 u(p_1) \times\bar{u}(p_4)[\gamma\cdot k\gamma\cdot(p_4-k_2)\gamma_{\mu_3} - \gamma_{\mu_3}\gamma\cdot(p_4-k_2^{\text{shift}})\gamma\cdot k]u(p_2) \sim(\bar{k}^2)k^2p_1\cdot p_4\ , \quad \text{(A12)}$$

which is negligible. Nor can we replace one of the k's in the first factor and the k_2 in the second factor by $\bar{k}$'s. For then we would have, for example,

$$N_{(1)}^{\text{2nd term}} \sim -(\bar{k}^2)\bar{u}(p_3)\gamma^{\mu_3}\gamma\cdot(p_1-k_1^{\text{shift}})\gamma^{\mu_4}u(p_1) \times\bar{u}(p_4)(\gamma\cdot k\gamma_{\mu_4}\gamma_{\mu_3} - \gamma_{\mu_3}\gamma_{\mu_4}\gamma\cdot k)u(p_2)\ . \quad \text{(A13)}$$

which again can be dropped since γ_{μ_3} and γ_{μ_4} are next to each other.

What is left is when all the k's in Eq. (A11) are replaced by shifts. In particular, k_2 in the first factor is $\sim p_2$. Some simple algebra gives us

$$N_1^{\text{2nd term}} \sim \alpha_2(p_1\cdot p_4)^2k^2\ . \quad \text{(A14)}$$

A direct analysis of the relevant integral shows that if we let $\alpha_1\sim\alpha_7\sim 0$, we do not have a singular configuration. Thus $N_1^{\text{2nd term}}\sim O(s\ln s)$, which we drop.

Similarly, $N^{\text{3rd term}}$ is negligible.

Lastly, we look at the fourth term of $N_{(1)}$,

$$N_{(1)}^{\text{4th term}} \cong 4\bar{u}(p_3)\gamma\cdot k\gamma\cdot(p_1-k_1)\gamma^{\mu_3}u(p_1) \times\bar{u}(p_4)\gamma_{\mu_3}\gamma\cdot(p_4-k_2)\gamma\cdot k u(p_2)\ . \quad \text{(A15)}$$

If we replace k_1 and k_2 by $\bar{k}$'s, we cannot have s^2 for it. If we substitute k_1 and k_2 by their shifts, we can at best obtain $(k^2)^2s$. Consequently, $N_{(1)}^{\text{4th term}}$ can be discarded.

We conclude then $iT_b^{(1)}$ does not contribute to $O(s\ln^2s)$, nor does $iT_b^{(2)}$.

APPENDIX B

We want to show that $iT_2^{(4)} - i\bar{T}^{(4)}$ does not contribute to order $s\ln^2 s$. We look at Eq. (4.26). Combining denominators, we rewrite it as

$$iT_2^{(4)} - i\bar{T}^{(4)} = -\left(\frac{g}{2}\right)^5 g\,2!\int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3) \times\int\frac{d^4\bar{k}_1}{(2\pi)^4}\int\frac{d^4\bar{k}_2}{(2\pi)^4}N_{(4)}\frac{1}{d_{(4)}{}^3}\frac{1}{m^2+(p_1-k_2)^2}\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}\frac{1}{m^2+(p_4-k_2)^2} \quad \text{(B1)}$$

where with all isospin structure understood,

$$N_{(4)} \cong \frac{2p'_{\mu_1}p'_{\mu_2}}{m}\bar{u}(p_3)[\gamma^{\mu_1}\gamma\cdot(p_1-k_2)\gamma\cdot p_1\gamma\cdot(p_1-k_1)\gamma^{\mu_2} - 2\gamma^{\mu_1}\gamma\cdot(p_1-k_2)\gamma\cdot k_2\gamma\cdot(p_1-k_1)\gamma^{\mu_2} + 2\gamma^{\mu_1}\gamma\cdot(p_1-k_2)\gamma^{\mu_2}\gamma\cdot(p_1-k_1)\gamma\cdot k_2 - \gamma^{\mu_1}k_2^{\mu_2}\gamma\cdot(p_1-k_2)\gamma^\lambda\gamma\cdot(p_1-k_1)\gamma_\lambda]u(p_1) \quad \text{(B2)}$$

$d_{(4)}$ and $\bar{k}_1$, etc. were introduced after Eq. (5.22).

Since k_1 appears only once for each factor in Eq. (B2), we can substitute $x_1p_1+x_2k_2$ for it. This is because a shift of the integration variable, $k_1=\bar{k}_1+x_2k_2$, is needed to bring the denominator (with Feynman parameters) into diagonalized form [see Eq. (5.22)]. After using $\gamma\cdot p_1$ to act on $u(p_1)$ and $\gamma\cdot p_3$ to act on $\bar{u}(p_3)$ and dropping m whenever it appears, we have

$$N_{(4)} \cong \frac{2p'_{\mu_1}p'_{\mu_2}}{m}\bar{u}(p_3)[x_2\gamma^{\mu_1}\gamma\cdot k_2\gamma\cdot p_1\gamma\cdot k_2\gamma^{\mu_2} + 4(1-x_1)\gamma^{\mu_1}p_1^{\mu_2}(k_2)^2 + 4(1-x_1)p_1\cdot k_2\gamma^{\mu_1}\gamma\cdot k_2\gamma^{\mu_2} + 2x_2\gamma^{\mu_1}k_2^{\mu_2}k_2{}^2 - 4x_2\gamma^{\mu_1}k_2^{\mu_2}p_1\cdot k_2]u(p_1)\ . \quad \text{(B3)}$$

We carry out the $\bar{k}_1$ integration and combine denominators to prepare for k_2 integration. This lead to

$$iT_2^{(4)} - i\bar{T}^{(4)} \cong -\left(\frac{g}{2}\right)^5 g \frac{i}{16\pi^2} \int dx_1 dx_2 dx_3 \delta(1-x_1-x_2-x_3)\, 4! \int \prod_{i=1}^{5} d\alpha_i \delta\left(1-\sum_{j=1}^{5}\alpha_j\right) \int \frac{d^4\bar{k}_2}{(2\pi)^4} \frac{N_{(4)}}{D_{(4)}{}^5}, \tag{B4}$$

where for easier access, we copy

$$\lambda \equiv \alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5 x_2(1-x_2),$$

$$D_{(4)} = \lambda \bar{k}_2{}^2 + \lambda^{-1} B_{(4)}{}^2, \tag{B5}$$

$$k_2 = \bar{k}_2 + \lambda^{-1}[p_1(\alpha_1+x_1x_2\alpha_5)+p_4\alpha_3 - k\alpha_2], \tag{B6}$$

$$B_{(4)}{}^2 = (s-2m^2-k^2)(\alpha_1+x_1x_2\alpha_5)\alpha_3 + m^2[(\alpha_1+x_1x_2\alpha_5)^2+\alpha_3{}^2+\alpha_5 x_1{}^2\lambda] + \mu^2[\alpha_2+\alpha_4+\alpha_5(x_2+x_3)]\lambda + k^2\alpha_2(\alpha_4+\alpha_5x_2x_3). \tag{B7}$$

Dismiss $N_{(4)}$ for the moment. After $\bar{k}_2$ integration in Eq. (B4), we must end up with an integral of the form

$$I \cong \int dx_1dx_2dx_3\delta(1-x_1-x_2-x_3) \times \int \prod_{i=1}^{5} d\alpha_i\, \delta\left(1-\sum_{j=1}^{5}\alpha_j\right) \times \frac{1}{[s(\alpha_1+x_1x_2\alpha_5)\alpha_3+a^2]^3} \tag{B8}$$

in which a^2 is what is left in Eq. (B7) after we take out $s(\alpha_1+x_1x_2\alpha_5)\alpha_3$. The way to extract the dominant behavior of I is to scale

$$\alpha_1 = \alpha_1' x_1x_2 \text{ and } \alpha_1'+\alpha_5=\rho, \tag{B9}$$

then

$$I \cong \int d\alpha_2 d\alpha_4 \delta(1-\alpha_2-\alpha_4) \times dx_1dx_2d\alpha_3 d\rho \frac{x_1x_2\rho}{(sx_1x_2\alpha_3\rho+a^2)^3}. \tag{B10}$$

Now, let us incorporate $N_{(4)}$ and consider the case when k_2's in Eq. (B3) are due to the shifted piece of Eq. (B6). A moment of reflection convinces us that we should have at least one $k_2 \sim \lambda^{-1}\alpha_3 p_4$. Then, to obtain $s\ln^2 s$ for $iT_2^{(4)} - i\bar{T}^{(4)}$, we can allow $N_{(4)} \sim m^{-2}s^3\alpha_3$ $(x_1, x_2, \alpha_3$, or $\rho)$. We run down every term in Eq. (B3) and find none so constituted.

What about replacing two of the k_2's with $\bar{k}_2$? After averaging, Eq. (B4) becomes

$$N_{(4)} \cong \frac{2(p_1\cdot p_4)^2}{2m^2} \times \bar{k}_2{}^2[2+2x_1-6x_2+3x_2\lambda^{-1}(\alpha_1+x_1x_2\alpha_5)]. \tag{B11}$$

This can be shown not to yield $s\ln^2 s$ behavior, when plugged into Eq. (B4).

APPENDIX C

We want to show that $iT^{(5)} - i\bar{T}_a^{(5)} - i\bar{T}_b^5 \sim O(s\ln s)$. We analyze Eqs. (4.32) and (4.33).

As usual, all denominators with k_1 are combined. Then

$$iT^{(5)} - i\bar{T}_a^{(5)} - i\bar{T}_b^{(5)} = -\left(\frac{g}{2}\right)^2 g\, 3! \int dx_1dx_2dx_3dx_4\delta(1-x_1-x_2-x_3-x_4) \times \int \frac{d^4\bar{k}_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} N_{(5)}^{\mu_1\mu_2} \epsilon_{a_2a_3a_4} \bar{u}(p_4)\gamma_{\mu_2}[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2) \times \frac{1}{d_{(5)}{}^4}\frac{1}{m^2+(p_4-k_2)^2}\frac{1}{k_2{}^2+\mu^2}\frac{1}{(k+k_2)^2+\mu^2}, \tag{C1}$$

with

$$d_{(5)} = x_1[m^2+(p_3-k_1)^2] + x_2[m^2+(p_1-k_1-k_2)^2] + x_3[(k_1+k_2)^2+\mu^2] + x_4(k_1{}^2+\mu^2) = \bar{k}_1{}^2 + b_{(5)}{}^2,$$

$$k_1 = \bar{k}_1 + k_1^{\text{shift}},$$

$$k_1^{\text{shift}} = p_1(x_1+x_2) + kx_1 - k_2(x_2+x_3), \tag{C2}$$

$$b_{(5)}{}^2 = m^2(x_1+x_2)^2 + k_2{}^2(x_1+x_4)(x_2+x_3) + k^2x_1x_2 + 2p_1\cdot k_2(x_1x_3-x_2x_4) + 2k\cdot k_2x_1(x_2+x_3) + (x_3+x_4)\mu^2. \tag{C3}$$

$N_{(5)}^{\mu_1\mu_2}$ in Eq. (C1) has terms with or without $\bar{k}_1{}^2$ dependence. Either way, after the $\bar{k}_1$ integration, we shall introduce α parameters to form a common denominator

$$D_{(5)}=\alpha_1 b_{(5)}{}^2+\alpha_2[(k+k_2)^2+\mu^2]+\alpha_3[(p_4-k_2)^2+m^2]+\alpha_4(k_2{}^2+\mu^2)=\lambda_{(5)}\bar{k}_2{}^2+\lambda_{(5)}{}^{-1}B_{(5)}{}^2 \ ,$$

in which

$$\begin{aligned}
&\lambda_{(5)}=\alpha_1(x_1+x_4)(x_2+x_3)+\alpha_2+\alpha_3+\alpha_4 \ ,\\
&B_{(5)}{}^2=s\alpha_1\alpha_3(-x_1x_3+x_2x_4)+m^2[(\alpha_1(x_1x_3-x_2x_4)+\alpha_3)^2+\alpha_1(x_1+x_2)^2\lambda_{(5)}]+\mu^2[\alpha_1(x_3+x_4)+\alpha_2+\alpha_4]\lambda_{(5)}\\
&\qquad +k^2[\alpha_2\alpha_4+\alpha_1\alpha_4x_1x_2+\alpha_1\alpha_2x_3x_4-\alpha_1\alpha_3x_2x_4] \ ,\\
&k_2=\bar{k}_2+k_2^{\text{shift}} \ ,
\end{aligned} \tag{C4}$$

and

$$k_2^{\text{shift}}=\lambda_{(5)}{}^{-1}\{p_1\alpha_1(-x_1x_3+x_2x_4)-k[\alpha_1x_1(x_2+x_3)+\alpha_2]+p_4\alpha_3\} \ . \tag{C5}$$

Equation (C4) implies that the high-energy behavior of Eq. (C1) is controlled by $\alpha_1\sim\alpha_3\sim 0$ and/or a pair of $x_{1,2,3,4}$ which makes $x_1x_3-x_2x_4\sim 0$.

Consider first the case when all the k_1's in Eq. (4.33) are replaced by k_1^{shift} of Eq. (C2). Then, after the $\bar{k}_1$ integration,

$$\begin{aligned}
(iT^{(5)}-i\bar{T}_a^{(5)}-i\bar{T}_b^{(5)})(k_1=k_1^{\text{shift}})=-\left(\frac{g}{2}\right)^5 g\,\frac{i}{16\pi^2}\int dx_1dx_2dx_3dx_4\,\delta\left(1-\sum_{i=1}^4 x_i\right)\\
\times 4!\int d\alpha_1d\alpha_2d\alpha_3d\alpha_4\,\delta\left(1-\sum_{j=1}^4\alpha_j\right)\\
\times\int\frac{d^4\bar{k}_2}{(2\pi)^4}\,\alpha_1 N_{(5)}^{\mu_1\mu_2}(k_1=k_1^{\text{shift}})\epsilon_{a_2a_3a_4}\\
\times\bar{u}(p_4)\gamma_{\mu_2}[m-\gamma\cdot(p_4-k_2)]\gamma_{\mu_1}\tau_{a_2}\tau_{a_1}u(p_2)\,\frac{1}{D_{(5)}{}^5} \ .
\end{aligned} \tag{C6}$$

Note the appearance of the factor α_1 in the numerator.

We consider separately two possibilities:

(i) $k_2=k_2^{\text{shift}}$: In order to reach a $s\ln^2 s$ behavior for Eq. (C6) it is necessary for $N_{(5)}^{\mu_1\mu_2}(k_1=k_1^{\text{shift}},\ k_2=k_2^{\text{shift}})\sim\alpha_3 sp^{\mu_1}p^{\mu_2}/m$. After another round of tedious but straightforward scrutiny, we find that the best we can do is

$$N_{(5)}^{\mu_1\mu_2}(k_1=k_1^{\text{shift}},\ k_2=k_2^{\text{shift}})\sim\alpha_1\alpha_3(-x_1x_3+x_2x_4)sp^{\mu_1}p^{\mu_2}/m. \tag{C7}$$

(ii) Two of the k_2's are equated with $\bar{k}_2$: It is quite clear that this will not do. Even if we do not pick up other damping factors, the optimal behavior is

$$N_{(5)}^{\mu_1\mu_2}\sim\bar{k}_2{}^2p^{\mu_1}p^{\mu_2}/m \ . \tag{C8}$$

This, when inserted into Eq. (C6), will not lead to the required magnitude.

Finally, we look into the case when $k_1=\bar{k}_1$. Once again, some algebra gives

$$N_{(5)}^{\mu_1\mu_2}(k_1=\bar{k}_1)\sim\bar{k}_1{}^2\bar{u}(p_3)\gamma^{\mu_2}\gamma^{\mu_1}\gamma\cdot k_2u(p_1), \quad \text{etc.} \tag{C9}$$

which can be dropped.

This completes the demonstration that

$$iT^{(5)}-i\bar{T}_a^{(5)}-i\bar{T}_b^{(5)}<O(s\ln^2 s) \ .$$

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