Effective potential of a non-Abelian supersymmetric theory*

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The effective potential of a non-Abelian supersymmetric theory with internal isospin symmetry is considered. In the one-loop approximation, it is shown that the potential is finite without renormalization, and that where it is not zero, it is complex. In two loops, as in one loop, it is shown that the effective potential of the scalar fields alone vanishes identically. Implications of these results are discussed with reference to possible spontaneous symmetry breaking.

I. INTRODUCTION

Progress in the construction of supersymmetric Lagrangian field theories has reached the state where such theories can be constructed possessing a non-Abelian gauge symmetry.[1] However, the requirement of fermion number conservation and the consequent complexification of the fermion fields forbids the presence of mass terms in the Lagrangian. Hence, it would be useful if spontaneous symmetry breaking occurred so that masses could be generated.[2] Up till now, no loop corrections have been computed, but it has been hoped[3] that they may induce breakdown of symmetry in the manner of Coleman and Weinberg.[4]

In this paper, the techniques recently developed[5] to study the effective potential are used to evaluate the effective potential for the supersymmetric gauge theory of Ferrara and Zumino and of Salam and Strathdee.[1] To reduce the number of fields, the gauge group is taken to be SU(2), although this can be generalized to SU(N).[6]

The effective potential is first calculated in the one-loop approximation as a function of all the spin-zero fields, and remarkably it is found to be finite before renormalization. Even more remarkable, it is zero at those points where it is zero in the tree approximation, and elsewhere it is complex.

The whole two-loop calculation is difficult because of nondiagonal propagators, but it has been done as a function of just the scalar fields. It vanishes. The complex region is expected to stay, since some propagators will have imaginary masses in that region.

II. THE MODEL AND TREE APPROXIMATION TO THE EFFECTIVE POTENTIAL

The theory[1] we consider involves the vector fields A_μ^k, the (complex) spinor fields χ_α^k, the scalar fields A^k, and the pseudoscalar fields B^k $(k=1,2,3)$. All are in the adjoint representation of the internal-symmetry group, which we take to be SU(2).

The Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}(A_{\mu\nu})^2 + \tfrac{1}{2}(D_\mu A)^2 + \tfrac{1}{2}(D_\mu B)^2 + i\bar{\chi}\gamma_\mu D^\mu \chi + ig\epsilon^{klm}\bar{\chi}^l(A^k - iB^k\gamma_5)\chi^m - \tfrac{1}{2}g^2(\epsilon^{klm}A^lB^m)^2,$$

where $D_\mu A^k = \partial_\mu A^k + g\epsilon^{klm}A_\mu^l A^m$, etc., and Bjorken-Drell conventions are used.

This is the most general supersymmetric and gauge-invariant model containing the given fields. No mass terms or any other interactions are permitted.

We proceed now to study the effective potential of the theory to see if spontaneous symmetry breaking can occur, allowing masses to be generated. So we shift the fields A^k and B^k as follows: $A^k \to A^k + \phi^k$, $B^k \to B^k + \sigma^k$. We choose isospin axes so that $\phi \equiv (0, 0, \phi_3)$ and $\sigma \equiv (\sigma_1, 0, \sigma_3)$. Then in the tree approximation, the contribution to the effective potential is $V_0(\phi, \sigma) = \frac{1}{2}g^2\phi_3^2\sigma_1^2$.

It is obvious that the smallest value of V_0 is zero and this is the value of V_0 along the axes of ϕ_3 and σ_1. Before coming to any conclusions, we have to consider higher orders. We start by considering the $O(\hbar)$ contribution.

III. $O(\hbar)$ CALCULATION OF THE EFFECTIVE POTENTIAL

The effective potential to $O(\hbar)$ receives contributions from a tree approximation counterterm as well as from the spinor, vector, and scalar loops. We work in the Landau gauge to avoid ghost contributions in this order. (For some cases, the calculations have been repeated using general "V gauges."[5])

A. χ loop

We write $\mathcal{L}_\chi$ for the part of $\mathcal{L}(\phi, \sigma; \chi, A_\mu, A, B)$ quadratic in χ. Then

$$\mathcal{L}_\chi = i\bar{\chi}^k\gamma_\mu\partial^\mu\chi^k + ig\epsilon^{klm}\bar{\chi}^l(\phi^k - i\sigma^k\gamma_5)\chi^m.$$

The $O(\hbar)$ contribution to the potential is then

$$V_1^{(\chi)}(\phi,\sigma)=4i\hbar\int\frac{d^4k}{(2\pi)^4}\ln[k^2-g^2(\phi_3{}^2+\sigma^2)].$$

B. A_μ loop

We write $\mathcal{L}_{A_\mu}$ for the part of $\mathcal{L}(\phi,\sigma;\chi,A_\mu,A,B)$ quadratic in A_μ, including gauge terms:

$$\mathcal{L}_{A_\mu}=-\tfrac{1}{4}(\partial_\mu A_\nu^k-\partial_\nu A_\mu^k)^2-\tfrac{1}{2}\xi(\partial^\mu A_\mu)^2+\tfrac{1}{2}g^2[A_\mu{}^2(\phi_3{}^2+\sigma^2)-A_\mu^lA^{\mu m}(\phi^l\phi^m+\sigma^l\sigma^m)],$$

where $\xi\to\infty$.

Thus, the $O(\hbar)$ contribution to the $O(\hbar)$ effective potential is

$$V_1^{(A_\mu)}(\phi,\sigma)=-\frac{3i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln[k^4-k^2g^2(\phi_3{}^2+\sigma^2)+g^4\phi_3{}^2\sigma_1{}^2]-\frac{3i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln[k^2-g^2(\phi_3{}^2+\sigma^2)].$$

C. A loop

Let $\mathcal{L}_A$ be the part of $\mathcal{L}(\phi,\sigma;\chi,A_\mu,A,B)$ quadratic in A. Then

$$\mathcal{L}_A=\tfrac{1}{2}(\partial_\mu A)^2-\tfrac{1}{2}g^2[A^2\sigma^2-(A\cdot\sigma)^2],$$

which implies a contribution to the $O(\hbar)$ effective potential of

$$V_1^{(A)}(\phi,\sigma)=-i\hbar\int\frac{d^4k}{(2\pi)^4}\ln(k^2-g^2\sigma^2).$$

D. B^2 and AB contribution

We have to include the contribution from the nondiagonal AB terms as well as the BB terms.

If $\mathcal{L}_B$ is the part of $\mathcal{L}(\phi,\sigma;\chi,A_\mu,A,B)$ quadratic in B, and $\mathcal{L}_{AB}$ is the part involving AB terms, then

$$\mathcal{L}_B=\tfrac{1}{2}(\partial_\mu B)^2-\tfrac{1}{2}g^2[B^2\phi^2-(B\cdot\phi)^2],$$

$$\mathcal{L}_{AB}=-g^2[2\phi_3\sigma_1A_3B_1-\phi_3\sigma_1A_1B_3-\phi_3\sigma_3(A_1B_1+A_2B_2)],$$

which gives a contribution to the $O(\hbar)$ effective potential of

$$\begin{aligned}V_1^{(B,AB)}(\phi,\sigma)=&-\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln[k^4-k^2g^2(\phi_3{}^2+\sigma^2)+g^4\phi_3{}^2\sigma_1{}^2]\\&+\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln[(k^2-g^2\sigma_1{}^2)(k^2-g^2\sigma_3{}^2)(k^2-g^2\sigma^2)]\\&-\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\{k^8-k^6g^2(\phi_3{}^2+\sigma^2)-k^4g^4(4\phi_3{}^2\sigma_1{}^2-\sigma_3{}^2\sigma_1{}^2)\\&+k^2g^6[\phi_3{}^2\sigma_1{}^2(\sigma^2+\phi_3{}^2)+4\sigma_1{}^2\sigma_3{}^2\phi_3{}^2]+3g^8\sigma_1{}^4\phi_3{}^4\}.\end{aligned}$$

Thus the total loop contribution to the $O(\hbar)$ potential is

$$\begin{aligned}V_i(\phi,\sigma)=&+\frac{5i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln[k^2-g^2(\phi_3{}^2+\sigma^2)]-\frac{4i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln[k^4-k^2g^2(\phi_3{}^2+\sigma^2)+g^4\phi_3{}^2\sigma_1{}^2]\\&-\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\left[\frac{(k^2-g^2\sigma^2)}{(k^2-g^2\sigma_1{}^2)(k^2-g^2\sigma_3{}^2)}\right]\\&-\frac{i\hbar}{2}\int\frac{d^4k}{(2\pi)^4}\ln\{k^8-k^6g^2(\phi_3{}^2+\sigma^2)-k^4g^4(4\phi_3{}^2\sigma_1{}^2-\sigma_3{}^2\sigma_1{}^2)\\&+k^2g^6[\phi_3{}^2\sigma_1{}^2(\sigma^2+\phi_3{}^2)+4\sigma_1{}^2\sigma_3{}^2\phi_3{}^2]+3g^8\sigma_1{}^4\phi_3{}^4\}.\end{aligned}$$

It is unnecessary to evaluate the integrals to understand the structure of $V_1(\phi,\sigma)$. It is easy to see that $V_1(\phi,\sigma_1=0,\sigma_3)=0$ and $V_1(0,\sigma)=0$. But elsewhere it is complex, since the argument of the last logarithm is a quartic polynomial in k^2 which has all positive roots only if the coefficient of k^2 is negative. Furthermore, it is easy to see that $V_1(\phi,\sigma)$ is finite by looking at the sum of the squares and fourth powers of the masses. Hence, only a finite counterterm is added to $V_1(\varphi,\sigma)$ to complete the $O(\hbar)$ calculation.

The results here to some extent have an analog with the dilaton model studied by Drummond.[7] There too the complex region could occupy almost all space in the one-loop approximation. However, in our case, because no counterterms are available to cancel off any ultraviolet divergence, elsewhere $V_1(\phi,\sigma)$ has to be finite.

IV. $O(\hbar^2)$ CALCULATION AND HIGHER ORDERS

The two-loop calculation of the effective potential to $O(\hbar^2)$, $V_2(\varphi, \sigma_1, \sigma_3)$, is made difficult by the fact that in general the propagators are nondiagonal. This problem is less severe in the calculation of $V_2(\varphi, 0)$, and this has been done. The details are in the Appendix. The answer is identically zero.

This is a very interesting result in the light of recent work on supersymmetric vacua by Zumino.[8] He has shown that the physics of supersymmetry lies in the absence of free zero-point energy for a supermultiplet, and that in a supersymmetric theory, the sum of vacuum graphs vanishes in each order. Thus, it is assured that the two-loop sum, with unshifted fields, vanishes. The fact that it vanishes with some fields shifted also is an indication of the extremely tight constraints placed on the model.

Despite the zero measure of the real region, it may be possible for the vacuum to lie in that region, as happens in Ref. 7. However, in view of the peculiar nature of the effective potential, no firm statement can be made. Thus, mass generation remains the most important problem for supersymmetric gauge theories of the kind examined. Some degree of soft breaking may be inevitable.[9]

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APPENDIX

In this appendix we evaluate the effective potential in two loops as a function of the shifted value φ^k of the scalar field A^k. We work in the Landau gauge for simplicity. In the more general "V gauges,"[5] the vector and scalar propagators are nondiagonal.

According to Ref. 5, we have to sum the two-loop vacuum graphs using the shifted fields. In addition to vector, spinor, and spin-zero loops, we have to include a ghost-loop contribution.

In the following calculation, no integrals are explicitly evaluated since it is the purpose of the exercise to show that the integrands sum to zero via mutual cancellation. Hence, each graph is expressed as a linear combination of the following basic integrals:

$$I_1 = \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{(k+l)^2}\,\frac{1}{(k^2-g^2\varphi^2)}\,\frac{1}{(l^2-g^2\varphi^2)},$$

$$I_2 = \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{(k+l)^2}\,\frac{1}{(k^2-g^2\varphi^2)}\,\frac{1}{(l^2)},$$

$$K_1 = \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{(k^2-g^2\varphi^2)}\,\frac{1}{(l^2-g^2\varphi^2)},$$

$$K_2 = \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{(k^2-g^2\varphi^2)}\,\frac{1}{(l^2)},$$

$$C_1 = g^4\varphi^4 \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{k^2(k^2-g^2\varphi^2)l^2(l^2-g^2\varphi^2)},$$

$$C_2 = g^2\varphi^2 \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{l^2k^2(k+l)^2},$$

$$C_3 = g^4\varphi^4 \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{l^2k^2(k+l)^2(l^2-g^2\varphi^2)},$$

$$D_1 = g^6\varphi^6 \int \frac{d^4k\,d^4l}{(2\pi)^8}\,\frac{1}{(k+l)^2k^2(k^2-g^2\varphi^2)l^2(l^2-g^2\varphi^2)},$$

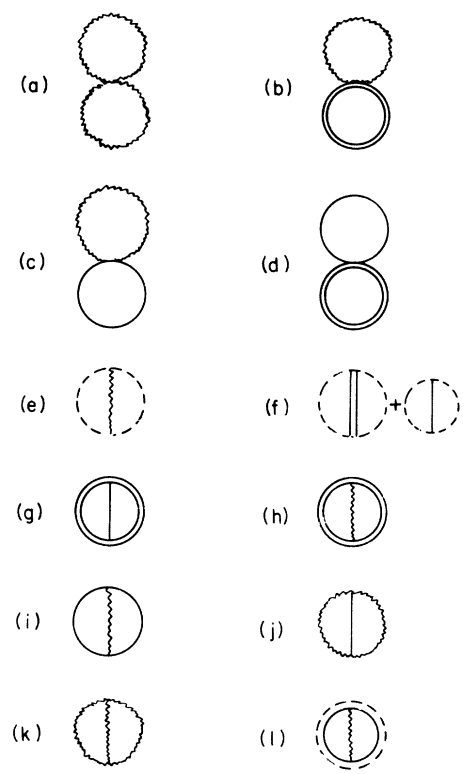

FIG. 1. Two-loop graphs contributing to the effective potential. A wavy line represents a vector propagator, a dashed line represents a fermion, a solid line represents a scalar, a double solid line represents a pseudoscalar, and a solid-dashed double line represents a ghost.

$$D_2 = g^6\phi^6 \times \int \frac{d^4k\, d^4l}{(2\pi)^8} \times \frac{1}{[(k+l)^2 - g^2\phi^2]\, k^2(k^2 - g^2\phi^2) l^2 (l^2 - g^2\phi^2)},$$

$$D_3 = g^6\phi^6 \times \int \frac{d^4k\, d^4l}{(2\pi)^8} \times \frac{k\cdot l}{[(k+l)^2 - g^2\phi^2](k^2)^2(k^2 - g^2\phi^2) l^2 (l^2 - g^2\phi^2)},$$

$$D_4 = g^6\phi^6 \int \frac{d^4k\, d^4l}{(2\pi)^8} \frac{k\cdot l}{(k+l)^2 l^2 (l^2 - g^2\phi^2)(k^2 - g^2\phi^2)(k^2)^2}.$$

The contribution from each graph shown in Fig. 1 is as follows:

Fig. 1(a) gives $\frac{-27}{8} K_1 - \frac{27}{4} K_2$,
Fig. 1(b) gives $-3K_1 - 6K_2$,
Fig. 1(c) gives $-6K_2$,
Fig. 1(d) gives $-2K_2$,
Fig. 1(e) gives $16K_2 - 8g^2\phi^2 I_1 - 4g^2\phi^2 I_2$,
Fig. 1(f) gives $8K_2 + 8g^2\phi^2 I_1 + 4g^2\phi^2 I_2$,
Fig. 1(g) gives $-2g^2\phi^2 I_1 - g^2\phi^2 I_2$,
Fig. 1(h) gives $\frac{3}{2}K_1 - K_2 + 2g^2\phi^2 I_1 + g^2\phi^2 I_2$,
Fig. 1(i) gives $2K_2 - g^2\phi^2 I_2$,
Fig. 1(j) gives $\frac{-13}{2} g^2\phi^2 I_1 - \frac{3}{4} g^2\phi^2 I_2 - 2K_2 + K_1 + (C_1 - C_2)/4 - D_4/2$,
Fig. 1(k) gives $\frac{31}{8} K_1 - \frac{5}{4} K_2 + 6g^2\phi^2 I_1 + 2g^2\phi^2 I_2 + \frac{1}{4}[(C_3 - C_1) + (D_1 + D_2) + 2D_3]$,
Fig. 1(l) gives $-K_2 + \frac{1}{2} g^2\phi^2 I_2$.

The sum of all these contributions vanishes identically.

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