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Boson-fermion and boson-boson scattering in a Yang-Mills theory at high energy: Sixth-order perturbation theory*

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We extend our previous study of Yang-Mills fields by calculating the high-energy behavior of the boson-fermion and of the boson-boson amplitude in sixth-order perturbation theory. In the isovector and isoscalar channels of both these processes the behavior of the amplitude is the same as that found in fermion-fermion scattering.

I. INTRODUCTION

Over a year ago, Nieh and Yao[1] pointed out that it would be of fundamental importance to study the high-energy behavior of non-Abelian gauge fields in general, and the Yang-Mills field[2] in particular. To avoid problems of infrared divergence, so that this high-energy behavior has an unambiguous meaning, it is essential to introduce, for the case of the Yang-Mills field, a complex scalar doublet and then use the Higgs mechanism[3] to give masses to all the vector mesons.

In a recent paper[4] we solved the problem of the high-energy behavior in sixth-order perturbation theory of fermion-fermion scattering in a Yang-Mills theory. The purpose of this present paper is to extend those calculations to boson-fermion and boson-boson scattering.

The kinematics of the two processes are given in Fig. 1. We let m be the mass of the fermion and λ be the mass of the boson, and we define the usual Mandelstam variables

$$s=(r_2+r_3)^2 \tag{1.1a}$$

and

$$t=(2r_1)^2=-\vec{\Delta}^2\ . \tag{1.1b}$$

Here $\vec{\Delta}$ is the (two-dimensional) momentum transfer and our metric is $(+\,-\,-\,-)$. Our results are that as $s\to\infty$ with $t\leq 0$ fixed, the leading real and leading imaginary parts of the amplitudes are given by

$$\mathfrak{M}_{BF}^{(6)}\sim -g^6m^{-1}\delta_{1,1'}\delta_{\nu_1,\nu_2}s\{\tfrac{1}{8}(\tau_a\tau_b-\tau_b\tau_a)(\lambda^2+\vec{\Delta}^2)(\ln^2 s-\pi i\ln s)K_2{}^2+\pi i\,\delta_{ab}\ln s[(\tfrac{5}{4}\lambda^2+\vec{\Delta}^2)K_2{}^2-K_3]\} \tag{1.2}$$

and[5]

$$\begin{aligned}\mathfrak{M}_{BB}^{(6)}\sim g^6\delta_{\mu_1,\mu_2}\delta_{\nu_1,\nu_2}s\{&-T^{(1)}_{ab,cd}(\lambda^2+\vec{\Delta}^2)(\ln^2 s-\pi i\ln s)K_2{}^2+T^{(2)}_{ab,cd}\pi i\ln s[(2\lambda^2+\vec{\Delta}^2)K_2{}^2-4K_3]\\&-T^{(0)}_{ab,cd}\tfrac{16}{3}\pi i\ln s[(\tfrac{5}{4}\lambda^2+\vec{\Delta}^2)K_2{}^2-K_3]\}\ ,\end{aligned} \tag{1.3}$$

where

$$K_2=\int\frac{d^2k_\perp}{(2\pi)^3}\,[(\vec{k}_\perp-\tfrac{1}{2}\vec{\Delta})^2+\lambda^2]^{-1}[(\vec{k}_\perp+\tfrac{1}{2}\vec{\Delta})^2+\lambda^2]^{-1}\ , \tag{1.4a}$$

$$K_3=\int\frac{d^2k_{1\perp}}{(2\pi)^3}\frac{d^2k_{2\perp}}{(2\pi)^3}(\vec{k}_{1\perp}{}^2+\lambda^2)^{-1}(\vec{k}_{2\perp}{}^2+\lambda^2)^{-1}[(\vec{k}_{1\perp}+\vec{k}_{2\perp}-\vec{\Delta})^2+\lambda^2]^{-1}\ . \tag{1.4b}$$

In (1.3) the isospin factors are

$$T^{(0)}_{ab,cd}=\delta_{ac}\delta_{bd}\ , \tag{1.5a}$$

$$T^{(1)}_{ab,cd}=\delta_{ad}\delta_{bc}-\delta_{ab}\delta_{cd}\ , \tag{1.5b}$$

$$T^{(2)}_{ab,cd}=\delta_{ad}\delta_{bc}+\delta_{ab}\delta_{cd}-\tfrac{2}{3}\delta_{ac}\delta_{bd}\ . \tag{1.5c}$$

Here τ_a are the Pauli matrices for the isospin of the fermion, $\delta_{1,1'}$ is 1 if the spin of the fermion is not flipped and zero if the spin is flipped, and we restrict the vector-meson polarization to be transverse ($\nu_1,\nu_2=1,2$).

For comparison the corresponding second- and fourth-order amplitudes are (Fig. 2)

$$\mathfrak{M}_{BF}^{(2)}\sim -g^2m^{-1}\delta_{1,1'}\delta_{\nu_1,\nu_2}\tfrac{1}{4}(\tau_a\tau_b-\tau_b\tau_a)s\,\frac{1}{\vec{\Delta}^2+\lambda^2}\ , \tag{1.6a}$$

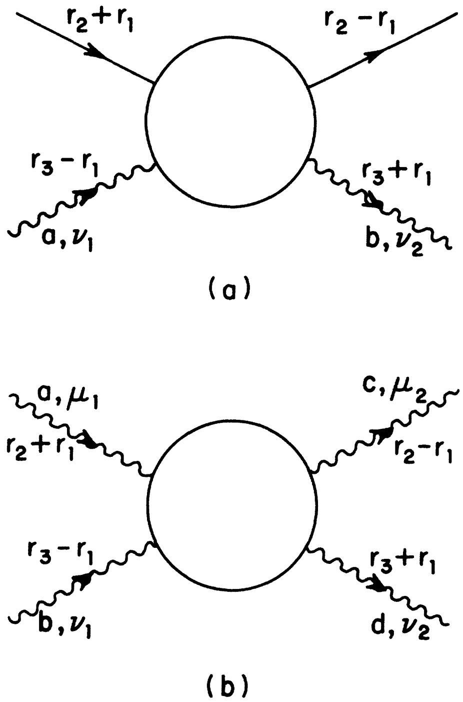

FIG. 1. (a) Kinematics for boson-fermion scattering. (b) Kinematics for boson-boson scattering.

$$\mathfrak{M}^{(4)}_{BF} \sim g^4 m^{-1} \delta_{1,1'} \delta_{\nu_1,\nu_2} \times s\left[\tfrac{1}{4}(\tau_a \tau_b - \tau_b \tau_a)(\ln s - \tfrac{1}{2}\pi i) K_2 + \tfrac{1}{2}\delta_{ab}\,\pi i K_2\right] , \tag{1.6b}$$

$$\mathfrak{M}^{(2)}_{BB} \sim -g^2 T^{(1)}_{ab,cd} \delta_{\mu_1,\mu_2} \delta_{\nu_1,\nu_2} 2s \frac{1}{\vec{\Delta}^2 + \lambda^2} , \tag{1.7a}$$

$$\mathfrak{M}^{(4)}_{BB} \sim g^4 \delta_{\mu_1,\mu_2} \delta_{\nu_1,\nu_2} \times 2s\left[T^{(1)}_{ab,cd}(\ln s - \tfrac{1}{2}\pi i) K_2 + \pi i \left(\tfrac{1}{2} T^{(2)}_{ab,cd} + \tfrac{4}{3} T^{(0)}_{ab,cd}\right) K_2\right] . \tag{1.7b}$$

From a comparison of these results with the corresponding results for fermion-fermion scattering we make the following two remarks:

(1) In the isovector channel of all three processes the amplitude in leading log approximation, at least up to sixth order, can be written in the Regge-pole form of

$$\text{const} \times (\vec{\Delta}^2 + \lambda^2)^{-1} s^{1+\alpha(t)} , \tag{1.8}$$

where

$$\alpha(t) = -g^2(\vec{\Delta}^2 + \lambda^2) K_2 . \tag{1.9}$$

(2) In the isoscalar channel of all three processes the sixth-order amplitude involves K_2 and K_3 in the universal combination

$$f_2 = (\tfrac{5}{4}\lambda^2 + \vec{\Delta}^2) K_2^{\,2} - K_3 . \tag{1.10}$$

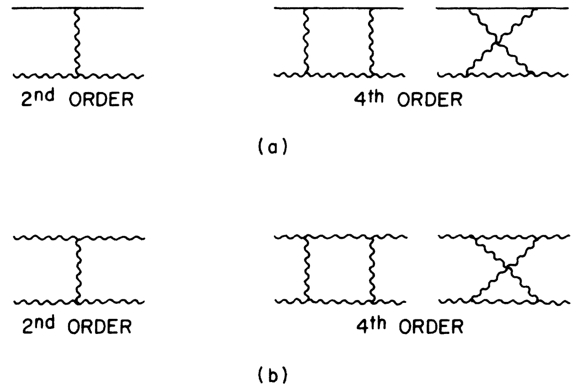

FIG. 2. (a) The Feynman diagrams that contribute to the high-energy behavior of boson-fermion scattering in second and fourth order. (b) The Feynman diagrams that contribute to the high-energy behavior of boson-boson scattering in second and fourth order.

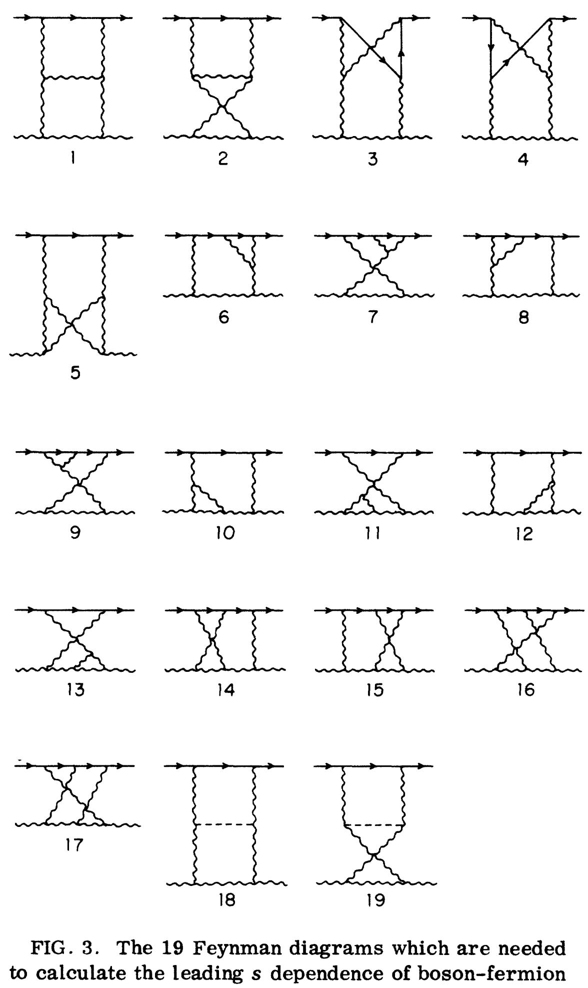

FIG. 3. The 19 Feynman diagrams which are needed to calculate the leading s dependence of boson-fermion scattering in sixth-order perturbation theory.

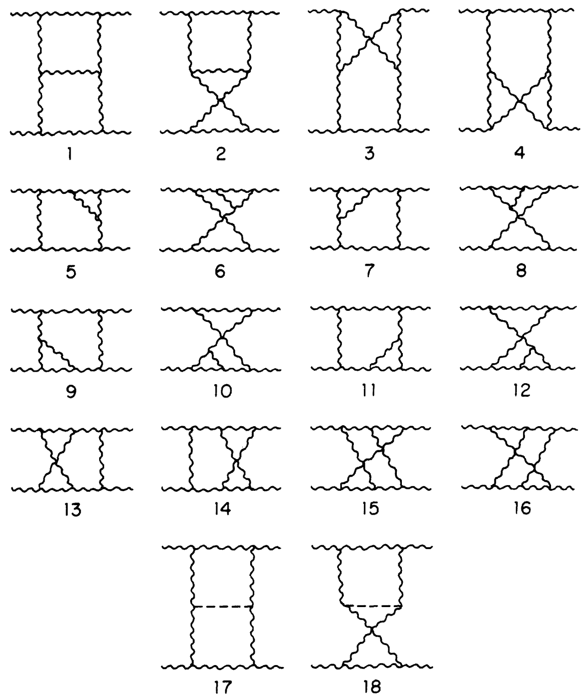

FIG. 4. The 18 Feynman diagrams which are needed to calculate the leading s dependence of boson-boson scattering in sixth-order perturbation theory.

The calculations which lead to these results are very similar to the corresponding calculations for fermion-fermion scattering. In particular we use momentum-space techniques throughout. Accordingly, our presentation will be brief and will concentrate on the modifications of the fermion-fermion calculation which must be made to obtain the present results. For boson-fermion scattering we must consider the 19 Feynman diagrams of Fig. 3. This is done in Sec. II. For boson-boson scattering we must consider the 18 Feynman diagrams of Fig. 4. This is carried out in Sec. III.

II. BOSON-FERMION SCATTERING

The analysis of the 19 sixth-order Feynman diagrams of Fig. 3 is very close to the analysis of the corresponding 20 diagrams of fermion-fermion scattering. Indeed, the diagrams of Fig. 3 have an obvious one-to-one correspondnce with the diagrams of Fig. 4 of Ref. 4 with the one exception that both Feynman diagrams 5 and 6 of fermion-fermion scattering correspond to Feynman diagram 5 of boson-fermion scattering. By use of this correspondence we may take over the results of the fermion-fermion analysis provided we make modifications in the following:

(1) the factor in front of the integral which comes from the charges and the factors of $\pm i$;

(2) the isospin factor;

(3) the momentum-dependent factors in the numerator.

In Table I we show the fermion-fermion diagrams, the prefactors, and the isospin factor for the 7 boson-fermion diagrams which must be independently studied. The rest of the 19 amplitudes are obtained from these 7 either by symmetry considerations or $s \leftrightarrow u$ crossing. In particular

$$\begin{aligned}(1)\ & \mathfrak{M}^{(6)}_{6BF} \sim \mathfrak{M}^{(6)}_{8BF}, \quad \mathfrak{M}^{(6)}_{10BF} \sim \mathfrak{M}^{(6)}_{12BF}, \\ & \mathfrak{M}^{(6)}_{14BF} \sim \mathfrak{M}^{(6)}_{15BF}, \quad \mathfrak{M}^{(6)}_{16BF} \sim \mathfrak{M}^{(6)}_{17BF};\end{aligned} \tag{2.1}$$

(2) $\mathfrak{M}^{(6)}_{2BF}$, $\mathfrak{M}^{(6)}_{4BF}$, $\mathfrak{M}^{(6)}_{7BF}$, $\mathfrak{M}^{(6)}_{9BF}$, $\mathfrak{M}^{(6)}_{11BF}$, $\mathfrak{M}^{(6)}_{13BF}$,

and $\mathfrak{M}^{(6)}_{19BF}$ are obtained from $\mathfrak{M}^{(6)}_{1BF}$, $\mathfrak{M}^{(6)}_{3BF}$, $\mathfrak{M}^{(6)}_{6BF}$, $\mathfrak{M}^{(6)}_{8BF}$, $\mathfrak{M}^{(6)}_{10BF}$, $\mathfrak{M}^{(6)}_{12BF}$, and $\mathfrak{M}^{(6)}_{18BF}$, respectively, by the replacements

$$a \leftrightarrow c\ , \tag{2.2a}$$

$$\ln^2 s - 2\pi i \ln s \leftrightarrow -\ln^2 s\ , \tag{2.2b}$$

and

(3) $\mathfrak{M}^{(6)}_{10BF}$ and $\mathfrak{M}^{(6)}_{17BF}$ are obtained from $\mathfrak{M}^{(6)}_{14BF}$ and $\mathfrak{M}^{(6)}_{15BF}$ by (2.2a) above.

TABLE I. The isospin factor, the prefactor, and the corresponding fermion-fermion diagrams for the 7 boson-fermion diagrams which must be studied independently.

Boson-fermion diagram	Fermion-fermion diagram	Prefactor	Isospin factor
1	1	$\frac{1}{4}$	$\tau_b \tau_a + 3\delta_{ab}$
3	3	$(\frac{1}{2}i)^3$	$-4i\delta_{ab}$
5	5, 6	$\frac{1}{4}$	$2\delta_{ab}$
6	7	$(\frac{1}{2}i)^3$	$-2i\tau_a\tau_b + 6i\delta_{ab}$
10	11	$(\frac{1}{4})$	$-\tau_a\tau_b + 3\delta_{ab}$
14	15	$(\frac{1}{2}i)^3$	$-2i\tau_b\tau_a - 3\tau_g\epsilon_{gab}$
18	19	$\frac{1}{4}\lambda^2$	$\tau_a\tau_b - 3\delta_{ab}$

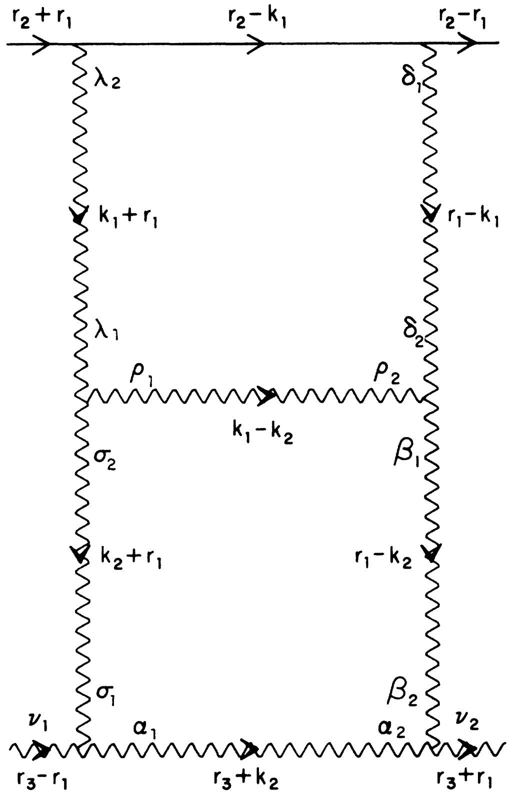

FIG. 5. Feynman diagram 1 for boson-fermion scattering.

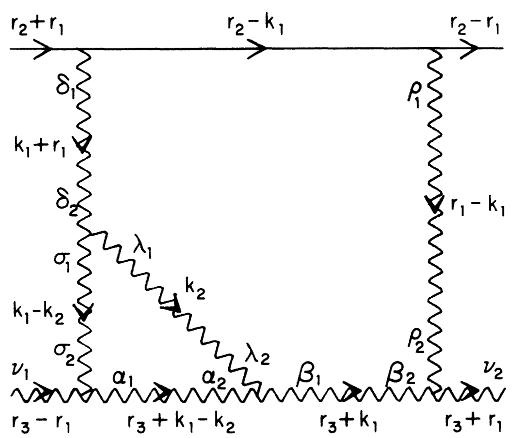

FIG. 6. Feynman diagram 10 for boson-fermion scattering.

It remains to discuss the momentum-dependent part of the numerators. Consider first Feynman diagram 1 (Fig. 5). There are now 81 terms in the numerator instead of the 9 terms we had for fermion-fermion scattering. These additional terms arise from the fact that at each of the two 3-boson vertices at the bottom of the diagram there are now 3 terms instead of 1. However, of these 9 possible combinations only the term

$$(-2r_3-k_2+r_1)_\sigma\, g_{\nu_1\alpha_1}(-2r_3-k_2-r_1)_\beta\, g_{\alpha_1\alpha_2} g_{\nu_2\alpha_2}$$
$$=-\delta_{\nu_1,\nu_2}(-2r_3-k_2+r_1)_\sigma(-2r_3-k_2-r_1)_\beta \quad (2.3)$$

contributes to the leading log. Therefore, the final replacement needed is to replace the factor

$$\bar{u}(r_3+r_1)\gamma_\beta(\not{r}_3+\not{k}_2+m)\,\gamma_\sigma\, u(r_3-r_1)$$

with (2.3) and the rest of the analysis of diagram 1 of the boson-fermion case is identical with the fermion-fermion case. The identical replacement reduces diagrams 3, 4, 6, and 18 to the corresponding fermion-fermion cases.

In Feynman diagram 10 (Fig. 6) there are 27 terms in the numerator which arise from the boson vertices on the lower boson line, but only the single term

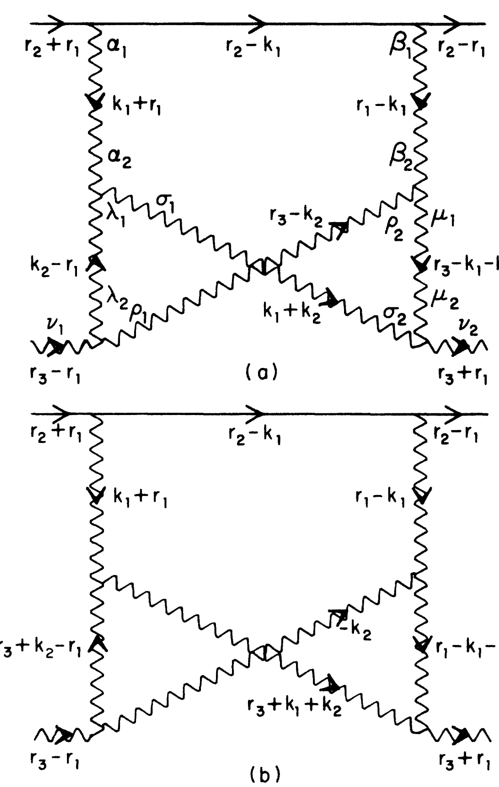

FIG. 7. Feynman diagram 5 for boson-fermion scattering with the two different choices of momentum variables discussed in the text.

$$(-2r_3-k_1+k_2+r_1)_\sigma g_{\nu_1\alpha_1}(-2r_3-2k_1+k_2)_\lambda g_{\alpha_1\alpha_2}g_{\alpha_2\beta_1}g_{\beta_1\beta_2}(-2r_3-k_1-r_1)_\rho g_{\beta_2\nu_2}$$
$$=-\delta_{\nu_1,\nu_2}(-2r_3-k_1+k_2+r_1)_\sigma(-2r_3-2k_1+k_2)_\lambda(-2r_3-k_1-r_1)_\rho \quad (2.4)$$

contributes to leading order. Thus, if we replace the factor

$$\bar{u}(r_3+r_1)\gamma_\rho(\not{r}_3+\not{k}_1-\not{k}_2+m)\gamma_\lambda(\not{r}_3+\not{k}_2-\not{k}_2+m)\gamma_\sigma u(r_3-r_1)$$

by (2.4), the analysis of the boson-fermion case is again reduced to the corresponding fermion-fermion case and we find

$$\mathfrak{M}^{(6)}_{10\,BF}\sim\mathfrak{M}^{(6)}_{6\,BF}. \quad (2.5)$$

The replacement of the spinor factors by the analog of (2.4) reduces Feynman diagram 14 to the corresponding fermion-fermion case.

It remains to discuss the momentum-dependent factors for Feynman diagram 5 (Fig. 7). Here there are two ways to reduce the diagram to a fermion-fermion diagram instead of only one because the large r_3 momenta can flow through the cross in the bottom half of the diagram in two different ways. It is convenient, therefore, to treat these two cases separately by introducing the two different sets of momentum coordinates given in Fig. 7 and imposing the restriction that for each set of coordinates

$$k_2=o(\omega)\ . \quad (2.6)$$

Then in Fig. 7 Feynman diagram 5(a) will correspond to the fermion-fermion diagram 5, and 5(b) corresponds to fermion-fermion diagram 6. For 5(a) replace the factor

$$\bar{u}(r_3+r_1)\gamma_\sigma(\not{r}_3-\not{k}_1-\not{k}_2+\not{r}_1+m)\gamma_\beta(\not{r}_3-\not{k}_2+m)\gamma_\lambda u(r_3-r_1)$$

of fermion-fermion diagram 5 by

$$(-2r_3+k_3+r_1)_\lambda g_{\nu_1\rho_1}(-2r_3+k_1+2k_2-r_1)_\beta g_{\rho_1\rho_2}g_{\rho_2\mu_1}g_{\mu_1\mu_2}(2r_3-k_1-k_2+2r_1)_\sigma g_{\mu_2\nu_2}$$
$$=-\delta_{\nu_1,\nu_2}(-2r_3+k_3+r_1)_\lambda(-2r_3+k_1+2k_2-r_1)_\beta(2r_3-k_1-k_2+2r_1)_\sigma \quad (2.7)$$

and we make the analogous replacement for Feynman diagram 5(b). Then the analysis of the fermion-fermion case shows that

$$\mathfrak{M}^{(6)}_{3\,BF}+\mathfrak{M}^{(6)}_{4\,BF}\sim\mathfrak{M}^{(6)}_{5\,BF}\ . \quad (2.8)$$

After making all of the above replacements the remainder of the analysis is identical with the fermion-fermion case and we obtain (1.2).

III. BOSON-BOSON SCATTERING

The 18 Feynman diagrams which must be considered for sixth-order boson-boson scattering are shown in Fig. 4. Of these 18 only diagrams 1, 3, 5, 13, and 17 need be independently studied and the correspondence of these five diagrams with the fermion-fermion diagrams of Ref. 4 is shown in Table II. Also shown are the relevant prefactors

TABLE II. The isospin factor, the prefactor, and the corresponding fermion-fermion diagrams for the 5 boson-boson diagrams which must be studied independently.

Boson-boson diagram	Fermion-fermion diagram	Prefactor	Isospin factor
1	1	1	$\delta_{ad}\delta_{bc}-3\delta_{ac}\delta_{bd}$
3	3, 4	1	$\delta_{ab}\delta_{cd}+\delta_{bc}\delta_{ad}-2\delta_{bd}\delta_{ac}$
5	7	1	$-\delta_{ac}\delta_{bd}-\delta_{ab}\delta_{cd}$
13	15	1	$\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}-2\delta_{ab}\delta_{cd}$
17	19	$-\lambda^2$	$\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}$

and isospin factors. We also note that in the boson-boson diagram 3 (and 4) there are two distinct paths the large momentum can follow through the diagram. The replacement of the spinor factors of the fermion-fermion case by the corresponding factors for the boson-boson case follows exactly as it did for the boson-fermion case of the last section. Using these replacements and the results of fermion-fermion scattering, we obtain (1.3).

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[1]H. T. Nieh and Y. P. Yao, Phys. Rev. Lett. 32, 1074 (1974).

[2]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[3]P. W. Higgs, Phys. Lett. 12, 132 (1964); Phys. Rev. Lett. 13, 508 (1964); F. Englert and R. Brout, *ibid.* 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, *ibid.* 13, 585 (1964).

[4]B. M. McCoy and T. T. Wu, Phys. Rev. D 12, 3257 (1975).

[5]While this manuscript was in preparation, we received a paper by L. N. Lipatov on vector-meson Reggeization and the vacuum singularity in a non-Abelian gauge theory [Leningrad Nuclear Physics Institute, Leningrad, U.S.S.R., report (unpublished)]. In this paper, the boson-boson scattering amplitude is studied in sixth-order perturbation theory by a method different from ours. The final result there given by Eq. (43) agrees with Eq. (1.3) here.